Legged Locomotion: Trajectory Optimization, Machine Learning, and Bio-Inspired Control

Guillaume Bellegarda

28.10.2025

Skild AI, 2025

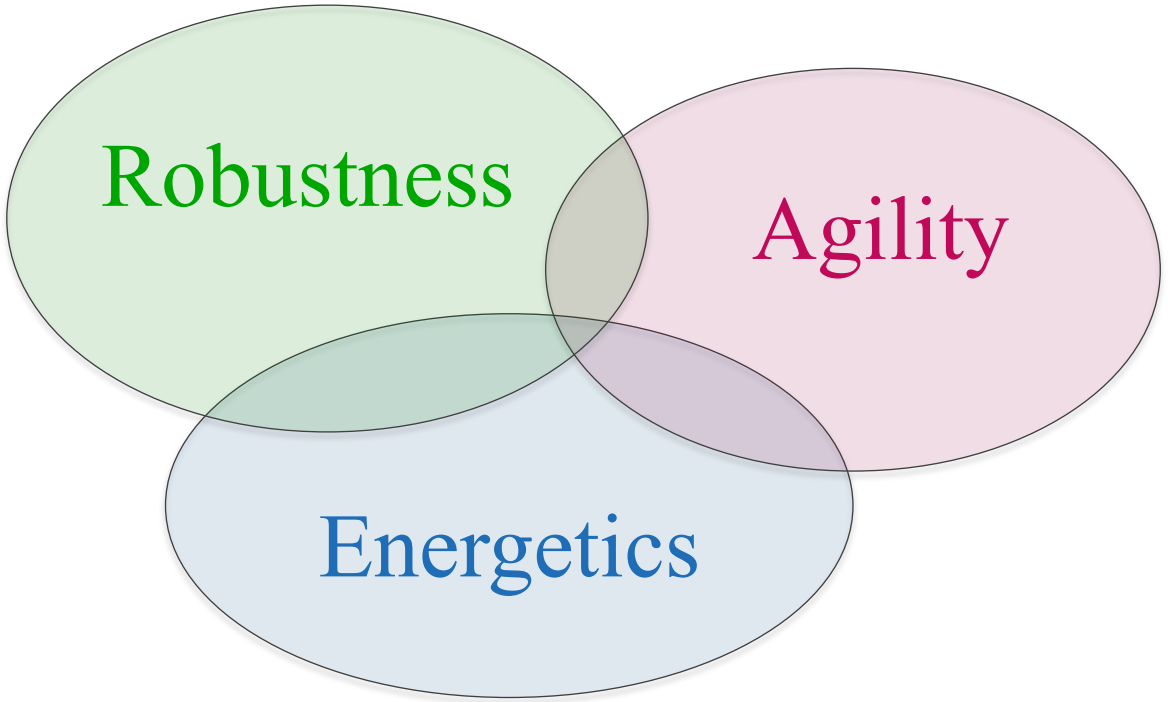
Kim et al., 2025

Is Legged Robots “Solved”?

Unitree, 2025

Yang et al., 2025

Locomotion Goals



…in stochastic environments (variability)

Robustness

unknown variability

Agility

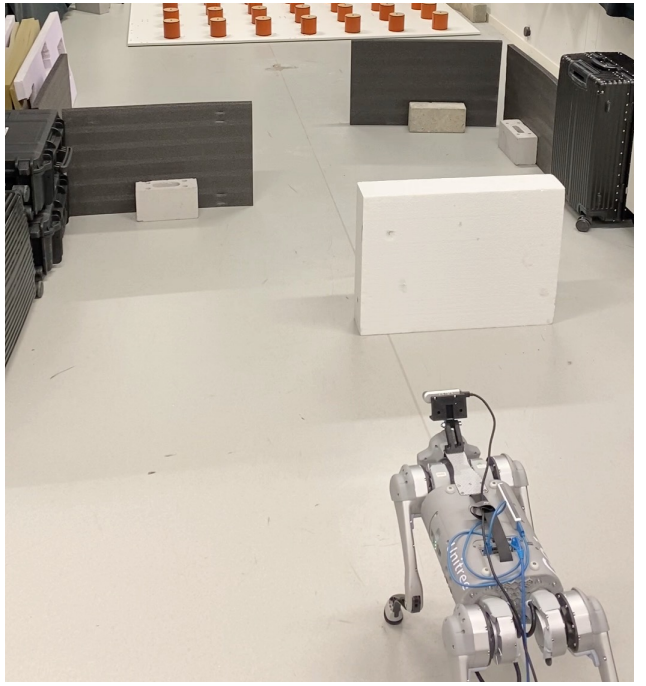
Energetics

G. Bellegarda, A. Ijspeert. “CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion,” RA-L 2022

G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. “Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning,” IROS 2022

Robustness

unknown variability



Agility

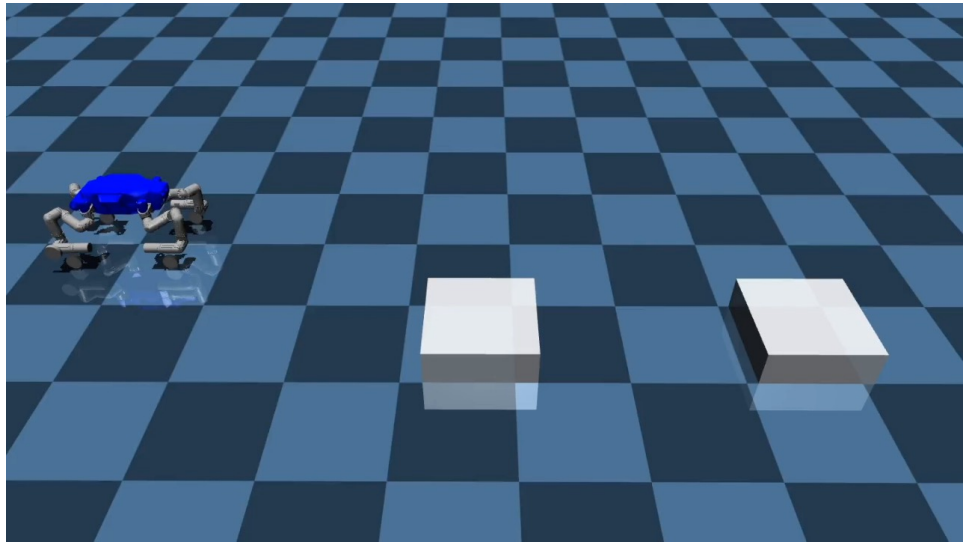
known variability

Energetics

M. Shafiee, **G. Bellegarda**, A. Ijspeert. “Viability Leads to the Emergence of Gait Transitions in Learning Agile Quadrupedal Locomotion on Challenging Terrains,” Nature Communications 2024.

G. Bellegarda, M. Shafiee, A. Ijspeert. “Visual CPG-RL: Learning Central Pattern Generators for Visually-Guided Quadruped Locomotion,” ICRA 2024.

G. Bellegarda, K. Byl. “Versatile Trajectory Optimization Using a LCP Wheel Model for Dynamic Vehicle Maneuvers,” ICRA 2020.



Robustness

unknown variability

Agility

known variability

Energetics

as efficient as practical

G. Bellegarda, K. Byl. "Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip," CDC 2019.
G. Bellegarda, K. Byl. "Design and Evaluation of Skating Motions for a Dexterous Quadruped," ICRA 2018.

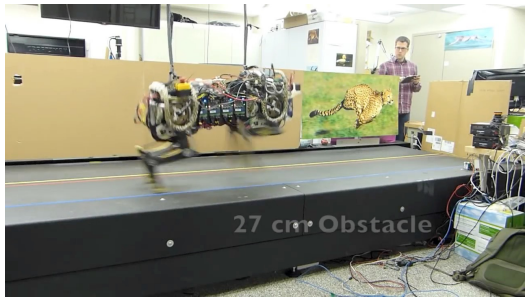
Robotics Approaches

Model-based methods

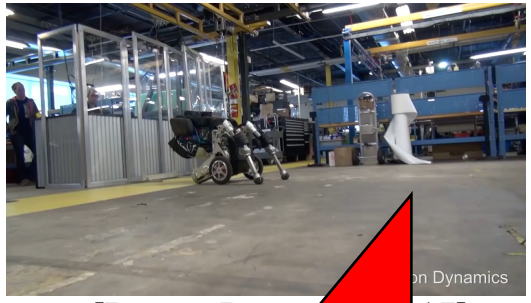
i.e. trajectory optimization, model-predictive control (MPC), Central Pattern Generators (CPGs)

Learning

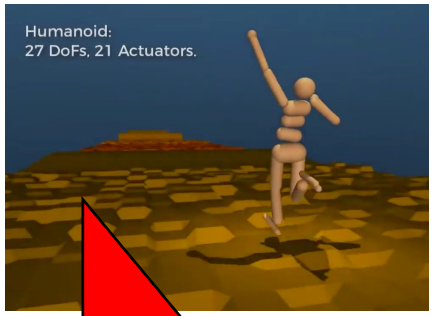
i.e. reinforcement learning, imitation learning



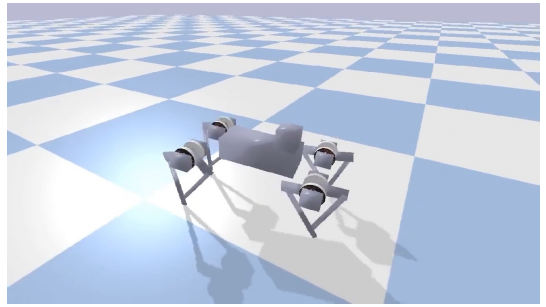
[Park et al. 2017]



[Boston Dynamics 2017]

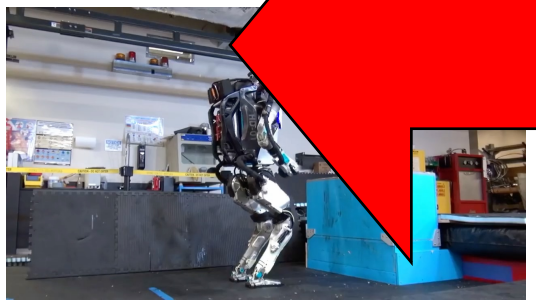


[Tan et al. 2017]

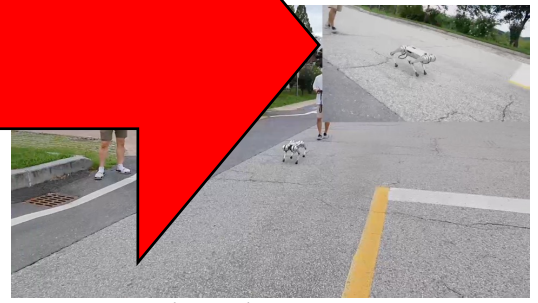


[Tan et al. 2018]

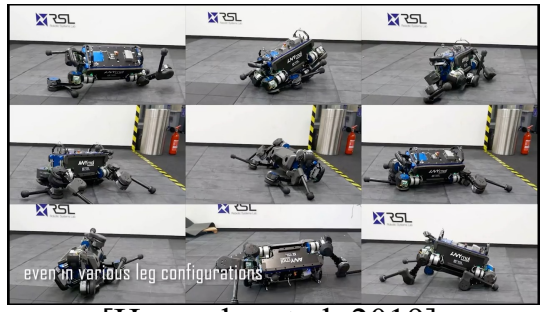
[Kim et al. 2019]



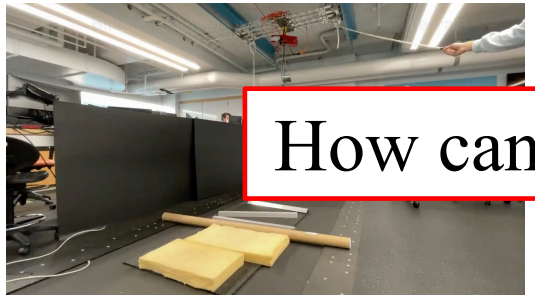
[Boston Dynamics 2018]



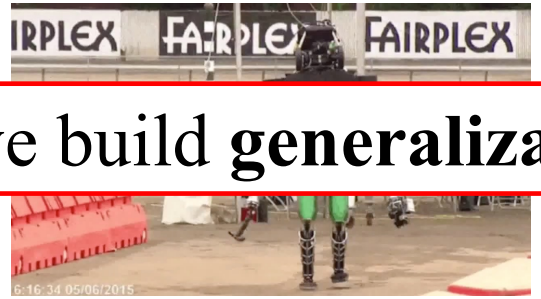
[Ji et al. 2022]



[Hwangbo et al. 2019]



[Xiong et al. 2021]



[DRC 2015]

[Lee et al. 2020]

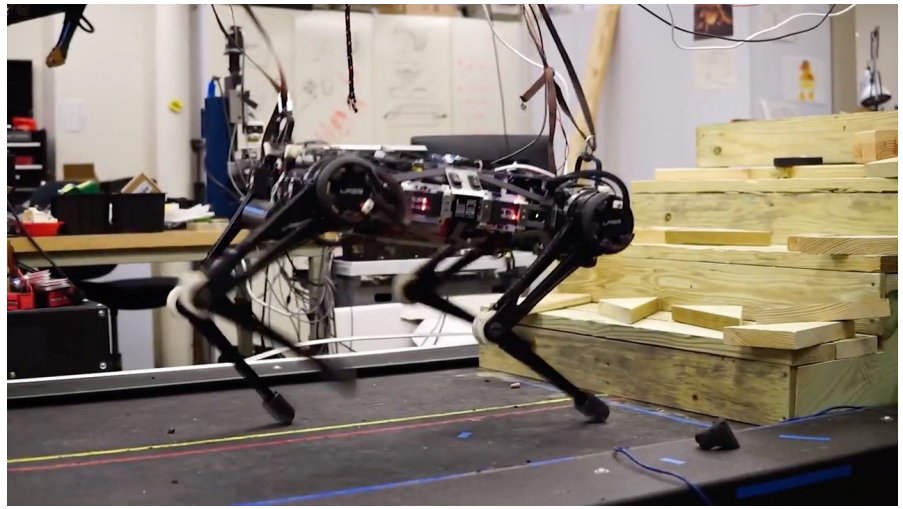
[Yang et al. 2021]

How can we build **generalizable**, agile, robust, and safe systems?

How to create locomotion controllers?

1. Trajectory optimization (model-predictive control)

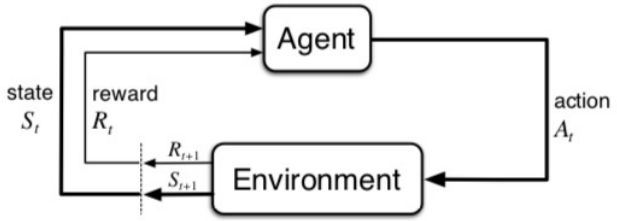
$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{u}} \quad & \sum_{i=0}^{k-1} ||\mathbf{x}_{i+1} - \mathbf{x}_{i+1,\text{ref}}||_{\mathbf{Q}_i} + ||\mathbf{u}_i||_{\mathbf{R}_i} \\
\text{subject to} \quad & \mathbf{x}_{i+1} = \mathbf{A}_i \mathbf{x}_i + \mathbf{B}_i \mathbf{u}_i, i = 0 \dots k-1 \\
& \underline{\mathbf{c}}_i \leq \mathbf{C}_i \mathbf{u}_i \leq \overline{\mathbf{c}}_i, i = 0 \dots k-1 \\
& \mathbf{D}_i \mathbf{u}_i = 0, i = 0 \dots k-1
\end{aligned}
$$

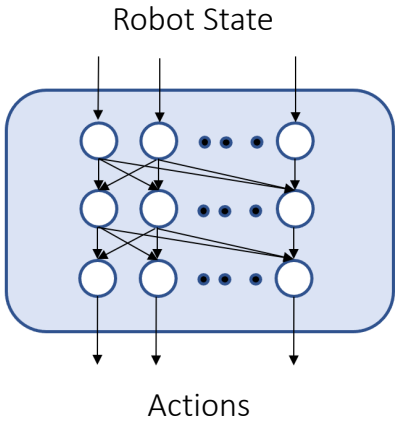


G. Bledt, P. Wensing, S. Ingersoll, S. Kim. *Contact Model Fusion for Event-Based Locomotion in Unstructured Terrains.* ICRA 2018
G. Bledt, M. Powell, B. Katz, J. D. Carlo, P. Wensing, S. Kim. *MIT Cheetah 3: Design and Control of a Robust, Dynamics Quadruped Robot.* IROS 2018

How to create locomotion controllers?

1. Trajectory optimization (model-predictive control)
2. Deep reinforcement learning / imitation learning

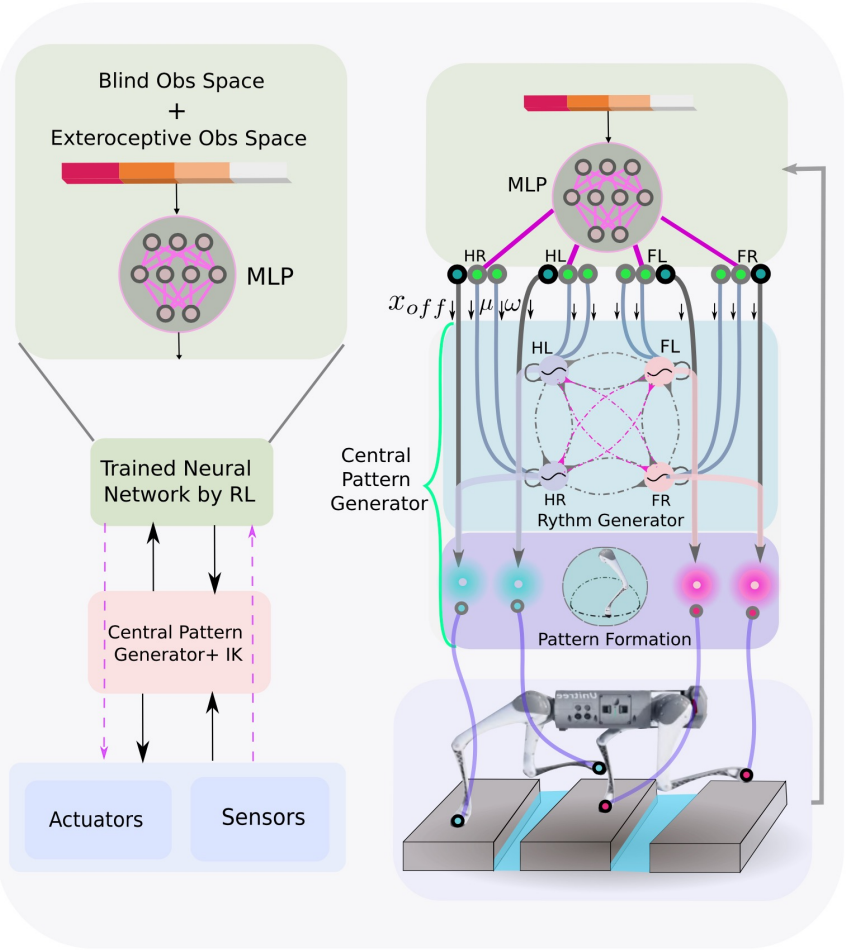




R. Sutton and A. Barto. Introduction to Reinforcement Learning. MIT Press 1998
J. Lee, J. Hwangbo, L. Wellhausen, V. Koltun, M. Hutter. Learning Quadrupedal Locomotion over Challenging Terrain. Science Robotics 2020

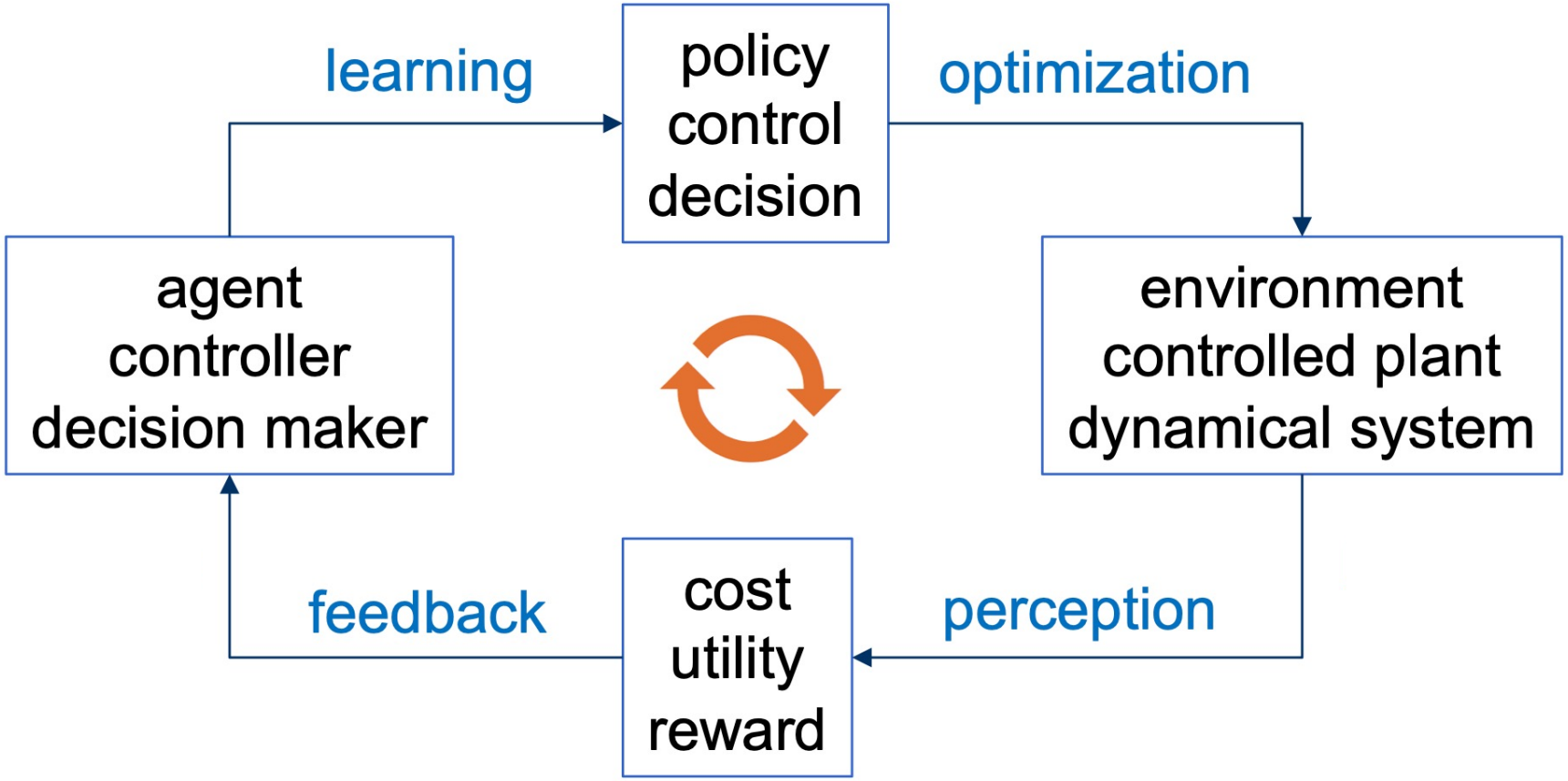
How to create locomotion controllers?

1. Trajectory optimization (model-predictive control)
2. Deep reinforcement learning / imitation learning
3. Bio-inspired learning



G. Bellegarda, M. Shafiee, A. Ijspeert. *AllGaits: Learning and Quadruped Gaits and Transitions.* ICRA 2025
M. Shafiee, **G. Bellegarda**, A. Ijspeert. *Viability Leads to the Emergence of Gait Transitions in Learning Agile Quadrupedal Locomotion on Challenging Terrains.* Nature Communications 2024

Common Setting: Closed-Loop Autonomous System



Trajectory Optimization

$$\underset{t_0, t_F, x(t), u(t)}{\text{minimize}} \quad J\big(t_0, t_F, x(t_0), x(t_F)\big) + \int_{t_0}^{t_F} w\big(\tau, x(\tau), u(\tau)\big) d\tau$$

subject to	$\dot{x}(t) = f\big(t, x(t), u(t)\big)$	System Dynamics
	$h\big(t, x(t), u(t)\big) \leq 0$	Path Constraint
	$g\big(t_0, t_F, x(t_0), x(t_F)\big) \leq 0$	Boundary Constraint
	$x_{low} \leq x(t) \leq x_{upp}$	Path Bound on State
	$u_{low} \leq u(t) \leq u_{upp}$	Path Bound on Control
	$t_{low} \leq t_0 < t_F \leq t_{upp}$	Bounds on Initial and Final Times
	$x_{0,low} \leq x(t_0) \leq x_{0,upp}$	Bound on Initial State
	$x_{F,low} \leq x(t_F) \leq x_{F,upp}$	Bound on Final State

M. Kelly. An Introduction to Trajectory Optimization: How to Do Your Own Direct Collocation. SIAM 2017

Direct Collocation

$$\underset{t_0, t_F, x(t), u(t)}{\text{minimize}} \quad J\big(t_0, t_F, x(t_0), x(t_F)\big) + \int_{t_0}^{t_F} w\big(\tau, x(\tau), u(\tau)\big) d\tau$$

subject to	$\dot{x}(t) = f\big(t, x(t), u(t)\big)$	System Dynamics
	$h\big(t, x(t), u(t)\big) \leq 0$	Path Constraint
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	$x_{0,low} \leq x(t_0) \leq x_{0,upp}$	Bound on Initial State
	$x_{F,low} \leq x(t_F) \leq x_{F,upp}$	Bound on Final State

$$\underset{x_k, u_k;\ k=1\ldots N}{\text{minimize}} \quad J\big(x_N\big) + h \sum_{k=1}^{N} w\big(x_k, u_k\big)$$

$$\text{subject to} \quad d\big(x_k, u_k, x_{k+1}, u_{k+1}\big) = 0,\ k = 1, \ldots, N-1$$

$$\phi\big(x_k, u_k\big) = 0,\ k = 1, \ldots, N$$

$$\psi\big(x_k, u_k\big) \geq 0,\ k = 1, \ldots, N$$

Backward Euler integration:

$$d\big(x_k, u_k, x_{k+1}, u_{k+1}\big) := x_{k+1} - \big(x_k + h f(x_{k+1}, u_{k+1})\big)$$

M. Posa, C. Cantu, R. Tedrake. A Direct Method for Trajectory Optimization of Rigid Bodies Through Contact. IJRR 2013
M. Kelly. An Introduction to Trajectory Optimization: How to Do Your Own Direct Collocation. SIAM 2017

Applying Trajectory Optimization to JPL's RoboSimian

- Four identical limbs, each with 7 degrees of freedom (DOFs)
- Joint motor speeds peak at 1 (rad/s)
  - RoboSimian is stable, but very slow
- Passive single-wheel skate mounted at each forearm
  - Increase efficiency, speed
- 6 actuated DOFs available to set 6-DOF pose of each skate
- No sensing available at skate contact

G. Bellegarda, K. van Teeffelen, K. Byl. "Design and Evaluation of Skating Motions for a Dexterous Quadruped," ICRA 2018

Trajectory Optimization Framework

find $q, \dot{q}, u, F_n, F_{fric}$ at discrete timesteps $k = 1...N$

subject to minimize cost J

- State Constraints:

$$\phi(q, \dot{q}, u, F_n) = 0$$

$$\psi(q, \dot{q}, u, F_n) \geq 0$$

- Dynamics Constraints:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + A(q)^T\lambda = B(q)u + F_n + J(q)^T F_{fric}$$

- Friction Constraints (for each contact i):

$$\gamma e + D^T v^{k+1} \geq 0, \quad \beta \geq 0$$

$$\mu F_{n_i} - e^T\beta \geq 0, \quad \gamma \geq 0$$

$$(\gamma e + D^T v^{k+1})^T \beta = 0$$

$$(\mu F_{n_i} - e^T\beta)\gamma = 0$$

$$F_{fric_i} = D\beta$$

- ZMP Constraints (RoboSimian Only)
- Gait Constraints (RoboSimian Only)

End Location Power

$$J = J_F(q_N, \dot{q}_N) + h\sum_{k=1}^{N}\sqrt{\left(\dot{q}_k^T U_k\right)^2 + \epsilon}$$

with

- $U = [0, 0, 0, 0, u_{x_i}, u_{y_i}, u_{z_i}, u_{\theta_i}]^T \in \mathbb{R}^{20}$ for $i \in \{1, 2, 3, 4\}$
- $\epsilon = 10^{-6}$ is a regularization term to help smooth the cost function
- $h = \Delta t$ is the time interval between time steps
- $J_F(q_N, \dot{q}_N)$ is a distance measure to a set of goal coordinates, such as:

$$J_F = \alpha_x(x_g - x_N)^2 + \alpha_y(y_g - y_N)^2 + \alpha_\theta(\theta_g - \theta_N)^2$$

M. Srinivasan, *Why walk and run: energetic costs and energetic optimality in simple mechanics-based models of a bipedal animal.* Cornell University, Ithaca, NY, 2006.

