Microtechnique: 2025-2026 Master Semester 1
Fundamentals of Integrated Photonic Components

Assignment #3
Gratings on waveguides

Problem 1 (30 points):

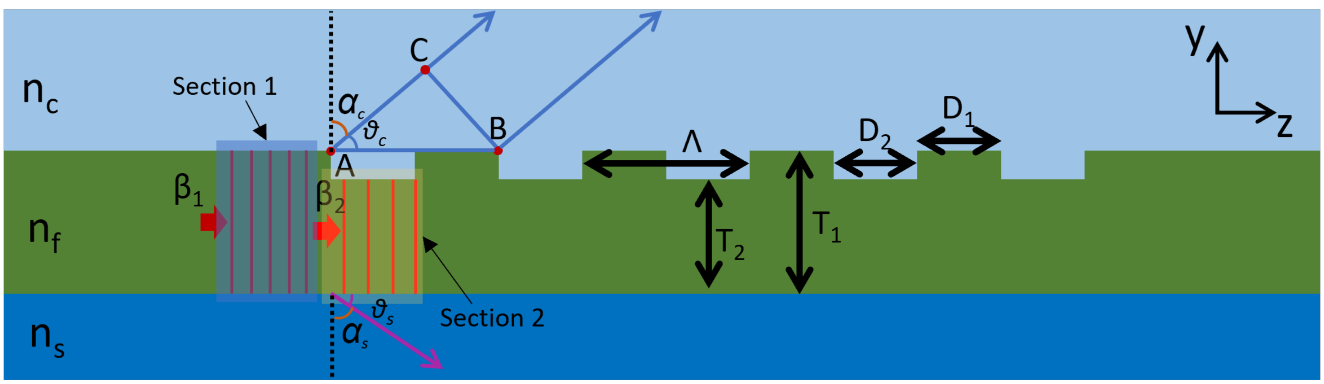

Figure 1: Sketch of a grating coupler realized on a slab waveguide. The grating itself can be regarded as made of different sections of a slab waveguide (namely 1 and 2), with different thicknesses T_1 and T_2, stacked together so as to realize a periodic pattern. The light propagating into the original waveguide (left side of the drawing), is then coupled out by the grating along the escaping angles θ_c and θ_s, directed towards the cladding and the substrate, respectively.

A grating structure is realized by partially etching the core layer of a slab waveguide (green part) along the propagation direction (z-axis), as represented in Fig.1. The definition of main parameters indicated in the figure are here listed:

T_1: thickness of core layer
n_{eff1}: effective refractive index of section 1
T_2: thickness of remaining etched core
n_{eff2}: effective refractive index of section 2
β_1 and β_2: propagation constant for section 1 and 2
Λ: grating period
D_1 : section of the period unetched (section 1)
D_2 : section of the period etched (section 2)
$d_c = D_1/\Lambda = (\Lambda - D_2)/\Lambda$: duty cycle
n_c: cladding refractive index

n_s: substrate refractive index
n_f: core refractive index

i) Express analytically the phase difference accumulated by the light rays emitted from the two points A and B, using the parameters given above (see Fig. 1). (**6 points**)
ii) What is the condition that the phase difference at point i) has to satisfy in order to give rise to constructive interference between the two rays in the far field? Derive the analytical expression for the angle θ_c from this condition. **(6 points)**
iii) Consider the case of a 50% duty cycle and shallow grating $T_1 \approx T_2$. How would you then simplify the expression for θ_c at point ii) in this specific case? **(3 points)**
iv) What and how many values can θ_c exhibit as a function of the order m? [Hint: evaluate the value m_{max} from the grating equation at point iii), as a function of all the other parameters. Take into account that the inverse function of a co/sinusoidal function admits real values only if the modulus of its argument is not larger than the unity]. Comment your answer. **(9 points)**
v) Repeat the considerations at point i)-ii)-iii) and derive the expression for the escaping angle θ_s through the substrate. **(6 points)**

Solution 1 (Analytical):

