Lecture 2 – Exercises

Exercise 2.1

An incident wave propagating in air is impinging on an optical system lying in the xy-plane. The wave propagates with angles $\theta = 30°$ and $\phi = 60°$ (spherical coordinate system). For simplicity, assume that the wavenumber is $k = 1$.

1. Find the wavevector $\mathbf{k}$.
2. Find the electric field vector $\mathbf{E}$ for TE, TM, LCP and RCP polarizations. How would you find the corresponding magnetic field $\mathbf{H}$?

Exercise 2.2

Consider an optical system lying in the xy-plane. An incident wave propagating in air impinges on this device with the following parameters:

$$\mathbf{E} = \begin{bmatrix} 2 \\ 4 \\ ? \end{bmatrix} \quad \text{and} \quad \mathbf{k} = \frac{\pi\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \\ \sqrt{6} \end{bmatrix}.$$

Find the following quantities:

1. the wavenumber k and the wavelength λ.
2. the incident angles θ and ϕ.
3. the electric field component E_z and the magnetic field $\mathbf{H}$.
4. the electric field intensity for the TE, TM, LCP, RCP polarizations.
5. the time-average Poynting vector $\langle \mathbf{S} \rangle$.

Exercise 2.3

A TM polarized plane wave is incident from medium 1 into medium 2. Both media are lossless. The angle of propagation of the wave, θ, is such that it is larger than the critical angle, i.e., $\theta > \theta_{\mathrm{c}}$, leading to an evanescent wave in medium 2. Compute the transmitted fields $\mathbf{E}$ and $\mathbf{H}$ in medium 2. Consider that the complex amplitude of the electric field is E_{t}, also replace the wavevector component normal to the interface by an attenuation constant α, e.g., $k_n = \pm j\alpha$.

What is the phase shift between the electric and magnetic field components and what does it imply? Compute the complex Poynting vector defined as $\mathbf{S} = (\mathbf{E} \times \mathbf{H}^*)/2$. What do you conclude about the nature of the power flow in the tangential and normal directions?

Exercise 2.4

A half-wave plate is rotated by an arbitrary angle θ. Find the Jones matrix $\overline{\overline{\mathrm{T}}}_{\mathrm{rot}}$ of this rotated wave plate. Now, an incident LCP polarized wave transmits through this wave plate, find the Jones vector of the transmitted wave. Describe how the transmitted differs from the incident one.

Exercise 2.5

Consider an ideal metal for which $\epsilon_{\mathrm{r}} = -10$ at a wavelength of 600 nm. The medium above the metal is air. Find the propagation constant β of the corresponding surface plasmon. A material with refractive index $n = 2$ is brought very close to the metal layer leaving only a few micron-thick air gap. A plane wave is propagating inside this material and reflects (total internal reflection) at the material-air interface creating an evanescent wave in the air gap. What should be the incidence angle of this wave such that the propagation constant of the evanescent wave matches β? What should be the polarization of the wave so that it couples to the plasmon mode?