G. Bellegarda and K. Byl. "Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip," CDC 2019

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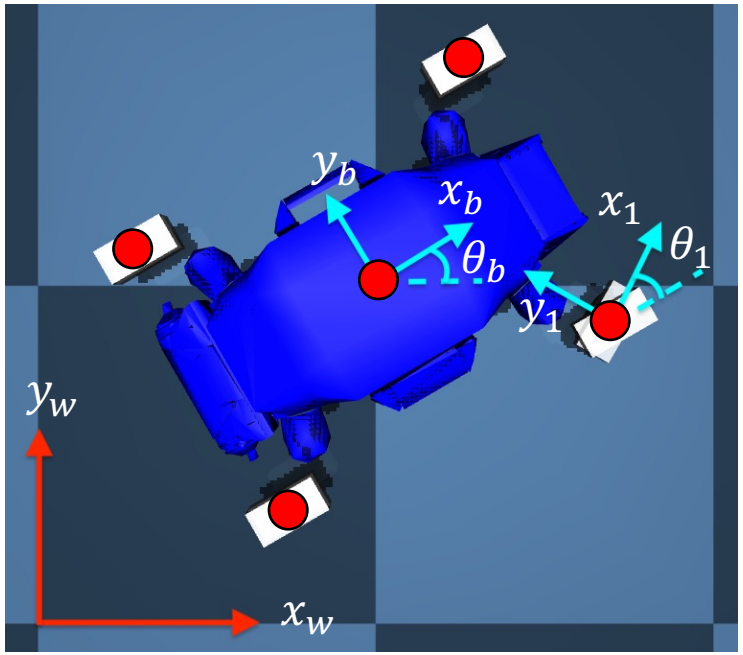
$F_{fric_i} = D\beta$

- ZMP Constraints (RoboSimian Only)
- Gait Constraints (RoboSimian Only)

- $q, \dot{q}$ are bounded by ranges for placing each skate with inverse kinematics, as well as by joint limits
- u is bounded explicitly, as well as implicitly by $\dot{q}$ ranges
- Known contact sequence, where $F_n \geq 0$ for each skate in contact, and $F_n = 0$ for skates not in contact



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- ZMP Constraints (RoboSimian Only)

- Gait Constraints (RoboSimian Only)

$$q_{k+1} = q_k + h\dot{q}_{k+1}$$
$$\dot{q}_{k+1} = \dot{q}_k + h\ddot{q}_{k+1}$$

with

$$\ddot{q}_{k+1} = M_{k+1}^{-1}(B_{k+1}u_{k+1} + F_{n_{k+1}} + J_{k+1}^T F_{fric_{k+1}} - C_{k+1}\dot{q}_{k+1} - G_{k+1})$$

G. Bellegarda and K. Byl. "Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip," CDC 2019

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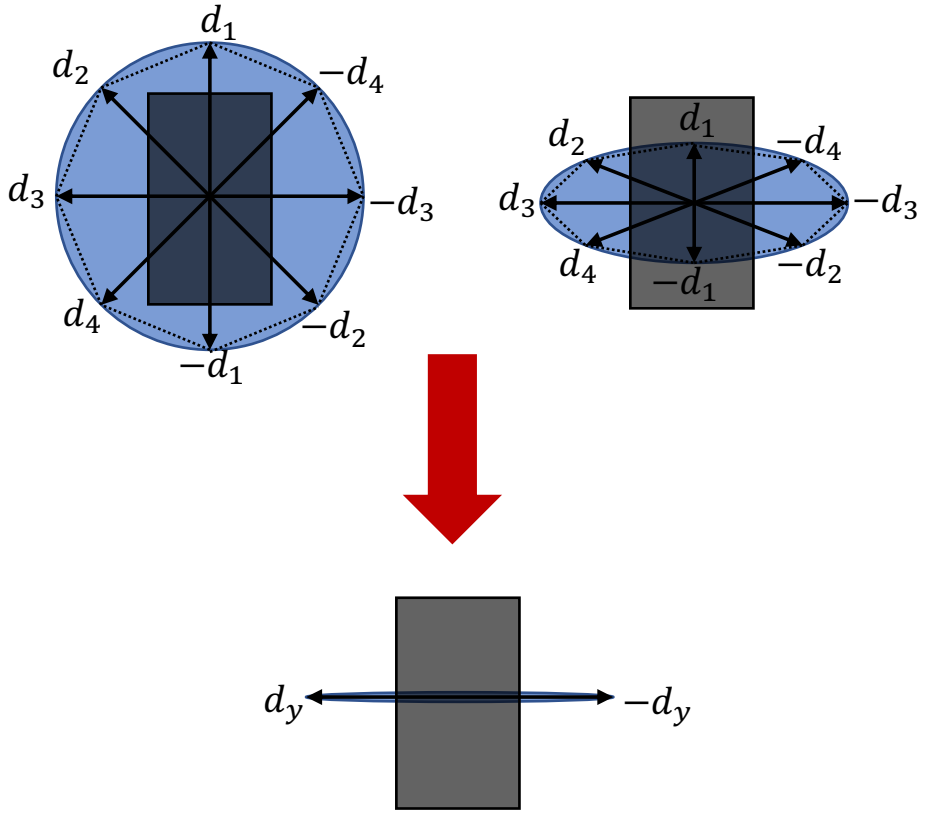
Friction as a Linear Complementarity Problem (LCP)



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G. Bellegarda and K. Byl. "Versatile Trajectory Optimization Using a LCP Wheel Model for Dynamic Vehicle Maneuvers," ICRA 2020

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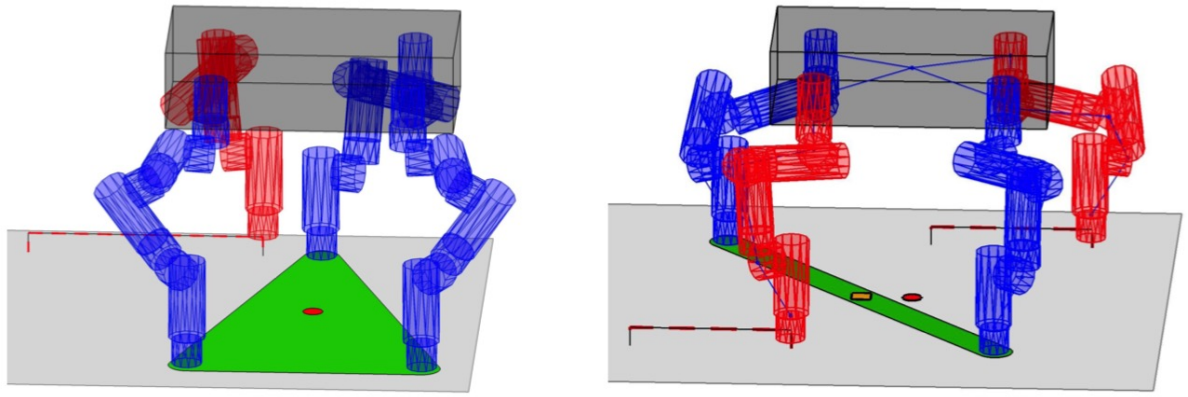
Zero-Moment Point (ZMP): point on ground where the sum of all moments of the active forces is equal to zero



M. Vukobratovic, B. A. Borovac. Zero-Moment Point – Thirty Five Years of its Life. IJHR 2004

P. Ha and K. Byl. Feasibility and Optimization of Fast Quadruped Walking with One- Versus Two-at-a-Time Swing Leg Motions for RoboSimian. 2014

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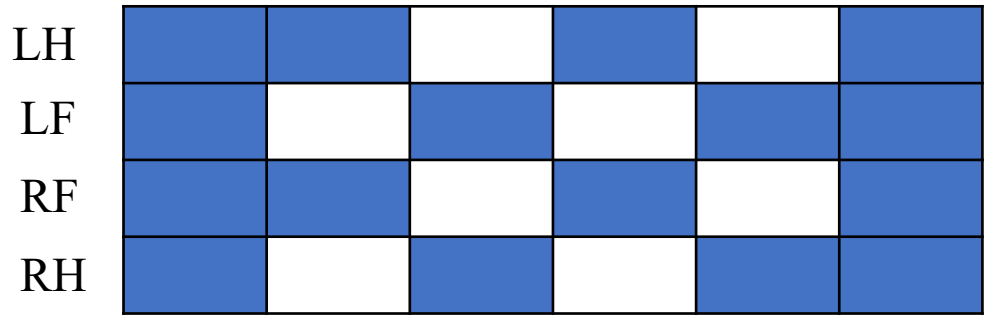
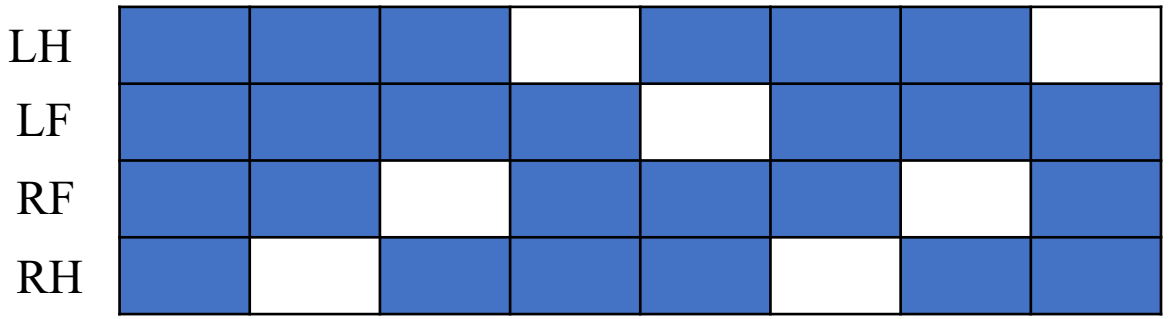
Static Trotting Gait

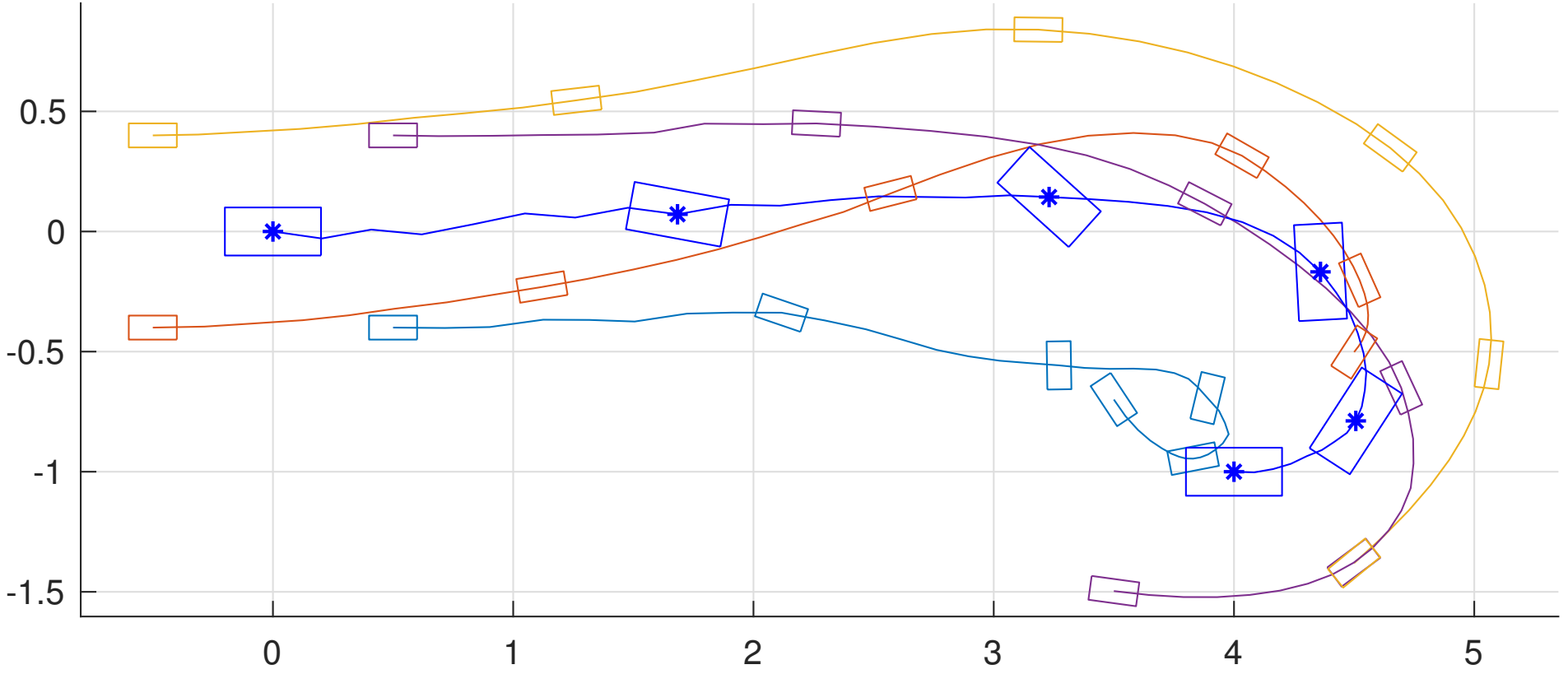


Static Walking Gait



Dynamic Parking Maneuver

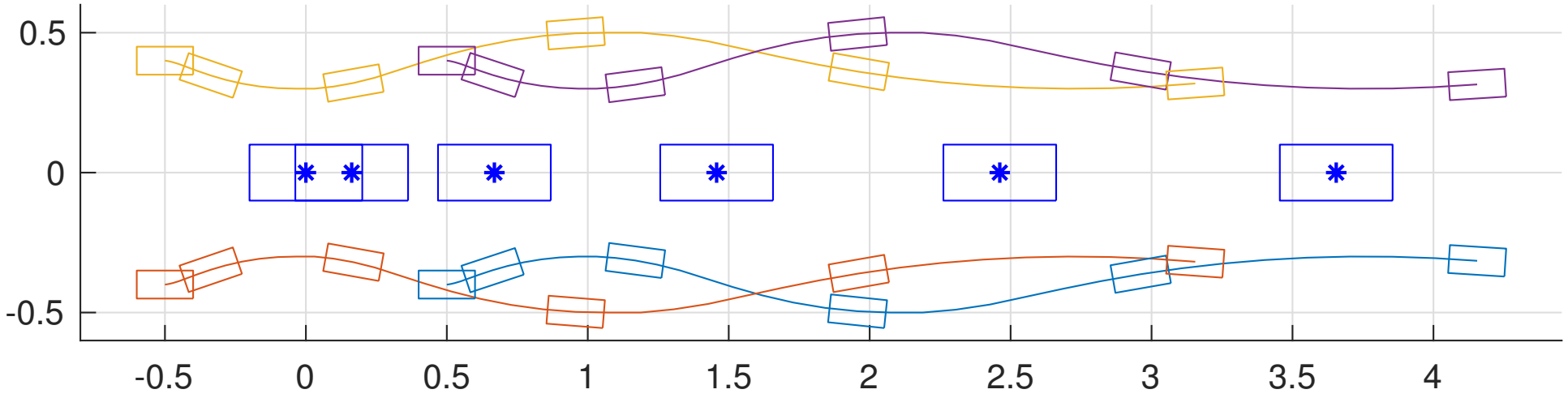
- Goal: "park" at $(x_b, y_b, \theta_b) = (4, -1, -\pi)$, initialized at (0,0,0) with $\dot{x}_b = 4$ (m/s)



G. Bellegarda and K. Byl. "Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip," CDC 2019

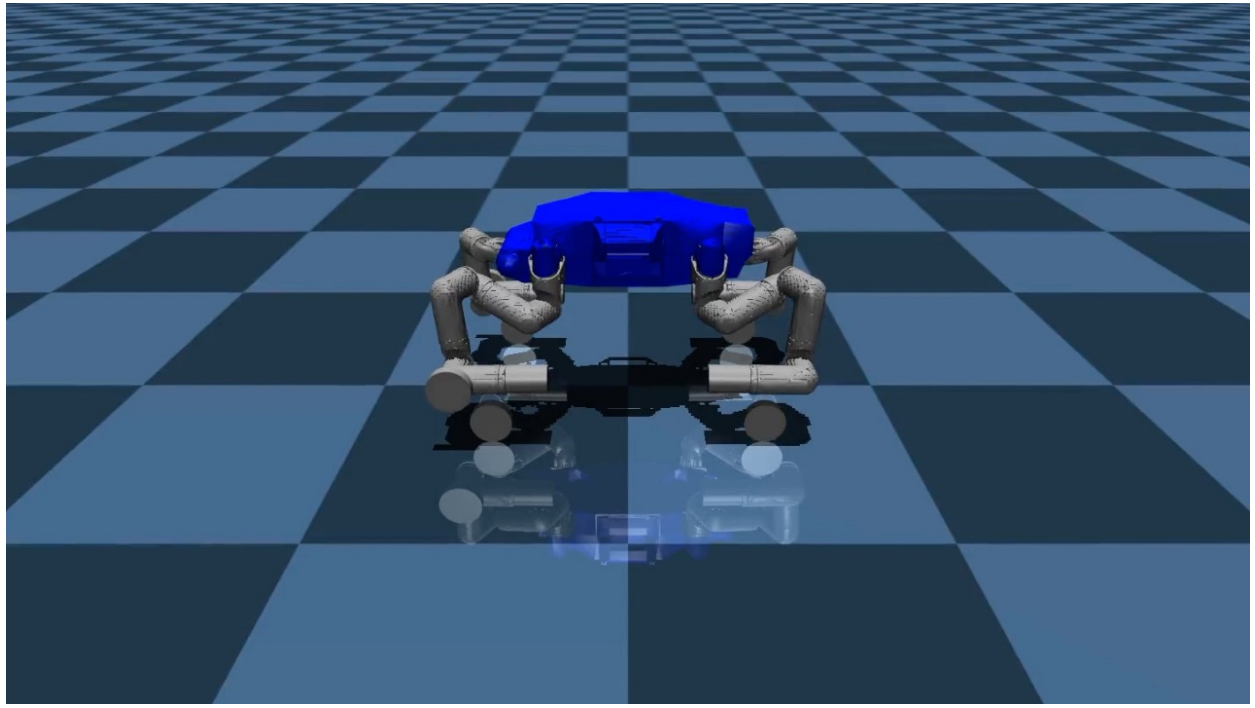
Energy-Efficient Forward Locomotion

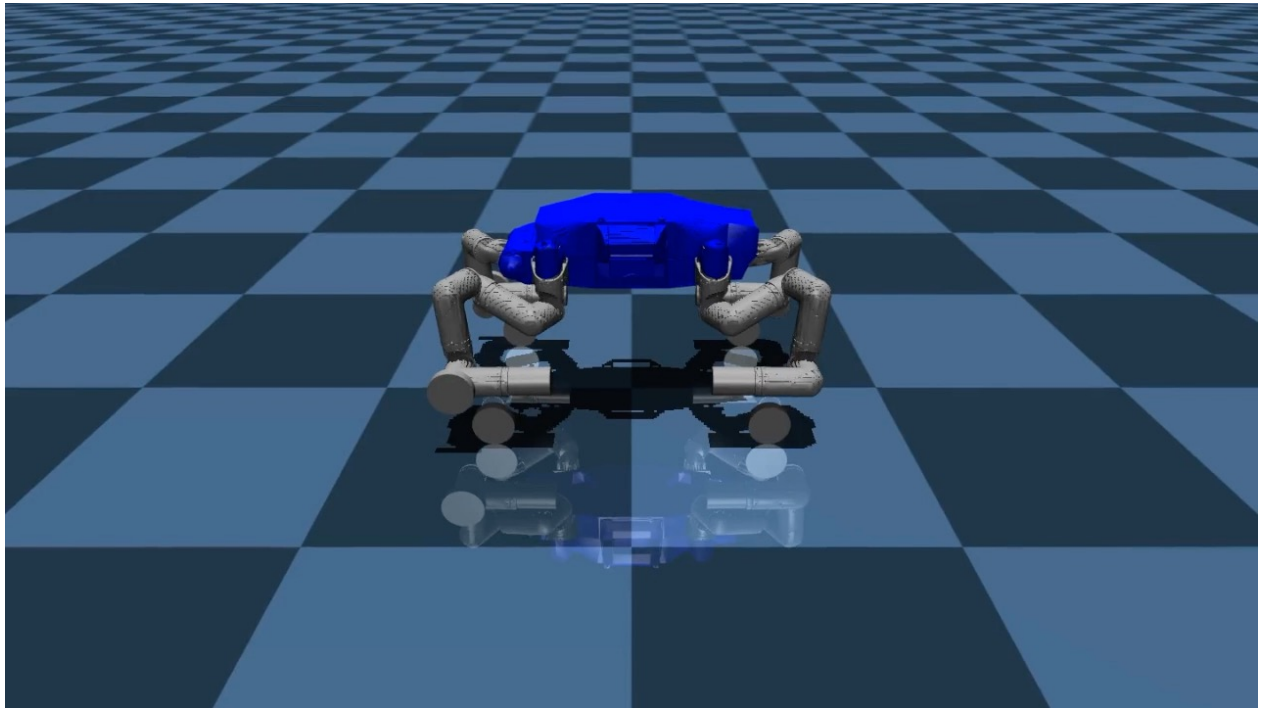
- Maximize final body position in the x-direction, while minimizing energy use



G. Bellegarda and K. Byl. "Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip," CDC 2019

Hybrid Rolling-Walking/Trotting





G. Bellegarda and K. Byl. “Trajectory Optimization for a Wheel-Legged System for Dynamic Maneuvers that Allow for Wheel Slip,” CDC 2019

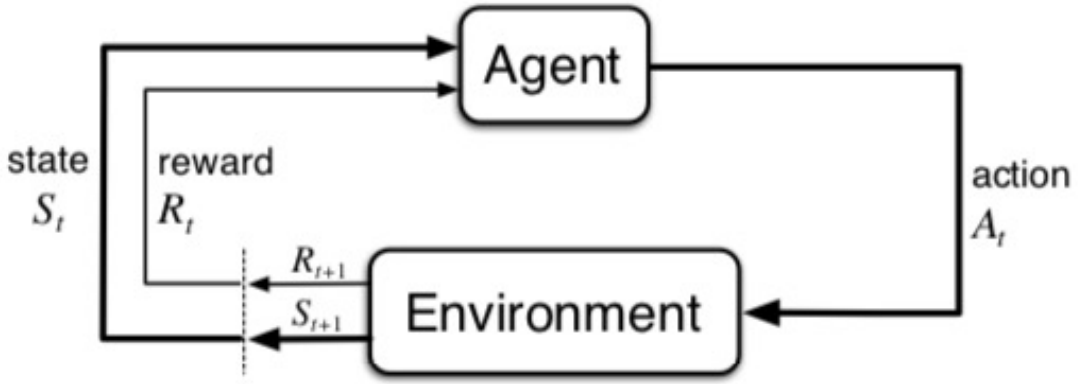
TO/MPC Limitations

- Very complex systems → cannot solve in real-time!
  - Model simplifications
- Uncertainty?
  - Typically assumptions on friction, actuation, etc.
  - Environment may be different than planned!
- Biases?
  - Gait pattern, constraints, cost function
- Integrate vision, hierarchical control, etc.

Reinforcement Learning as a Markov Decision Process (MDP)

An MDP is defined by:

- Set of states S
- Set of actions A
- Transition function $P(s' \mid s, a)$
- Reward function $R(s, a, s')$
- Start state s_0
- Discount factor γ
- Horizon H



- Return over a trajectory $\tau = (s_0, a_0, s_1, a_1, \ldots)$

$$R(\tau) = \sum_{t=0}^{\infty} \gamma^t r_t$$

- Policy $\pi(a_t|s_t)$ maps from states s_t to actions a_t (Goal: find policy maximizing above return)
- Value function: $V^{\pi}(s) = \mathbb{E}_{\tau\sim\pi}[R(\tau)|s_0 = s]$
- Action-value function: $Q^{\pi}(s, a) = \mathbb{E}_{\tau\sim\pi}[R(\tau)|s_0 = s, a_0 = a]$
- Advantage function: $A^{\pi}(s, a) = Q^{\pi}(s, a) - V^{\pi}(s)$

R. Sutton and A. Barto. Introduction to Reinforcement Learning. MIT Press 1998
Deep RL Bootcamp. Berkeley CA, August 2017

Many Existing Tools for Reinforcement Learning

- RL algorithm implementations
  - rsl_rl https://github.com/leggedrobotics/rsl_rl
  - stable-baselines3 https://github.com/DLR-RM/stable-baselines3
  - ray[rllib] https://github.com/ray-project/ray
  - skrl https://skrl.readthedocs.io/en/latest/
  - rl_games https://github.com/Denys88/rl_games
  - … many others!
- Physics simulators
  - Isaac Gym https://developer.nvidia.com/isaac-gym
  - Isaac Lab https://github.com/isaac-sim/IsaacLab
  - pybullet https://github.com/bulletphysics/bullet3
  - MuJoCo https://mujoco.org
  - RaiSim https://raisim.com
  - … and others!

Reinforcement Learning Considerations

Algorithm

- On/off policy
- Hyperparameters
- Network architecture
- Random seeds/trials

…implementation dependent!

MDP Design Decisions

- Observation space
- Action space
- Reward function

Environment Parameters

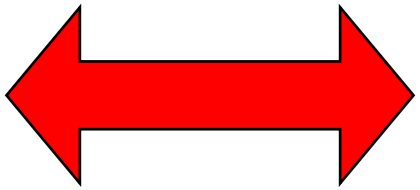
- Simulator dynamics
- Control gains – joint/Cartesian
- Control/environment time step
- Noise, latency

P. Henderson et al. *Deep Reinforcement Learning that Matters*. arXiv:1709.06560, 2017

Learning in Simulation → Deploy in real world (Why?)

Real World

- Slow
- Unsafe
- Expensive
- Human Supervision



Simulation

- Fast
- Safe
- Cheap
- Scalable

BUT: "Sim-to-real" gap → Simulation != Reality

- Unmodeled dynamics
- Wrong simulation parameters
- Inaccurate contact models
- Latency
- Actuator dynamics
- Sensor noise
- Stochastic real world
- …

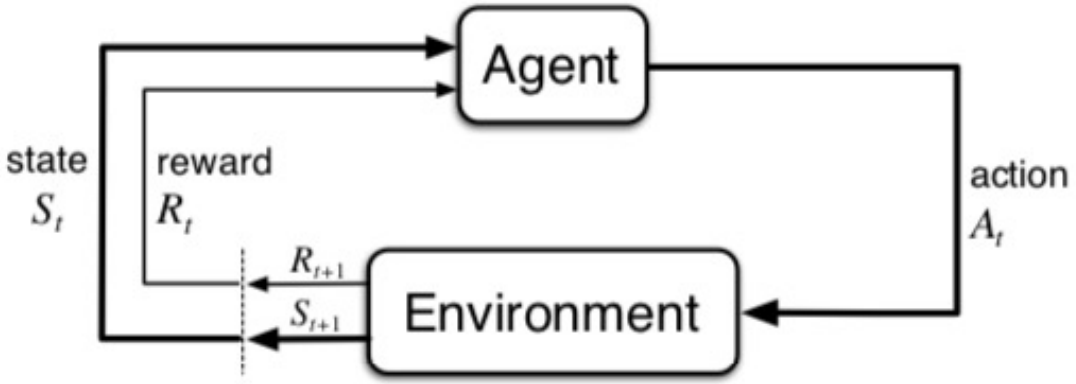
General RL Techniques for Locomotion

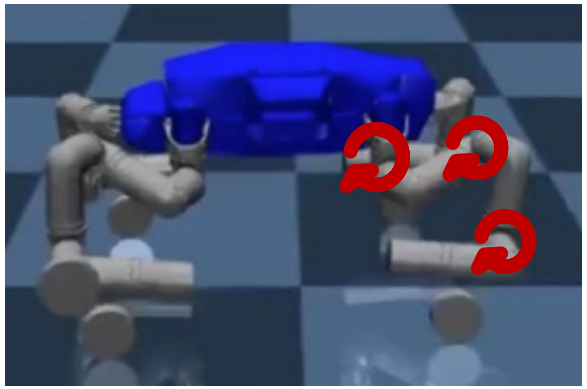
- System identification
  - Add real-world identified actuator dynamics to simulator
- Domain randomization
  - Randomize geometries, mass, inertia, friction, perturbations, etc.
- Curriculum learning
  - To learn a “difficult skill”, first start with an easier version of the task, then make it harder as the agent gets better
- Teacher-Student distillation
  - Train a teacher policy with privileged information (i.e. not known in real world)
  - Train a student policy to imitate the teacher action with supervised learning
- Reward shaping
  - Change or tune the reward function over time to prioritize different task-related components

N. Rudin, D. Hoeller, P. Reist, M. Hutter. *Learning to Walk in Minutes Using Massively Parallel Deep Reinforcement Learning.* CORL 2021

State/Action/Reward Space?

s_t ? i.e.
-body (z, r, p, y)
-body velocities
-joint states





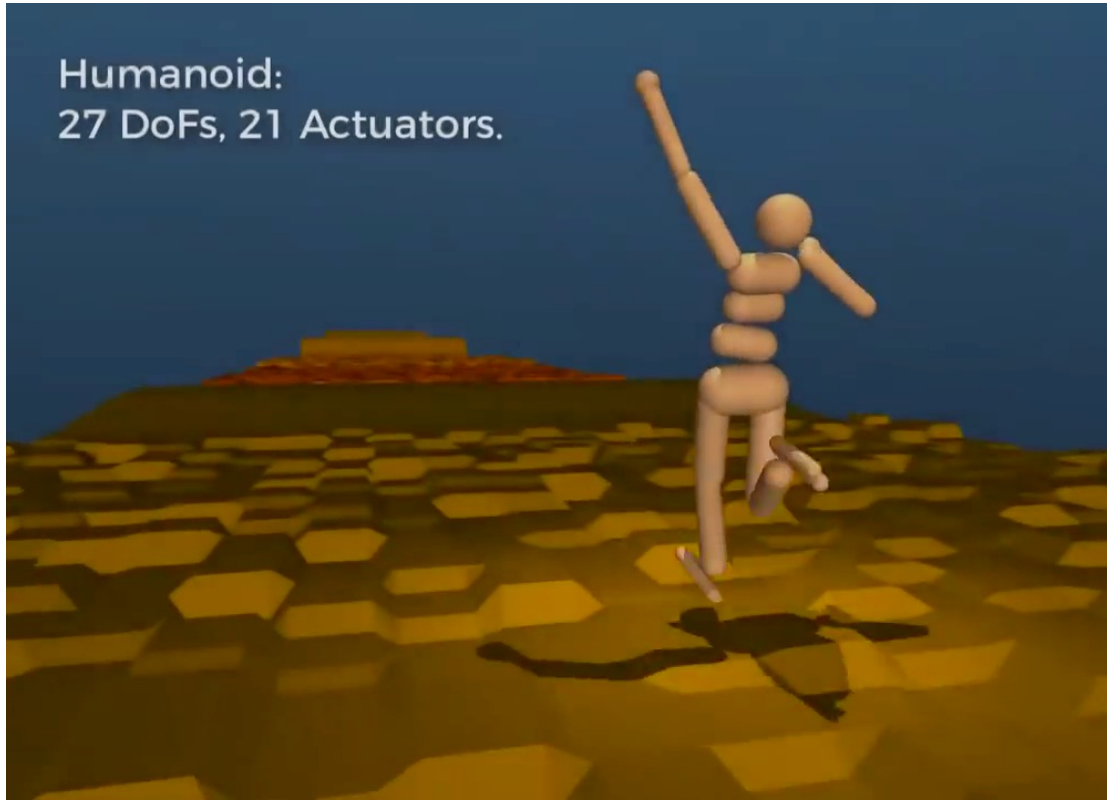
a_t ?
-motor positions/torques

r_t ? i.e.
-body linear velocity
-energy penalty

G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

Training in Joint Space (PPO)

Action Space: $a_t = \tau_{1\ldots N}$

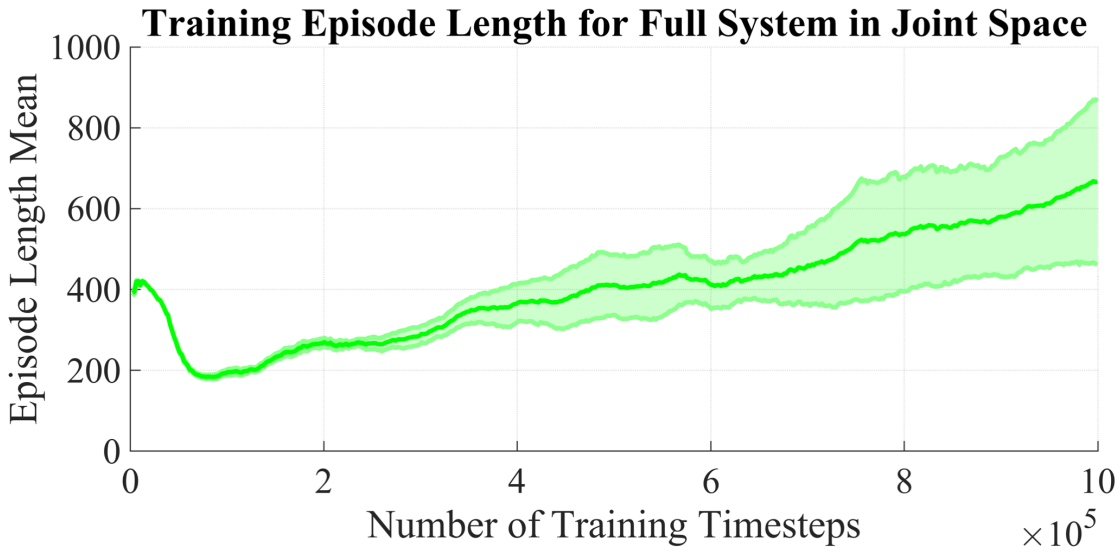


N. Heess et al. Emergence of Locomotion Behaviours in Rich Environments. *arXiv:1707.02286*, 2017
J. Schulman et al. Proximal policy optimization algorithms. *arXiv:1707.06347*, 2017
G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

Training in Joint Space (PPO)

Action Space: $a_t = \tau_{1\ldots N}$

- Agent is essentially forced to learn forward and inverse kinematics!