Let us consider the following pictorial representation: a light beam with wavelength λ is guided from the left to the right (positive direction along the z-axis) with propagation constant β_{N1} (corresponding to an effective refractive index N_1). It encounters the grating on point A, which represents the first emitting element for the radiation out-coupled from the waveguide. The perturbation induced by the grating structure on those regions of the waveguide as large as D_2 is accounted through a different propagation constant β_{N2} (corresponding to an effective refractive index N_2). Continuing in the propagation, light is again emitted at point B, and so on.
i) We note that in order for the out-coupled beam to form a wavefront (i.e., a plane perpendicular to propagation direction where the phase associated to each point is the same), all points belonging to the segment CB must exhibits the same phase retardation. In particular let us focus the attention on points B and C. The phase accumulated by the ray traveling along the AC path (i.e., in the cladding material) can be written as:

$$\Delta\phi_{AC} = n_c k_0 \Lambda \cos\theta_c \tag{1}$$

while the phase accumulated by the other ray travelling the distance AB is:

$$\Delta\phi_{AB} = \beta_2 D_2 + \beta_1 D_1 = \beta_2(\Lambda - D_1) + \beta_1 D_1 = \beta_2\Lambda + D_1(\beta_1 - \beta_2) = k_0[n_{eff2}\Lambda + D_1(n_{eff1} - n_{eff2})] \tag{2}$$

The phase difference accumulated by the light rays emitted from A and B is thus $\Delta\phi_{AB} - \Delta\phi_{AC}$.

ii) Now, if the light emitted at points A and B has to be in-phase, then the contributions in Eqs.1 and 2 must differ by a multiple of 2π, that is:

$$\Delta\phi_{AB} - \Delta\phi_{AC} = 2\pi m \tag{3}$$

where $m = \pm1, \pm2, ...$ is an integer. Working on Eq. 3, we obtain:

$$\begin{aligned} & k_0[n_{eff2}\Lambda + D_1(n_{eff1} - n_{eff2})] - n_c k_0 \Lambda\cos\theta_c = 2\pi m \\ \Rightarrow\ & \frac{n_{eff2}\Lambda + D_1(n_{eff1} - n_{eff2})}{\lambda} - \frac{n_c\Lambda\cos\theta_c}{\lambda} = m \\ \Rightarrow\ & \cos\theta_c = \frac{n_{eff2}\Lambda + D_1(n_{eff1} - n_{eff2})}{n_c\Lambda} - \frac{m\lambda}{n_c\Lambda} \end{aligned} \tag{4}$$

iii) Let us now consider the case of 50% duty cycle. This means that $D_1 = D_2 = \Lambda/2$, which leads to:

$$\cos\theta_c = \frac{n_{eff1} + n_{eff2}}{2n_c} - \frac{m\lambda}{n_c\Lambda} \tag{5}$$

As a side note, we could do a further approximation valid in the case of shallow gratings. In this case $T_1 \approx T_2$, which also implies $n_{eff1} \approx n_{eff2} = n_{eff}$, thus leading to:

$$\cos\theta_c = \frac{n_{eff}}{n_c} - \frac{m\lambda}{n_c\Lambda} \Rightarrow \theta_c = \arccos\left(\frac{n_{eff}}{n_c} - \frac{m\lambda}{n_c\Lambda}\right) \tag{6}$$

One should pay attention to the fact that Eq.6 is exclusively valid for small perturbations, whereas it greatly fails to predict the behavior of etching depth approaching the entire core thickness.
iv) For simplicity let us refer to the case represented by Eq. 6, which could be easily extended to a more general formula with similar considerations. In order for the cosine function to admit real values, the right side of Eq. 6 has to be not larger than the unity in modulus, a condition that we can write as:

$$\left|\frac{n_{eff}}{n_c} - \frac{m\lambda}{n_c\Lambda}\right|_{max} = 1 \Rightarrow \frac{n_{eff}}{n_c} - \frac{m_{max}\lambda}{n_c\Lambda} = \pm 1 \tag{7}$$

Let us examine the two cases separately. Taking the positive case, we have:

$$\frac{n_{eff}}{n_c} - \frac{m\lambda}{n_c\Lambda} = 1 \Rightarrow m^+_{max} = \left\lfloor\left(\frac{n_{eff}}{n_c} - 1\right)\frac{n_c\Lambda}{\lambda}\right\rfloor. \tag{8}$$

