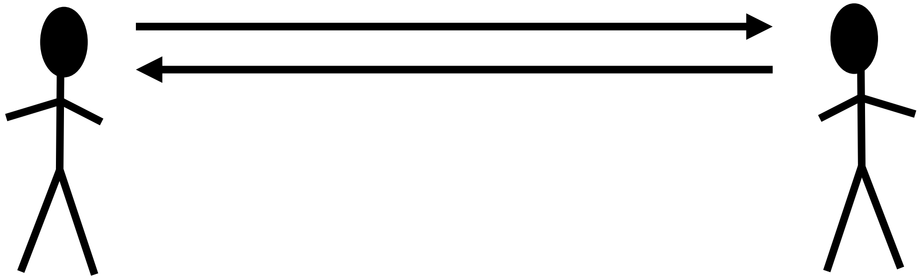
Lecture 8

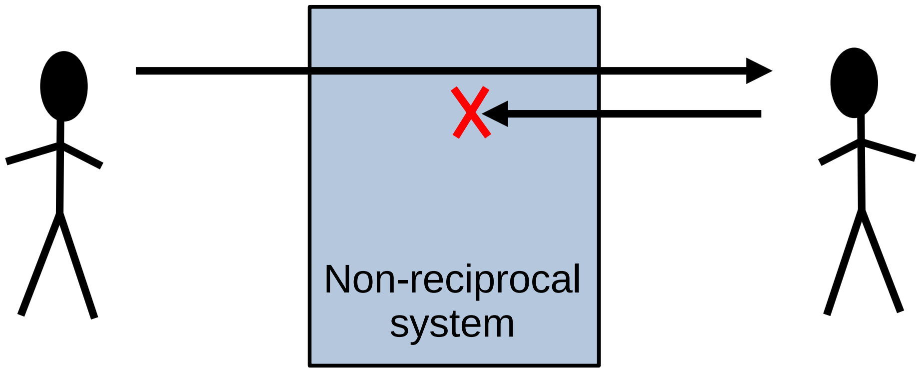
Electromagnetic Reciprocity

Naive Understanding of (Non-)reciprocity

Reciprocity: "I see you, you see me"



Non-reciprocity: "I see you, you don't see me"



Example of a Science Paper that got it Wrong

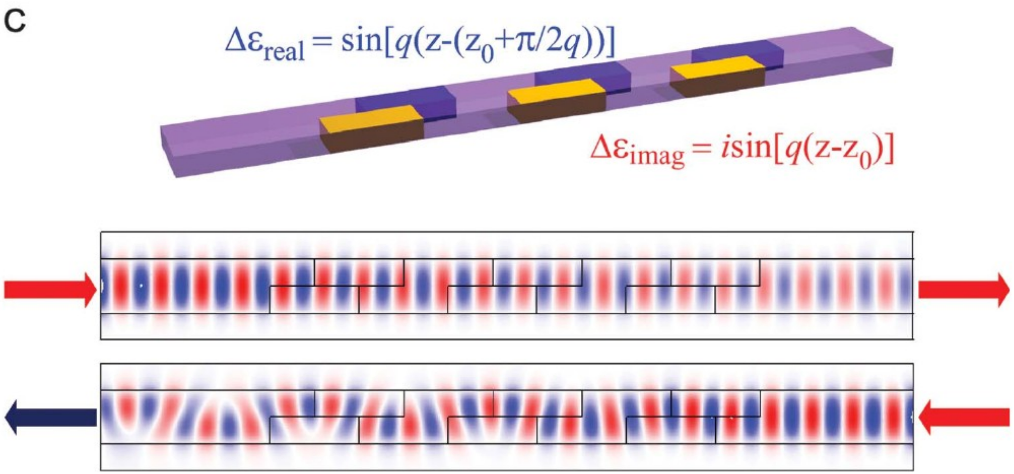
Nonreciprocal Light Propagation in a Silicon Photonic Circuit

Liang Feng,[1,2,4]*† Maurice Ayache,[3]* Jingqing Huang,[1,4]* Ye-Long Xu,[2] Ming-Hui Lu,[2] Yan-Feng Chen,[2]† Yeshaiahu Fainman,[3] Axel Scherer[1,4]†

Optical communications and computing require on-chip nonreciprocal light propagation to isolate and stabilize different chip-scale optical components. We have designed and fabricated a metallic-silicon waveguide system in which the optical potential is modulated along the length of the waveguide such that nonreciprocal light propagation is obtained on a silicon photonic chip. Nonreciprocal light transport and one-way photonic mode conversion are demonstrated at the wavelength of 1.55 micrometers in both simulations and experiments. Our system is compatible with conventional complementary metal-oxide-semiconductor processing, providing a way to chip-scale optical isolators for optical communications and computing.

https://doi.org/10.1126/science.1206038

Asymmetry = Non-reciprocity ?



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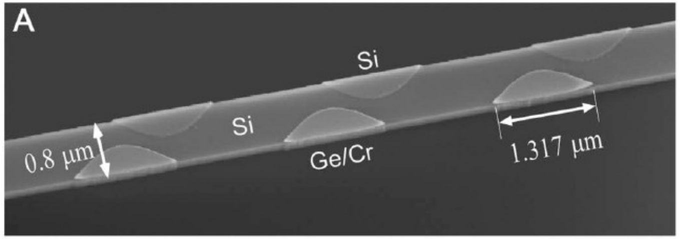
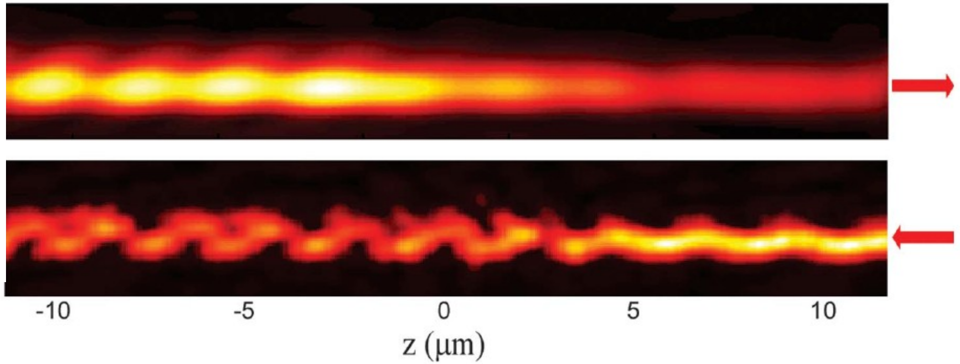
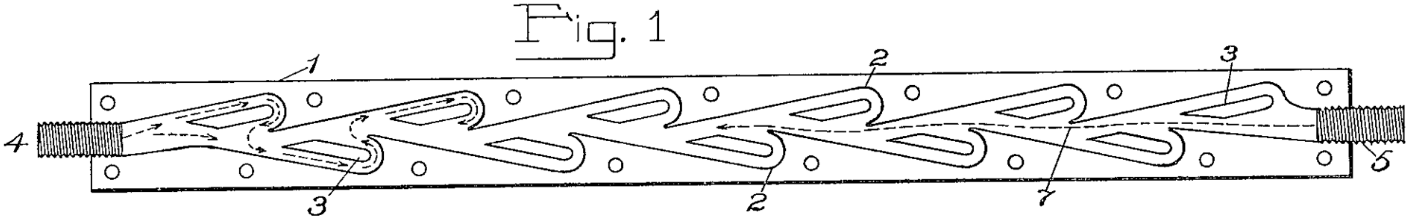
science.1206038



