Lecture 6 – Exercises

Exercise 6.1

Consider a wave coming from air onto a medium at normal incidence. The medium is described by a Lorentzian permittivity $\epsilon(\omega)$ with $\mu_\mathrm{r} = 1$ leading to a complex refractive index $n = n_0 + j\kappa$. Find the expression of the Fresnel field and power reflection and transmission coefficients in terms of n_0 and κ.

Now consider that the parameters of the Lorentzian resonance are $\omega_0 = 2$, $\omega_\mathrm{p} = 6$ and $\Gamma = 0.3$. For a frequency ranging between $\omega = [0, 10]$, plot the reflectance versus frequency ω as well as the variations of $n_0(\omega)$ and $\kappa(\omega)$. Highlight the regions of high and low transmittance/reflectance and absorptance.

Exercise 6.2

Design a lossless optical device of length d that creates a phase shift $\Delta\phi$ between two beams of frequencies ω_1 and ω_2, respectively. Consider that the material of the device is made with particles that exhibit a single Lorentzian response with resonance frequency ω_0 and plasma frequency ω_p. Assume that ω_0 is known, find ω_p. For simplicity, you may consider that $|\chi(\omega)| \ll 1$.

Exercise 6.3

Analyze the response of an Omega-shaped particle. Consider a plane wave propagating in the $+z$-direction and being x-polarized. The orientation of the Omega particle is the same as seen in the lecture. Express the induced electric and magnetic dipole moments in terms of both the exciting electric and magnetic fields. Do the same for an x-polarized plane wave propagating the in the $-z$-direction. How does the response of the particle change based on the direction of the illumination? What do you conclude? Note: you must consider that the particle is *reciprocal* meaning that the following relation must be satisfied: $\overline{\overline{\alpha}}_\mathrm{em} = \overline{\overline{\alpha}}_\mathrm{me}^T$

Referring to the classification of bi-anisotropic materials, what term would be used to refer to this particle? To help you with that classification, assume that its polarizabilities are all zeros ($\alpha \approx 0$) except those associated to the considered wave polarization and propagation direction.

Exercise 6.4

Show that a bi-isotropic medium is circularly birefringent. To help you with this, consider the case of a circularly-polarized plane wave propagating in the $+z$-direction in this medium such that

$$\mathbf{E} = E_0 \mathbf{p}_\pm e^{-jkz} \quad \text{where} \quad \mathbf{p}_\pm = (\hat{\mathbf{x}} \pm j\hat{\mathbf{y}})/\sqrt{2},$$

with $\mathbf{p}_\pm$ being the unit polarization vector in circular basis. With this field definition, we have the following identity

$$\nabla \times \mathbf{E} = -j\mathbf{k} \times \mathbf{E} = -jk(\hat{\mathbf{z}} \times \mathbf{E}) = -jk(\mp j\mathbf{E}) = \mp k\mathbf{E}.$$

What is the corresponding identity for $\nabla \times \mathbf{H}$? For a reciprocal bi-isotropic medium ($\zeta = -\xi$), we may conveniently express the bi-isotropic constitutive relations as

$$\mathbf{D} = \epsilon\mathbf{E} - j\kappa\sqrt{\epsilon\mu}\mathbf{H} \quad \text{and} \quad \mathbf{B} = \mu\mathbf{H} + j\kappa\sqrt{\epsilon\mu}\mathbf{E},$$

where κ is a chirality parameter ($\zeta = j\kappa\sqrt{\epsilon\mu}$). Combine these constitutive relations with the identities above to eliminate $\mathbf{H}$ and obtain the wave equation from which you will be able to deduce the refractive index of the medium for LCP and RCP waves, $n_\pm$. Consider now that an x-polarized plane wave propagates in this medium. What is the value of κ such that the wave becomes y-polarized after a propagation distance d?