Since the fraction n_{eff}/n_c is always greater than 1, the values of m must be taken with a positive sign in order to achieve an overall value not larger than the unity. Equation 8 indicates that the larger the refractive index contrast, the larger the number of modes that will be emitted by the grating. Also, higher-order modes ($m > 1$) exhibits larger angles of emission. Moreover, if the grating period is too small compared to the optical wavelength (i.e., $\Lambda << \lambda$), the structure might not be able to couple out any light, as no mode could exist.
Let us now consider the negative case:

$$\frac{n_{eff}}{n_c} - \frac{m\lambda}{n_c\Lambda} = -1 \Rightarrow m^-_{max} = \left\lfloor\left(\frac{n_{eff}}{n_c} + 1\right)\frac{n_c\Lambda}{\lambda}\right\rfloor. \tag{9}$$

We need to pay particular attention to Eq. 9. First of all, since n_{eff}/n_c is a positive quantity, the only way to make the first term overall negative is by taking positive values for m. Moreover, since negative cosine values correspond to angles greater than 90°, this condition predicts the existence of light beams out-coupled directly in the backward direction (negative z-axis) with respect to the guided propagation in the core. However, we also note that, compared to the case in Eq. 8, here the right side of Eq. 9 is of a much greater value. Since the light power associated to higher-order modes decreases dramatically with it, the only way to achieve backward out-coupling on the lower orders is to significantly decrease the values or the grating period Λ. Note that this condition collides with the establishment of modes in the forward direction. Thus, it is not possible to achieve both conditions simultaneously.
v) For the substrate case, we could simply redraw a similar geometry as in Fig. 1, yet with the escaping rays directed towards the substrate. The same considerations could be done by simply replacing n_c with n_s in Eq. 5. This way, we can write down:

$$\cos\theta_s = \frac{n_{eff1} + n_{eff2}}{2n_s} - \frac{m\lambda}{n_s\Lambda} \Rightarrow \theta_s = \arccos\left(\frac{n_{eff1} + n_{eff2}}{2n_s} - \frac{m\lambda}{n_s\Lambda}\right) \tag{10}$$

Sometimes n_s is larger than n_c, therefore θ_s is usually smaller than θ_c.

Problem 2 (30 points):

Consider the following values for the parameters of the structure drawn in Fig. 1 above:

$T_1 = 220\ nm$
$T_2 = 150\ nm$

$\Lambda = 610\ nm$
$n_c = 1.44$
$n_s = 1.44$
$n_f = 3.48$

Consider constant effective refractive indices across the entire wavelength range, having the following values: $n_{eff1} = 2.8514$ and $n_{eff2} = 2.54$.

i) Plot the values of angle $\alpha_c = \pi/2 - \theta_c$ and $\alpha_s = \pi/2 - \theta_s$ (measured with respect to the axis orthogonal to the propagation direction), as a function of the light wavelength, in the range $\lambda = 1500 - 1600\ nm$, with steps of 10 nm. Consider a duty cycle of $d_c = 50\%$. [Note: in practical applications, it is more common (and convenient) to refer to α as the coupling angle, rather than θ]. (**10 points**)
ii) Fix the value of the wavelength at 1550 nm and let the grating period be variable. Plot the values of angle $\alpha_c = \pi/2 - \theta_c$ and $\alpha_s = \pi/2 - \theta_s$ as a function of the grating period in the range $\Lambda = 200 - 2000\ nm$, with steps of 20 nm. Consider a duty cycle of $d_c = 50\%$. (**10 points**)
iii) Plot the values of angle $\alpha_c = \pi/2 - \theta_c$ and $\alpha_s = \pi/2 - \theta_s$ as a function of the grating duty cycle d_c, for $\lambda = 1550nm$, $\Lambda = 880nm$. Assume the following range of duty cycle $d_c = D_1/\Lambda = 10 - 90\%$, with 5% increment **(10 points)**.

Solution 2 (Numerical):

i) Dependence on the wavelength.

