Lecture 13 – Exercises

Exercise 13.1

Consider the case of the invisible slab discussed in lecture 3. The slab has a refractive index of $n = 2$ and is sandwiched on both sides by air with $n = 1$. The thickness of the slab is $d = 500$ nm. The slab is illuminated at normal incidence by an x-polarized plane wave. The scattering parameters of the slab are, from lecture 3, defined as

$$r = \frac{r_{12}\left(1 - e^{-j2\Psi}\right)}{1 - r_{12}^2 e^{-j2\Psi}} \quad \text{and} \quad t = \frac{\left(1 - r_{12}^2\right) e^{-j\Psi}}{1 - r_{12}^2 e^{-j2\Psi}},$$

where r_{12} is the reflection coefficient at the interface between air and glass and $\Psi = k_0 n_2 d$ with n_2 being the refractive index of glass. Use these scattering parameters to find the susceptibilities of a metasurface that would exhibit the same scattering response. What are the values of the metasurface electric and magnetic susceptibilities at the wavelength where the slab is invisible and how could such values be obtained if we assume that these susceptibilities are expressed as sums of several Lorentzian functions ?

Exercise 13.2

Find the susceptibilities of a chiral metasurface surrounded by air that transforms an x-polarized normally incident plane wave into a normally transmitted y-polarized plane wave. The metasurface must be reflectionless, reciprocal, passive and lossless.

Exercise 13.3

Design a metasurface wave combiner. This metasurface lies in the xy-plane at $z = 0$ and takes as an input two obliquely TM-polarized plane waves propagating at $\theta = \pm 45°$ within the xz-plane. These two plane waves have the same field amplitude $|\mathbf{E}| = 1$. The metasurface outputs an x-polarized plane wave propagating in the z-direction. Find the amplitude, A, of the transmitted wave by trying to achieve power conservation (equality of input and output Poynting vectors, you may need to spatially average them). Assume that the metasurface only possesses the susceptibilities χ_{ee}^{xx} and χ_{mm}^{yy}. Find these susceptibilities. Does this corresponds to a passive/lossless metasurface.