Germanium/
Chromium by-
layers

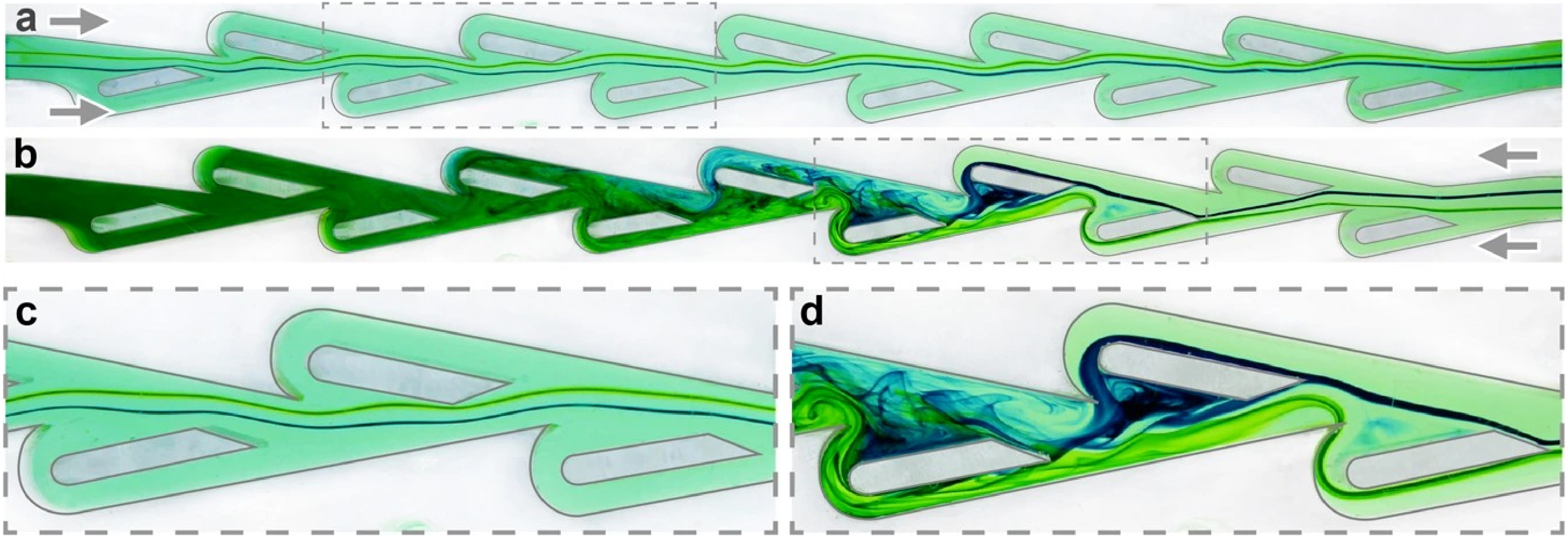


Reminiscent of the concept of the Tesla valve...



U.S. patent 1,329,559 (1920)

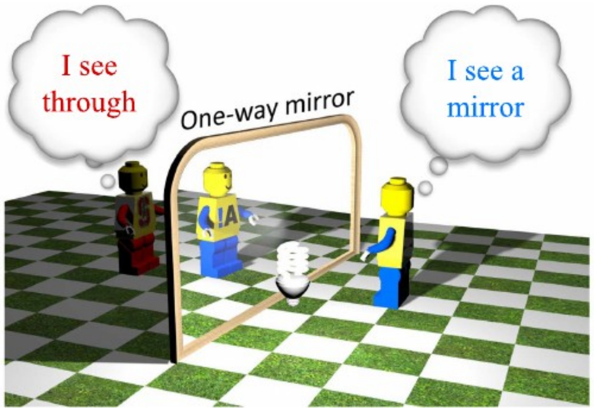
Flow Visualization of Tesla Valve

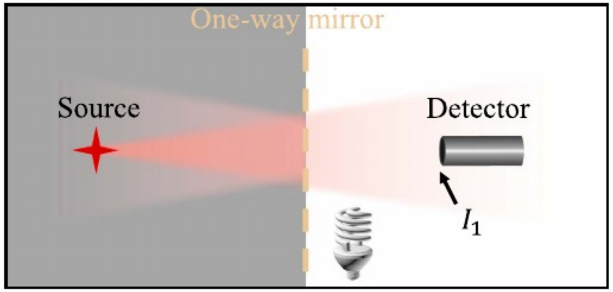


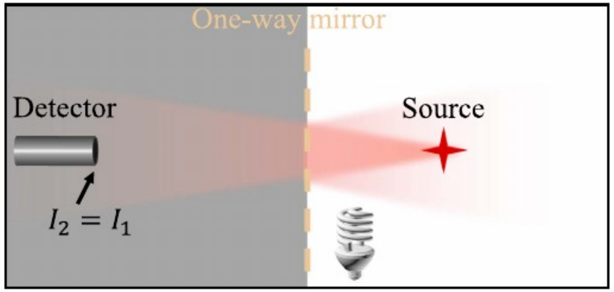
Streakline flow visualization at Re=200 using dye injected upstream: (a) Forward direction. Two adjacent filaments remain in the central corridor of the conduit with only small lateral deflections. (b) Reverse direction. The filaments ricochet off the periodic structures, deflecting increasingly sharply before being rerouted around the 'islands' and mixing. (c) and (d) are zoomed-in images

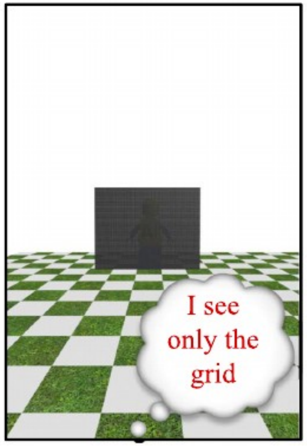
https://commons.wikimedia.org/wiki/File:Nguyen_Tesla_flow_visualization.webp#/media/File:Nguyen_Tesla_flow_visualization.webp

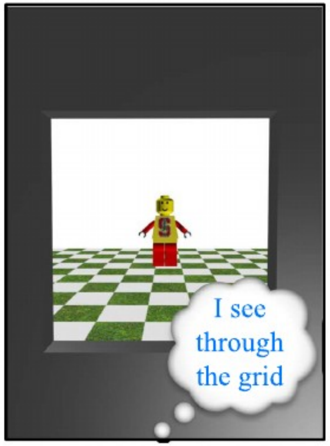
Example of Perfectly Reciprocal Situations

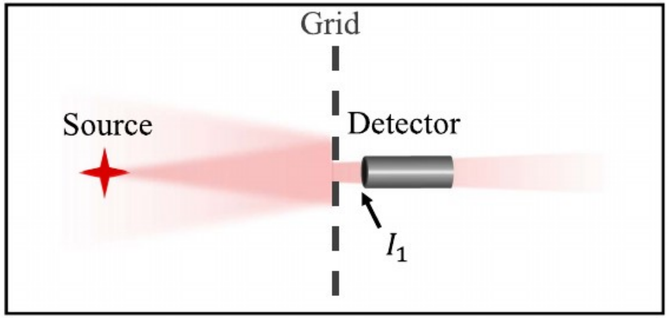


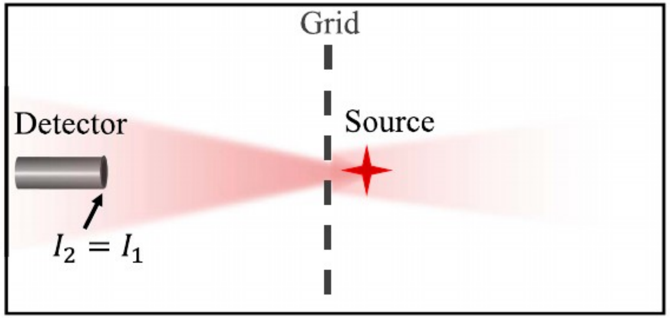












https://doi.org/10.1109/JPROC.2020.3012381

Nature Photonics Paper Needed to Explain Non-reciprocity

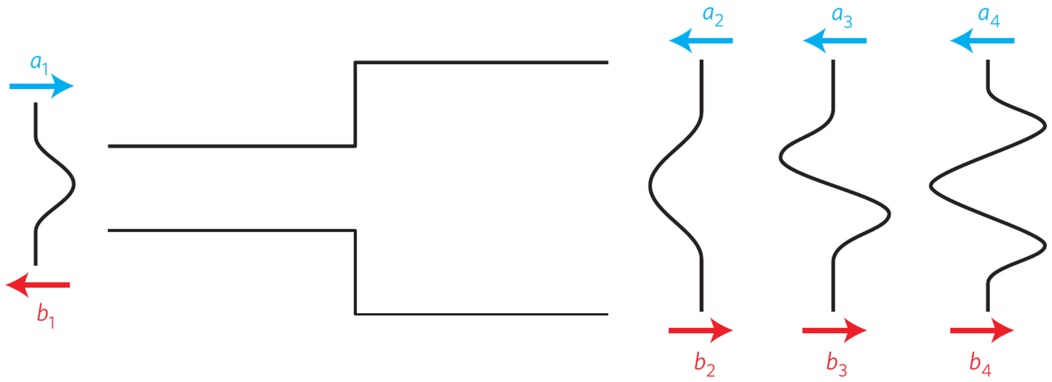
What is — and what is not — an optical isolator

The quest for on-chip optical isolators has recently spawned many new isolator structures. However, there has been some confusion about the requirement of nonreciprocity. Here, we review the essential characteristics of an isolator.

