Lecture 5 – Exercises

Exercise 5.1

We want to transfer power between two identical magnetic loop dipole antennas. The diameter of the loops is much smaller than the wavelength, i.e., $D \ll \lambda$. We consider an operating frequency of $f = 1$ MHz. We consider that the first loop is used as an emitter and is placed at the origin, i.e., $(0, 0, 0)$, and that it creates a dipole moment in the $\hat{\mathbf{z}}$-direction. The second loop is used as a receiver and is placed somewhere in the xz-plane at position $(d_x, 0, d_z)$.

1. What is the wavelength in the air and what is the size of the near-field region?
2. In the near-field region, what should be the orientation and the position of the receiver antenna with respect to the emitter to maximize power transfer?
3. Same question in the case where the receiver is in the far-field region.
4. In the far-field, what is the received power?

Exercise 5.2

Design a magnetic loop antenna with an efficiency of $\eta_{\text{eff}} = 50\%$. Consider an operating frequency of $f = 6$ MHz, a loop diameter of $D = 1$ cm, a copper wire radius of $r = 0.25$ mm. To increase efficiency, the loop is wound N times around a ferrite core with $\mu_{\text{r}} = 1000$. The loop resistance may be approximated as

$$R_{\text{loss}} \approx \frac{\rho N D}{2r}\sqrt{\frac{\omega\mu_0}{2\rho}},$$

where $\rho \approx 1.7 \cdot 10^{-8}\ \Omega \cdot \text{m}$ is the resistivity of copper.

Find the number of turns N such that $\eta_{\text{eff}} = 50\%$.

Exercise 5.3

Consider an antenna array with a period larger than the wavelength such that several main lobes exist within the visible region. We know that the value of Ψ is zero for the main central lobe. What is the value of Ψ for the other main lobes?

Design an antenna array that exhibits three main lobes at $\theta = [45°, 90°, 135°]$. What should be the element spacing, d, of this array?

Exercise 5.4

The electric field of an electric quadrupole, $\overline{\overline{\text{Q}}}$, in the far region is given by

$$\mathbf{E} \propto \hat{\mathbf{r}} \times \left[\hat{\mathbf{r}} \times \left(\overline{\overline{\text{Q}}} \cdot \hat{\mathbf{r}}\right)\right].$$

If we only consider the Q_{xz} component of the electric quadrupole, we have in spherical coordinates that

$$\overline{\overline{\text{Q}}} \cdot \hat{\mathbf{r}} = \begin{bmatrix} 2\sin\theta\cos\theta\cos\phi \\ \cos\phi\left(\cos^2\theta - \sin^2\theta\right) \\ -\sin\phi\cos\theta \end{bmatrix}.$$

Find the solid angle, directivity and effective aperture of this quadrupole. How does it compare to that of an electric dipole?