N. Heess et al. Emergence of Locomotion Behaviours in RichEnvironments.*arXiv:1707.02286*, 2017
J. Schulman et al. Proximal policy optimization algorithms. *arXiv:1707.06347*, 2017
G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

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s_t ? i.e.
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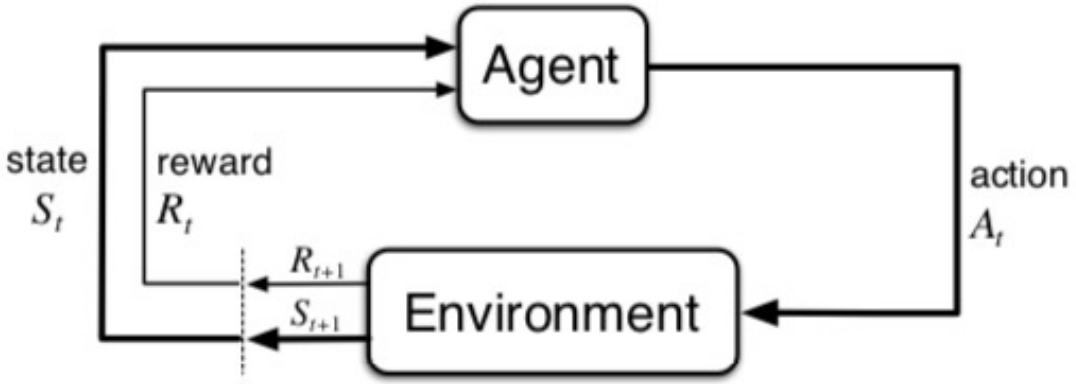
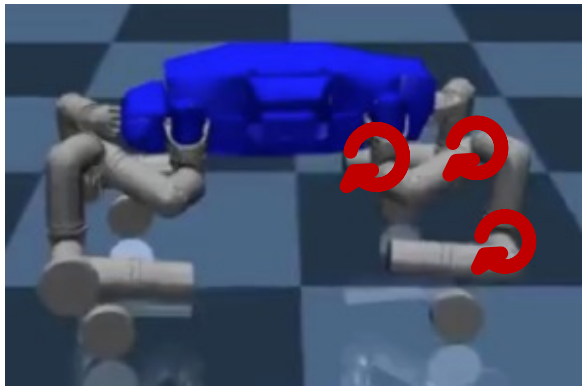
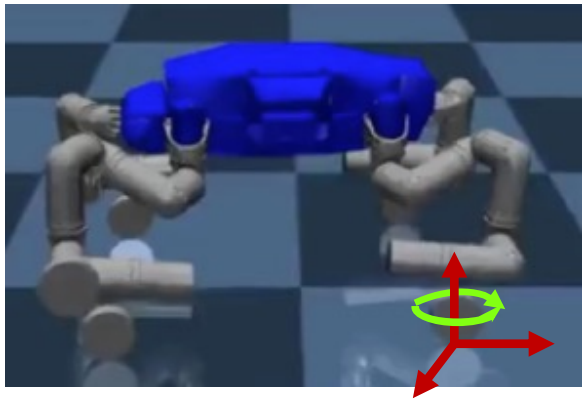


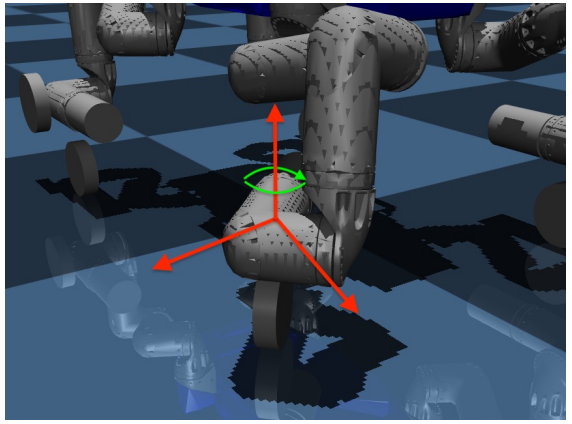


a_t ?
-motor positions/torques
-Cartesian coordinates

r_t ? i.e.
-body linear velocity
-energy penalty

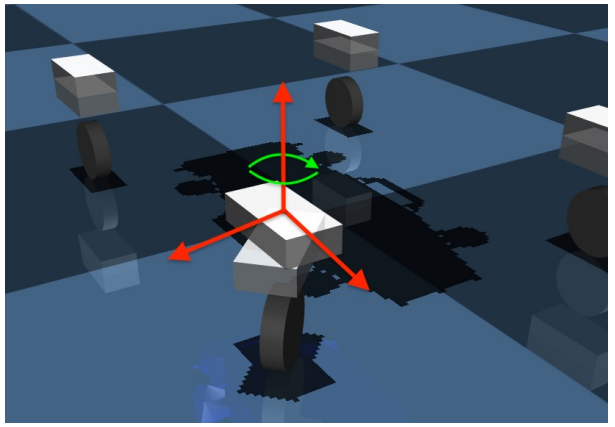


G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019



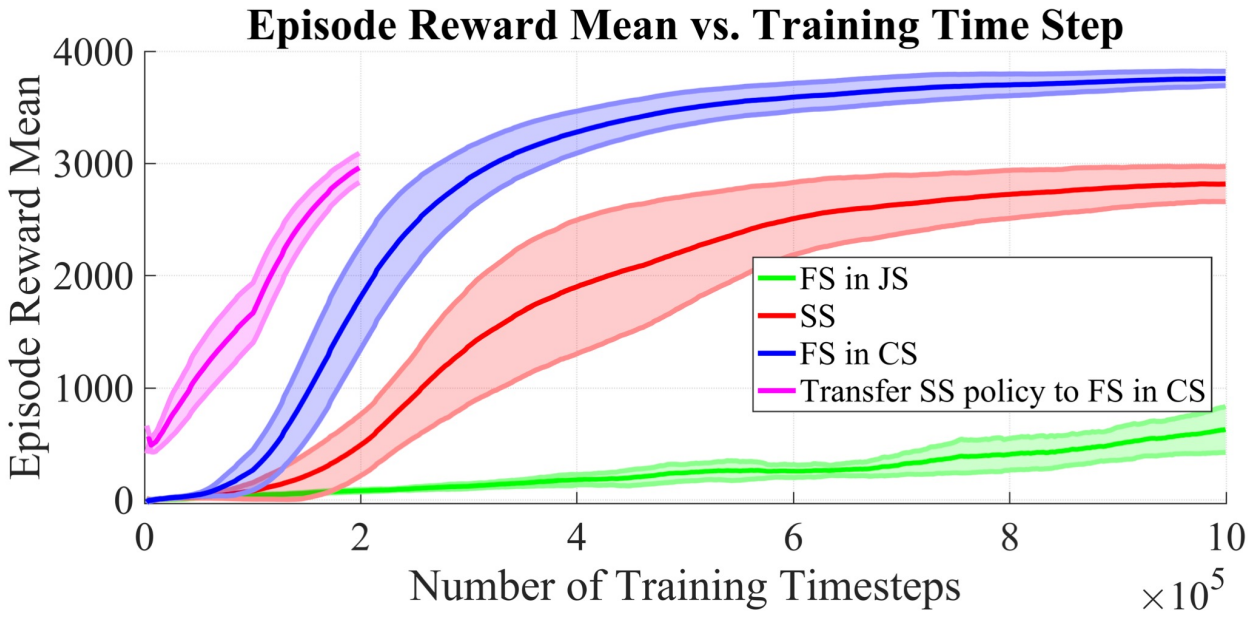
Training in Task Space (PPO)

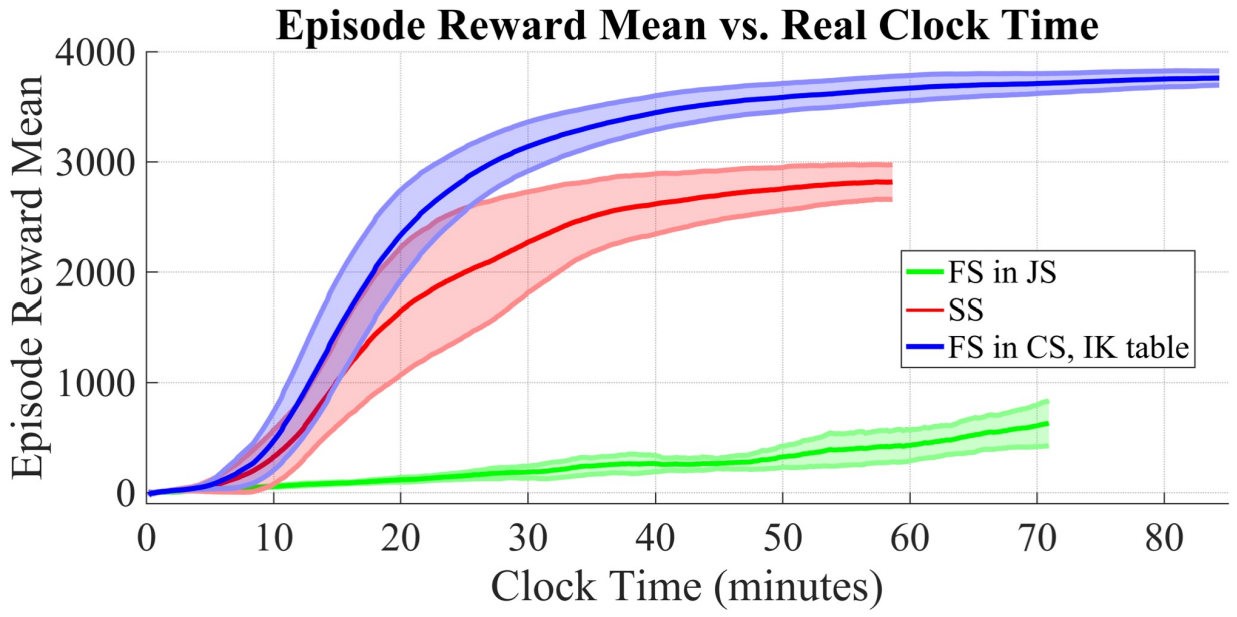
Action Space: $a_t = [y_{ee_i}, \phi_{ee_i}]$ for $i \in \{1,2,3,4\}$



G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

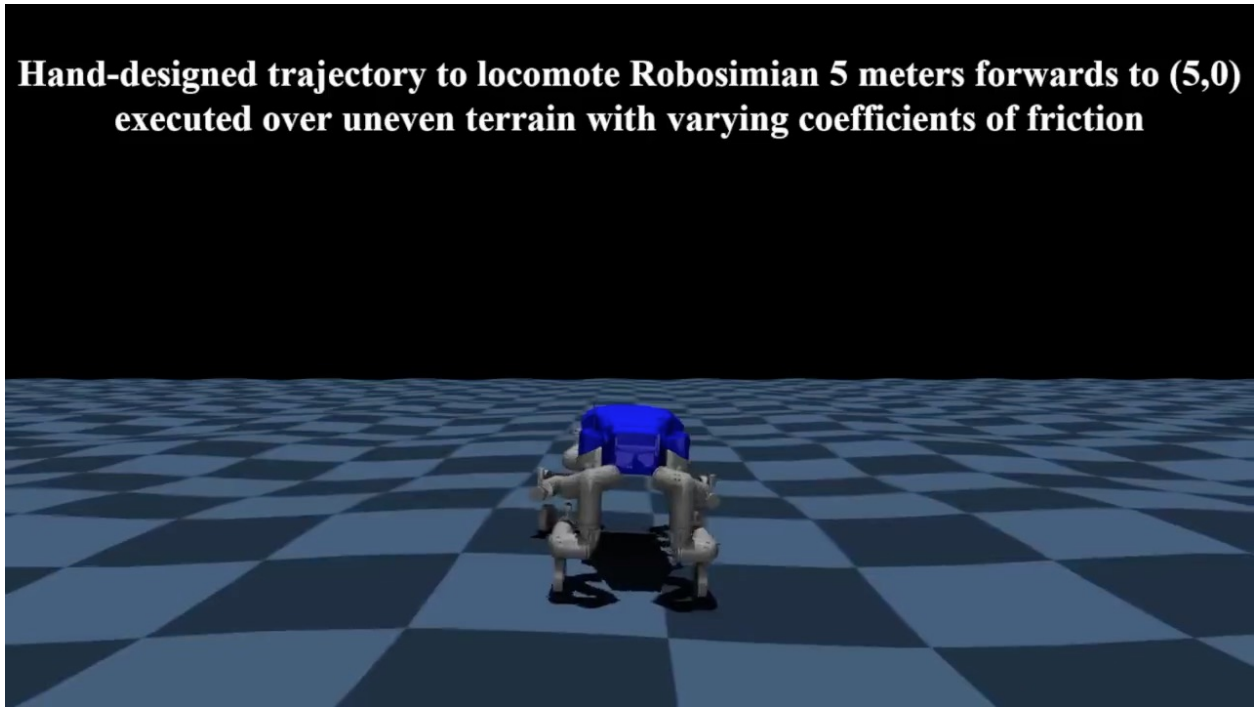
Training in Joint Space vs. Task Space

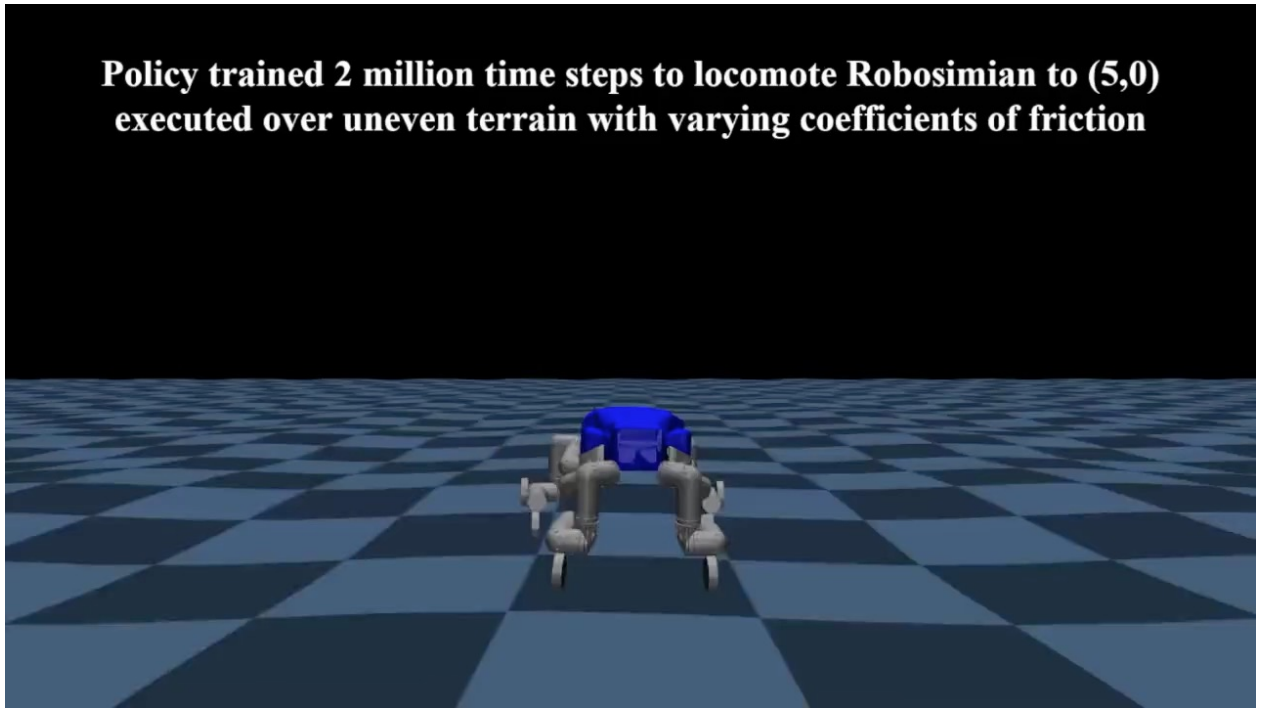




G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

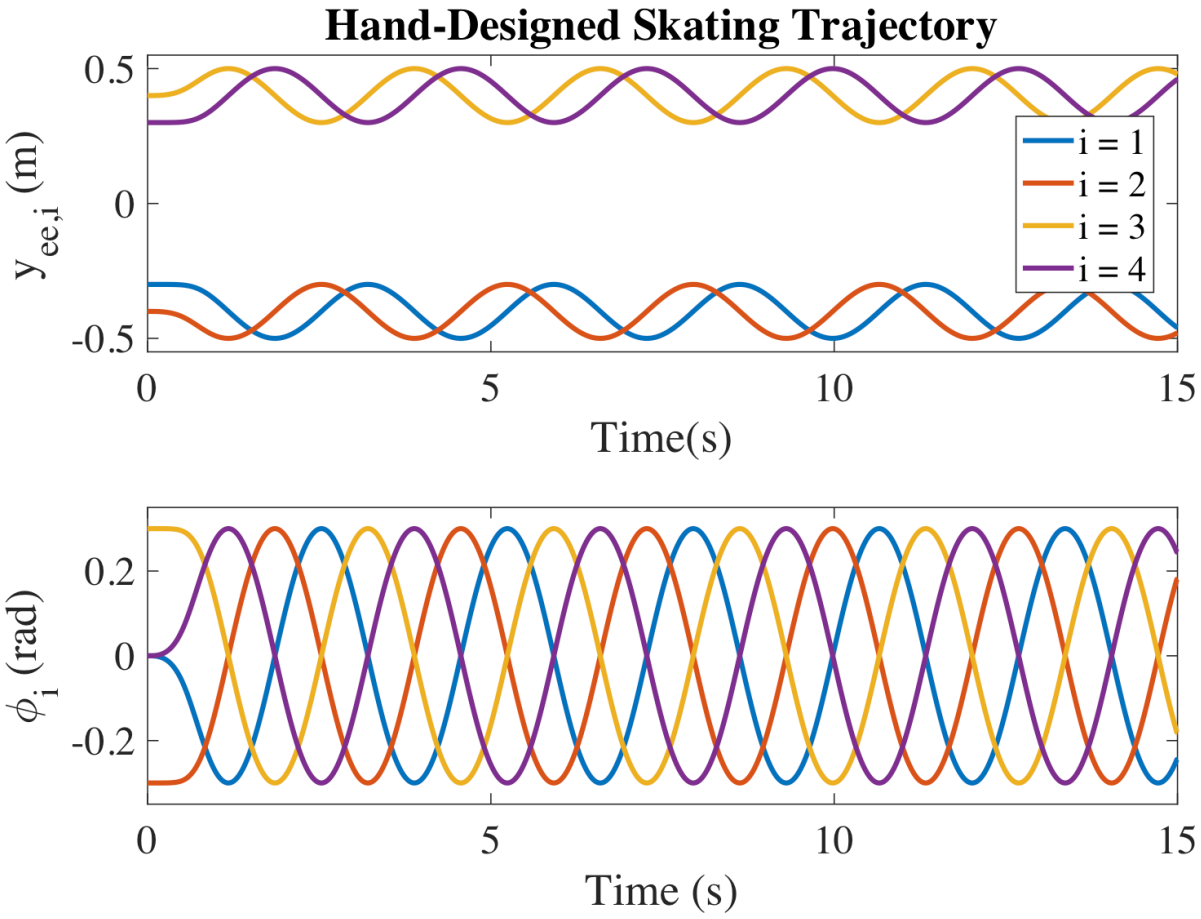
Hand-designed Trajectory vs. Trained Policy over uneven terrain with varying coefficients of friction

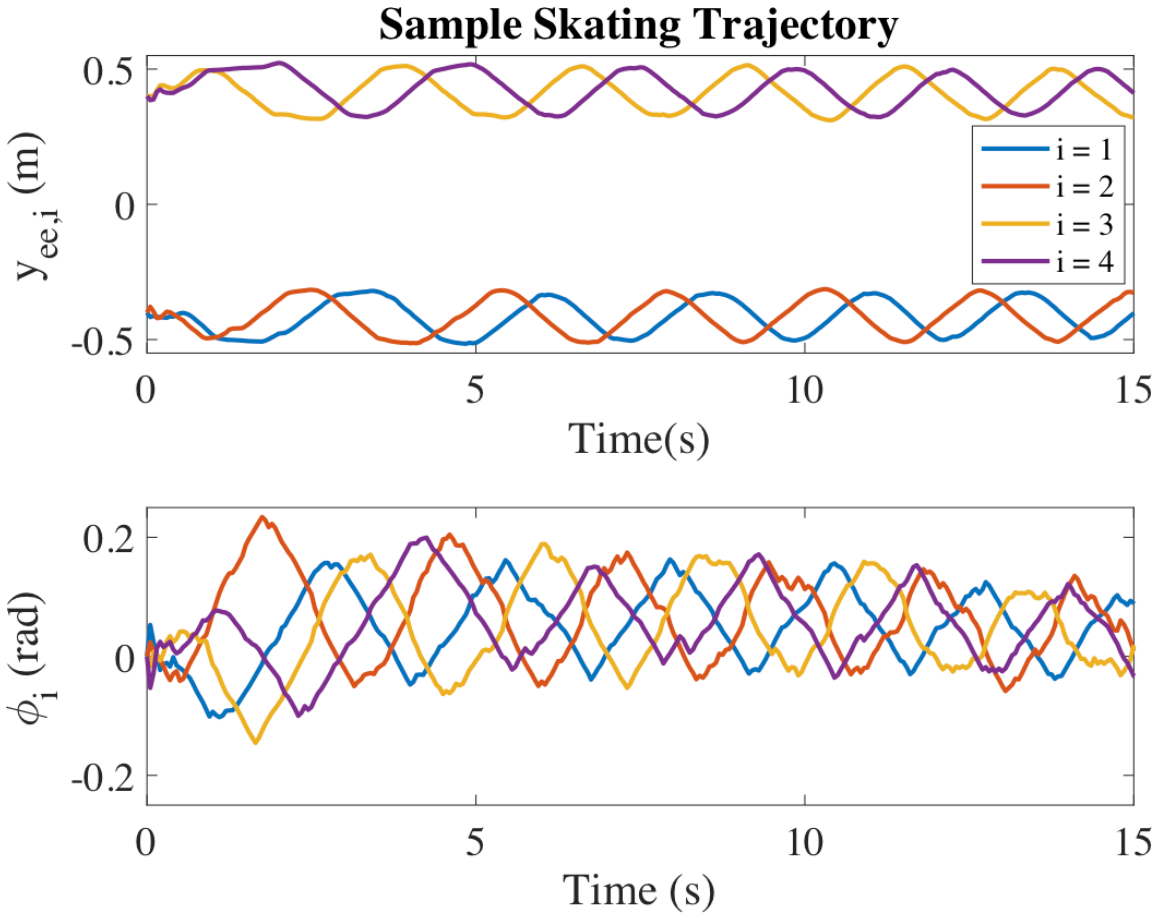




G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

Hand-designed Trajectory vs. Trained Policy

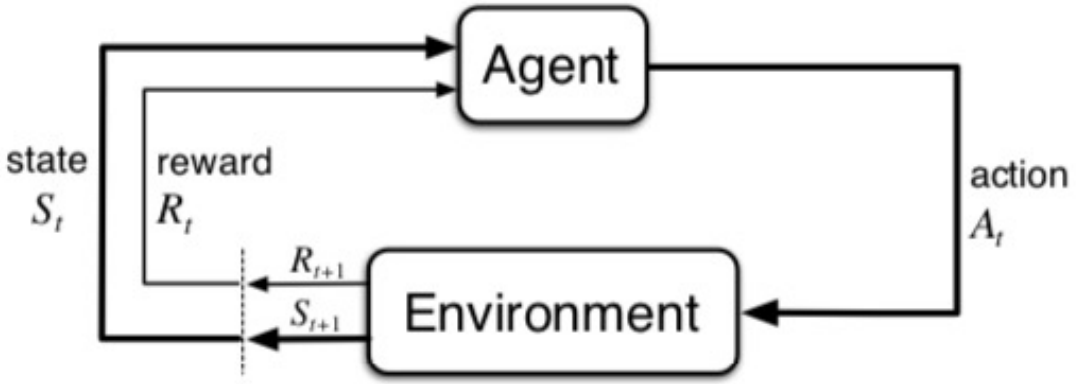


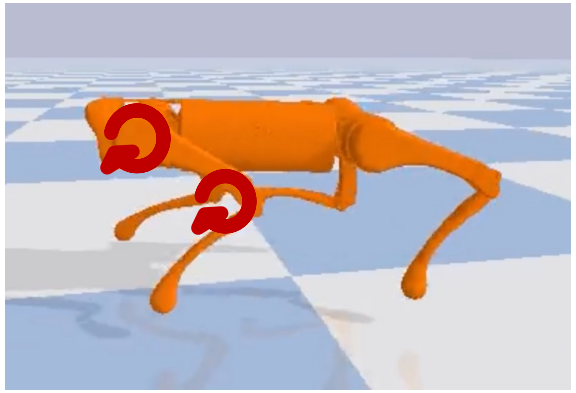


G. Bellegarda and K. Byl. "Training in Task Space to Speed Up and Guide Reinforcement Learning," IROS 2019

State/Action/Reward Space: A1

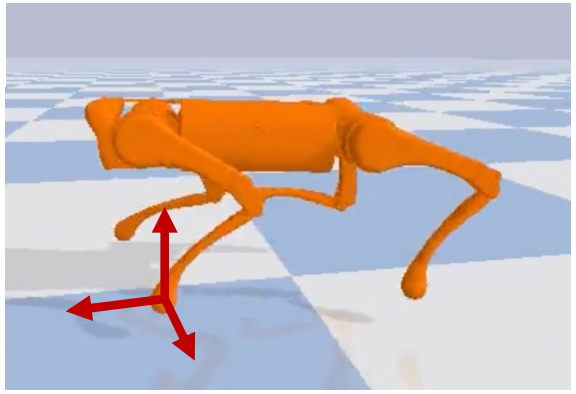
s_t ? i.e.
-body (z, r, p, y)
-body velocities
-joint states





a_t ?
-motor positions/torques
-Cartesian PD

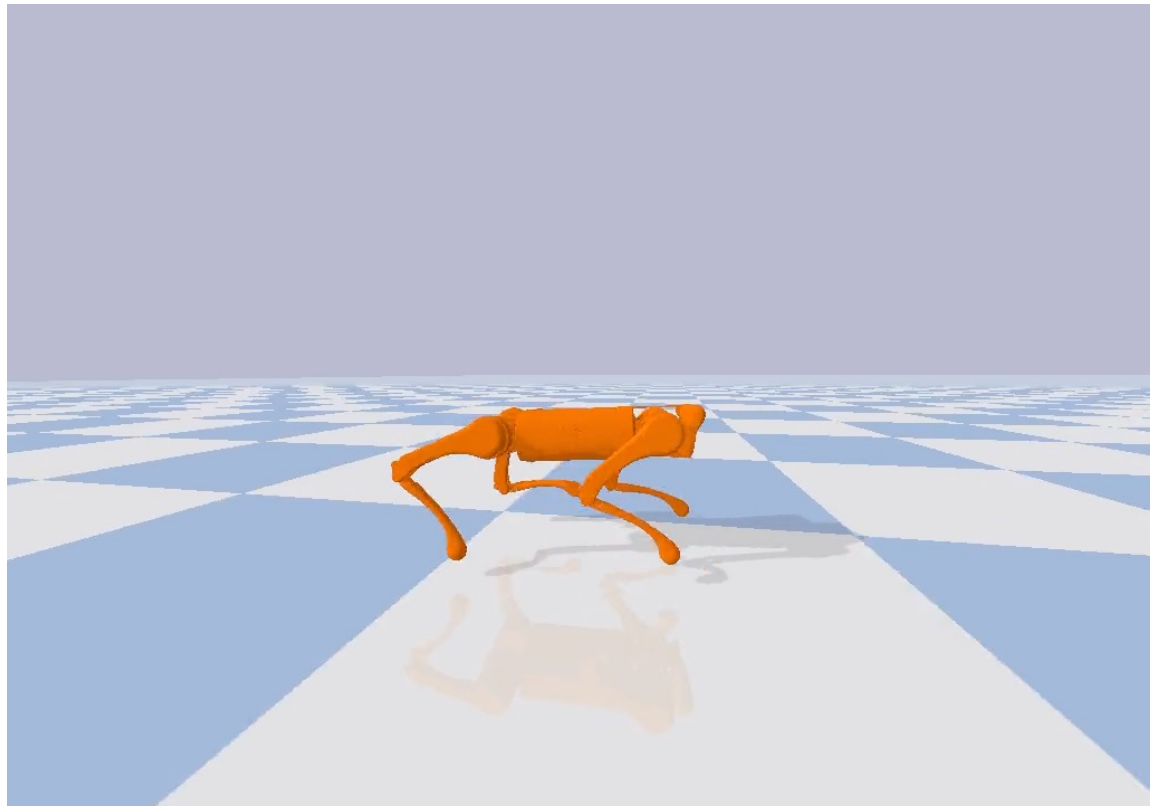
r_t ? i.e.
-body linear velocity
-energy penalty

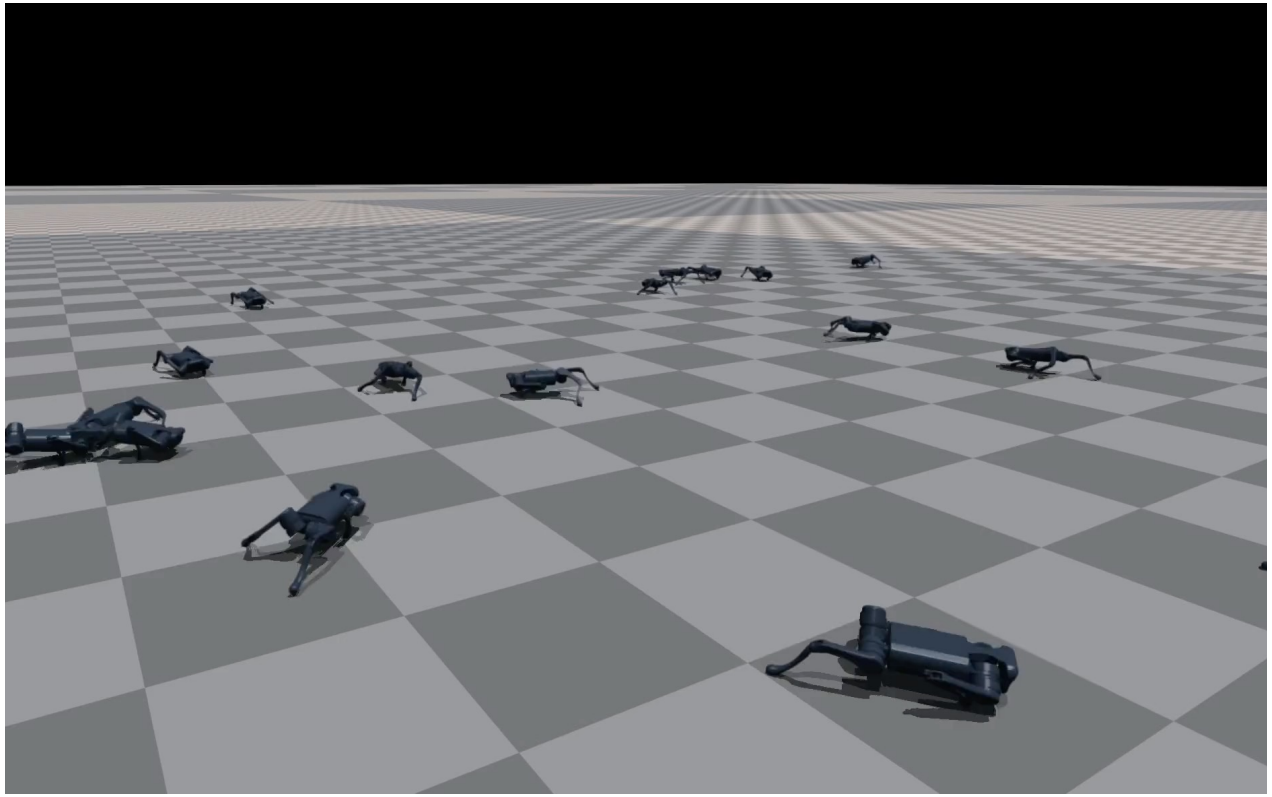


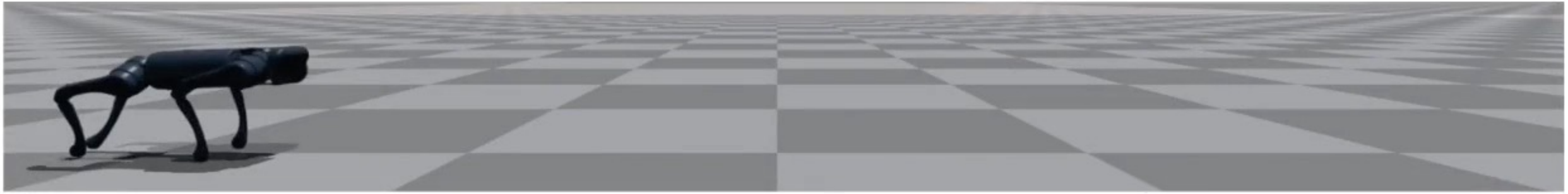
G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. "Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning," IROS 2022