Dirk Jalas, Alexander Petrov, Manfred Eich, Wolfgang Freude, Shanhui Fan, Zongfu Yu, Roel Baets, Miloš Popović, Andrea Melloni, John D. Joannopoulos, Mathias Vanwolleghem, Christopher R. Doerr and Hagen Renner

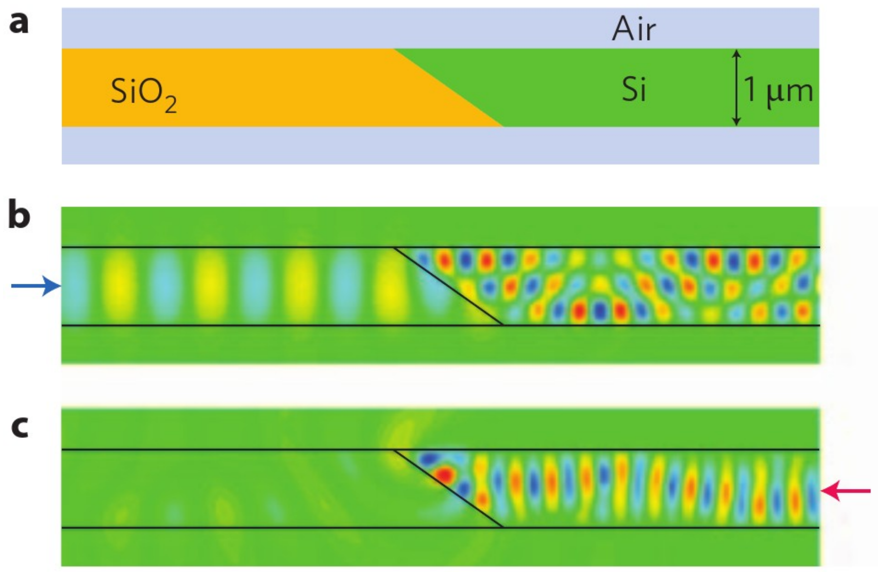
https://doi.org/10.1038/nphoton.2013.185

Nature Photonics Paper Needed to Explain Non-reciprocity



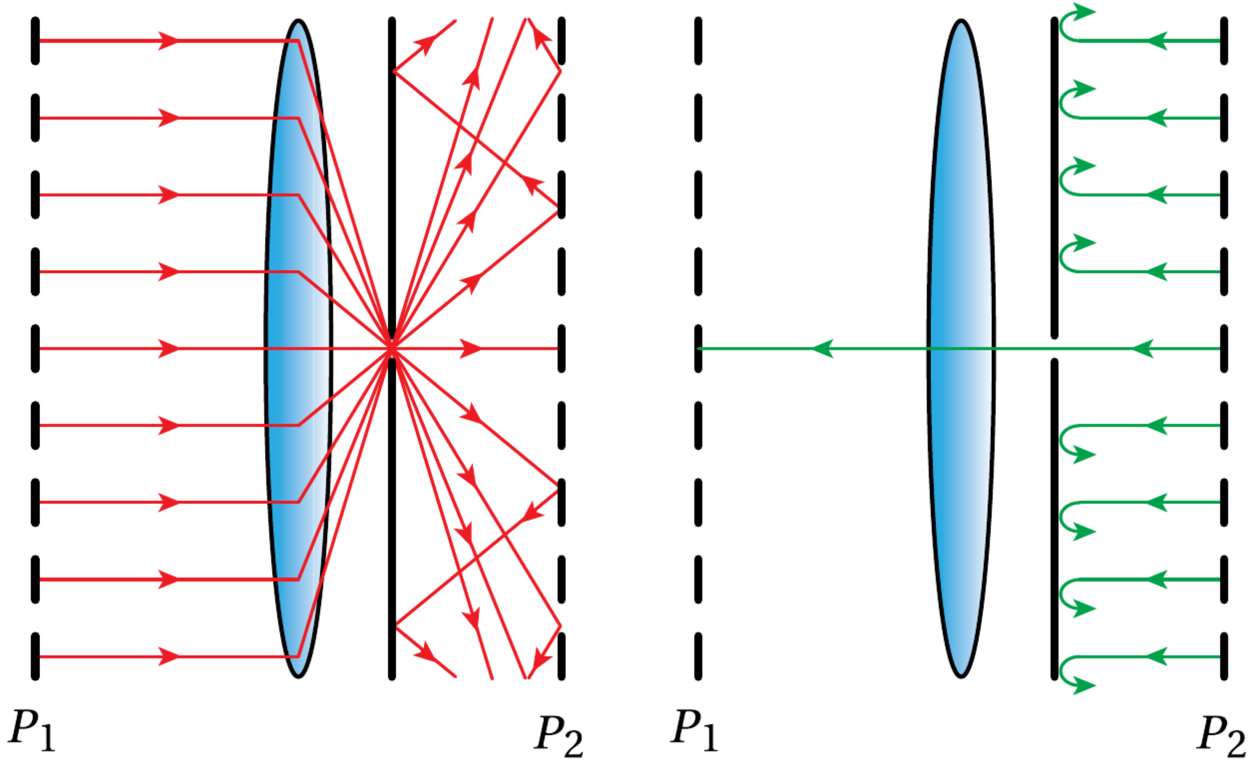
$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ \vdots \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & \cdots \\ S_{21} & S_{22} & S_{23} & S_{24} & \cdots \\ S_{31} & S_{32} & S_{33} & S_{34} & \cdots \\ S_{41} & S_{42} & S_{43} & S_{44} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \end{pmatrix}$$

$$S_{\mu\nu} = S_{\nu\mu}$$



https://doi.org/10.1038/nphoton.2013.185

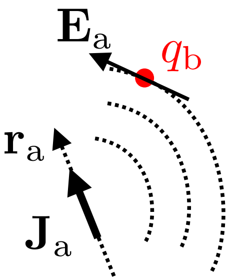
Other Example of Asymmetric Reciprocal System



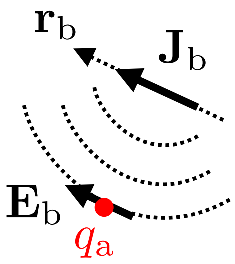
http://dx.doi.org/10.1103/PhysRevApplied.10.047001

Lorentz Reciprocity Theorem

Basic Concept of Reciprocity



$$\mathbf{F}_\mathrm{b} = q_\mathrm{b}\mathbf{E}_\mathrm{a}$$



$$\mathbf{F}_\mathrm{a} = q_\mathrm{a}\mathbf{E}_\mathrm{b}$$

$\mathbf{F}_\mathrm{a} \nparallel \mathbf{J}_\mathrm{a}$ → Reciprocity is not defined as $\mathbf{F}_\mathrm{a} = \mathbf{F}_\mathrm{b}$ but rather as $\mathbf{F}_\mathrm{a} \cdot \mathbf{r}_\mathrm{a} = \mathbf{F}_\mathrm{b} \cdot \mathbf{r}_\mathrm{b}$

$$\mathbf{F}_\mathrm{a} \cdot \frac{\partial}{\partial t}\mathbf{r}_\mathrm{a} = \mathbf{F}_\mathrm{b} \cdot \frac{\partial}{\partial t}\mathbf{r}_\mathrm{b}$$

$$\mathbf{E}_\mathrm{b} \cdot \frac{\partial}{\partial t}q_\mathrm{a}\mathbf{r}_\mathrm{a} = \mathbf{E}_\mathrm{a} \cdot \frac{\partial}{\partial t}q_\mathrm{b}\mathbf{r}_\mathrm{b}$$

$$\mathbf{E}_\mathrm{b} \cdot \mathbf{J}_\mathrm{a} = \mathbf{E}_\mathrm{a} \cdot \mathbf{J}_\mathrm{b}$$

