Lecture 8 – Exercises

Exercise 8.1

A small current source $\mathbf{J}_a = (1.7A, 2.5B, 0)$ at point $\mathbf{r}_a$ produces the field $\mathbf{E}_b = (1.2, 0.8, 0.2)$ at point $\mathbf{r}_b$. The source and receiver positions are exchanged. We now have $\mathbf{J}_b = (2, 0, 2)$ and $\mathbf{E}_a = (A, B, 0)$. If $B = 0.25$, what should be the value of A such that the system is reciprocal?

Exercise 8.2

A lossless/gainless metasurface lying in the xy-plane at $z = 0$ is designed to transform an incident wave propagating in the xz-plane with angle $\pm\theta$ into a normally propagating. All waves have the same polarization. The system works such that 100% of the incident power, incident either from $+\theta$ or $-\theta$, is normally transmitted from the metasurface. Determine if this system is reciprocal by writing down its S-matrix. Hint: the system may be modeled as a 3-port network.

Exercise 8.3

Consider the following material parameters or scattering matrices and determine if they correspond to reciprocal or non-reciprocal systems.

1. $\overline{\overline{\epsilon}} = \epsilon_0\epsilon_\mathrm{r}\overline{\overline{\mathrm{I}}},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \overline{\overline{\xi}} = 0$

2. $\overline{\overline{\epsilon}} = \begin{bmatrix} 3+0.2j & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \overline{\overline{\xi}} = 0$

3. $\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_{xx} & j\epsilon_{xy} & 0 \\ j\epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix},\ \overline{\overline{\mu}} = \begin{bmatrix} \mu_{xx} & j\mu_{xy} & 0 \\ -j\mu_{xy} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix},\ \overline{\overline{\zeta}} = \overline{\overline{\xi}} = 0$

4. $\overline{\overline{\epsilon}} = \epsilon_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \overline{\overline{\xi}} = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. $\overline{\overline{\epsilon}} = \epsilon_0\epsilon_\mathrm{r}\overline{\overline{\mathrm{I}}},\ \overline{\overline{\mu}} = \mu_0\overline{\overline{\mathrm{I}}},\ \overline{\overline{\zeta}} = \kappa\overline{\overline{\mathrm{I}}},\ \overline{\overline{\xi}} = -\kappa\overline{\overline{\mathrm{I}}}$

6. $\overline{\overline{\mathrm{S}}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

7. $\overline{\overline{\mathrm{S}}}_{11} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{22} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{21} = \begin{bmatrix} 0 & 0.7 \\ 0 & 0.2 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{12} = \begin{bmatrix} 0 & 0 \\ 0.7 & 0.2 \end{bmatrix}$

8. $\overline{\overline{\mathrm{S}}}_{11} = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.3 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{22} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{21} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{12} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$

9. $\overline{\overline{\mathrm{S}}}_{11} = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.3 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{22} = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{21} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix},\ \overline{\overline{\mathrm{S}}}_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$

Exercise 8.4

A reciprocal isolated particle is excited with plane waves propagating in different directions. For each plane wave, we compute the dipolar response of the particle and notice that it is varying with respect to the gradient of the excitation. What can we conclude about its multipolar profile?