Learning in Joint Space (PPO)

Action Space: $a_t = q_{1...N}$

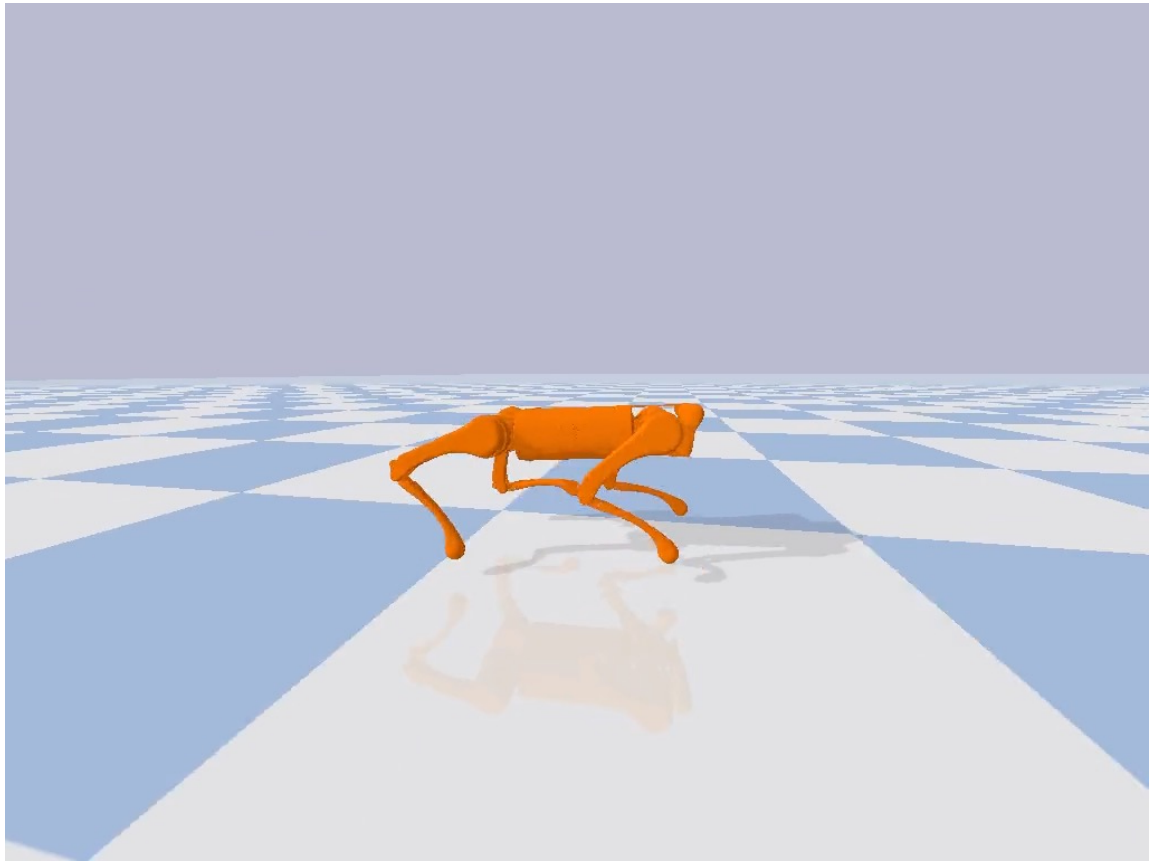




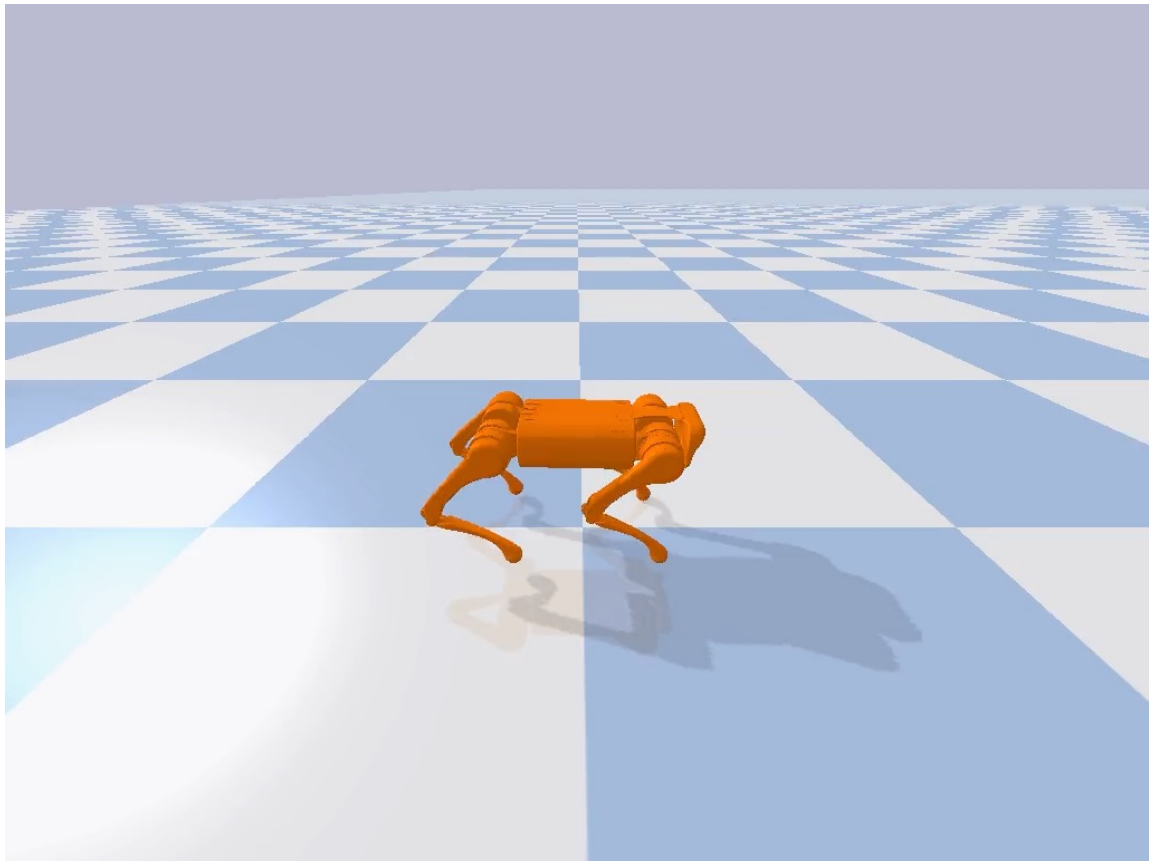


Position Control vs. Cartesian PD Control (PPO)

Action Space: $a_t = q_{1...N}$

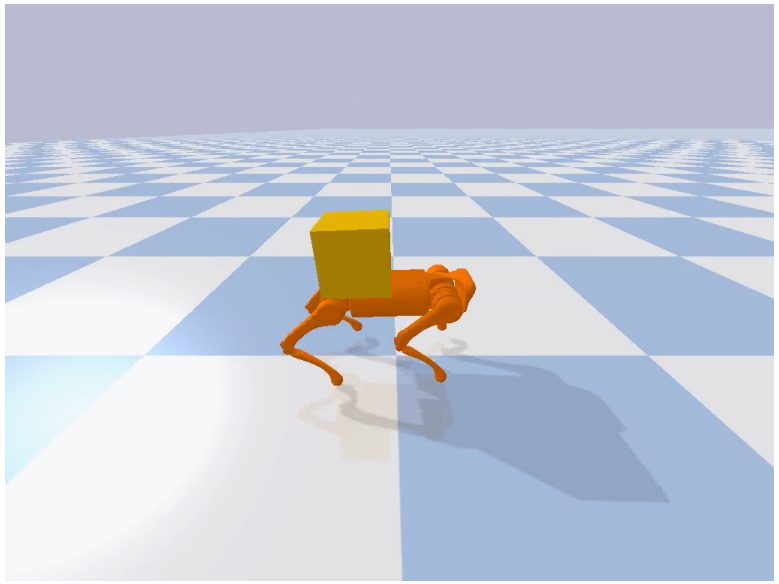


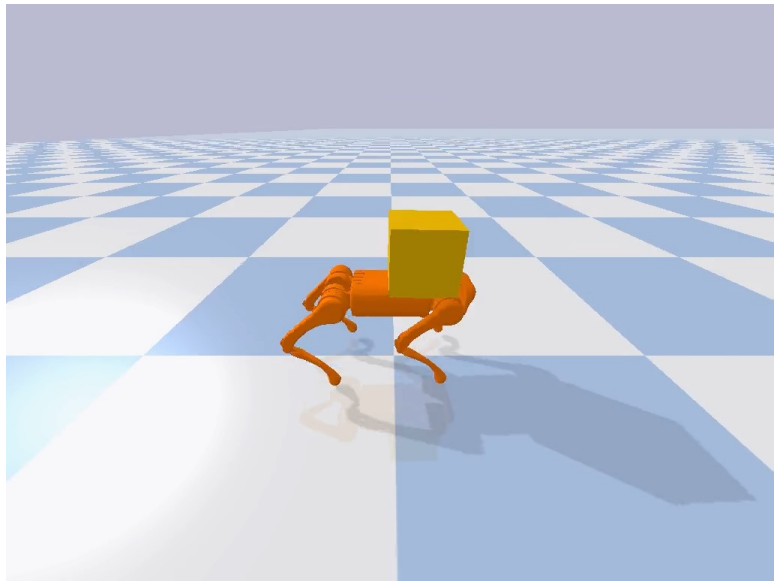
Action Space: $a_t = [x_{ee_i}, y_{ee_i}, z_{ee_i}]$



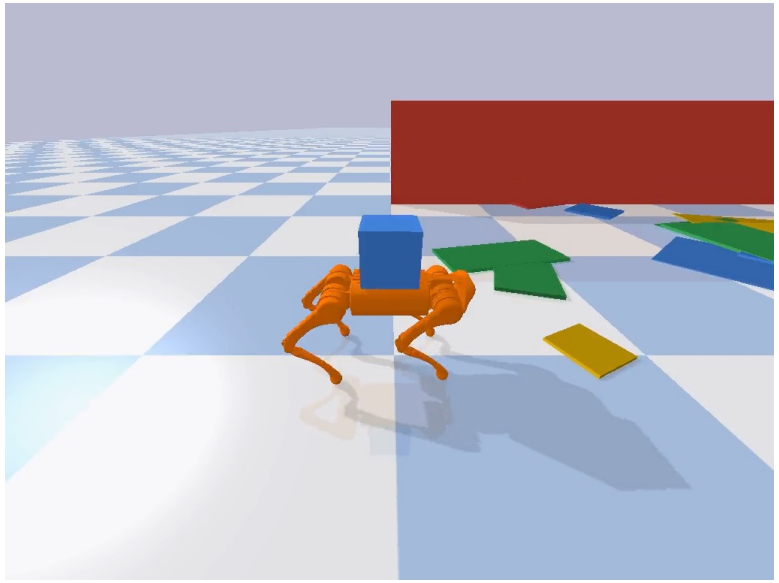
G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. "Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning," IROS 2022

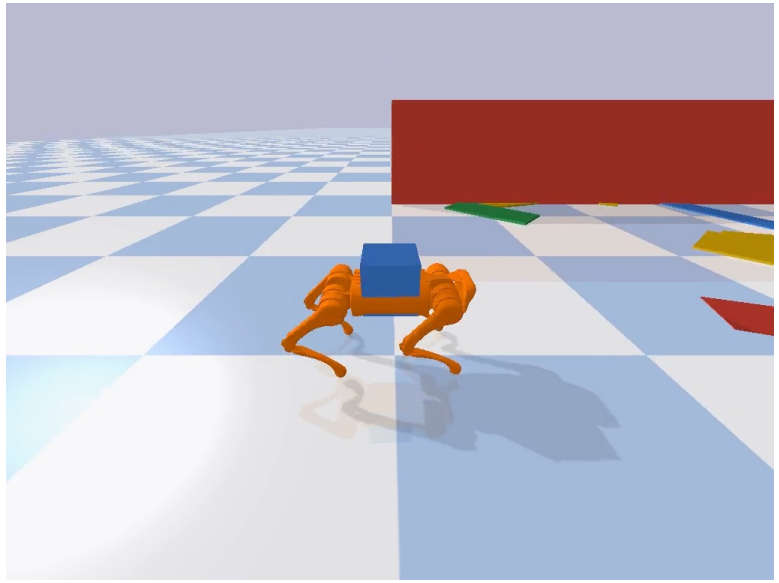
Robust to disturbances (10kg load, and 20% body mass/inertia variability)





Robust to disturbances (6kg load, rough terrain up to 0.04m in height)





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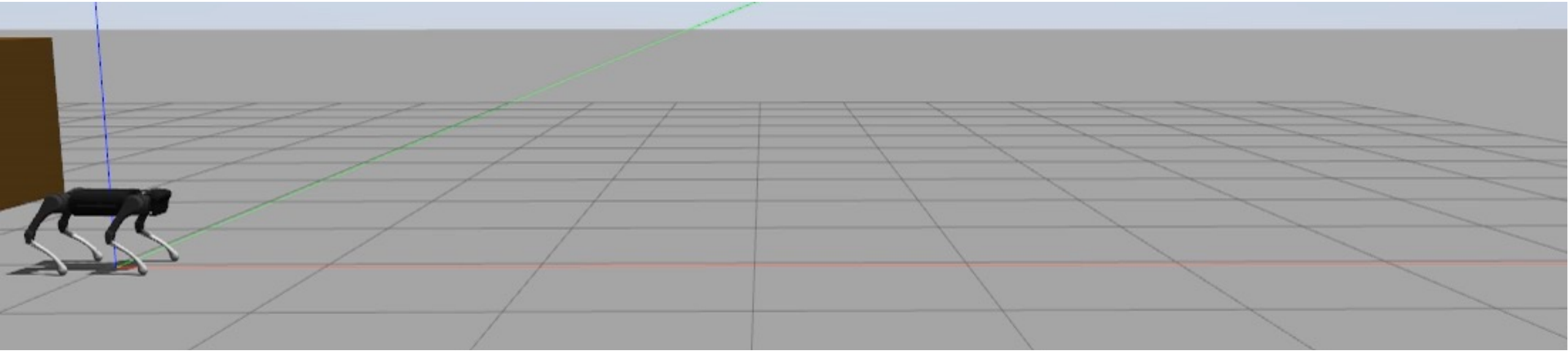
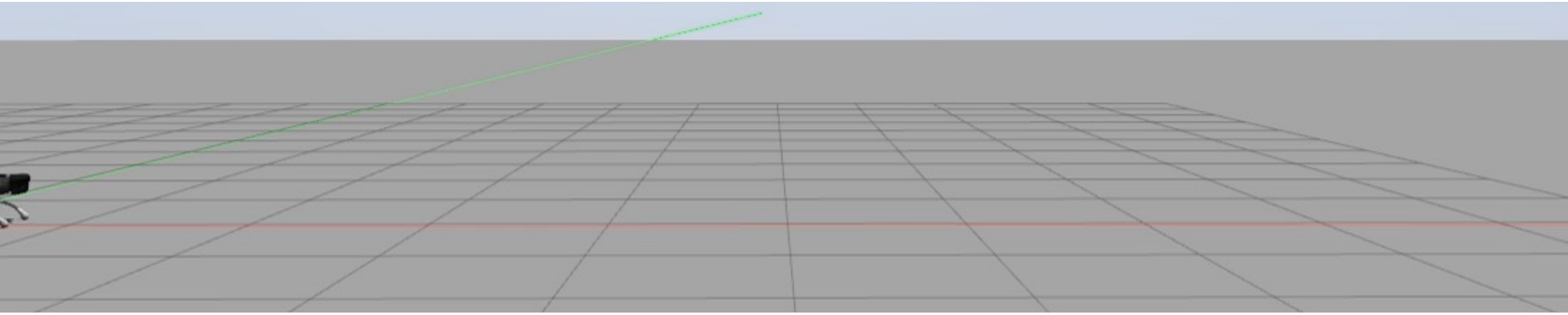
Sim-to-sim Transfer

Running at 4 m/s



Running at 3.5 m/s with a 10kg load



G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. “Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning,” IROS 2022

Sim-to-Real Additions

- Reward function terms:

Name	Formula	Weight
Velocity reward	$1 - \lvert v_{base} - v_{des} \rvert$	0.1
Feet swing reward	$\sum_{i=0}^{3}(t_i^k - t_i^{k-1} - 0.5)$	0.2
Energy penalty	$\int_t^{t+1} \lvert \boldsymbol{\tau} \cdot \dot{\boldsymbol{q}} \rvert dt$	-0.008
Orientation penalty	$\lVert \boldsymbol{\omega} - (0,0,0,1) \rVert$	-0.1
Lateral drift penalty	$\lvert y \rvert$	-0.1
Height penalty	$\lvert z - 0.3 \rvert$	-0.1

- Dynamics randomization:
  - Terrain randomization, observation noise

Parameter	Lower Bound	Upper Bound
Mass (each body link)	80%	120%
Added mass	0 kg	5 kg
Added mass base offset	[-0.15,-0.05,-0.05] m	[0.15,0.05,0.05] m
Coefficient of friction	0.5	1

G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. "Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning," IROS 2022

Experimental Results

0kg load

3kg load

5kg load

5kg load

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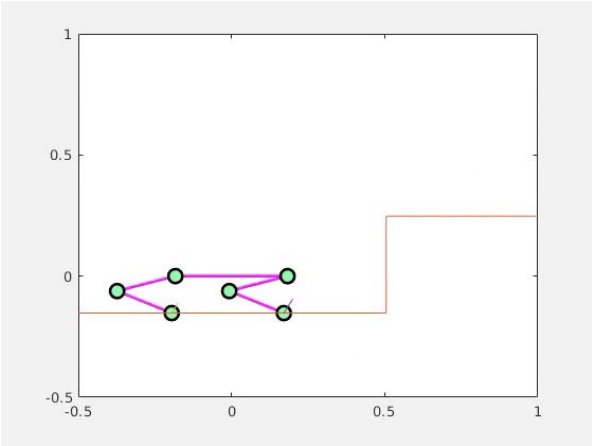
Running Over Different Terrains

G. Bellegarda, Y. Chen, Z. Liu, Q. Nguyen. "Robust High-Speed Running for Quadruped Robots via Deep Reinforcement Learning," IROS 2022

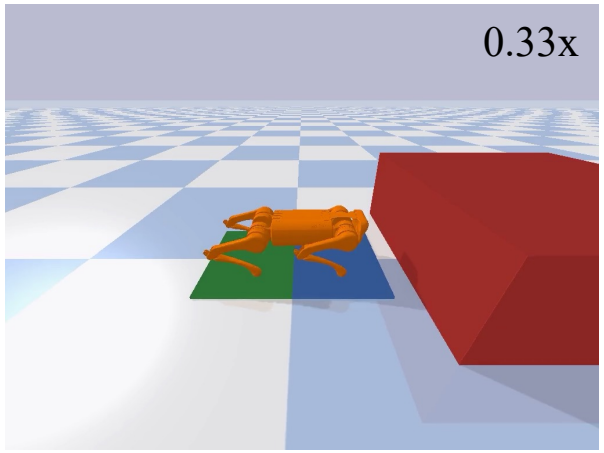
How can we combine trajectory optimization and deep learning?

- Use learning to improve trajectory optimization

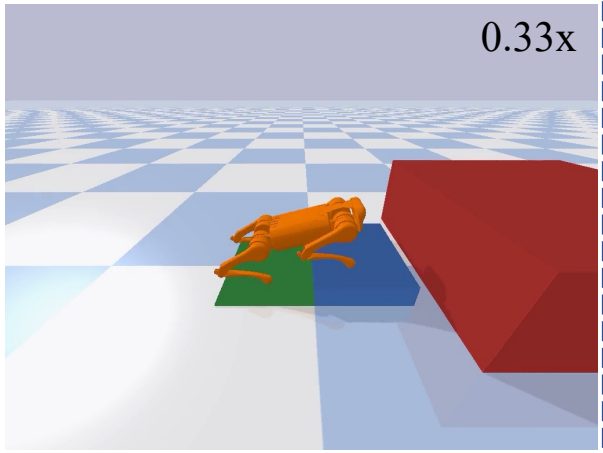
Feedforward trajectory optimization



Nonlinear trajectory optimization generates jumping motions

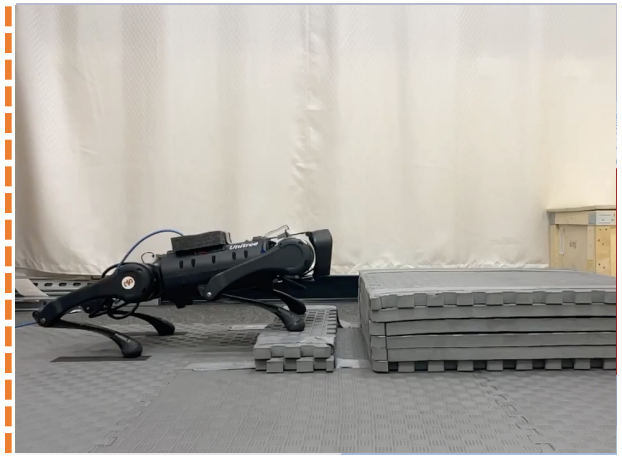


Feedforward execution



Feedforward execution **with disturbances** – doesn't work!

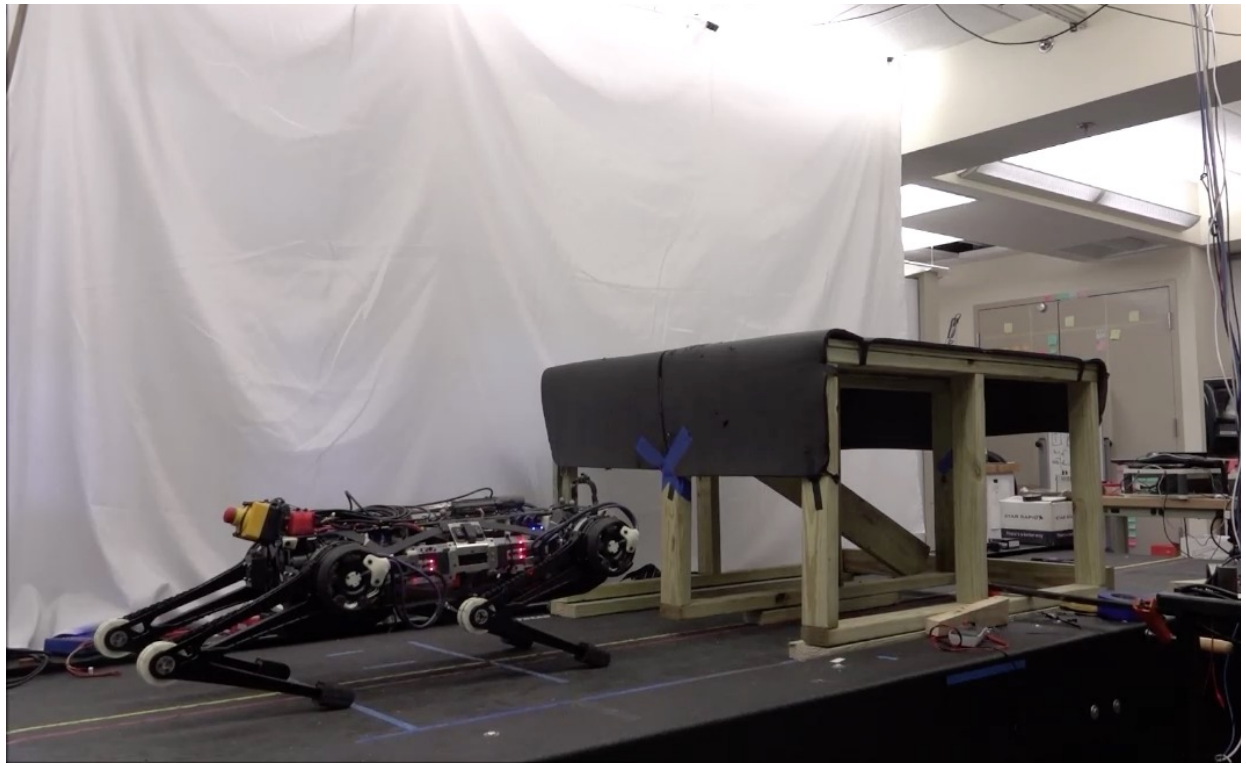
TO + Learning

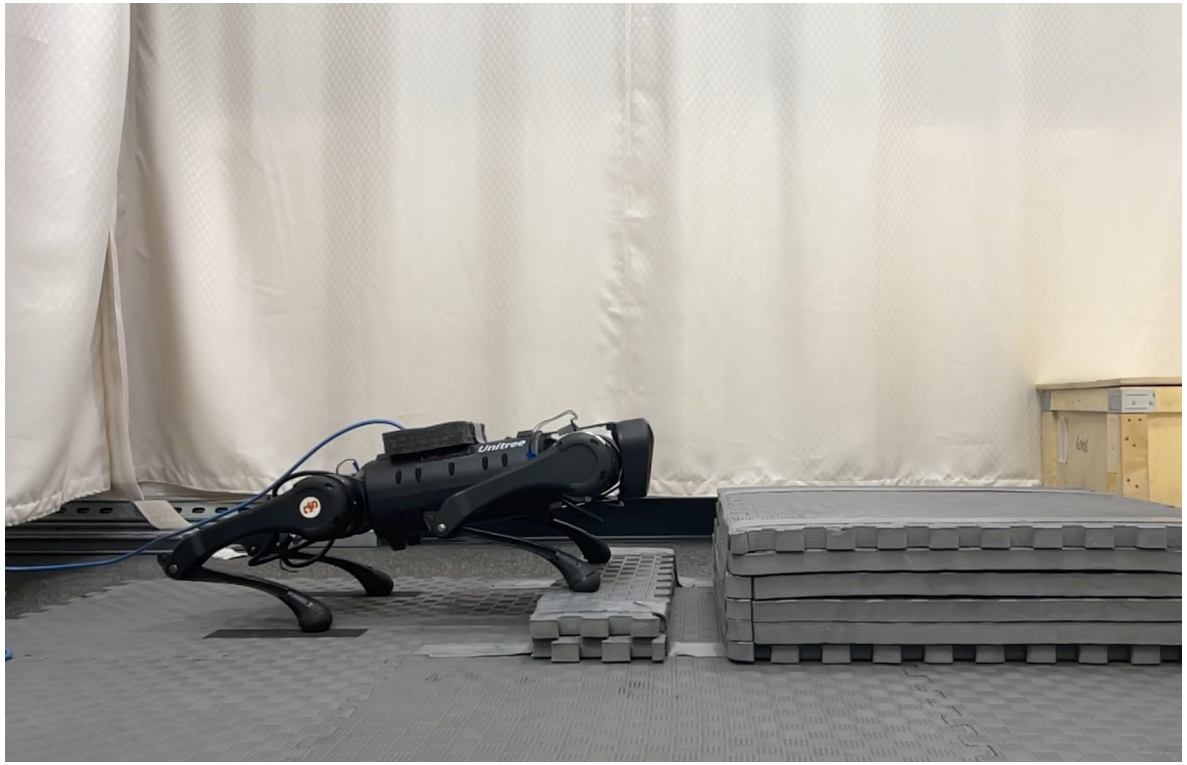


Our method:
Trajectory Optimization + DRL

G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

Jumping Task (Robust?)

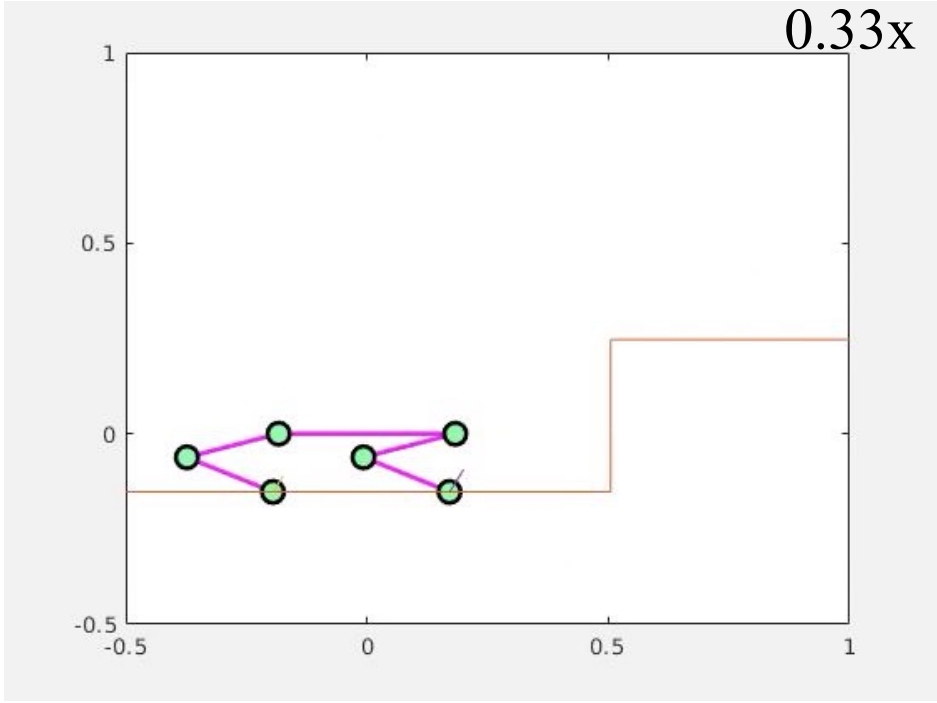




Q. Nguyen, M. Powell, B. Katz, J. D. Carlo, S. Kim. *Optimized Jumping on the MIT Cheetah 3 Robot.* ICRA 2019
G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

Optimization

Nonlinear trajectory optimization generates jumping motions



$\{\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \boldsymbol{\tau}_d\}$

$\{\boldsymbol{p}_d, \boldsymbol{v}_d\}$

Joint PD Controller

Cartesian PD Controller

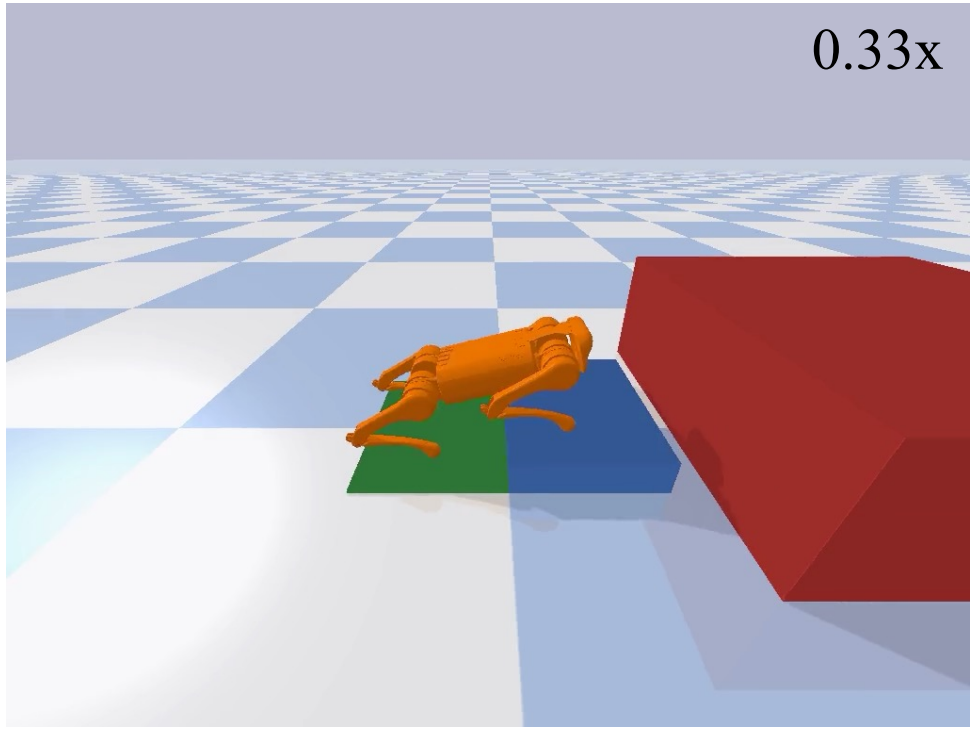
$\boldsymbol{\tau}$

Robot

$\{\boldsymbol{q}, \dot{\boldsymbol{q}}\}$

$\{\boldsymbol{p}, \boldsymbol{v}\}$

G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.