Eletromagnetic Reaction and Lorentz Theorem

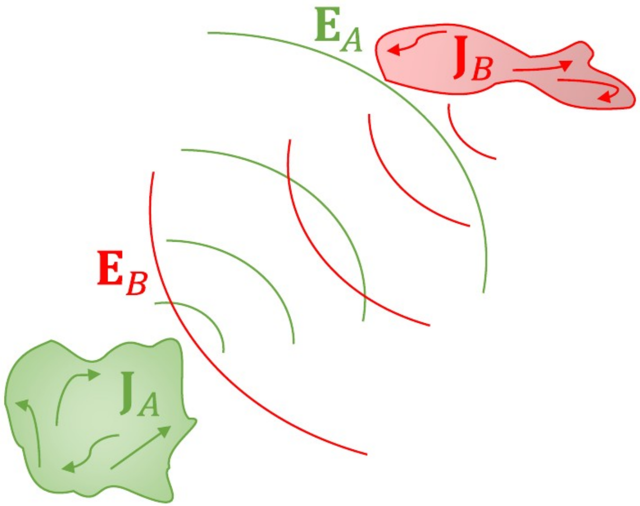
$$\mathbf{E}_\mathrm{b} \cdot \mathbf{J}_\mathrm{a} = \mathbf{E}_\mathrm{a} \cdot \mathbf{J}_\mathrm{b}$$

Electromagnetic reaction

$$\langle a, b \rangle = \int \mathbf{E}_\mathrm{a} \cdot \mathbf{J}_\mathrm{b} dV$$

Lorentz reciprocity theorem

$$\langle a, b \rangle = \langle b, a \rangle \longrightarrow \int \mathbf{E}_\mathrm{a} \cdot \mathbf{J}_\mathrm{b} dV = \int \mathbf{E}_\mathrm{b} \cdot \mathbf{J}_\mathrm{a} dV$$



https://doi.org/10.1109/
JPROC.2020.3012381

Reciprocity in terms of Green function

E-field from Green function

$$\mathbf{E}(\mathbf{r}) = -j\omega\mu \int \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dV'$$

→

For a point current source

$$\mathbf{E}(\mathbf{r}) = -j\omega\mu \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')$$

$$\mathbf{E}_{\mathrm{a}} \cdot \mathbf{J}_{\mathrm{b}} = \mathbf{E}_{\mathrm{b}} \cdot \mathbf{J}_{\mathrm{a}}$$

$$\mathbf{E}_{\mathrm{a}}(\mathbf{r}_{\mathrm{b}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}}) = \mathbf{E}_{\mathrm{b}}(\mathbf{r}_{\mathrm{a}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{a}})$$

$$\overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{b}}, \mathbf{r}_{\mathrm{a}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{a}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}}) = \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{a}}, \mathbf{r}_{\mathrm{b}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{a}})$$

$$\mathbf{J}(\mathbf{r}_{\mathrm{a}}) \cdot \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{b}}, \mathbf{r}_{\mathrm{a}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}}) = \mathbf{J}(\mathbf{r}_{\mathrm{a}}) \cdot \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{a}}, \mathbf{r}_{\mathrm{b}}) \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}})$$

$$\mathbf{J}(\mathbf{r}_{\mathrm{a}}) \cdot \left[\overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{b}}, \mathbf{r}_{\mathrm{a}}) - \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{a}}, \mathbf{r}_{\mathrm{b}})\right] \cdot \mathbf{J}(\mathbf{r}_{\mathrm{b}}) = 0$$

← Ge is symmetric

$$\overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}, \mathbf{r}') = \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}, \mathbf{r}')^{\mathrm{T}}$$

$$\overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{b}}, \mathbf{r}_{\mathrm{a}}) = \overline{\overline{\mathrm{G}}}_{\mathrm{e}}(\mathbf{r}_{\mathrm{a}}, \mathbf{r}_{\mathrm{b}})$$

source and observer positions can be exchanged.

Let's Develop the Equation

$$\langle a, b\rangle = \langle b, a\rangle \longrightarrow \int \mathbf{E}_{\rm a} \cdot \mathbf{J}_{\rm b} dV = \int \mathbf{E}_{\rm b} \cdot \mathbf{J}_{\rm a} dV$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$$

$$\mathbf{J} = \nabla \times \mathbf{H} - j\omega\mathbf{D}$$

$$\langle a, b\rangle - \langle b, a\rangle = \int \mathbf{E}_{\rm a} \cdot \mathbf{J}_{\rm b} dV - \int \mathbf{E}_{\rm b} \cdot \mathbf{J}_{\rm a} dV$$

$$= \int \mathbf{E}_{\rm a} \cdot \left(\nabla \times \mathbf{H}_{\rm b} - j\omega\mathbf{D}_{\rm b}\right) dV - \int \mathbf{E}_{\rm b} \cdot \left(\nabla \times \mathbf{H}_{\rm a} - j\omega\mathbf{D}_{\rm a}\right) dV$$

$$= \int \left[\mathbf{E}_{\rm a} \cdot (\nabla \times \mathbf{H}_{\rm b}) - j\omega\mathbf{E}_{\rm a} \cdot \mathbf{D}_{\rm b} - \mathbf{E}_{\rm b} \cdot (\nabla \times \mathbf{H}_{\rm a}) + j\omega\mathbf{E}_{\rm b} \cdot \mathbf{D}_{\rm a}\right] dV$$

$$= \int \left[\nabla \cdot (\mathbf{H}_{\rm b} \times \mathbf{E}_{\rm a}) + \mathbf{H}_{\rm b} \cdot (\nabla \times \mathbf{E}_{\rm a}) - j\omega\mathbf{E}_{\rm a} \cdot \mathbf{D}_{\rm b} - \nabla \cdot (\mathbf{H}_{\rm a} \times \mathbf{E}_{\rm b}) - \mathbf{H}_{\rm a} \cdot (\nabla \times \mathbf{E}_{\rm b}) + j\omega\mathbf{E}_{\rm b} \cdot \mathbf{D}_{\rm a}\right] dV$$

$$= \int \left[\nabla \cdot (\mathbf{H}_{\rm b} \times \mathbf{E}_{\rm a}) - j\omega\mathbf{H}_{\rm b} \cdot \mathbf{B}_{\rm a} - j\omega\mathbf{E}_{\rm a} \cdot \mathbf{D}_{\rm b} - \nabla \cdot (\mathbf{H}_{\rm a} \times \mathbf{E}_{\rm b}) + j\omega\mathbf{H}_{\rm a} \cdot \mathbf{B}_{\rm b} + j\omega\mathbf{E}_{\rm b} \cdot \mathbf{D}_{\rm a}\right] dV$$

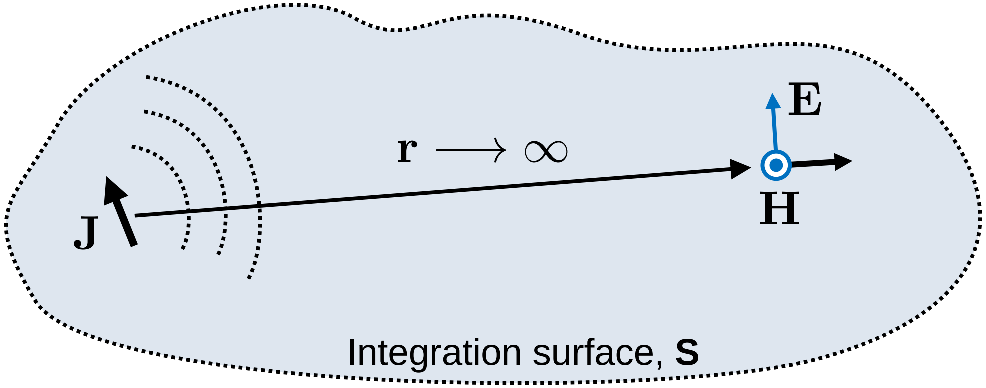
$$= \int \left[\nabla \cdot (\mathbf{H}_{\rm b} \times \mathbf{E}_{\rm a} - \mathbf{H}_{\rm a} \times \mathbf{E}_{\rm b}) + j\omega\left(\mathbf{E}_{\rm b} \cdot \mathbf{D}_{\rm a} - \mathbf{E}_{\rm a} \cdot \mathbf{D}_{\rm b} + \mathbf{H}_{\rm a} \cdot \mathbf{B}_{\rm b} - \mathbf{H}_{\rm b} \cdot \mathbf{B}_{\rm a}\right)\right] dV$$

Divergence theorem

$$= \oint \left(\mathbf{E}_{\rm b} \times \mathbf{H}_{\rm a} - \mathbf{E}_{\rm a} \times \mathbf{H}_{\rm b}\right) \cdot d\mathbf{S} + j\omega \int \left(\mathbf{E}_{\rm b} \cdot \mathbf{D}_{\rm a} - \mathbf{E}_{\rm a} \cdot \mathbf{D}_{\rm b} + \mathbf{H}_{\rm a} \cdot \mathbf{B}_{\rm b} - \mathbf{H}_{\rm b} \cdot \mathbf{B}_{\rm a}\right) dV = 0.$$

