Lecture 4 – Exercises

Exercise 4.1

Consider an electric dipole with $\mathbf{p} = p_0\hat{\mathbf{x}}$ and a magnetic dipole with $\mathbf{m} = m_0\hat{\mathbf{y}}$. Both dipoles are placed at the origin, i.e., $(0,0,0)$, and radiate at the same frequency ω. By superposition, the total electric field in the far-field is the sum of the far-field radiated by each dipoles. We next consider that $m_0 = cp_0e^{-j\alpha}$, where α is an arbitrary phase shift and c is the speed of light.

1. Assuming that $\alpha = 0$, make a sketch in the xz-plane of the electric far-field radiated by each dipoles separately. Use small arrows to indicate the direction of the field polarization.

2. Compute the **total** electric field in the $\pm\hat{\mathbf{z}}$ directions. How does that field vary when α is equal to 0 and π ? Illustrate the two cases with sketches of the total electric field.

3. Compute the time-average Poynting vector, $\langle\mathbf{S}\rangle = \mathrm{Re}\left\{\mathbf{E}\times\mathbf{H}^*\right\}/2$, of the far-field of the electric dipole only.

4. Compute the power scattered by the electric dipole. To do so, integrate the Poynting vector over a sphere. Hint: from the solution of point 3, you should have that

$$\langle\mathbf{S}\rangle\cdot\hat{\mathbf{r}} = |p_0|^2\frac{\mu^2\omega^4}{32\pi^2r^2\eta}\left[\cos(\theta)^2\cos(\phi)^2 + \sin(\phi)^2\right].$$

Exercise 4.2

Use the python script provided in the course that implements the angular spectrum representation technique to simulate the Fourier filtering effect of a 4f optical system. To do so, use the following package and code

```
from PIL import Image
im = Image.open("star.jpg").convert('L').resize((N,N))
```

to load the provided `star.png` file. Here `N` is a variable corresponding to the number of pixels as already used in the provided script. Consider the following parameters: the wavelength is 500 nm, the two lenses forming the system have a focal length of 25 cm, the dimensions of the image is 10 mm x 10 mm, use at least 200 pixels of padding on each side of the image. To simulate a 4f system: propagate the field of the image to the first lens, apply the transfer function of the lens as seen in the lecture, propagate to the Fourier plane, apply a filter (for instance a low pass filter with a radius of 0.4 mm), propagate to the second lens, apply the lens transfer function, and propagate one last time to get the final reconstructed image. Plot the final image and compare with the initial one.

Remember to use the right convention for the sign of the imaginary part of the lens transfer function.