Optimization

$\{\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \boldsymbol{\tau}_d\}$

$\{\boldsymbol{p}_d, \boldsymbol{v}_d\}$

Joint PD Controller

Cartesian PD Controller

$\{\boldsymbol{q}, \dot{\boldsymbol{q}}\}$

$\{\boldsymbol{p}, \boldsymbol{v}\}$

$\boldsymbol{\tau}$

Robot

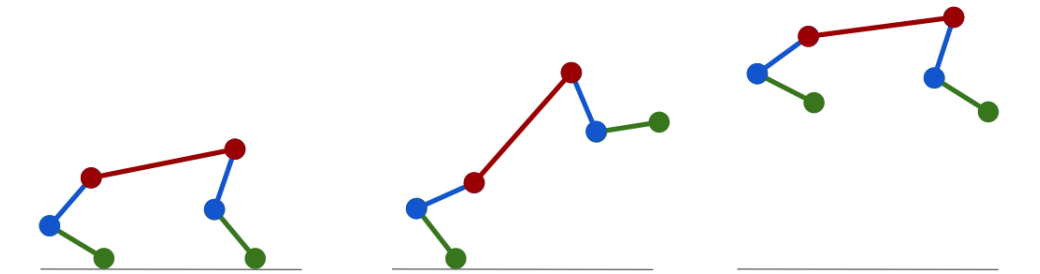
Feedforward controller tracks jumping motions…

…but only works under ideal conditions!

G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

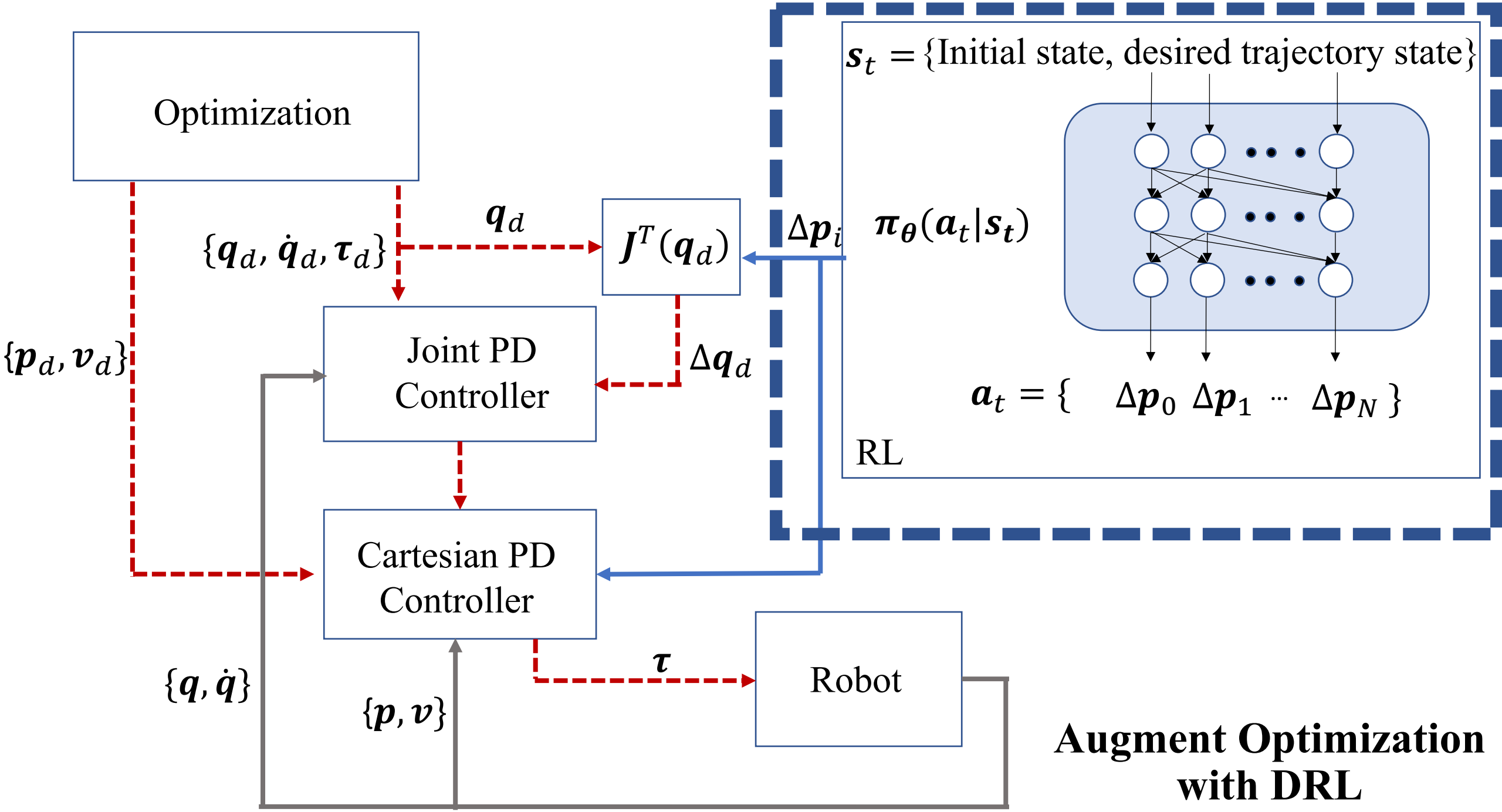
How to improve jumping with learning?

- Jumping involves very specific joint trajectories

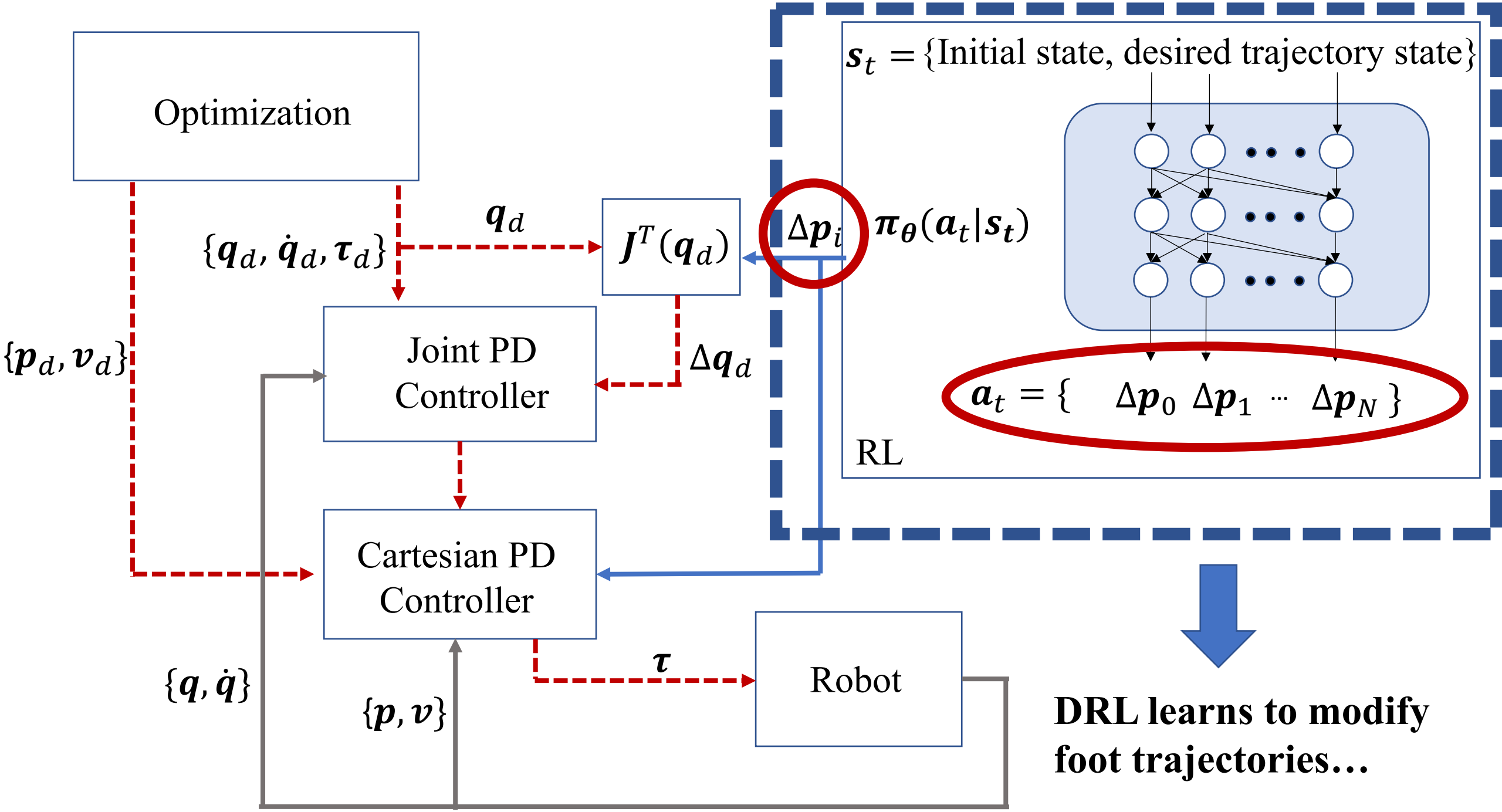


- Difficult to learn from scratch..
  - Imitate trajectory?
  - Reward function tuning..
- Already have an approximate solution in ideal case
  - Learn to modify existing trajectories for robustness (residual learning)

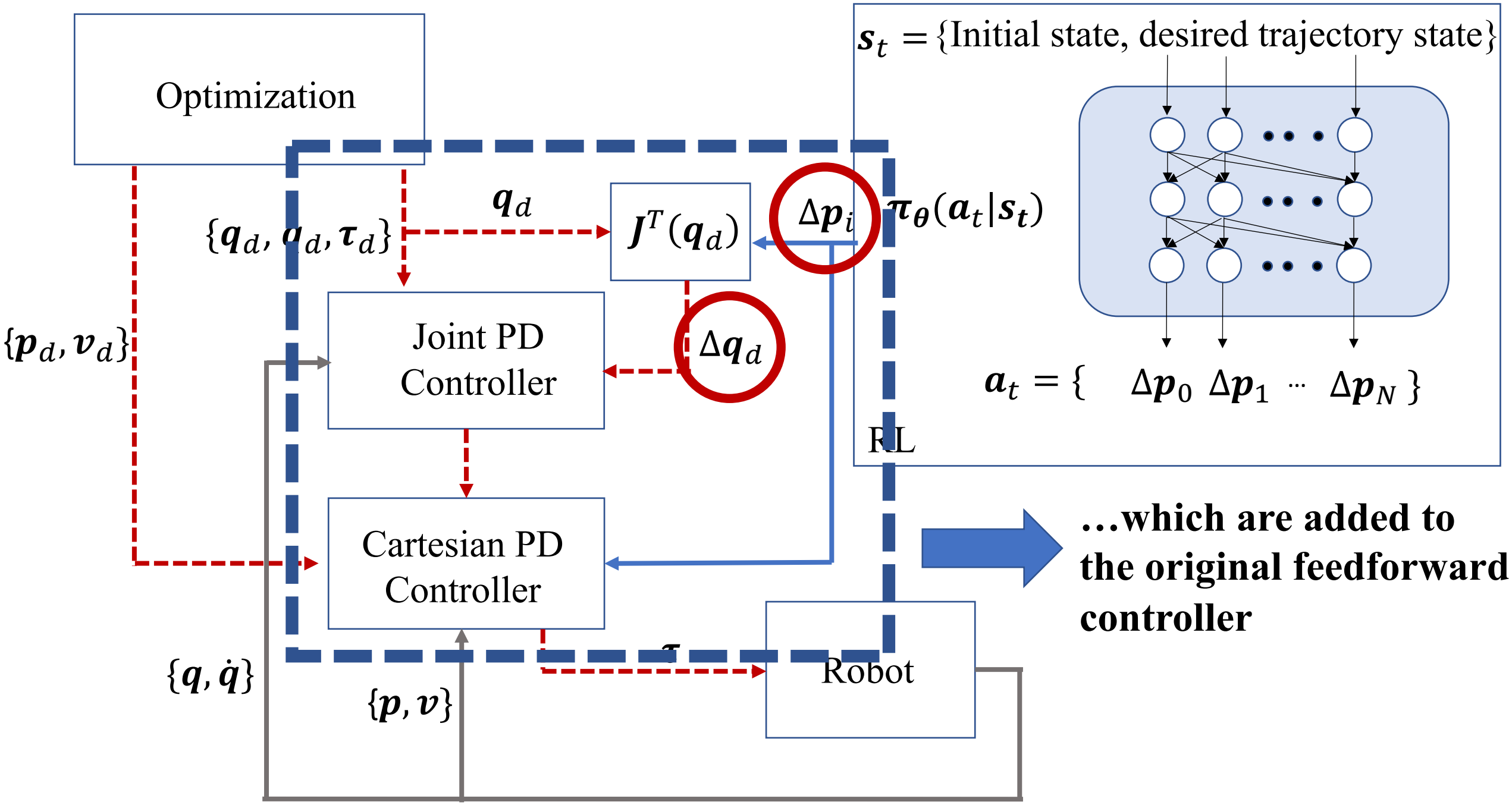
G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.



G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.



G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

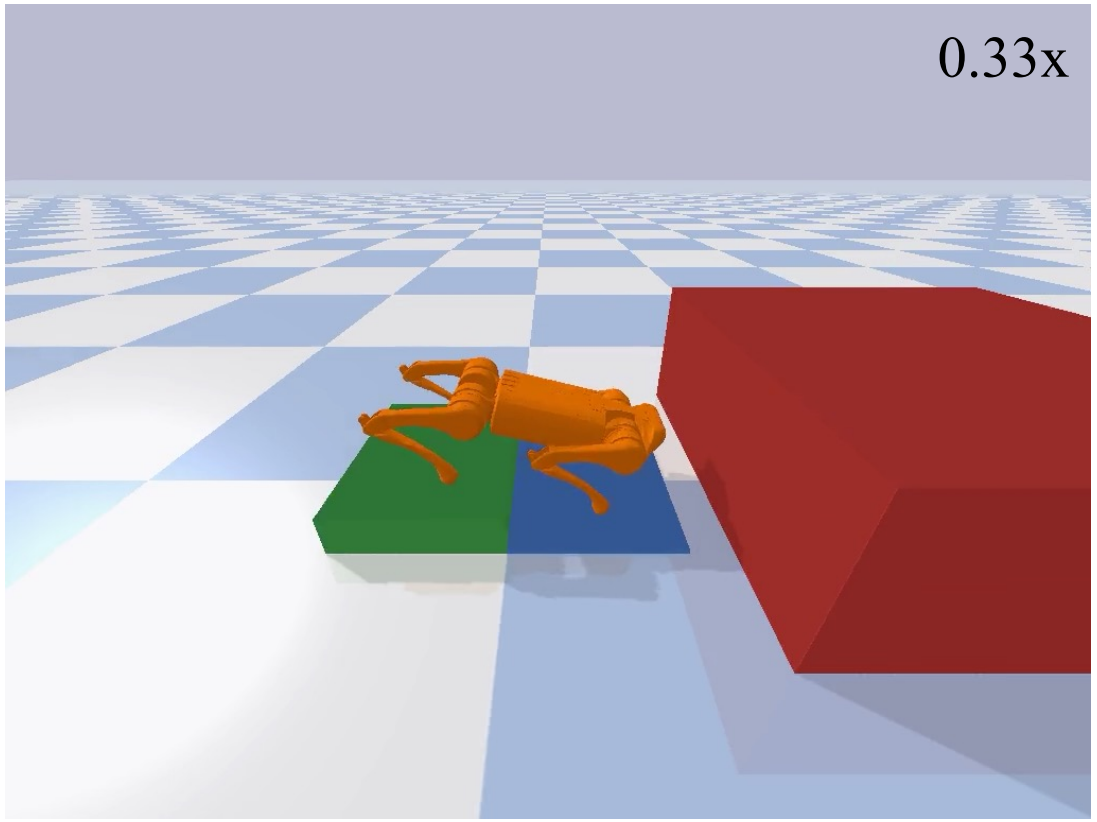


G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

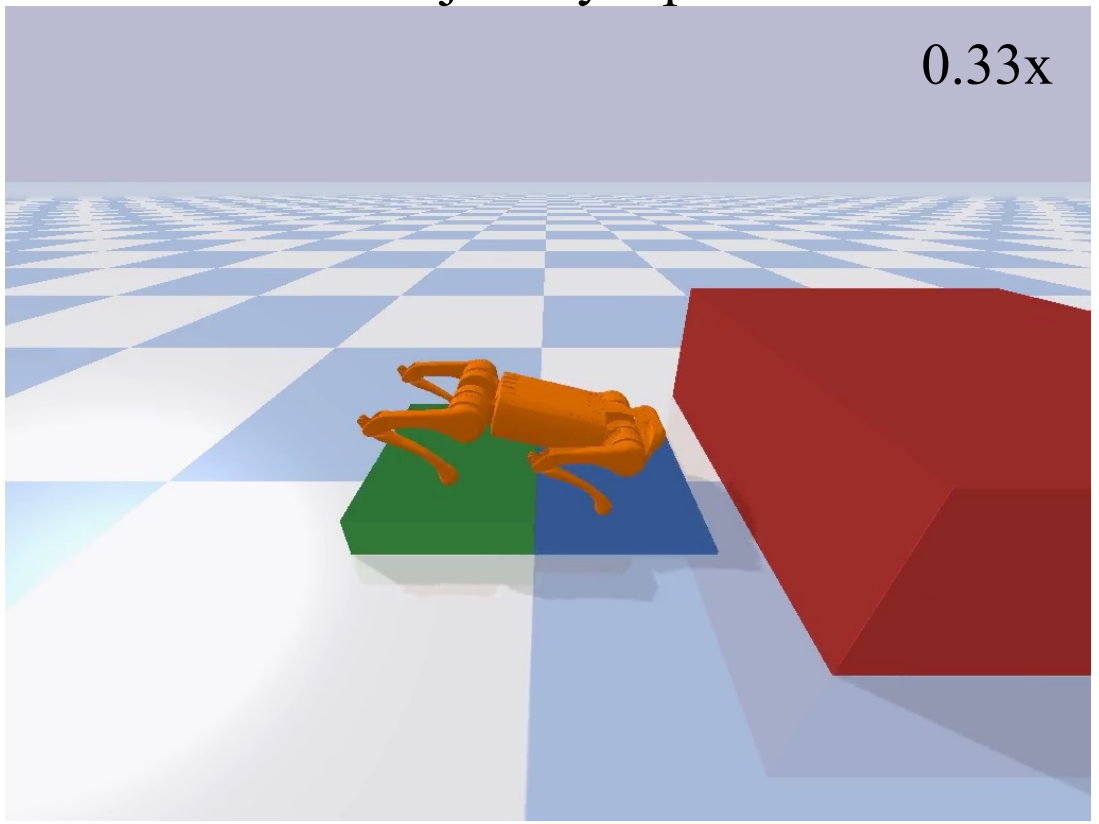
Example Simulation Result

- Jumping height and distance: **(0.4, 0.7) m**
- Mass and inertia uncertainty: **5%**
- Ground uncertainty: **0.1 m** block under rear feet

Baseline - Pure Trajectory Optimization



Our method – Trajectory Optimization + DRL

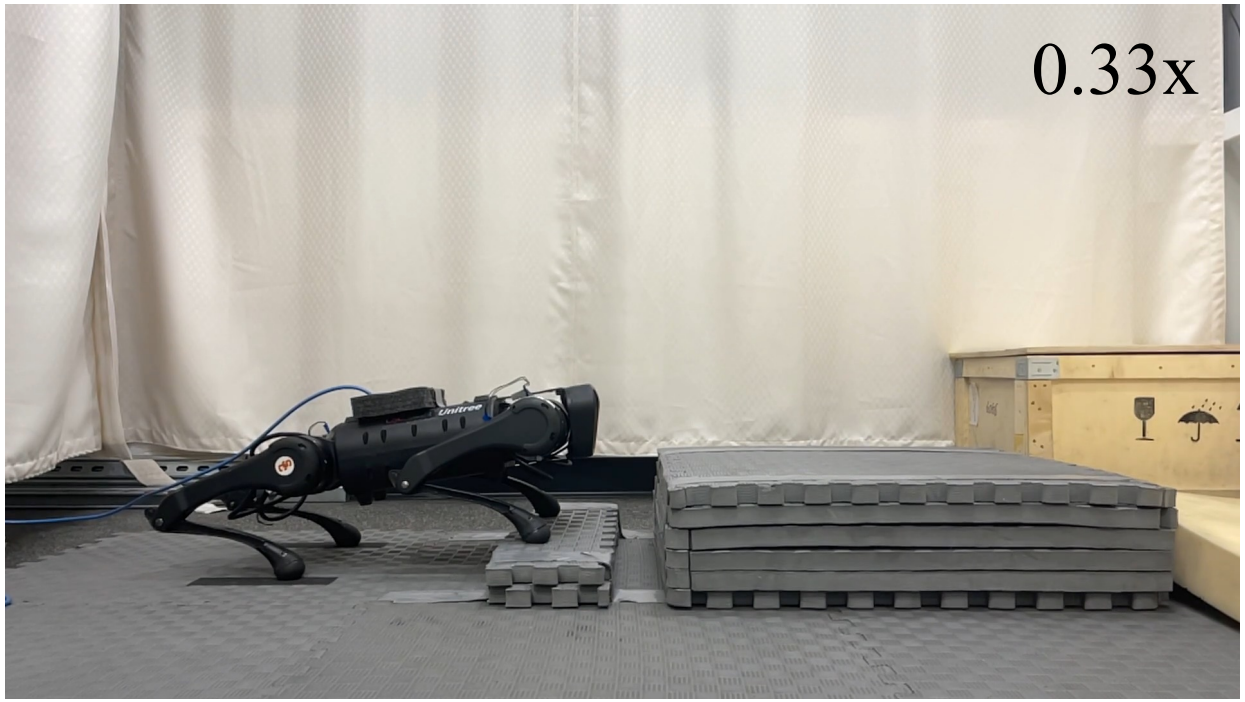


G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

Example Hardware Result

- Jumping height and distance: **(0.2, 0.6) m**
- Ground uncertainty: **0.06 m** block under front feet

Baseline - Pure Trajectory Optimization



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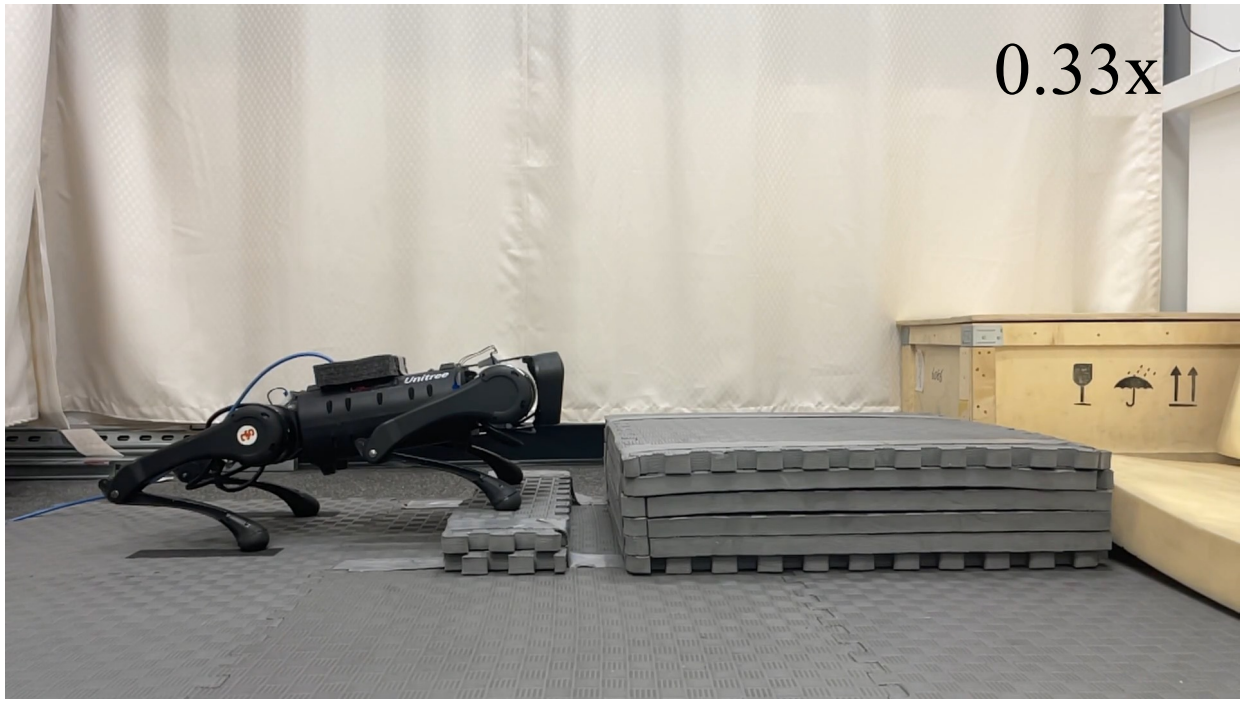


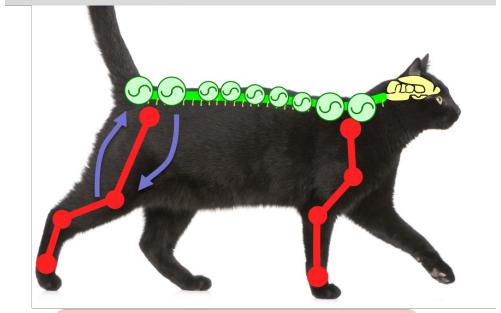
G. Bellegarda, C. Nguyen, Q. Nguyen. "Robust Quadruped Jumping via Deep Reinforcement Learning," Robotics and Autonomous Systems 2024.

From Lecture 4

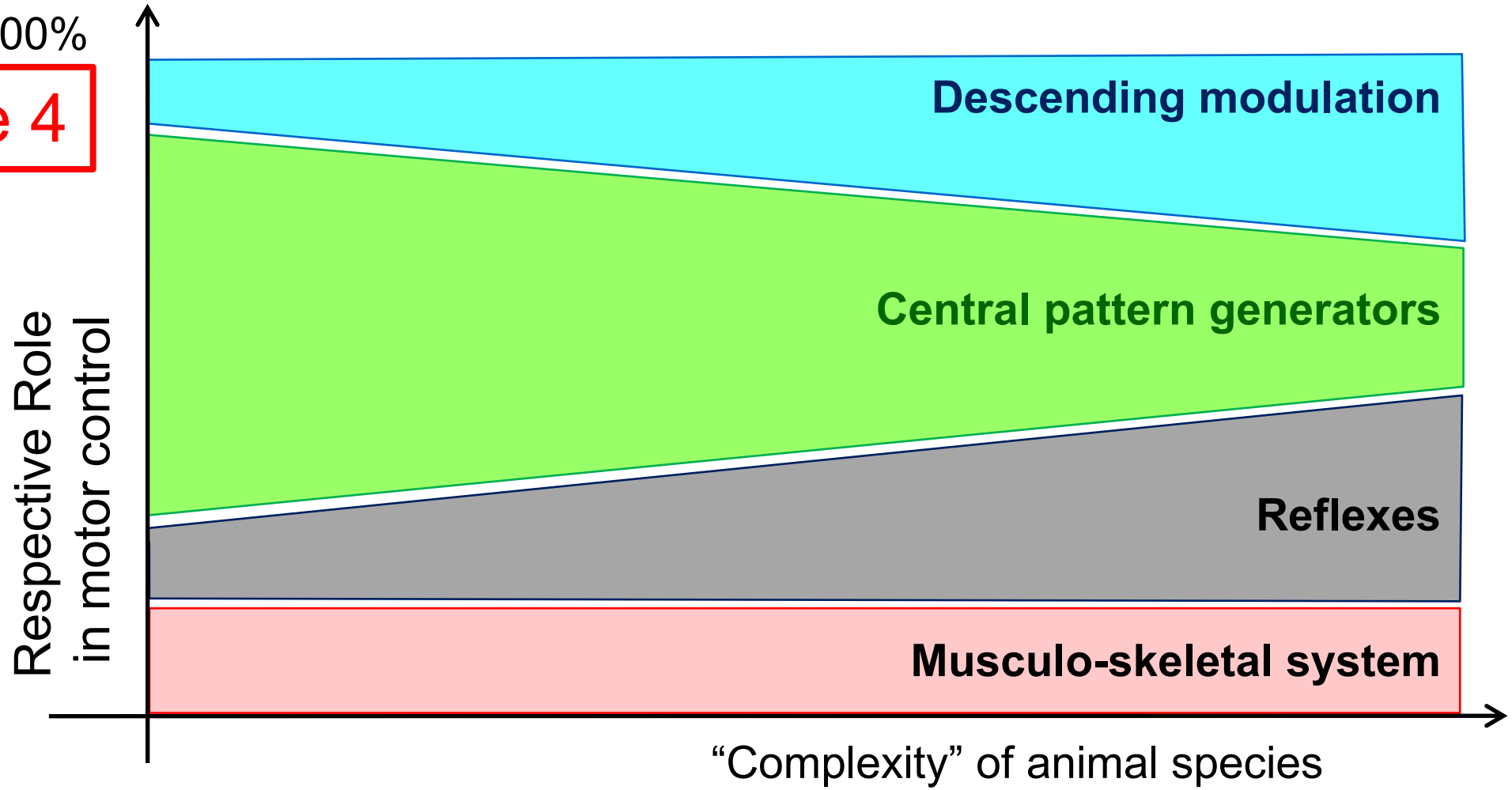
Descending modulation

Spinal cord

Reflexes | Central pattern generators



Musculoskeletal system



Ryczko, Simon, Ijspeert, Trends in Neuroscience, 2020

Minassian et al Neuroscientist 2017

lamprey

salamander

cat

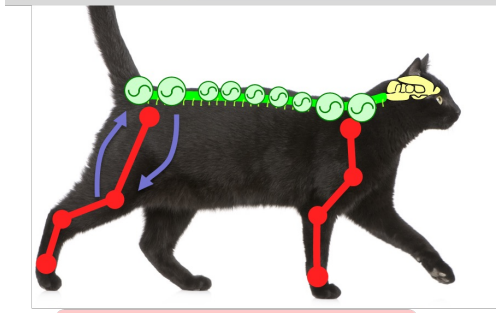
human

From Lecture 4

Descending modulation

Spinal cord

Reflexes | Central pattern generators



Musculoskeletal system

100%

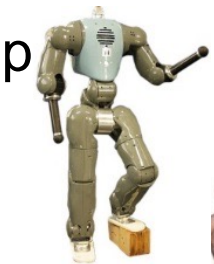
Respective Role in motor control

lamprey

salamander

“Comp ıimal sp

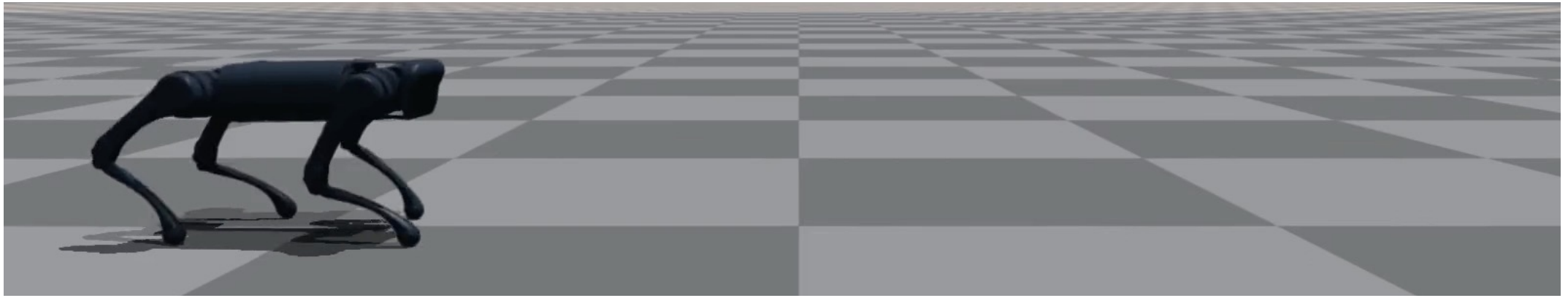
cat



human

Traditional Central Pattern Generators for Quadruped Locomotion

Trot



Amplitude:

$$\ddot{r}_i = a\left(\frac{a}{4}(\mu_i - r_i) - \dot{r}_i\right)$$

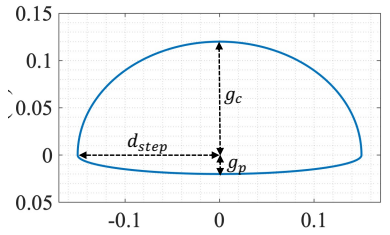
Phase:

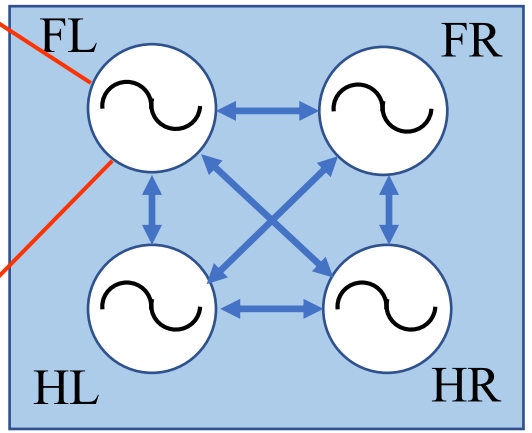
$$\dot{\theta}_i = \omega_i + \sum_j r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij}), \quad \omega_i = \begin{cases} \omega_{\text{swing}} & \text{if } 0 \le \theta_i < \pi \\ \omega_{\text{stance}} & \text{if } \pi \le \theta_i < 2\pi \end{cases}$$

Output:

$$x_{\text{foot},i} = -d_{step} r_i \cos(\theta_i)$$

$$z_{\text{foot},i} = \begin{cases} -h + g_c \sin(\theta_i) & \text{if } \sin(\theta_i) > 0 \\ -h + g_p \sin(\theta_i) & \text{otherwise} \end{cases}$$





A. Ijspeert, A. Crespi, D. Ryczko, J.M. Cabelguen. *From Swimming to Walking with a Salamander Robot Driven by a Spinal Cord Model.* Science 2007

Traditional Central Pattern Generators for Quadruped Locomotion

- Require careful parameter tuning (optimization-based or hand-tuned)
- Result in a single fixed gait (must re-tune for each desired gait and speed)
- Difficult to specify omni-directional motion (i.e. add control like VMC to turn)

Can we improve the general capabilities of CPGs with deep reinforcement learning?