Getting Rid of the Surface Integral

$$\langle a,b\rangle - \langle b,a\rangle = \oint (\mathbf{E}_\mathrm{b}\times\mathbf{H}_\mathrm{a} - \mathbf{E}_\mathrm{a}\times\mathbf{H}_\mathrm{b})\cdot d\mathbf{S} + j\omega\int(\mathbf{E}_\mathrm{b}\cdot\mathbf{D}_\mathrm{a} - \mathbf{E}_\mathrm{a}\cdot\mathbf{D}_\mathrm{b} + \mathbf{H}_\mathrm{a}\cdot\mathbf{B}_\mathrm{b} - \mathbf{H}_\mathrm{b}\cdot\mathbf{B}_\mathrm{a})\,dV = 0$$



In the far-field, we have

$$\mathbf{E}\cdot\hat{\mathbf{r}} = 0$$

$$\mathbf{H} = \hat{\mathbf{r}}\times\mathbf{E}/\eta$$

$$\mathbf{E}_\mathrm{b}\times\mathbf{H}_\mathrm{a} - \mathbf{E}_\mathrm{a}\times\mathbf{H}_\mathrm{b} = \mathbf{E}_\mathrm{b}\times\hat{\mathbf{r}}\times\mathbf{E}_\mathrm{a}/\eta - \mathbf{E}_\mathrm{a}\times\hat{\mathbf{r}}\times\mathbf{E}_\mathrm{b}/\eta = (\mathbf{E}_\mathrm{b}\cdot\mathbf{E}_\mathrm{a})\hat{\mathbf{r}}/\eta - (\mathbf{E}_\mathrm{a}\cdot\mathbf{E}_\mathrm{b})\hat{\mathbf{r}}/\eta = 0$$

Using the identity

$$\mathbf{A}\times\mathbf{B}\times\mathbf{C} = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

This leads to $\oint \cdots d\mathbf{S} \longrightarrow 0$

The Lorentz theorem becomes

$$\int(\mathbf{E}_\mathrm{b}\cdot\mathbf{D}_\mathrm{a} - \mathbf{E}_\mathrm{a}\cdot\mathbf{D}_\mathrm{b} + \mathbf{H}_\mathrm{a}\cdot\mathbf{B}_\mathrm{b} - \mathbf{H}_\mathrm{b}\cdot\mathbf{B}_\mathrm{a})\,dV = 0$$

Reciprocity Conditions on Dipolar Material Parameters

$$\int \underbrace{(\mathbf{E}_\mathrm{b} \cdot \mathbf{D}_\mathrm{a} - \mathbf{E}_\mathrm{a} \cdot \mathbf{D}_\mathrm{b}}+ \mathbf{H}_\mathrm{a} \cdot \mathbf{B}_\mathrm{b} - \mathbf{H}_\mathrm{b} \cdot \mathbf{B}_\mathrm{a})\, dV = 0$$

Doing it only for the permittivity

$$\mathbf{E}_\mathrm{b} \cdot \overline{\overline{\epsilon}} \cdot \mathbf{E}_\mathrm{a} - \mathbf{E}_\mathrm{a} \cdot \overline{\overline{\epsilon}} \cdot \mathbf{E}_\mathrm{b} = 0$$

$$\mathbf{E}_\mathrm{b} \cdot \overline{\overline{\epsilon}} \cdot \mathbf{E}_\mathrm{a} - \mathbf{E}_\mathrm{b} \cdot \overline{\overline{\epsilon}}^\mathrm{T} \cdot \mathbf{E}_\mathrm{a} = 0$$

$$\mathbf{E}_\mathrm{b} \cdot \left(\overline{\overline{\epsilon}} - \overline{\overline{\epsilon}}^\mathrm{T} \right) \cdot \mathbf{E}_\mathrm{a} = 0$$

Bianisotropic constitutive relations

$$\mathbf{D}_\mathrm{a/b} = \overline{\overline{\epsilon}} \cdot \mathbf{E}_\mathrm{a/b} + \overline{\overline{\xi}} \cdot \mathbf{H}_\mathrm{a/b}$$

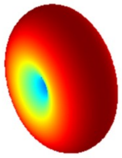
$$\mathbf{B}_\mathrm{a/b} = \overline{\overline{\zeta}} \cdot \mathbf{E}_\mathrm{a/b} + \overline{\overline{\mu}} \cdot \mathbf{H}_\mathrm{a/b}$$

Reciprocity conditions

$$\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^\mathrm{T} \qquad \overline{\overline{\mu}} = \overline{\overline{\mu}}^\mathrm{T} \qquad \overline{\overline{\xi}}^\mathrm{T} = -\overline{\overline{\zeta}}$$

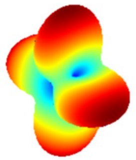
Reciprocity Conditions for Quadrupolar Responses

dipole



Quadrupolar constitutive relations

Quadrupole



$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} - \frac{1}{2}\nabla \cdot \overline{\overline{\mathrm{Q}}}$$ ← Electric quadrupole

$$\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M} - \frac{1}{2}\nabla \cdot \overline{\overline{\mathrm{S}}} \right)$$ ← Magnetic quadrupole

$$\begin{pmatrix} P_i \\ M_i \\ Q_{il} \\ S_{il} \end{pmatrix} \propto \begin{pmatrix} \chi_{\mathrm{ee},ij} & \chi_{\mathrm{em},ij} & \chi'_{\mathrm{ee},ijk} & \chi'_{\mathrm{em},ijk} \\ \chi_{\mathrm{me},ij} & \chi_{\mathrm{mm},ij} & \chi'_{\mathrm{me},ijk} & \chi'_{\mathrm{mm},ijk} \\ Q_{\mathrm{ee},ilj} & Q_{\mathrm{em},ilj} & Q'_{\mathrm{ee},iljk} & Q'_{\mathrm{em},iljk} \\ S_{\mathrm{me},ilj} & S_{\mathrm{mm},ilj} & S'_{\mathrm{me},iljk} & S'_{\mathrm{mm},iljk} \end{pmatrix} \cdot \begin{pmatrix} E_j \\ H_j \\ \nabla_k E_j \\ \nabla_k H_j \end{pmatrix}$$

Dipolar reciprocity conditions

$$\chi_{\mathrm{ee},ij} = \chi_{\mathrm{ee},ji} \qquad \overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^{\mathrm{T}}$$

$$\chi_{\mathrm{mm},ij} = \chi_{\mathrm{mm},ji} \quad \longleftrightarrow \quad \overline{\overline{\mu}} = \overline{\overline{\mu}}^{\mathrm{T}}$$

$$\chi_{\mathrm{em},ij} = -\chi_{\mathrm{me},ji} \qquad \overline{\overline{\xi}}^{\mathrm{T}} = -\overline{\overline{\zeta}}$$

Remember that

$$\overline{\overline{\epsilon}} = \overline{\overline{\mathrm{I}}} + \overline{\overline{\chi}}_{\mathrm{ee}}$$

Reciprocity Conditions for Quadrupolar Responses

$$\begin{pmatrix} P_i \\ M_i \\ Q_{il} \\ S_{il} \end{pmatrix} \propto \begin{pmatrix} \chi_{\mathrm{ee},ij} & \chi_{\mathrm{em},ij} & \chi'_{\mathrm{ee},ijk} & \chi'_{\mathrm{em},ijk} \\ \chi_{\mathrm{me},ij} & \chi_{\mathrm{mm},ij} & \chi'_{\mathrm{me},ijk} & \chi'_{\mathrm{mm},ijk} \\ Q_{\mathrm{ee},ilj} & Q_{\mathrm{em},ilj} & Q'_{\mathrm{ee},iljk} & Q'_{\mathrm{em},iljk} \\ S_{\mathrm{me},ilj} & S_{\mathrm{mm},ilj} & S'_{\mathrm{me},iljk} & S'_{\mathrm{mm},iljk} \end{pmatrix} \cdot \begin{pmatrix} E_j \\ H_j \\ \nabla_k E_j \\ \nabla_k H_j \end{pmatrix}$$

2nd rank tensors

$$\chi_{\mathrm{ee},ij} = \chi_{\mathrm{ee},ji}$$
$$\chi_{\mathrm{mm},ij} = \chi_{\mathrm{mm},ji}$$
$$\chi_{\mathrm{em},ij} = -\chi_{\mathrm{me},ji}$$

