Lecture 10 – Exercises

Exercise 10.1

Consider an asymmetric isolated particle being excited by an x-polarized plane wave. The resulting dipole moments excited in the particle are related to the exciting fields as

$$p_x = \alpha_{\mathrm{ee}}^{xx} E_x + \alpha_{\mathrm{em}}^{xy} H_y$$
$$m_y = \alpha_{\mathrm{em}}^{yx} E_x + \alpha_{\mathrm{mm}}^{yy} H_y.$$

What should be the spatial symmetries of this structure to exhibit such polarizabilities ? How would you find the four polarizabilities of this particle? Hint: consider the concept discussed on slide 18 of the lecture.

Exercise 10.2

Is it possible to create a structure with the following shape of the material parameter tensors:

$$\overline{\overline{\epsilon}}, \overline{\overline{\mu}} \propto \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \quad \text{and} \quad \overline{\overline{\zeta}}, \overline{\overline{\xi}} \propto \begin{bmatrix} 0 & D & 0 \\ E & 0 & F \\ 0 & G & 0 \end{bmatrix}.$$

In principle, you should find that it is not possible to exactly achieve this specification. Now, let's assume that we really we want to achieve the shape of $\overline{\overline{\zeta}}$ and $\overline{\overline{\xi}}$ given above and are more relaxed about the shape of $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$. What are the spatial symmetries that must be broken and not broken to do achieve that ? In this case, how is the resulting shape of $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$? Propose a geometry that would correspond to such a structure.

Exercise 10.3

Propose a metasurface structure that exhibits different angular transmission coefficients when illuminated in the xz-plane but identical angular transmission coefficients in the yz-plane.

Exercise 10.4

Consider an isolated flat spiral particle (no substrate) with spatial symmetries σ_z and C_{4z}. Assume that the particle is illuminated at normal incidence by an x-polarized wave. What type of response do you except this particle to exhibit ?

Hint: investigate its dipolar response in terms of both fields and field gradients and compare it to that of a flat square (with symmetries σ_x, σ_y, σ_z and C_{4z}).