Amplitude:

$$\ddot{r}_i = a\left(\frac{a}{4}(\mu_i - r_i) - \dot{r}_i\right)$$

Phase:

$$\dot{\theta}_i = \omega_i + \sum_j r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij}), \quad \omega_i = \begin{cases} \omega_{\text{swing}} & \text{if } 0 \le \theta_i < \pi \\ \omega_{\text{stance}} & \text{if } \pi \le \theta_i < 2\pi \end{cases}$$

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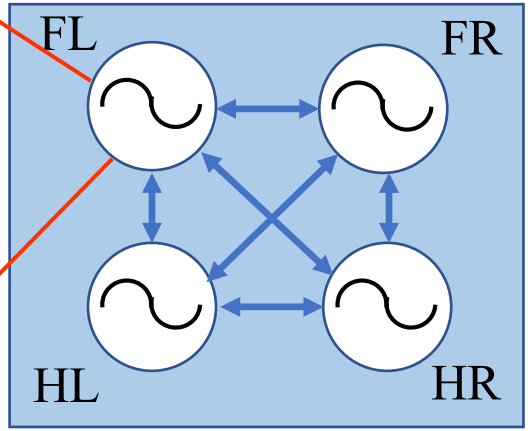
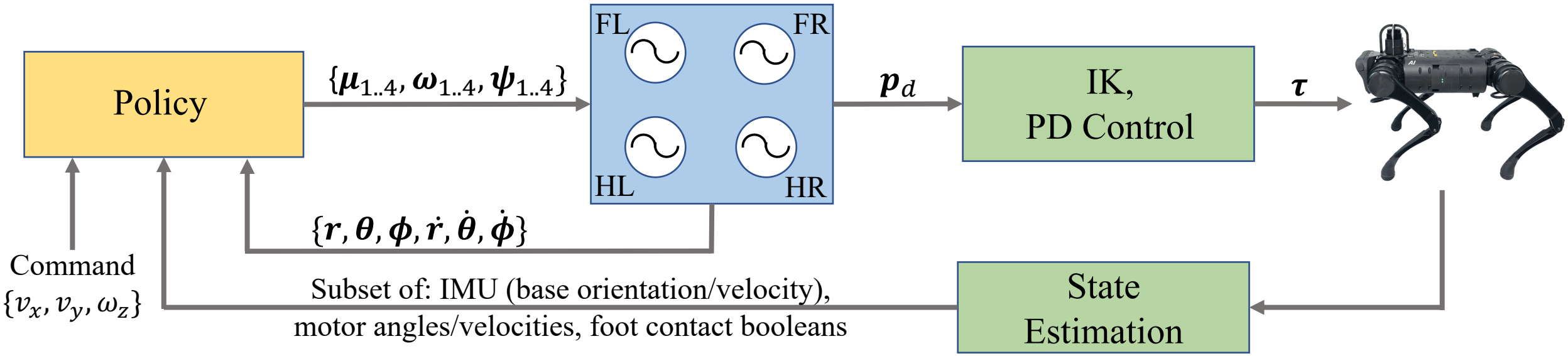


M. Ajallooeian, S. Pouya, A. Sproewitz, A. Ijspeert. *Central Pattern Generators Augmented with Virtual Model Control for Quadruped Rough Terrain Locomotion.* ICRA 2013

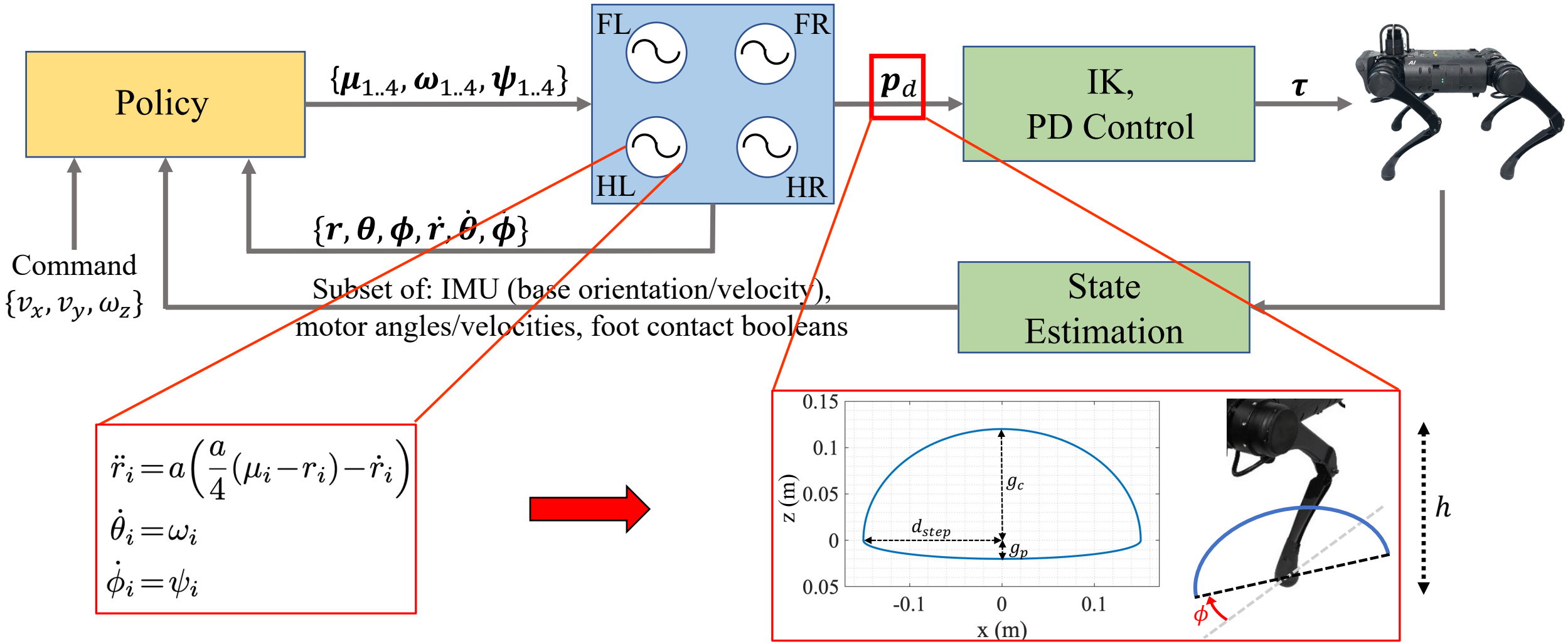
CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion



In this work, we propose to learn to modulate the neural oscillator intrinsic amplitude and frequency of each oscillator that together forms a Central Pattern Generator with deep reinforcement learning.

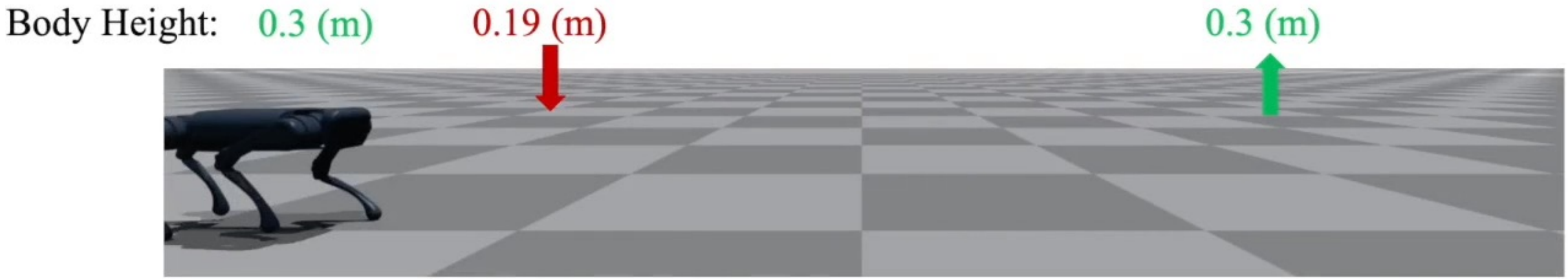
G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

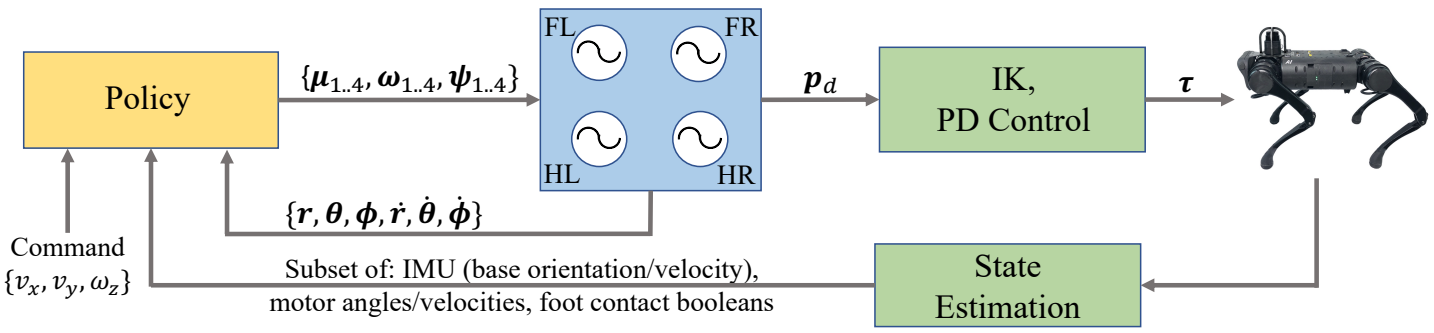
CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion

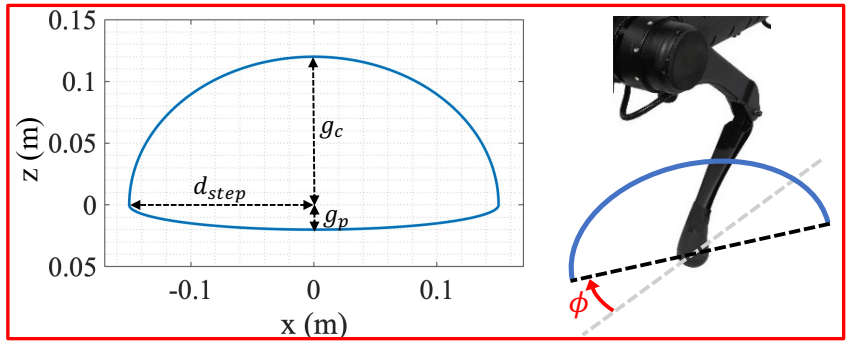


G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

Body Height and Swing Foot Height can be adjusted on the fly







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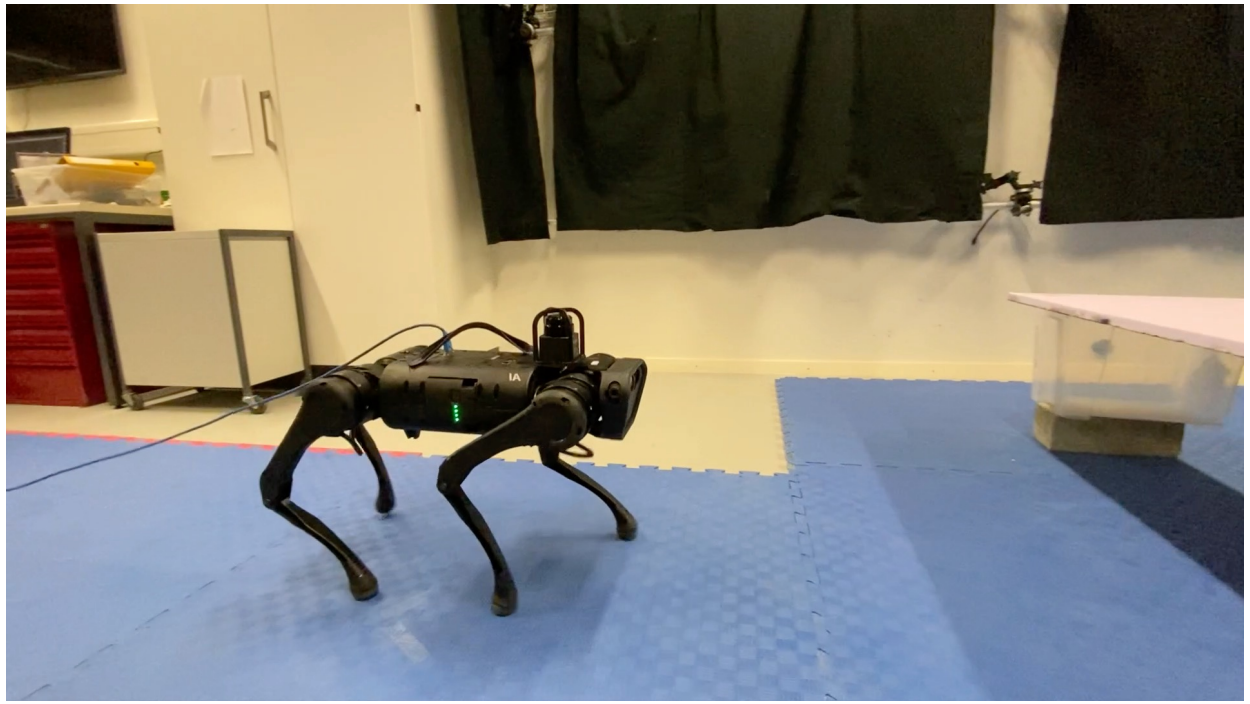
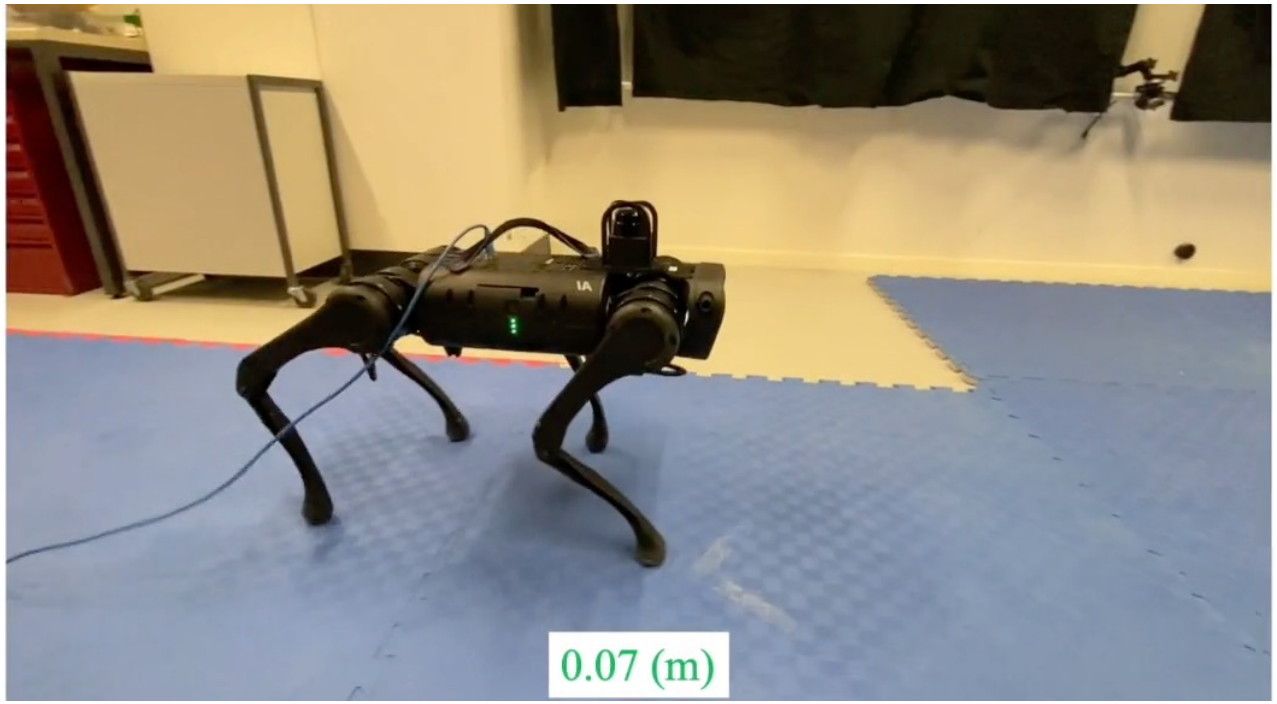
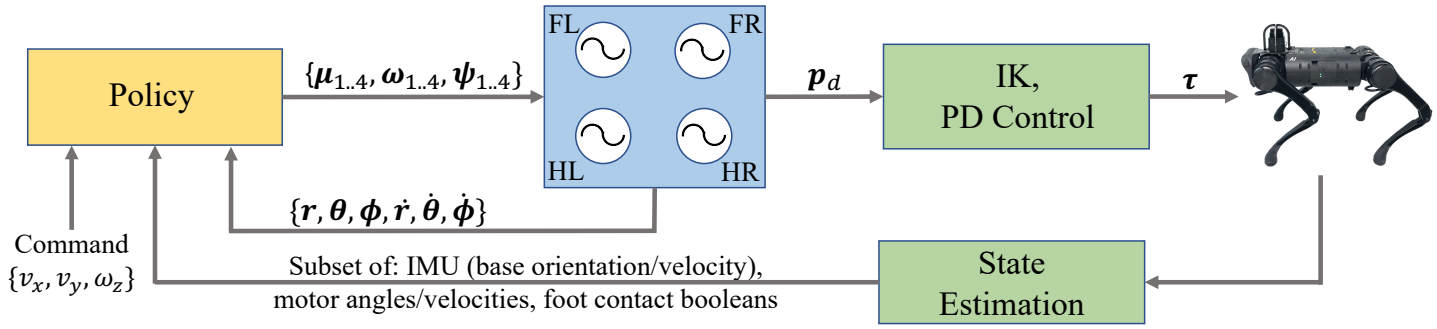
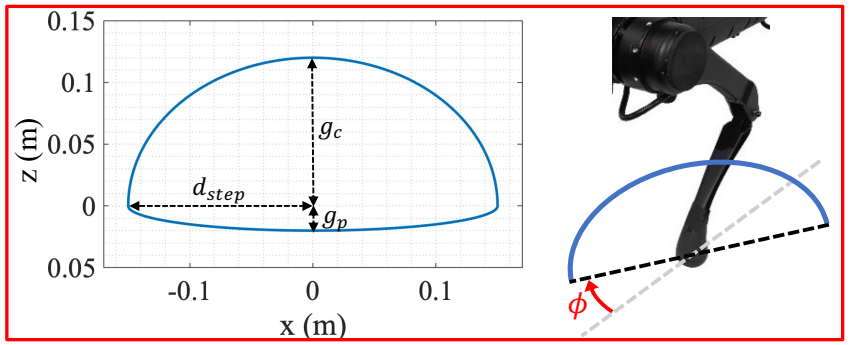


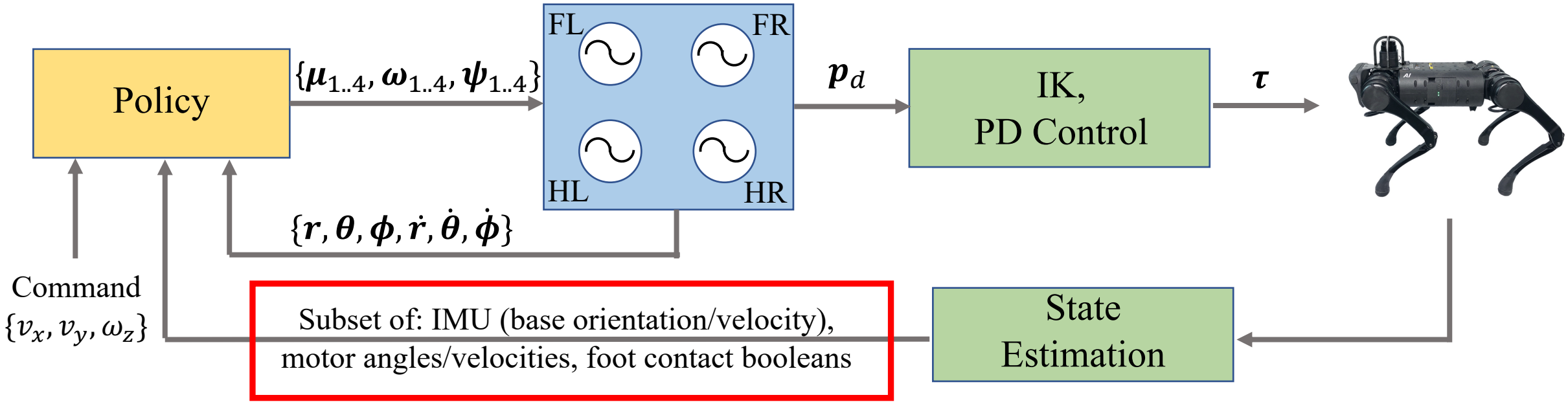






G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

What sensory information should be in the observation space?



G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

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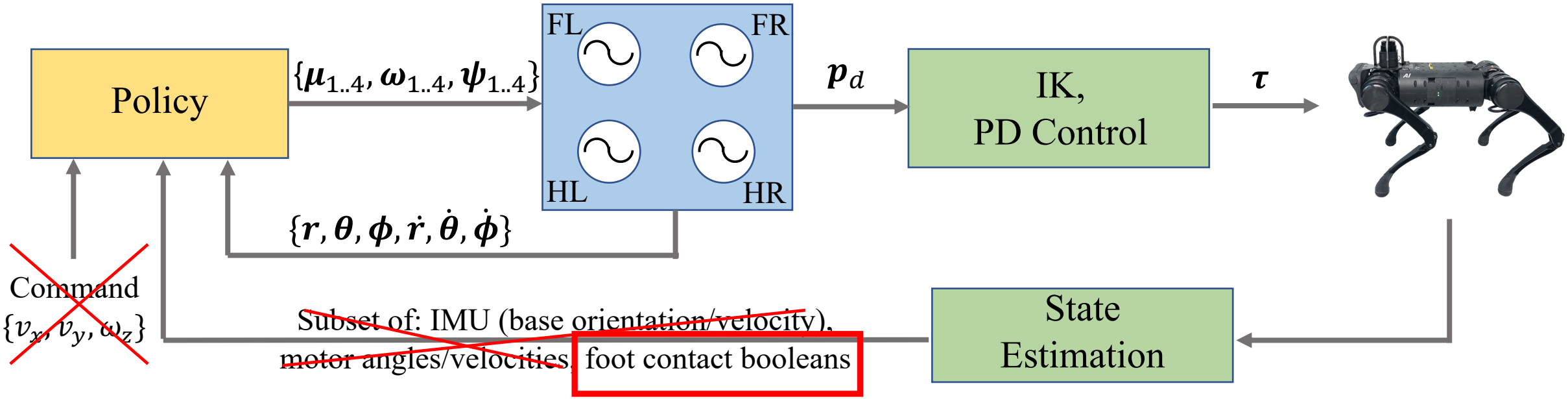


- Similar to the "Tegotae" feedback idea:

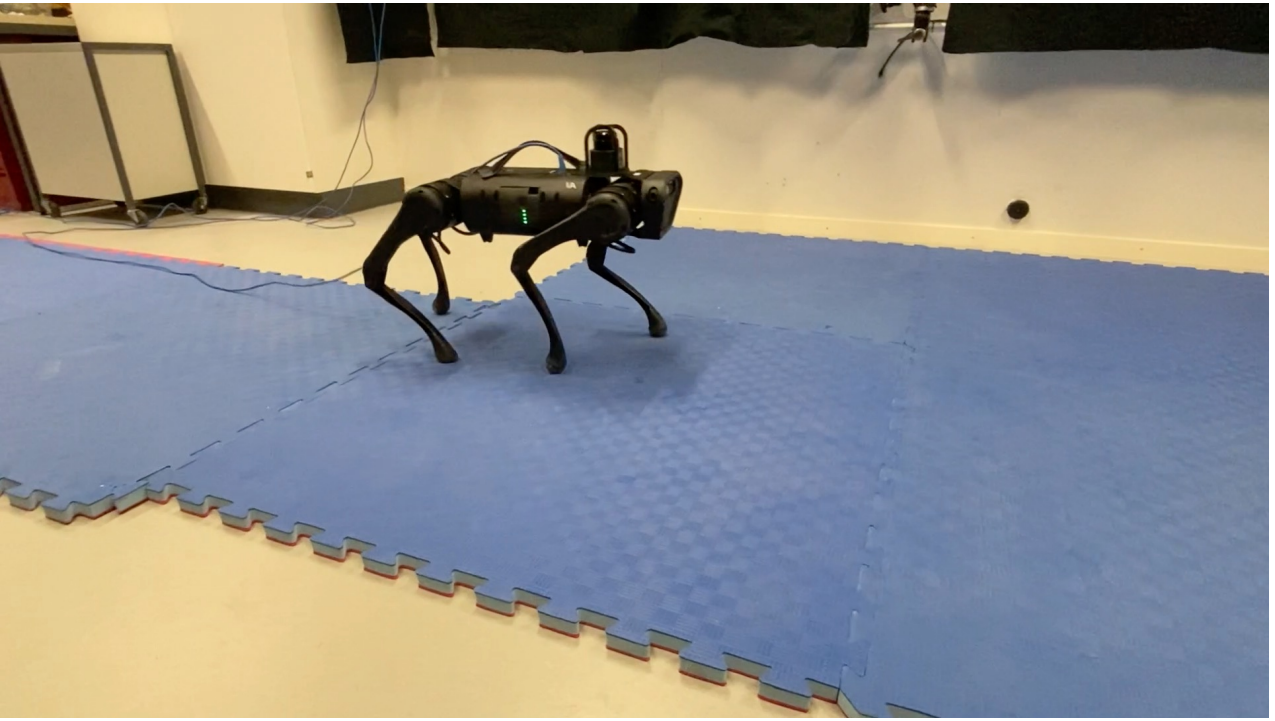
$$\dot{\theta}_i = \omega_i + \sum r_j w_{ij} \sin(\theta_j - \theta_i - \phi_{ij}) - \sigma N_i \cos\theta_i$$

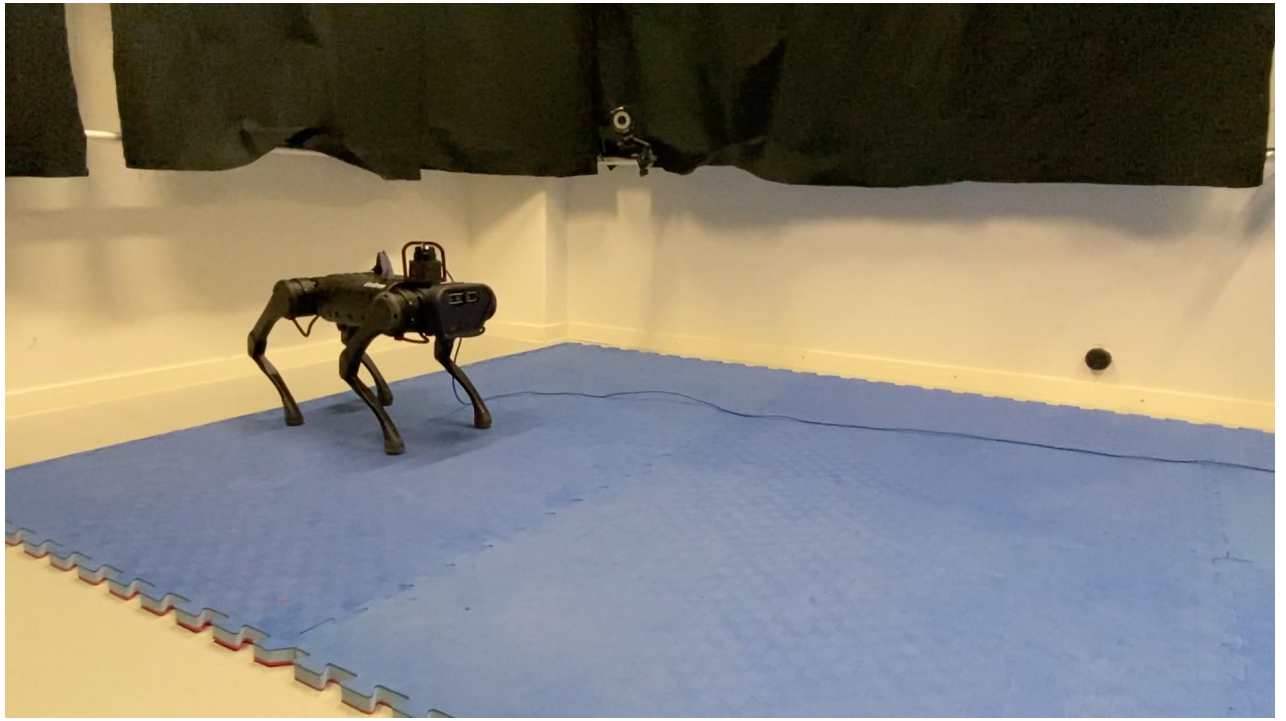
D. Owaki et al. *A Minimal Model Describing Hexapedal Interlimb Coordination: The Tegotae-Based Approach*