3rd rank tensors

$$\chi'_{\mathrm{ee},kji} = Q_{\mathrm{ee},ijk},$$
$$\chi'_{\mathrm{mm},kij} = S_{\mathrm{mm},ijk}$$
$$\chi'_{\mathrm{em},kij} = -S_{\mathrm{me},ijk}$$
$$\chi'_{\mathrm{me},kji} = -Q_{\mathrm{em},ijk}$$

4th rank tensors

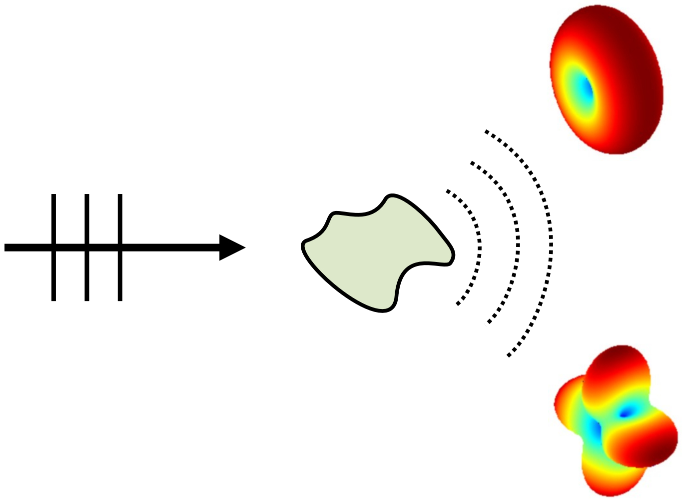
$$Q'_{\mathrm{ee},klij} = Q'_{\mathrm{ee},ijkl}$$
$$S'_{\mathrm{mm},klij} = S'_{\mathrm{mm},ijkl}$$
$$Q'_{\mathrm{em},klij} = -S'_{\mathrm{me},ijkl}$$

EM-ME tensors are connected to each other via minus their transpose.

For detailed derivation, see https://doi.org/10.1103/PhysRevB.104.165426

Reciprocity, Nonlocality and Multipolar Responses

$\chi_{\mathrm{ee},ij} = \chi_{\mathrm{ee},ji}$ Reciprocally connected to itself



$$\mathbf{P} \propto \overline{\overline{\chi}}_{\mathrm{ee}} \cdot \mathbf{E} + \overline{\overline{\chi}}'_{\mathrm{ee}} \cdot \nabla \mathbf{E}$$

Reciprocally connected together!

$$\chi'_{\mathrm{ee},kji} = Q_{\mathrm{ee},ijk}$$

$$\overline{\overline{\mathrm{Q}}} \propto \overline{\overline{\mathrm{Q}}}_{\mathrm{ee}} \cdot \mathbf{E} + \overline{\overline{\mathrm{Q}}}'_{\mathrm{ee}} \cdot \nabla \mathbf{E}$$

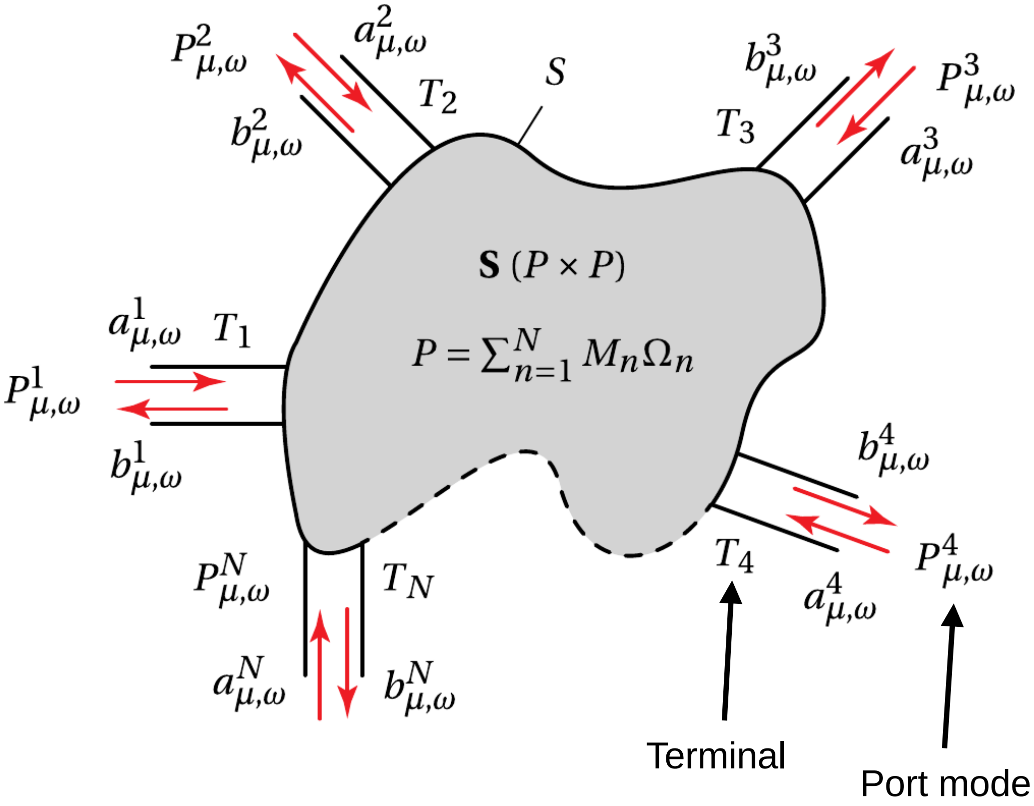
Reciprocally connected to itself $\mathrm{Q'}_{\mathrm{ee},klij} = Q'_{\mathrm{ee},ijkl}$

What Have We Learned So Far….

- Reciprocity is a confusing concept and it is many times confused with asymmetric scattering.
- Asymmetric LTI structures are always reciprocal even though their scattering effect may intuitively look non-reciprocal.
- Lorentz reciprocity is defined as an invariance of electromagnetic reaction when exchanging source and receiver. This translates to an equivalent invariance of the dyadic Green function.
- The Lorentz reciprocity theorem allows us to derive reciprocity relations for material parameters.
- Reciprocity creates connection between different multipolar material parameter tensors. For instance, a dipole moment related to field gradients is connected to quadrupolar responses related to fields.

Reciprocity & Scattering Parameters

Scattering Parameters Formalism



Tangential field components in each port (p)

$$\mathbf{E}_{\parallel,p} = (a_p e^{-j\beta_p z} + b_p e^{+j\beta_p z})\mathbf{e}_{\parallel,p}$$

$$\mathbf{H}_{\parallel,p} = (a_p e^{-j\beta_p z} - b_p e^{+j\beta_p z})\mathbf{h}_{\parallel,p}$$

Polarization vector

The system total scattering matrix is

$$\mathbf{b} = \overline{\overline{\mathrm{S}}} \cdot \mathbf{a}$$

where

$$\mathbf{a} = [a_1, a_2, \dots]$$

$$\mathbf{b} = [b_1, b_2, \dots]$$

μ: mode (e.g.,TE_{11})
ω: mode frequency

http://dx.doi.org/10.1103/PhysRevApplied.10.047001

Connection with Lorentz Theorem

$$\langle a,b\rangle - \langle b,a\rangle = \underbrace{\oint (\mathbf{E}_\mathrm{b}\times\mathbf{H}_\mathrm{a} - \mathbf{E}_\mathrm{a}\times\mathbf{H}_\mathrm{b})}\cdot d\mathbf{S} + j\omega\int \underbrace{(\mathbf{E}_\mathrm{b}\cdot\mathbf{D}_\mathrm{a} - \mathbf{E}_\mathrm{a}\cdot\mathbf{D}_\mathrm{b} + \mathbf{H}_\mathrm{a}\cdot\mathbf{B}_\mathrm{b} - \mathbf{H}_\mathrm{b}\cdot\mathbf{B}_\mathrm{a})}_{=0} dV = 0$$

For a reciprocal system

$$\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^\mathrm{T} \qquad \overline{\overline{\mu}} = \overline{\overline{\mu}}^\mathrm{T} \qquad \overline{\overline{\xi}}^\mathrm{T} = -\overline{\overline{\zeta}}$$

Must be satisfied if the system is reciprocal

$$\oint (\mathbf{E}'\times\mathbf{H}'' - \mathbf{E}''\times\mathbf{H}')\cdot d\mathbf{S} = 0 \quad \longrightarrow \quad \sum_p \left(b'_p a''_p - a'_p b''_p\right) = 0$$