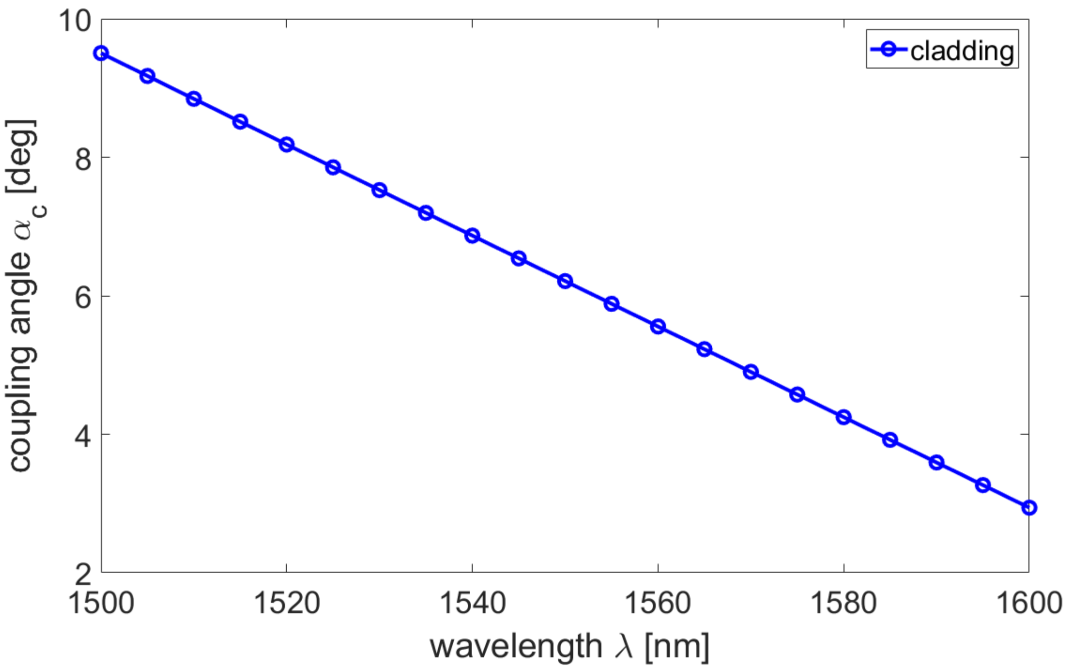

Figure 2: Escaping angle from the grating towards cladding and substrate as a function of the optical wavelength.

ii) Dependence on the grating period.

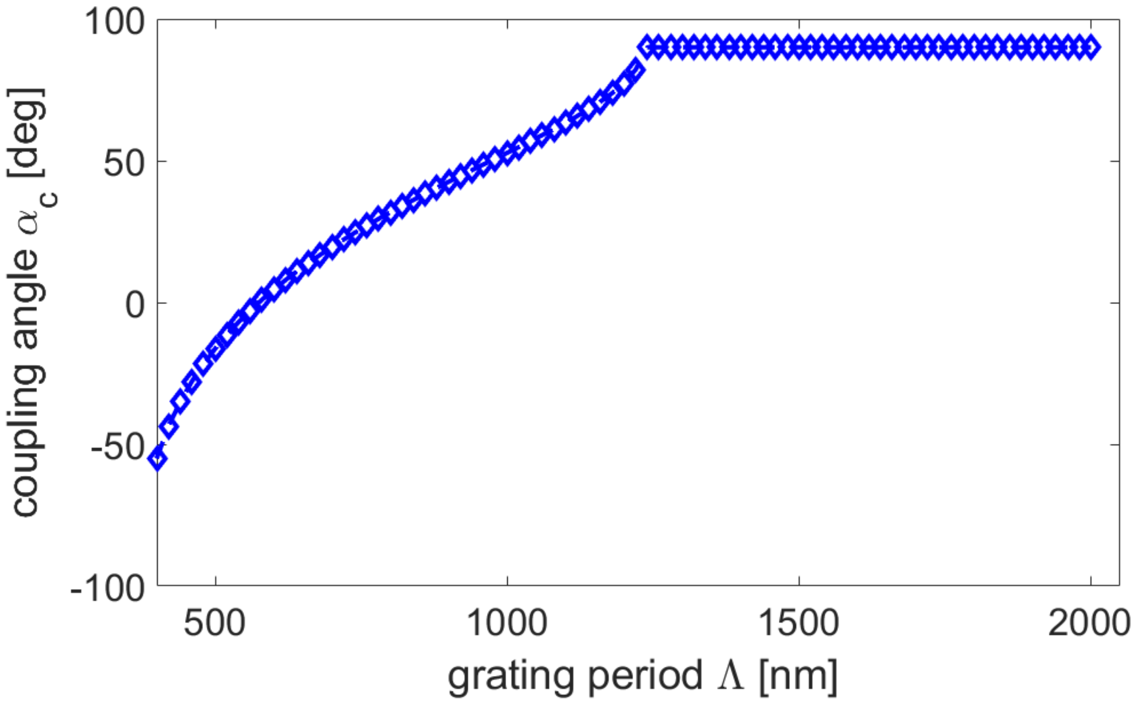

Figure 3: Escaping angle from the grating towards cladding and substrate as a function of the grating period.

iii) Dependence on the grating duty cycle.