G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

Observation space: {Foot contact booleans, CPG states}

- No domain randomization or noise during training





G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

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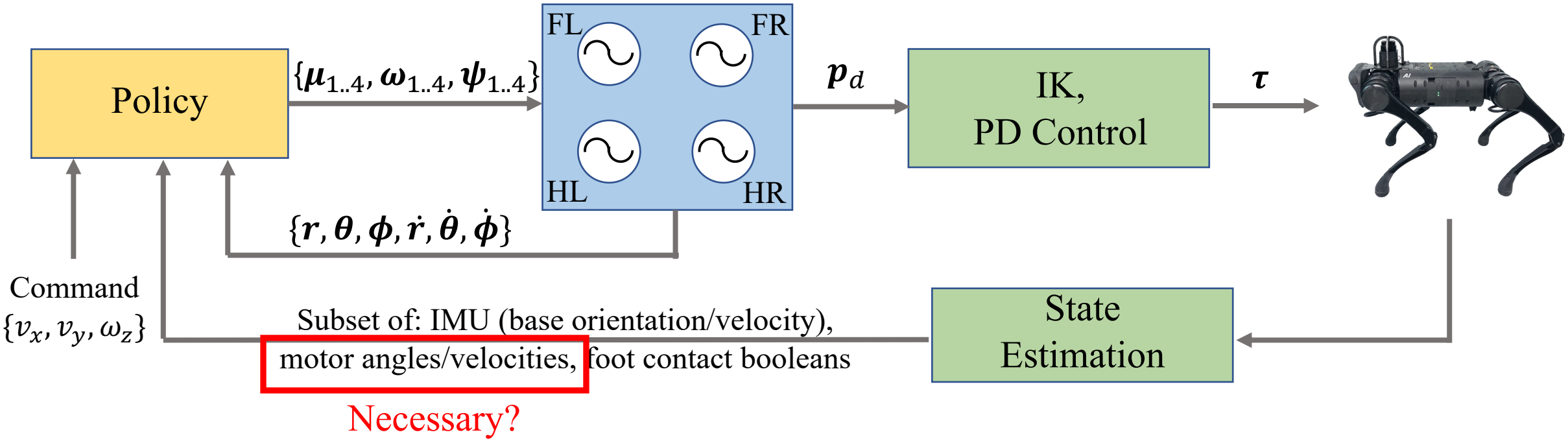
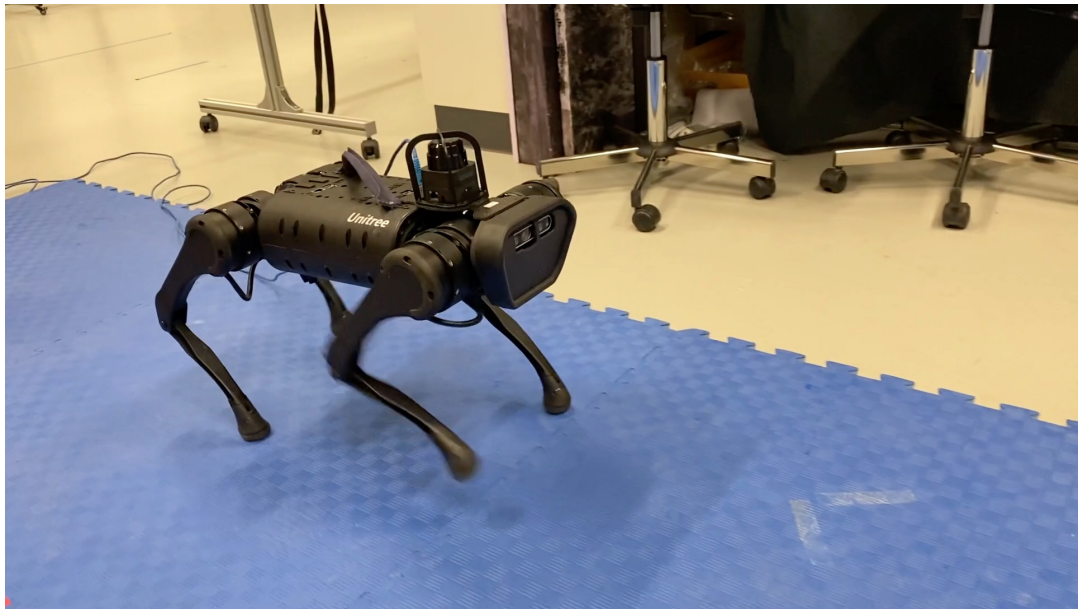
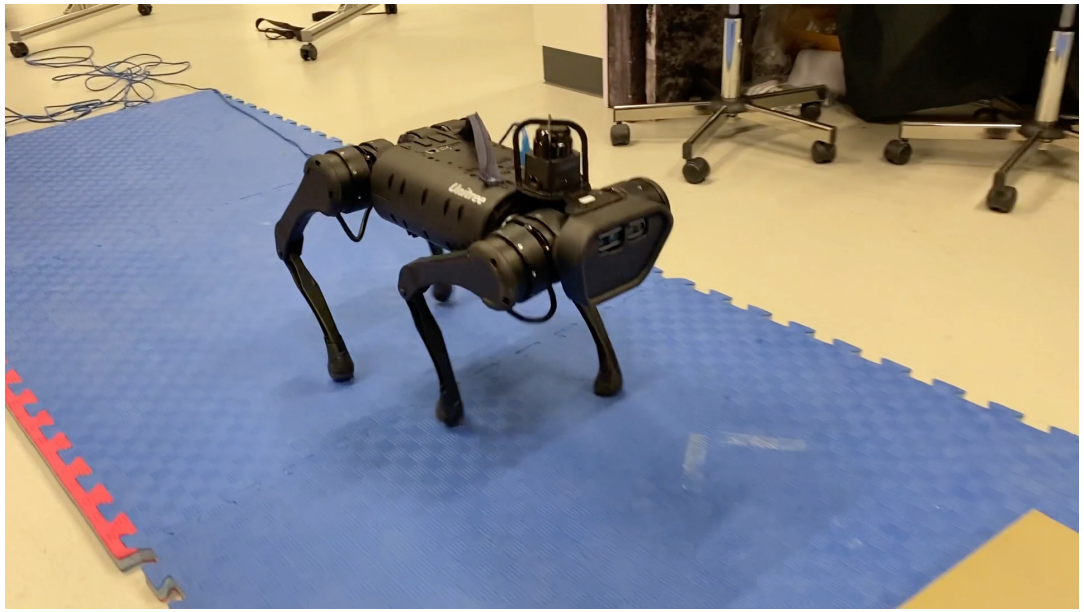


G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

Full Obs. Space, $v^*_{b,x} = 0.3$ [m/s]

Medium Obs. Space, $v^*_{b,x} = 0.8$ [m/s]
(No joint states in obs. space)





G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

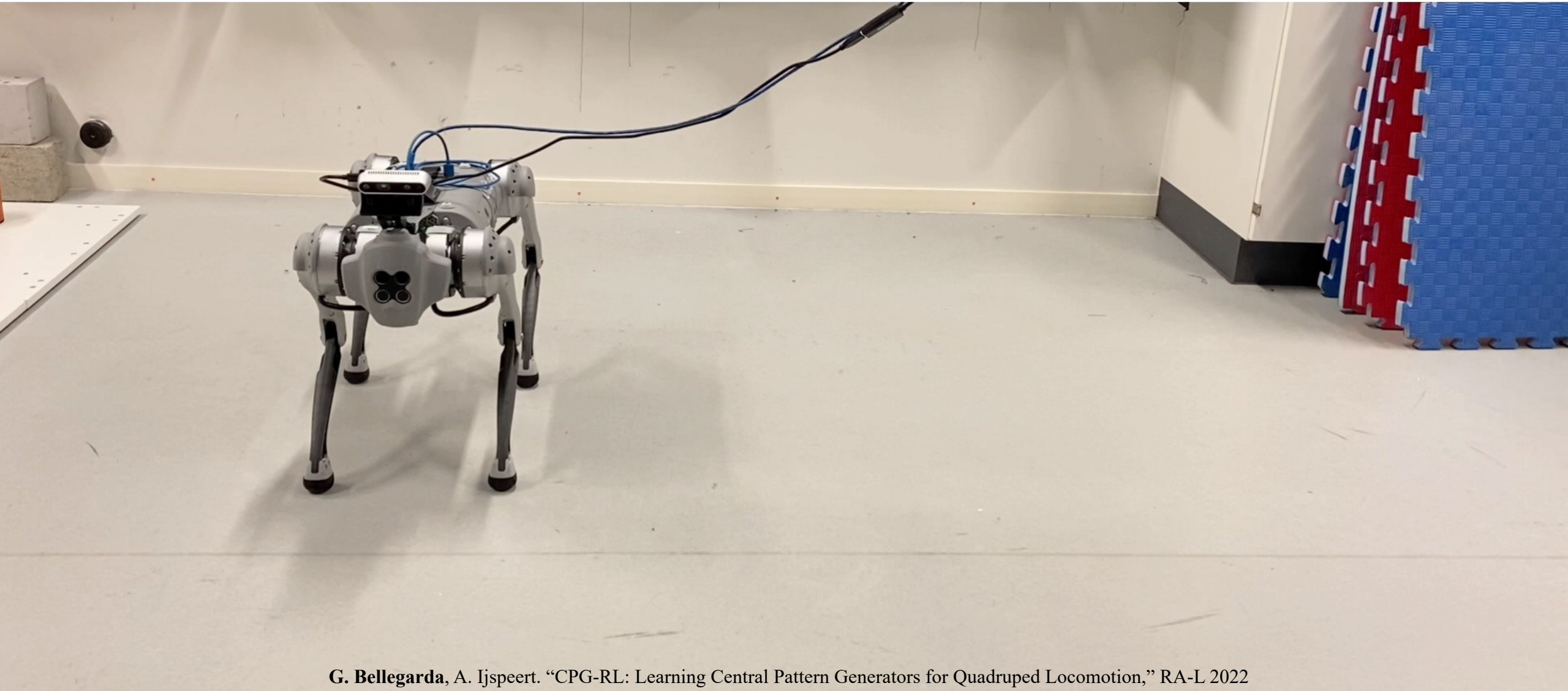
Dynamically Added Mass (5+5+2.5+1.25=13.75 kg)

Medium Observation Space, $v^*_{b,x} = 0.8$ [m/s]

G. Bellegarda, A. Ijspeert. "CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion," RA-L 2022

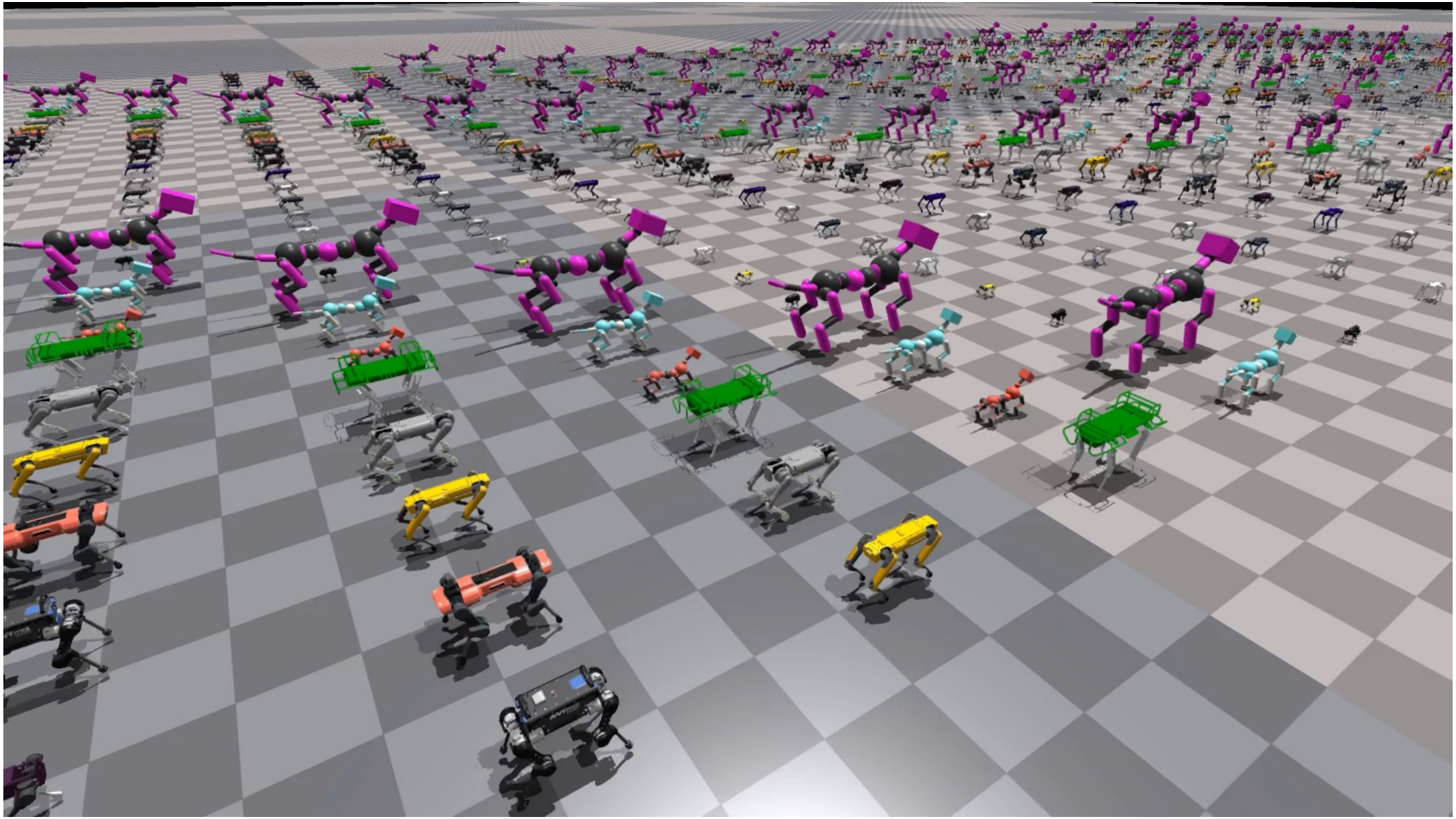
Omnidirectional Locomotion

- Command: $v^*_{b,y} = 0.4$ [m/s]



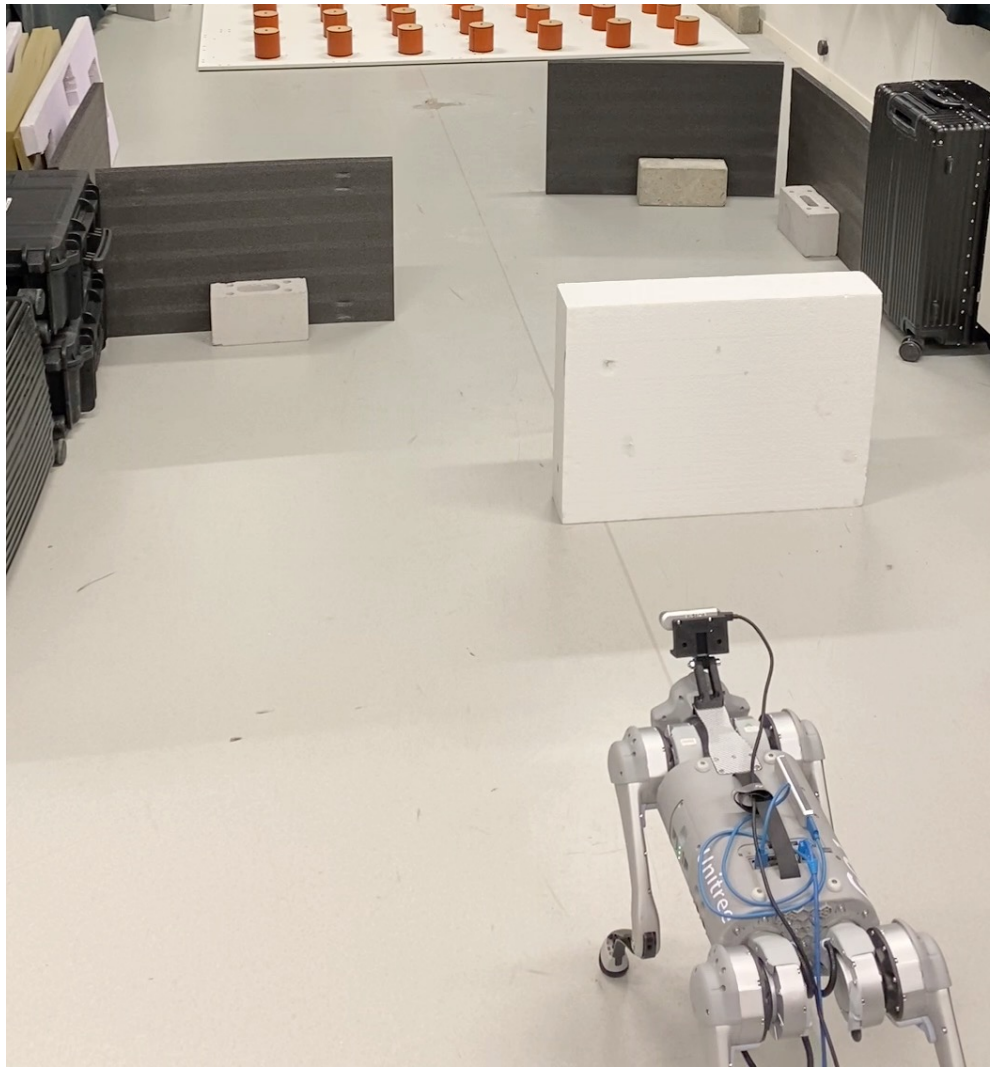
G. Bellegarda, A. Ijspeert. “CPG-RL: Learning Central Pattern Generators for Quadruped Locomotion,” RA-L 2022

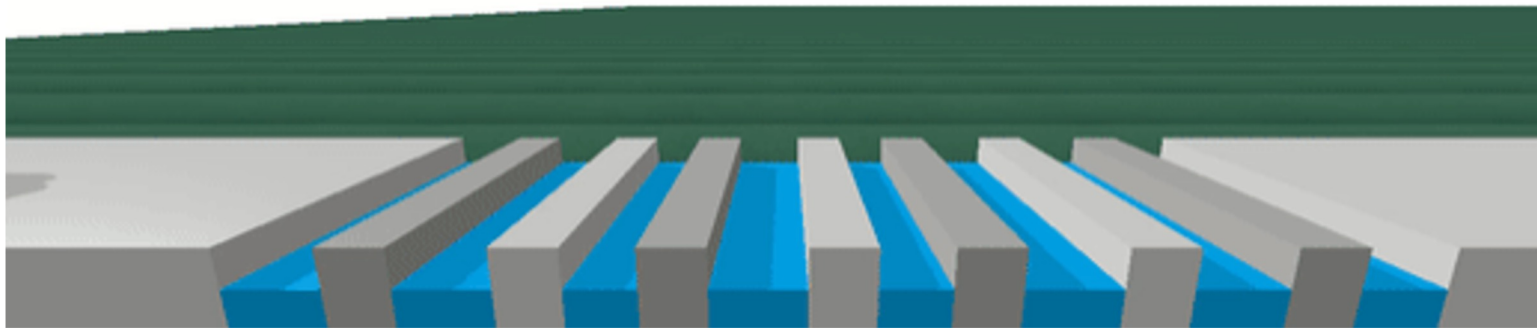
ManyQuadrupeds: A single locomotion policy capable of controlling diverse robots



M. Shafiee, **G. Bellegarda**, A. Ijspeert. “ManyQuadrupeds: Learning a Single Locomotion Policy for Diverse Quadruped Robots,” ICRA 2024

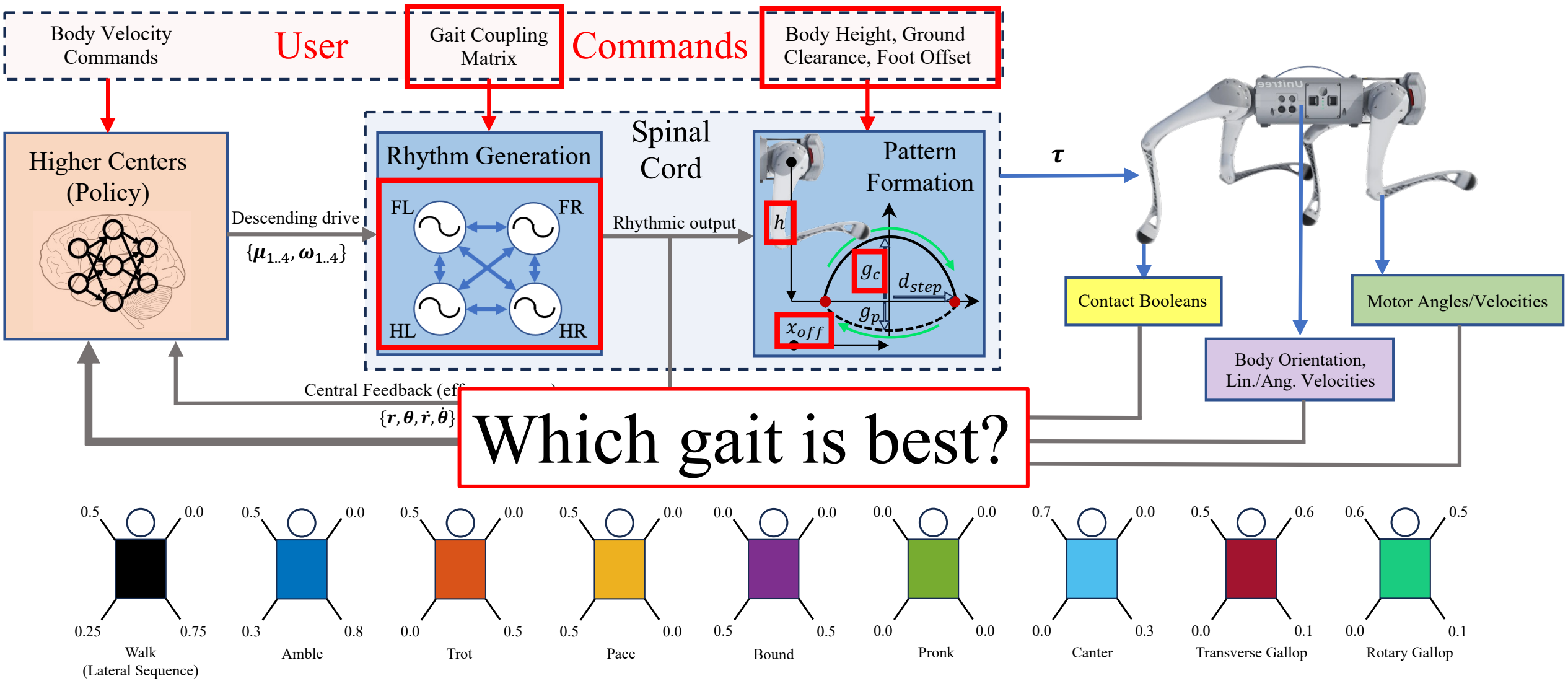
Visual CPG-RL





G. Bellegarda, M. Shafiee, A. Ijspeert. “Visual CPG-RL: Learning Central Pattern Generators for Visually-Guided Quadruped Locomotion,” ICRA 2024
M. Shafiee, **G. Bellegarda**, A. Ijspeert. “Viability Leads to the Emergence of Gait Transitions in Learning Anticipatory Quadrupedal Locomotion Skills,” Nature Communications 2024
M. Shafiee, **G. Bellegarda**, A. Ijspeert. “Learning Anticipatory Quadrupedal Locomotion Based on Interactions of a Central Pattern Generator and Supraspinal Drive,” ICRA 2023

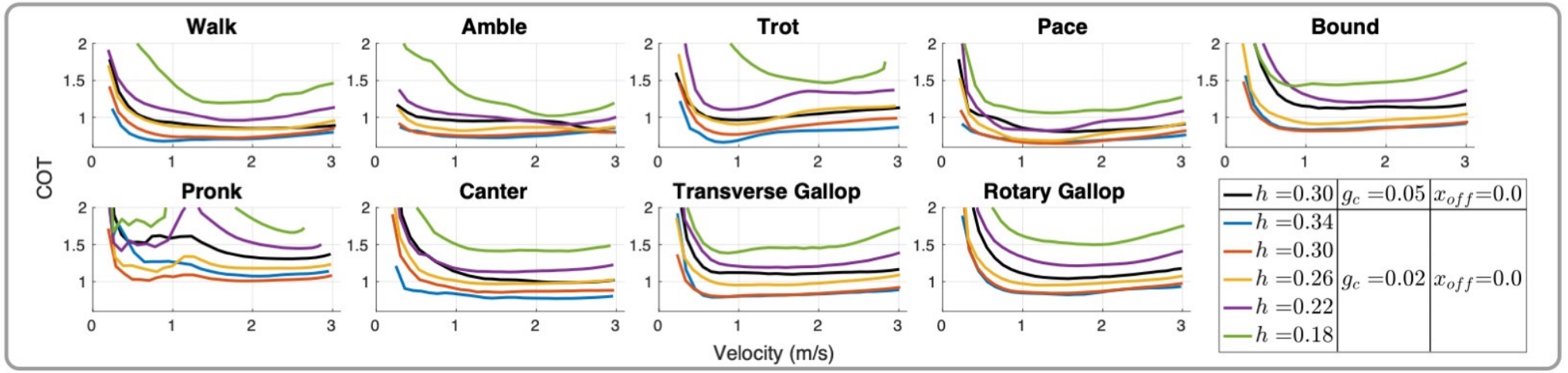
Learning All Gaits, Styles, and Transitions

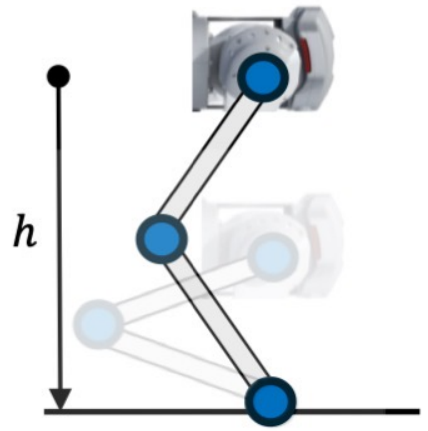


Bellegarda et al. *ICRA 2025*

We investigate the effects of different gait styles on the Cost of Transport (COT)

- **Changing body height (h):** more upright postures are more energy-efficient

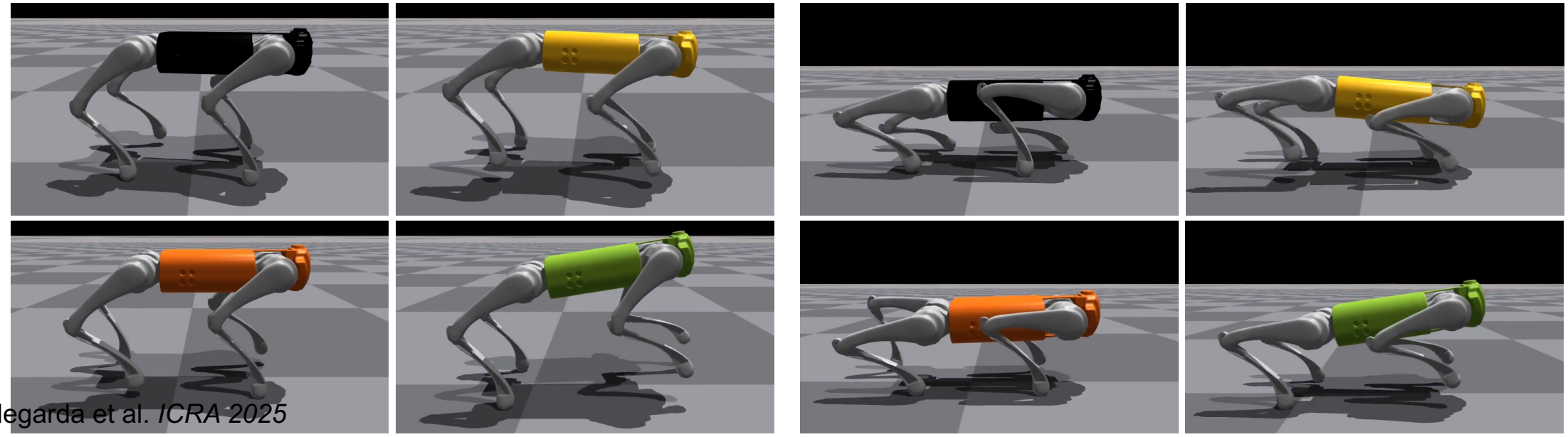




- **Examples:**

Higher Body Height (Lower COT)

Lower Body Height (Higher COT)



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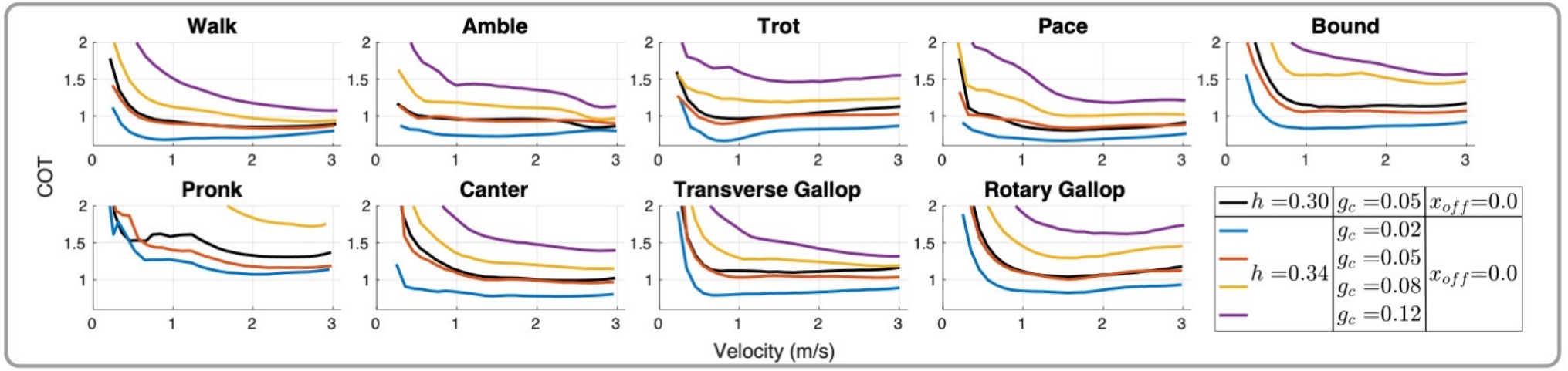
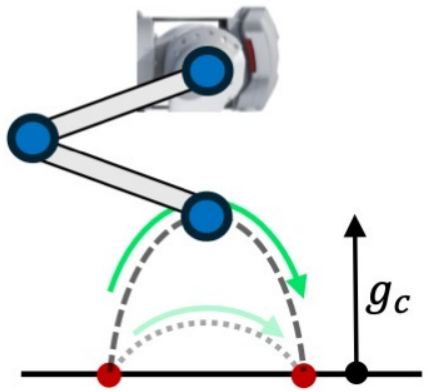
Bellegarda et al. *ICRA 2025*