After “a few” steps

Using the definition of the fields at the object (z=0)

$$\mathbf{E} = \sum_p (a_p + b_p)\mathbf{e}_{\|,p}$$

$$\mathbf{H} = \sum_p (a_p - b_p)\mathbf{h}_{\|,p}$$

And using the orthogonality $\oint \mathbf{e}''_{\|,p}\times\mathbf{h}'_{\|,q}\cdot d\mathbf{S} = 2\delta_{pq}$

Full derivation in appendix of http://dx.doi.org/10.1103/PhysRevApplied.10.047001

Scattering Matrix Reciprocity Condition

$$\sum_p \left(b'_p a''_p - a'_p b''_p \right) = 0$$

Could be written in matrix form as

$$[b'_1, b'_2, \dots][a''_1, a''_2, \dots]^T - [a'_1, a'_2, \dots][b''_1, b''_2, \dots]^T = \mathbf{b}' \cdot \mathbf{a}''^T - \mathbf{a}' \cdot \mathbf{b}''^T = 0$$

Remembering that $\mathbf{b} = \overline{\overline{\mathrm{S}}} \cdot \mathbf{a}$ we have that

$$\mathbf{b}' \cdot \mathbf{a}''^T - \mathbf{a}' \cdot \mathbf{b}''^T = \overline{\overline{\mathrm{S}}} \cdot \mathbf{a}' \cdot \mathbf{a}''^T - \mathbf{a}' \cdot (\overline{\overline{\mathrm{S}}} \cdot \mathbf{a}'')^T = \mathbf{a}' \cdot \mathbf{a}''^T \cdot (\overline{\overline{\mathrm{S}}} - \overline{\overline{\mathrm{S}}}^T) = 0$$

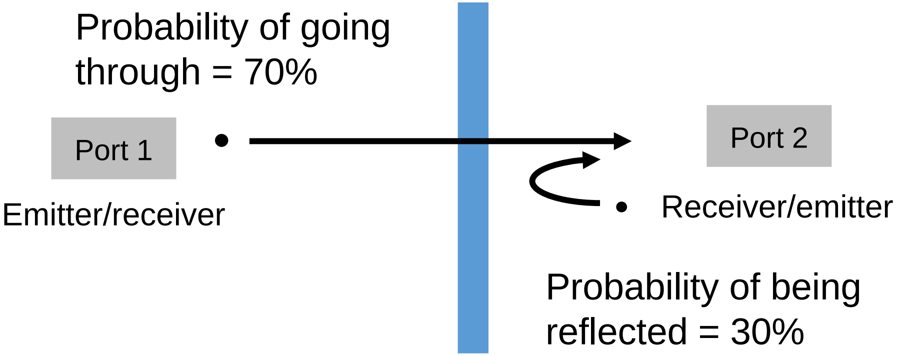
S-matrix reciprocity condition

$$\overline{\overline{\mathrm{S}}} = \overline{\overline{\mathrm{S}}}^T$$

Microscopic vs Macroscopic

Example: thick slab of glass. From Fresnel, we have: R = 30% and T=70%

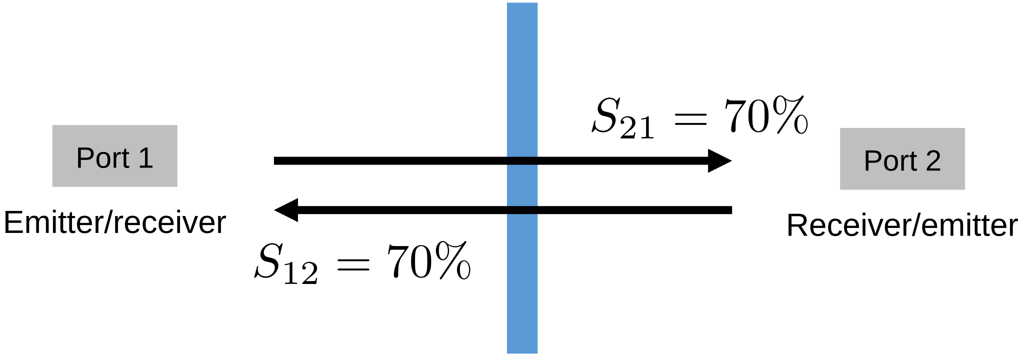
Microscopic case
(1 photon at a time)



If emitter/receiver are exchanged, transmission between the two is in general **nonreciprocal** due to the probabilistic scattering of individual photons.

Quantum optics

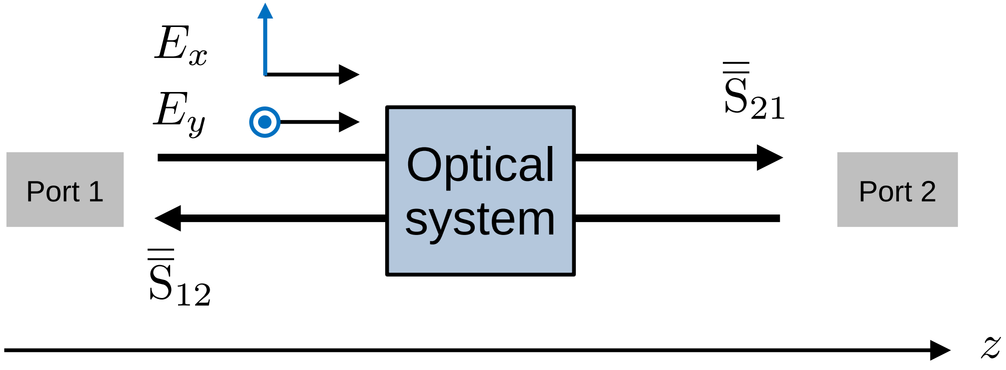
Macroscopic case
(many photons)



If emitter/receiver are exchanged, transmission between the two is in general **reciprocal** due to the averaging of probabilistic scattering of billions of photons.

Classical electromagnetics

Scattering Matrix Optical System Example



Transmission scattering matrix

$$\overline{\overline{\mathrm{S}}}_{21} = \begin{bmatrix} S_{21}^{xx} & S_{21}^{xy} \\ S_{21}^{yx} & S_{21}^{yy} \end{bmatrix}$$

Reciprocity conditions

$$\overline{\overline{\mathrm{S}}} = \overline{\overline{\mathrm{S}}}^T$$

$$\overline{\overline{\mathrm{S}}}_{21}^T = \overline{\overline{\mathrm{S}}}_{12}$$

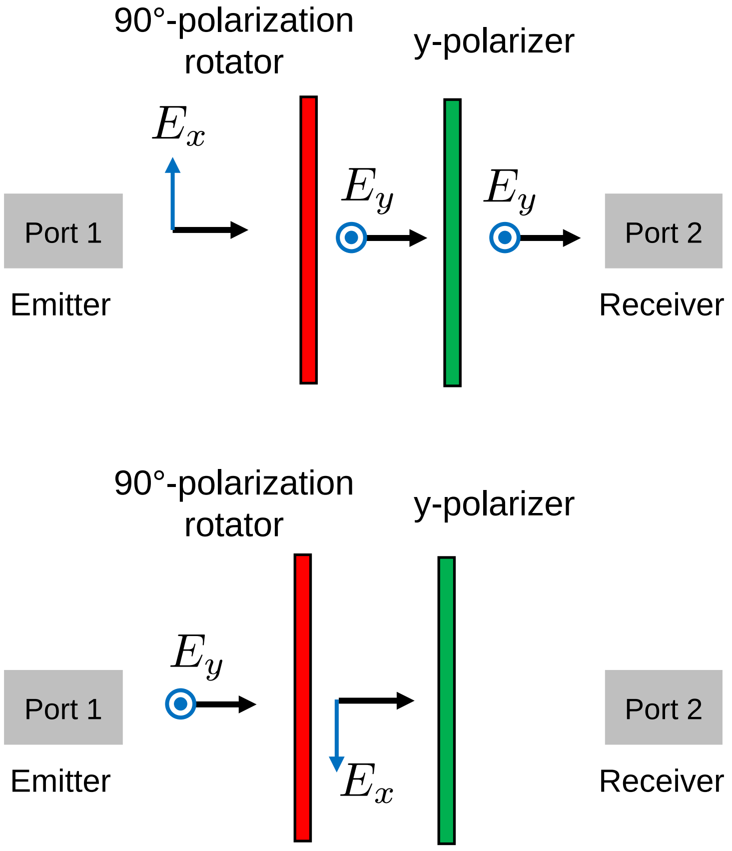
$$\overline{\overline{\mathrm{S}}}_{22}^T = \overline{\overline{\mathrm{S}}}_{22}$$

$$\overline{\overline{\mathrm{S}}}_{11}^T = \overline{\overline{\mathrm{S}}}_{11}$$

