

Lecture 7 – Exercises

Exercise 7.1

Using Mie theory, what should be the relative permittivity and permeability of a sphere of radius r such that its electric and magnetic anapoles coincide with respect to the wavelength?

Exercise 7.2

Consider a distribution of electric current density consisting of 2 Dirac sources of amplitude A and B in the x -direction separated by a distance $2d$ along the z -axis. It is mathematically given by

$$\mathbf{J} = A\hat{\mathbf{x}}\delta(x)\delta(y)\delta(z - d) + B\hat{\mathbf{x}}\delta(x)\delta(y)\delta(z + d).$$

Compute the corresponding electric and magnetic primitive dipole and quadrupole moments. Comment on how these multipoles vary with respect to A , B and d . Show that the magnetic quadrupole is reducible. Reduce it accordingly.

Exercise 7.3

Develop your own multipolar decomposition script in python. To do so, use the template script `multipolar_decomposition.py` provided in the `data.zip` archive. This template already loads all the required simulation data and computes the primitive Cartesian electric and magnetic dipole moments as an example of how to perform a multipolar decomposition.

Three problems have been simulated: (1) a 100-nm radius dielectric sphere, (2) a 100-nm radius gold sphere and (3) a subwavelength periodic array of dielectric cylinders. To load these different problems simply change the variable `PATH` at the beginning of the script. All the relevant data for these three cases is stored in the directories entitled: `isolated_dielectric`, `isolated_metallic` and `periodic_dielectric`, respectively. To complete this exercise, you only need to change the value of `PATH` to either of these three directories and implement the *spherical* multipole moments (at around line 105 in the script). The script will then automatically compute the scattering cross-sections (SCS) for the two isolated spheres or the reflectance and transmittance (RT) for the periodic array. The scripts will also allow you to compare your computed SCS/RT curves to the COMSOL simulated ones.

For these cases, compare how considering spherical multipoles instead of primitive ones and adding multipolar contributions improves the matching with the COMSOL simulated data.

Note that to achieve appropriate accuracy several tens of thousands of field/current points are required within the particle. That makes the multipolar decomposition rather slow. To speed it up, you can increase the value of the variable `S`, which is given just below the `PATH` variable. This allows you to reduce the number of points considered in the computation, making it faster but less accurate. The most motivated among you could try to parallelize the code.

Exercise 7.4

Compute the electric and magnetic polarizabilities of the two spheres of the previous exercise using the computed dipole moments. Consider that the excitation is a plane wave propagating in the $+z$ -direction with $E_x = 1$ at the position of the spheres.