Lecture 9 – Exercises

Exercise 9.1

During the lecture, we have seen what are the material parameter conditions for a structure that is reciprocal, gainless and lossless. What are these conditions if the structure is *non-reciprocal*, gainless and lossless ? Give an example of such a system.

Exercise 9.2

Consider two plane waves. One propagates in the x-direction and is z-polarized. The other propagates in the y-direction and is x-polarized. These two waves have the same wavelength and field amplitude $|E| = 1$. The background medium is vacuum. Express the electromagnetic fields of these two waves. Compute the total Poynting vector.

You should find something strange, the Poynting vector has an oscillating component in the z-direction even though the waves propagate in the xy-plane ! Let us now consider a point like particle placed at $(0, 0, 0)$. Compute the z-oriented force acting on this particle assuming that it may be modeled using $\mathbf{p} = \alpha_\mathrm{e}\mathbf{E}$ and $\mathbf{m} = \alpha_\mathrm{m}\mathbf{H}$. To do so, consider the following formula

$$\langle\mathbf{F}\rangle = \frac{1}{2}\mathrm{Re}\left\{(\nabla\mathbf{E}_\mathrm{i}^*)\cdot\mathbf{p} + (\nabla\mathbf{B}_\mathrm{i}^*)\cdot\mathbf{m}\right\} - \frac{k^4}{12\pi\epsilon_0 c}\mathrm{Re}\left\{\mathbf{p}\times\mathbf{m}^*\right\}.$$

You may ignore the two first terms that only give a force in the xy-plane. Instead, only consider the term associated with $\mathbf{p}\times\mathbf{m}^*$.

Exercise 9.3

Determine the properties of passivity/losslessness and reciprocity of the following systems described either in terms of material parameters or scattering matrices.

1. $\overline{\overline{\epsilon}} = \epsilon_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\mu}} = \begin{bmatrix} 2 & j0.3 & 0 \\ -j0.3 & 2 & 0 \\ 0 & 0 & 1.5 \end{bmatrix},\ \overline{\overline{\zeta}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},\ \overline{\overline{\xi}} = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

2. $\overline{\overline{\epsilon}} = \begin{bmatrix} 2-j0.2 & 0 & 0 \\ 0 & 2+j0.2 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \begin{bmatrix} 0 & 0.1 & 0 \\ -0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \overline{\overline{\xi}} = \begin{bmatrix} 0 & -0.1 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\overline{\overline{\epsilon}} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -j0.1 & 0 & 0 \end{bmatrix},\ \overline{\overline{\xi}} = \begin{bmatrix} 0 & 0 & j0.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4. $\overline{\overline{\epsilon}} = \epsilon_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \beta\overline{\overline{\mathrm{I}}},\ \overline{\overline{\xi}} = \beta\overline{\overline{\mathrm{I}}}$, where $\beta \in \mathbb{R}$

5. $\overline{\overline{\mathrm{S}}} = \begin{bmatrix} 0 & 1.5 \\ 0.3 & 0 \end{bmatrix}$

6. $\overline{\overline{\mathrm{S}}} = \begin{bmatrix} 0.2-0.5j & 1 \\ 1 & 0.2+0.5j \end{bmatrix}$

7. $\overline{\overline{\mathrm{S}}} = \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}$