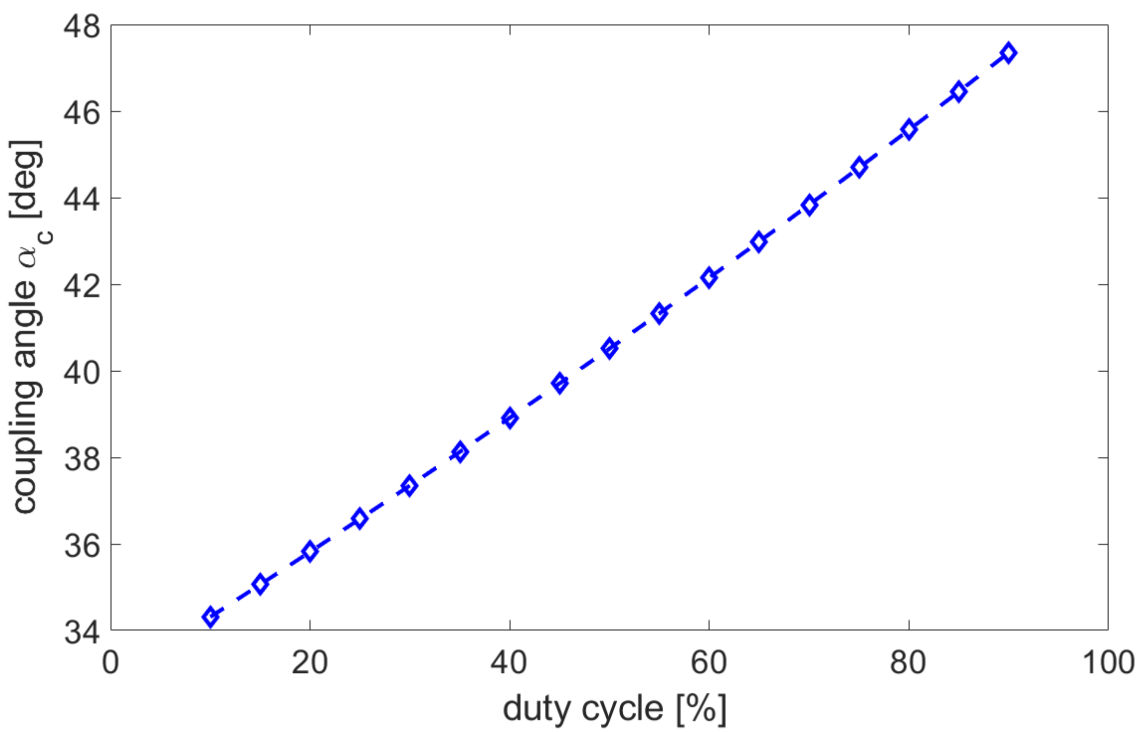

Figure 4: Escaping angle from the grating towards cladding and substrate as a function of grating duty cycle.

Problem 3 (40 points):

Consider the 3D grating structure realized on a silicon-on-insulator platform as depicted in Fig. 5. Implement the same structure on a CST file by using the following values for the parameters indicated in the figure:

T_1 = 220 nm (waveguide thickness, silicon material)
t_{etch} = 70 nm (etching depth)
$T_2 = T1 - t_{etch}$
D_1 = 305 nm (unetched section, silicon material)
D_2 = 305 nm (etched section, filled with silicon dioxide)
Λ = 610 nm (grating period)
W = 15 μm (waveguide width)
L_{WG_out} = 2 μm (length of the waveguide output)
L_{WG_in} = 5 μm (length of the waveguide input)
h_{clad} = 7 μm
h_{sub} = 5 μm
$L_{tot} = L_{WG\_in} + N\Lambda + D_2 + L_{WG\_out}$
W_{tot} = 25 μm
N = 11 (number of periods)
n_c = 1.44 (cladding refractive index, silicon dioxide)
n_s = 1.44 (substrate refractive index, silicon dioxide)
n_f = 3.48 (core refractive index, silicon)

IMPORTANT: Modify the background material properties to match those of the cladding material (SiO2,$n_s = n_c = 1.44$). To do so, go to "Simulation" tab, than select "Background". In the new material properties window, click on "Properties". A further new window will open with the a box indicating "Epsilon". By default this is set to one. For this exercise, change it to $n_c * n_c = 1.44 * 1.44 = 2.0736$.