We investigate the effects of different gait styles on the Cost of Transport (COT)

- **Changing swing foot height (g_c):** lower swing foot height is more energy-efficient

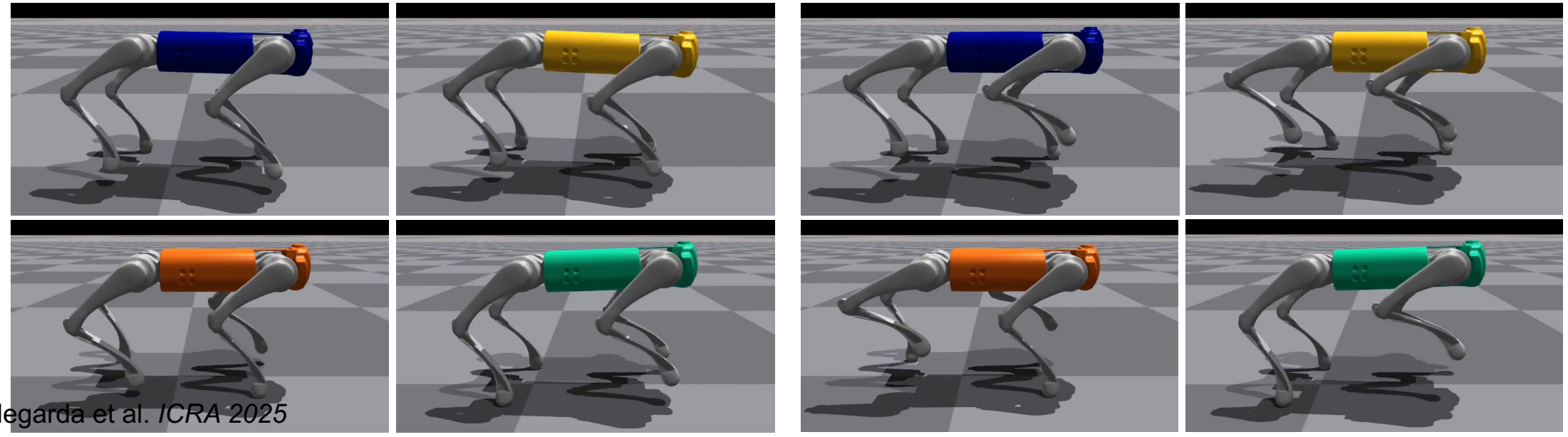




- **Examples:**

Lower Swing Foot Height (Lower COT)

Higher Swing Foot Height (Higher COT)



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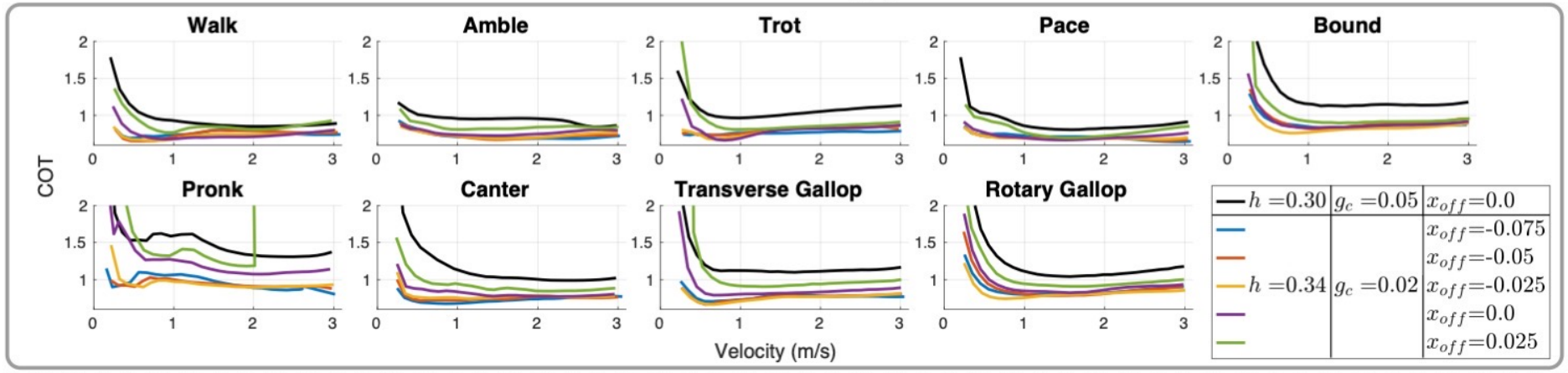
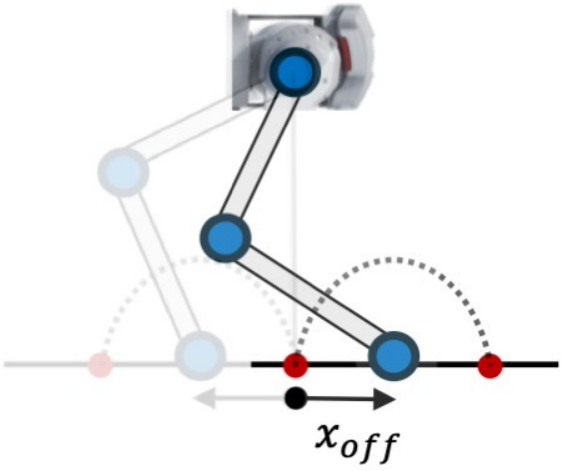
We investigate the effects of different gait styles on the Cost of Transport (COT)

- **Changing x foot offset (x_{off}):** variability based on velocity, but generally more energy-efficient to center the foot oscillations behind the nominal hip position

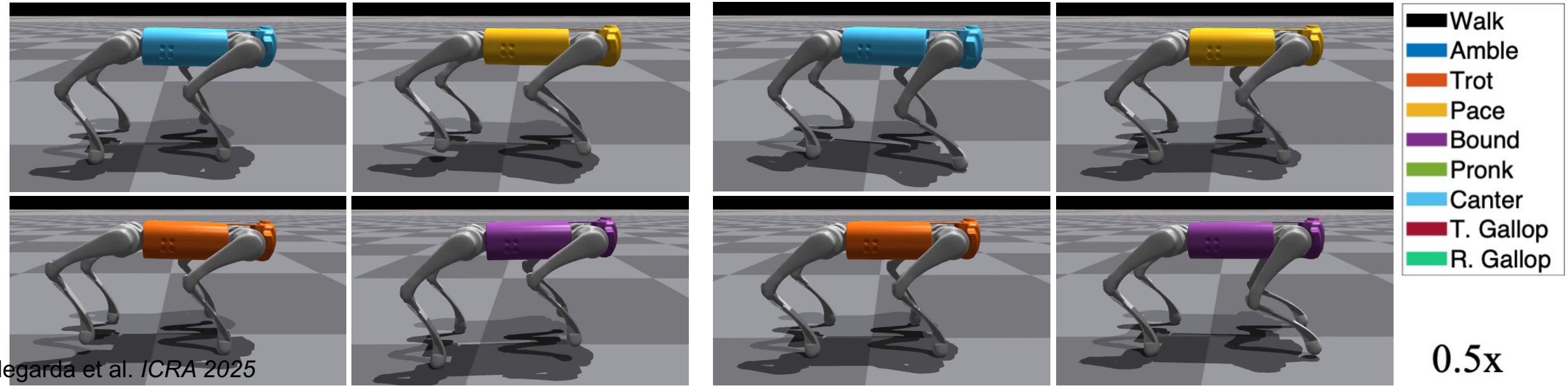




- **Examples:**

Foot offset behind hip (Lower COT)

Foot offset in front of hip (Higher COT)



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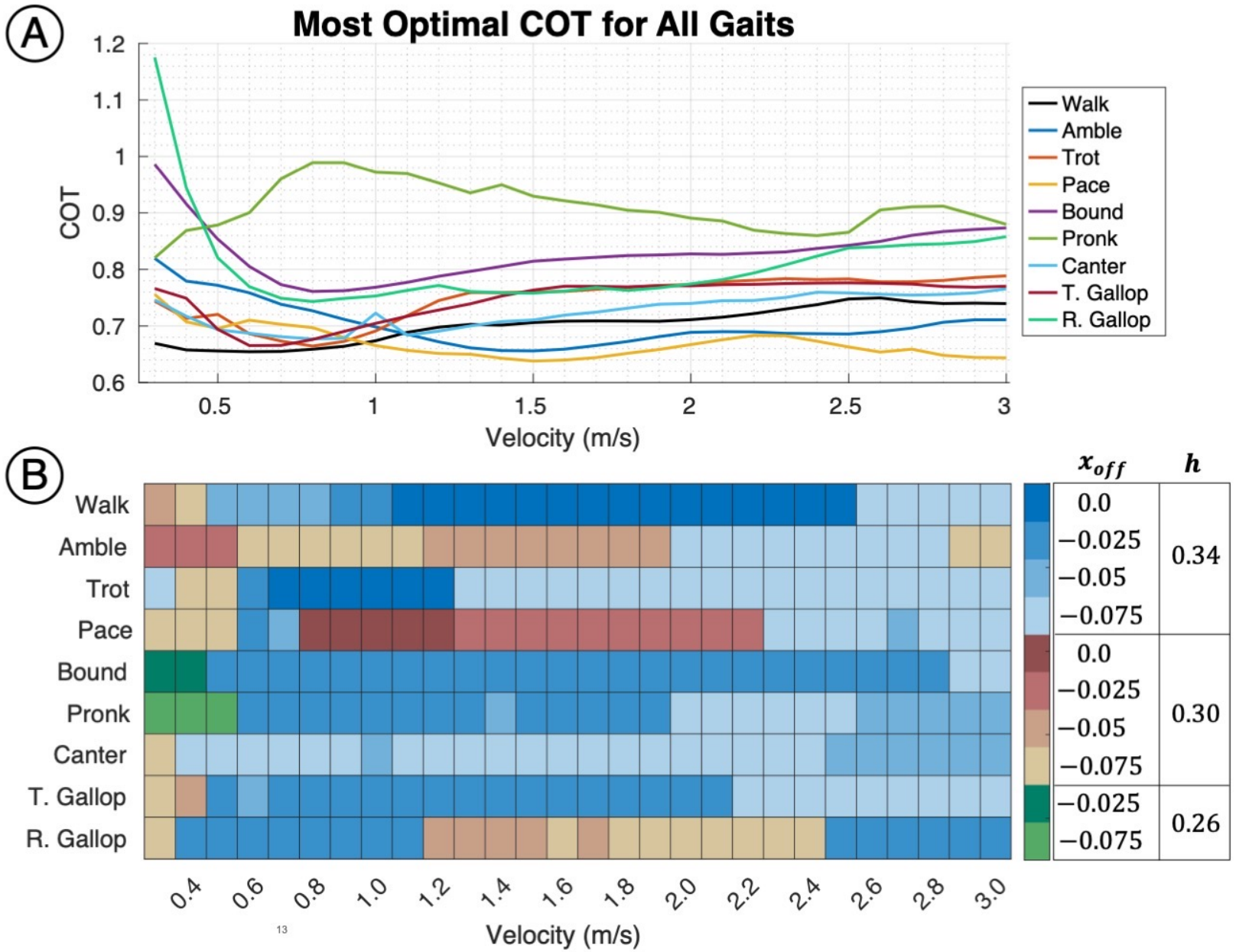
Most Energy-Efficient Gaits

- **Walk** gait is most energy-efficient at low velocities
- **Pace** gait is most efficient at higher velocities
- Most gaits minimize the COT with:
  - **high** body height
  - **low** swing foot ground clearance
  - foot offsets **behind** the hip



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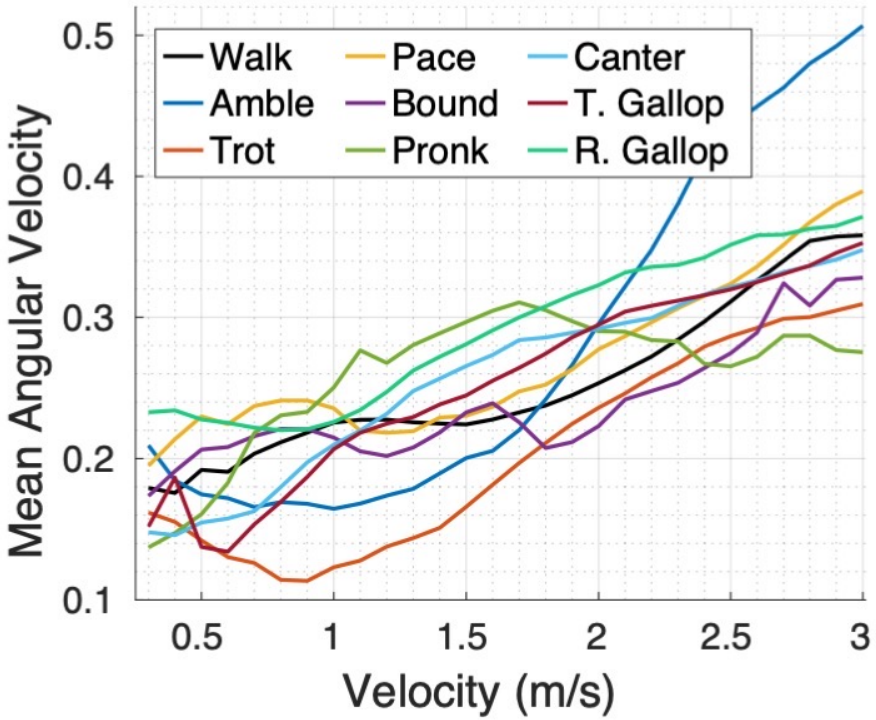
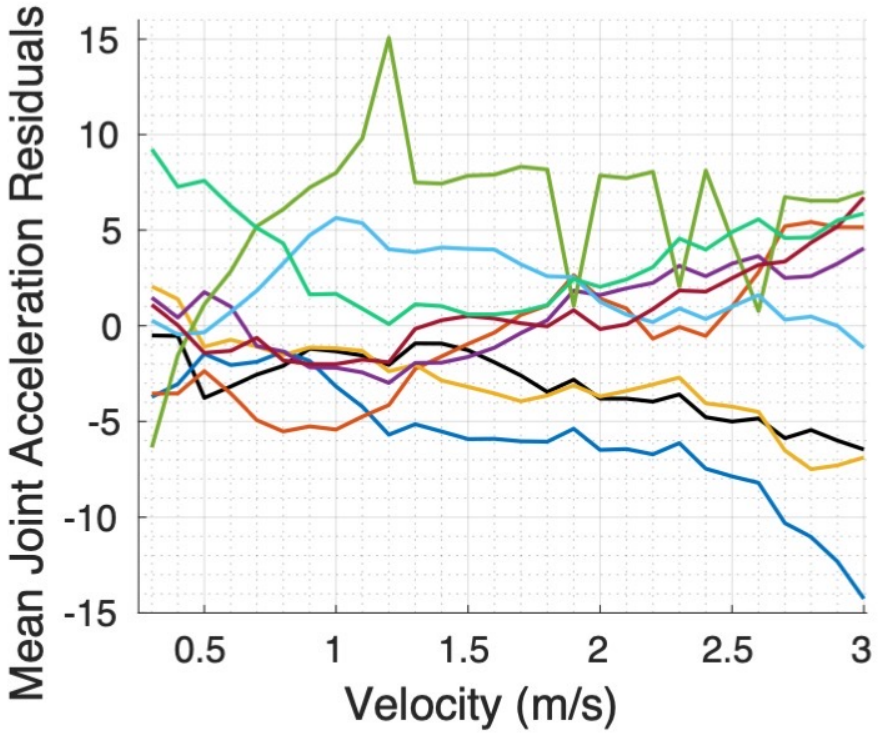
Considering Different Metrics Leads to Different 'Optimal' Gaits





- **Trot** gait minimizes base angular velocities at low and middle speeds
- **Bound**, and then **pronk**, are 'better' at higher velocities

- While **amble** results in high mean base angular velocities, it minimizes mean joint acceleration relative to other gaits at the same velocity

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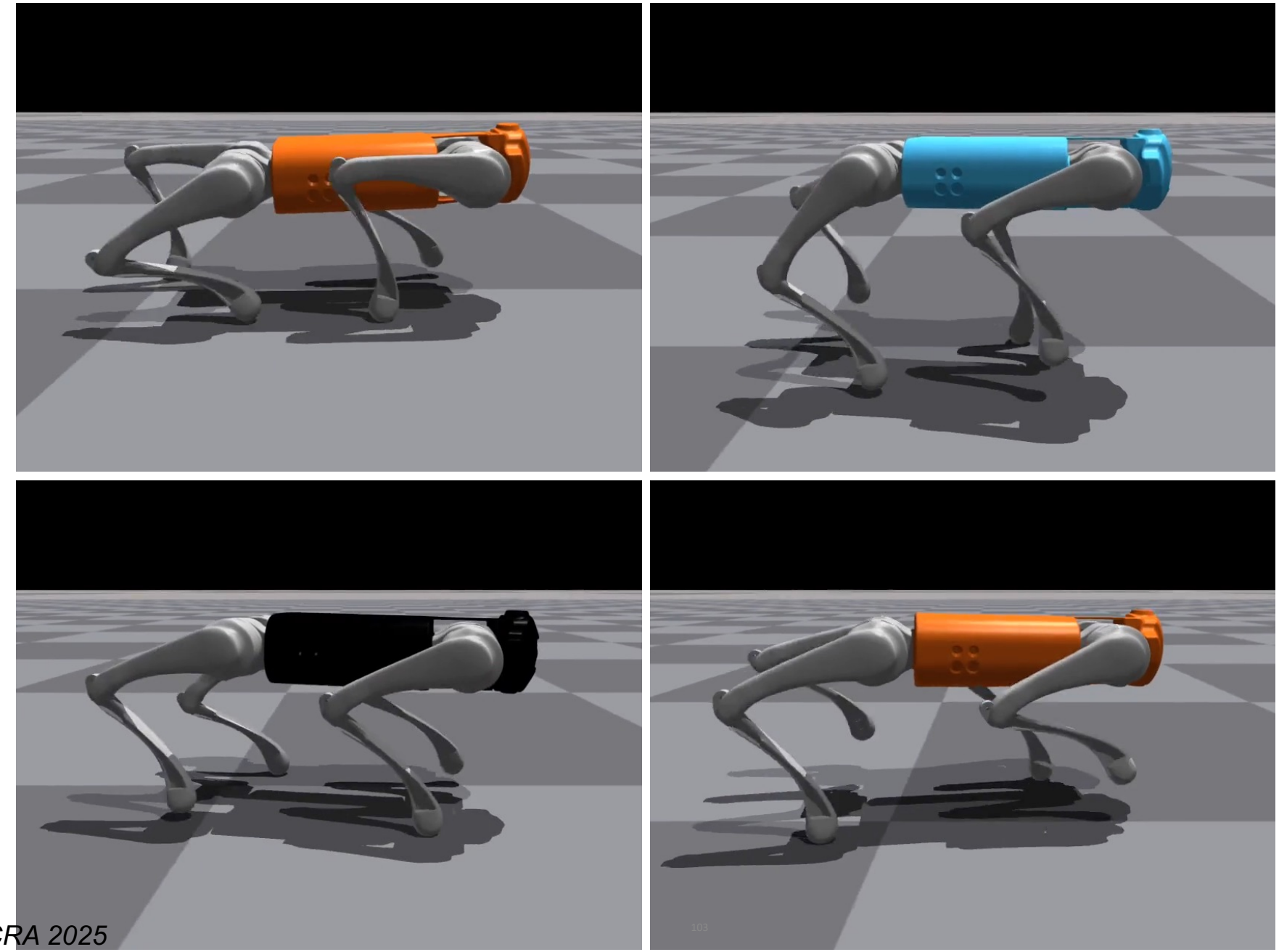
We can transition between any gait and style, at any velocity



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Sample Gait Transitions

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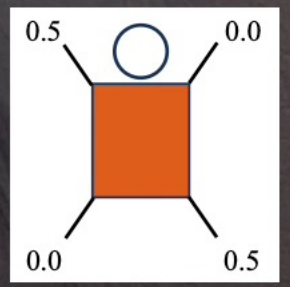
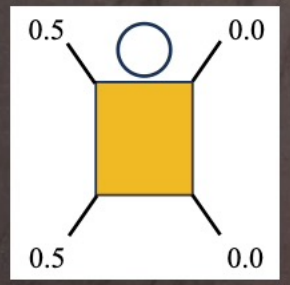
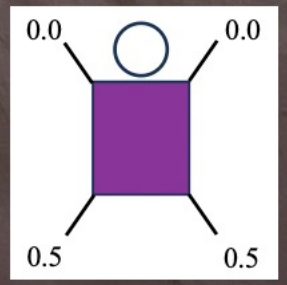


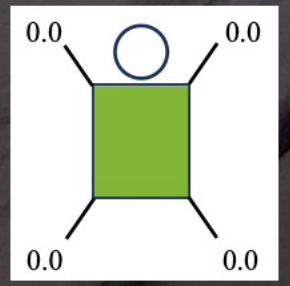
Trot → Pace



Pace→ Bound



Bound→ Pronk



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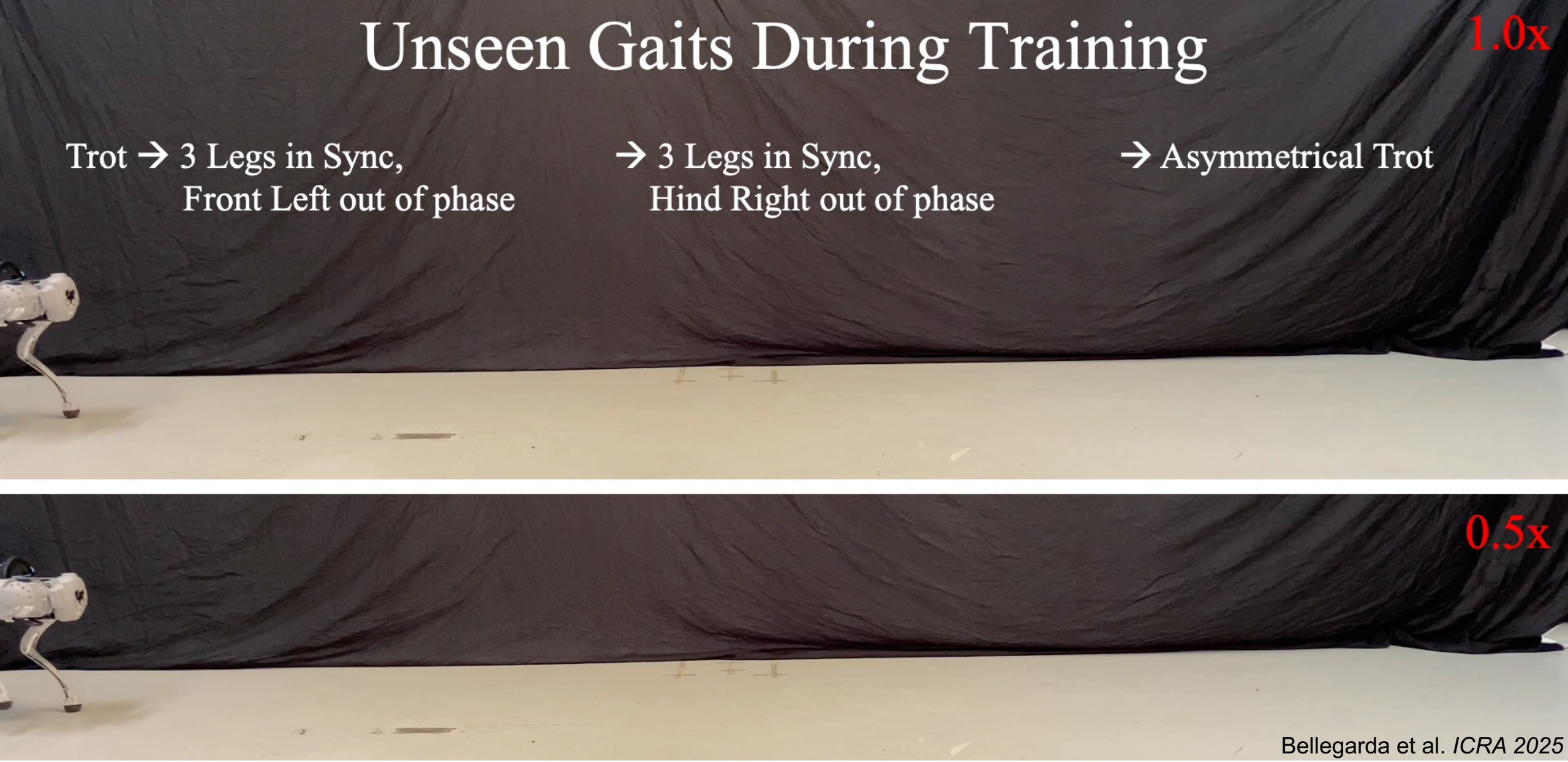
Bellegarda et al. *ICRA 2025*

Bellegarda et al. *ICRA 2025*

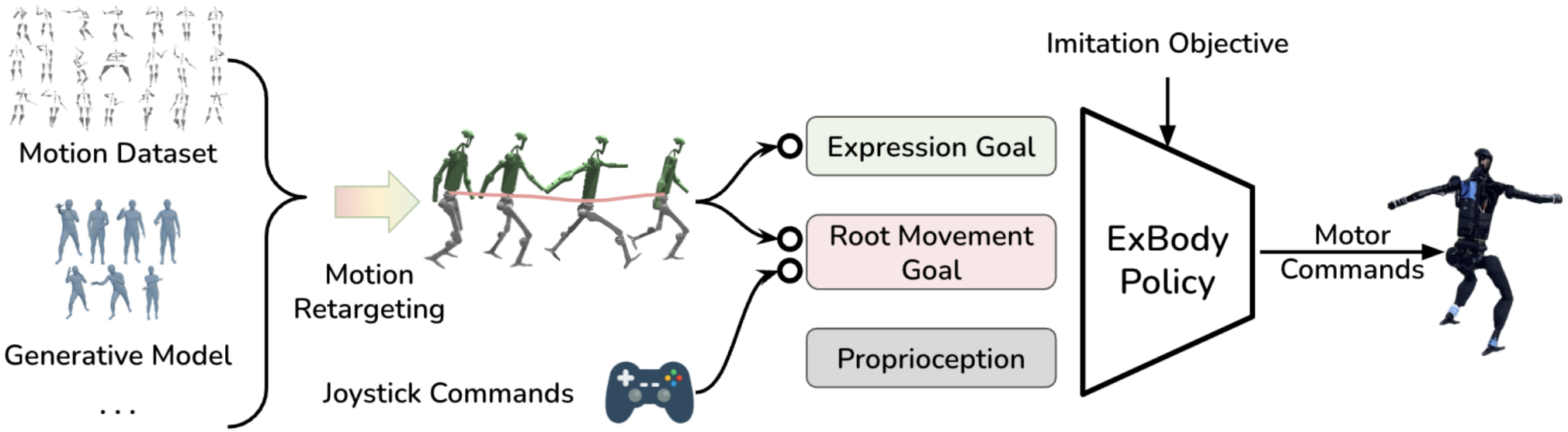


The policy generalizes to new gaits specified with novel coupling matrices directly in hardware experiments, despite never experiencing such coupling or observations during training.

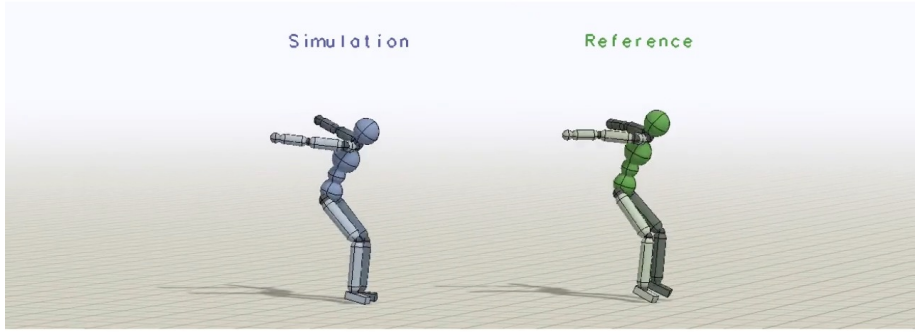
CPG-RL Summary

- CPG-RL: integrated abstract oscillators into the deep reinforcement learning framework to learn quadruped locomotion
- Online modulation of body height and swing foot height
- Robust sim-to-real transfer (115% nominal mass load, uneven terrain)
- Minimal proprioceptive sensing (only contact Booleans)
- Training without any dynamics randomization or noise
- No reward function tuning!
- Can learn a single policy for Many Quadrupeds!
- Can learn all the gaits, styles, and gait transitions!
- Vision in-the-loop for navigation and gap crossing

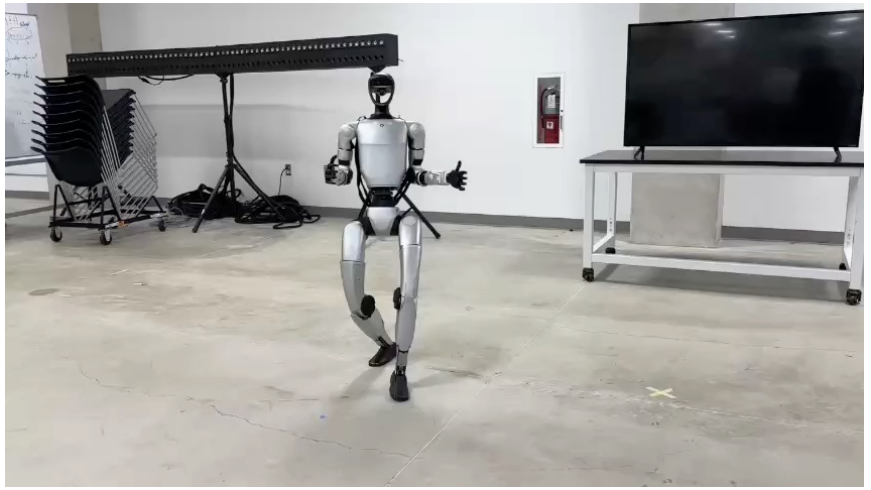
Current Humanoid Approaches – (Deep) imitation learning



RSI + ET (Our Method)



With RSI and ET, the policy learns to perform the flip.



Peng et al. *DeepMimic: Example-Guided Deep Reinforcement Learning of Physics-Based Character Skills.* SIGGRAPH 2018
Cheng et al. *Expressive Whole-Body Control for Humanoid Robots*. RSS 2024
Chen et al. *GMT: General Motion Tracking for Humanoid Whole-Body Control*. Arxiv 2025

Conclusion

- Trajectory optimization and deep reinforcement learning can both create locomotion controllers
- Both have pros and cons:
  - TO: model-based (explainable) and verifiable, but need a lot of online compute, and may not generalize to uncertainties
  - RL: can learn potentially anything, but “black-box”, and exploration / sim-to-real
- Abstractions and knowledge priors such as kinematics and Jacobians are very beneficial for learning
- The CPG is a powerful tool to combine with RL to simplify and robustify sim-to-real, and can be used to answer scientific questions

- At Inria we have several masters’ projects available, feel free to contact me

Possible Exam Questions

- How are model-based methods and learning-based methods similar? How are they different?
- When might you consider using a model-based controller over a learning-based controller, and vice-versa?
- Assume you need to design a locomotion controller with trajectory optimization. What would your cost function include? What kind of constraints do you need?
- Assume you need to train a locomotion controller with deep reinforcement learning. How will you structure the Markov Decision Process (i.e. observation space, action space, reward function)?
- What are some benefits of integrating CPGs with reinforcement learning?
- What makes reinforcement learning challenging? What might you consider when applying reinforcement learning to a new problem?