Lecture 1 – Exercises

Exercise 1.1

From the basic prescriptions of electrostatics, explain why it is possible to obtain an electric field $\mathbf{E}$ (which is a vector with three components) simply in terms of a *scalar* potential V. In other words, obtaining three quantities (field components) from a single one (scalar potential) must necessarily imply redundancy. Where does this redundancy come from?

Exercise 1.2

We have seen that an infinite current line, supporting the current I, produces a magnetic field

$$\mathbf{H}(r) = \frac{I}{2\pi r}\hat{\boldsymbol{\varphi}}.$$

Make a vector plot of $\mathbf{H}$ and compute the following quantities: $\nabla \cdot \mathbf{H}$, $\nabla \times \mathbf{H}$ and $\oint \mathbf{H} \cdot d\mathbf{l}$. Is this a conservative field? Is it irrotational/divergenceless?

There should be a paradox in your results. How to resolve it?

Instead of a current line, consider now the case of a wire of radius a carrying a uniform current I such that $\mathbf{J} = I/\left(\pi a^2\right)\hat{\mathbf{z}}$. Find the magnetic fields $\mathbf{H}_{\text{in}}(r)$ and $\mathbf{H}_{\text{out}}(r)$ that are inside and outside the wire, respectively. For both fields, compute $\nabla \cdot \mathbf{H}$, $\nabla \times \mathbf{H}$ and $\oint \mathbf{H} \cdot d\mathbf{l}$.

What do you conclude ?

Exercise 1.3

An uncharged spherical conductor of radius a, centered at the origin, has a cavity of arbitrary shape carved inside of it. Within the cavity, there is a charge q. What is the field outside the sphere?

If the charge q is placed outside the sphere, what is the field inside the cavity?

Exercise 1.4

In magnetostatics, the relation between current density and magnetic field is $\nabla \times \mathbf{H} = \mathbf{J}$. Clearly, the presence of a current leads to a rotating magnetic field. But is it possible to form a magnetic field (e.g., by arranging magnets in space) that has a non-zero curl leading to the formation of a current in a nearby conductive material?

Exercise 1.5

A parallel-plate capacitor with an air dielectric is charged up to 10 V. The voltage source is then disconnected. A dielectric slab having a permittivity of $2\epsilon_0$ is placed in between the plates. What is the new capacitor voltage? What about the stored energy?

Exercise 1.6

A rectangular wire loop having $0 \leq x \leq a$, $0 \leq y \leq b$ is in the $z = 0$ plane. A small gap with terminals AB is at $(x, y, z) = (a/2, 0, 0)$. A magnetic flux density $\mathbf{B} = \hat{\mathbf{z}} B_0 \cos(\omega t)$ is applied to the loop. Find the terminal voltage V_{AB}. If a resistor R is placed at the gap, find the current and its direction.