[Note: a clever way to draw the grating structure is the following: 1) Create a brick with the dimensions of the single element of the grating right aside of the input waveguide. 2) Use the command "Transform" in the Modeling Menu. Select "Translate" and "Copy", to maintain the original one at its first place. Select the axis along which you want to duplicate the item many times and the distance between consecutive copies.]

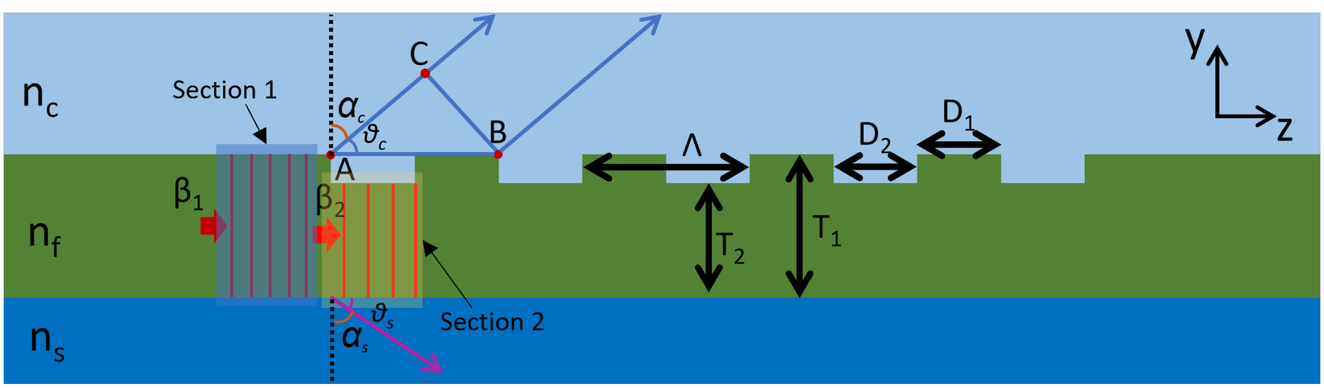

Figure 5: Schematic of structure.

a) Place a port at the waveguide input (the left side in Fig. 5(a)).

b) Consider a wavelength range of $1.5 - 1.6 \mu m$.

c) Define electric field monitors for the wavelengths 1.5, 1.55 and 1.6 μm.

d) Define far-field monitors for the wavelengths 1.5, 1.55 and 1.6 μm (use default values initially).

Simulate the grating in the time domain (set the accuracy to -40 dB) and generate the following results:

1. 2D electric field distribution of the guided electric field, on the z0x plane (i.e, passing for the center of the structure and developing along the propagating direction, refer to Fig. 5). In particular: i) report Abs values (*Contour, Abs*), ii) real values of the guided component *(Contour, Component)*. For both cases, use a linear scale. Report the results for all the three wavelengths indicated above. [Note: to run the simulation, in "Setup solver", the second menu is "Stimulation settings". Use as "Source type", "Port 1". And for "Mode" set 1. For the other options keep default values.] **(20 points)**
2. 1D far field polar distribution for the three wavelengths indicated above. What is the azimuth angle for which the radiation patter exhibit a maximum in each case? What is the angular width (calculated at -3 dB from the peak angle) around the main lobe? **(20 points)**

[Note 1: In order to create a "far-field monitor", go to "Navigation Tree" (left panel on your CST file) and then "Field Monitors". Right click on it and create a "New Field Monitor". From the dialog window choose, under "Type", "Farfield/RCS". Select the desired label and wavelength in the option below that. You can create multiple far field monitors at different wavelengths by following the same procedure. **Note 2: In order to correctly visualize the electric field being coupled out by the grating, the mesh size needs to be adequately set up. Consider using not less than 10 Million cells. Simulation times for this mesh size should not exceed 15 minutes for an accuracy of -40 dB on the VDI**.]

Solution 3 (Simulation):

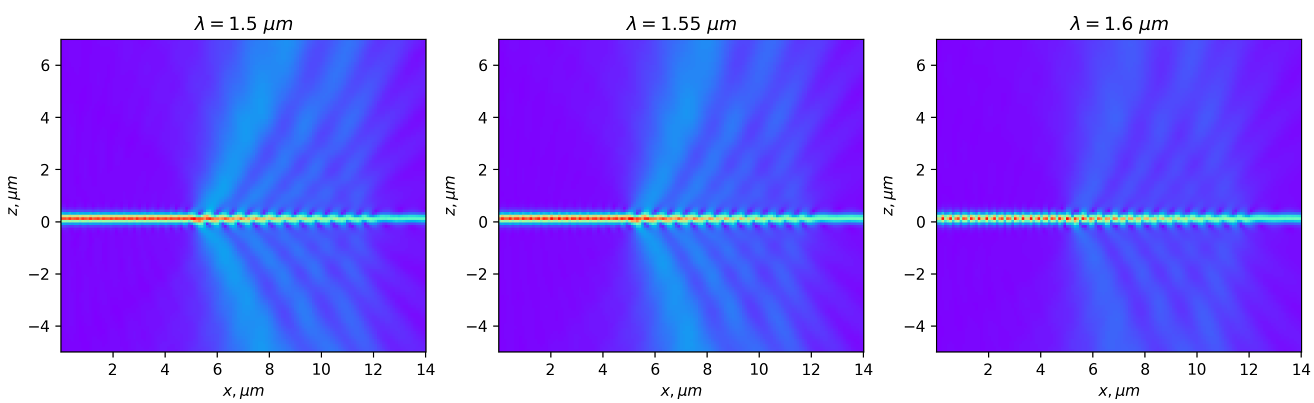

Figure 6: 2D distribution of the total amplitude of the electric field (Abs) on the plane cutting the structure on its center and running throughout its entire length for three different wavelengths: 1.5μm, 1.55μm and 1.6μm

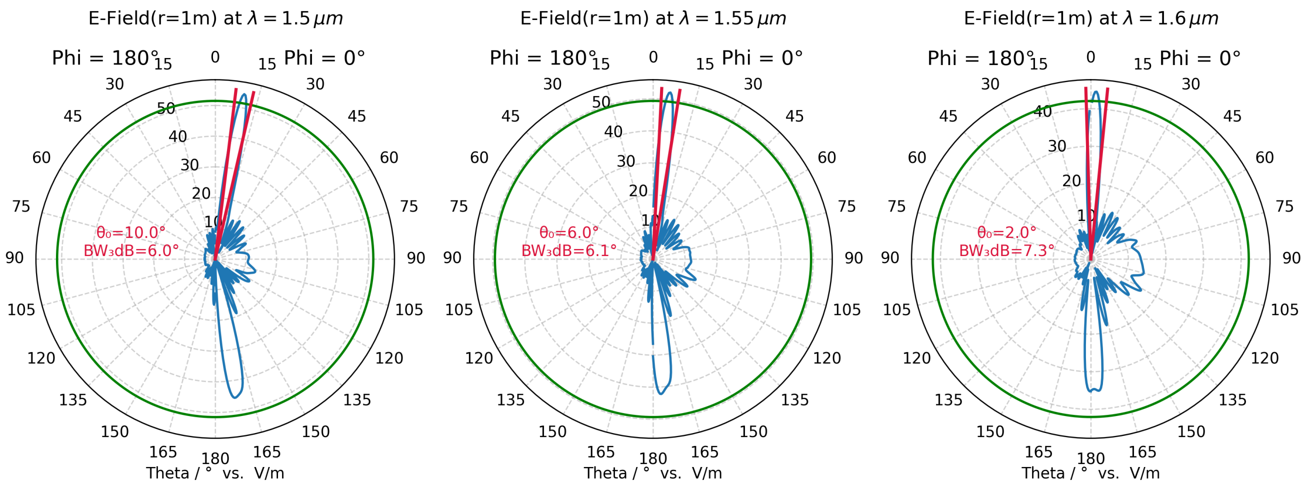

Figure 7: 1D far field distributions (1 m far from the structure) of the total electric field amplitude (Abs) for three different wavelengths: 1.5μm, 1.55μm and 1.6μm