System total scattering matrix

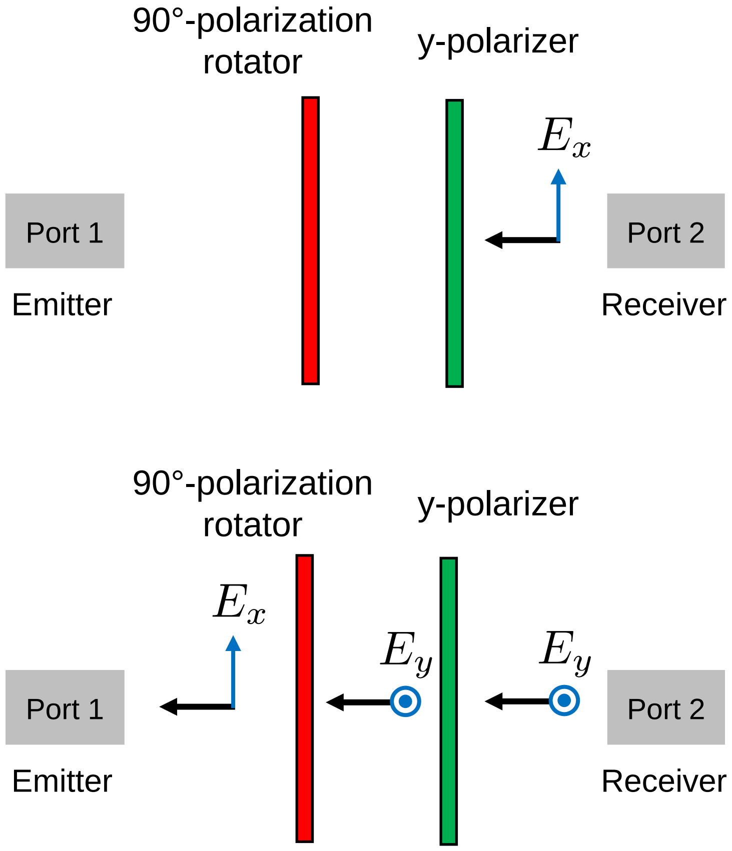
$$\overline{\overline{\mathrm{S}}} = \begin{bmatrix} \overline{\overline{\mathrm{S}}}_{11} & \overline{\overline{\mathrm{S}}}_{12} \\ \overline{\overline{\mathrm{S}}}_{21} & \overline{\overline{\mathrm{S}}}_{22} \end{bmatrix} = \begin{bmatrix} S_{11}^{xx} & S_{11}^{xy} & S_{12}^{xx} & S_{12}^{xy} \\ S_{11}^{yx} & S_{11}^{yy} & S_{12}^{yx} & S_{12}^{yy} \\ S_{21}^{xx} & S_{21}^{xy} & S_{22}^{xx} & S_{22}^{xy} \\ S_{21}^{yx} & S_{21}^{yy} & S_{22}^{yx} & S_{22}^{yy} \end{bmatrix}$$

Asymmetric or Non-Reciprocal ?



$$S_{21}^{xx} = 0$$
$$S_{21}^{yx} \neq 0$$

$$S_{21}^{xy} = 0$$
$$S_{21}^{yy} = 0$$



$$S_{12}^{yx} = 0$$
$$S_{12}^{xx} = 0$$

$$S_{12}^{xy} \neq 0$$
$$S_{12}^{yy} = 0$$

$$\overline{\overline{\mathrm{S}}}_{21} = \begin{bmatrix} S_{21}^{xx} & S_{21}^{xy} \\ S_{21}^{yx} & S_{21}^{yy} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ S_{21}^{yx} & 0 \end{bmatrix}$$

$$\overline{\overline{\mathrm{S}}}_{21} \neq \overline{\overline{\mathrm{S}}}_{21}^{\mathrm{T}}$$

$$\overline{\overline{\mathrm{S}}}_{12} = \begin{bmatrix} S_{12}^{xx} & S_{12}^{xy} \\ S_{12}^{yx} & S_{12}^{yy} \end{bmatrix} = \begin{bmatrix} 0 & S_{12}^{xy} \\ 0 & 0 \end{bmatrix}$$

What Have We Learned So Far….

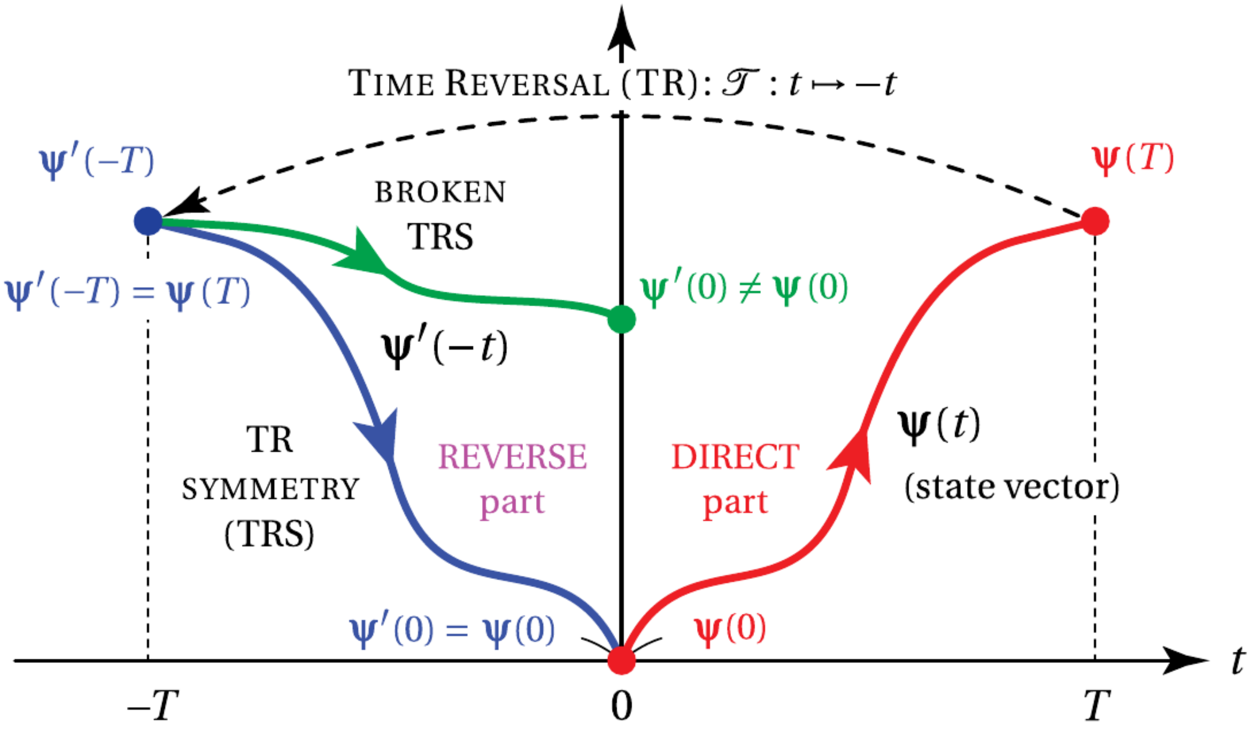
- As an alternative to the material parameters reciprocity relations, we can express reciprocity in terms of relations that apply on the scattering matrices.
- The global scattering matrix should be equal to its transpose.
- This implies that the scattering matrices in reflection (S_{11} and S_{22}) are equal to their own transpose by reciprocity (i.e., $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$). However, the transmission matrices (S_{21} and S_{12}) are connected to each other transpose by reciprocity (i.e., $S_{21} = S_{12}^T$).
- This means that the transmission may be asymmetric ($S_{21} \neq S_{21}^T$) while still being reciprocal.

Time-Reversal Symmetry

Time-Reversal Symmetry and Reciprocity

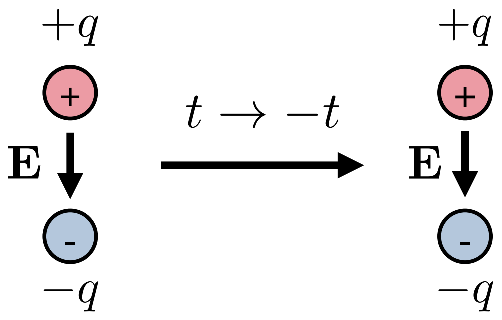
Time reversal: flip the sign of time:

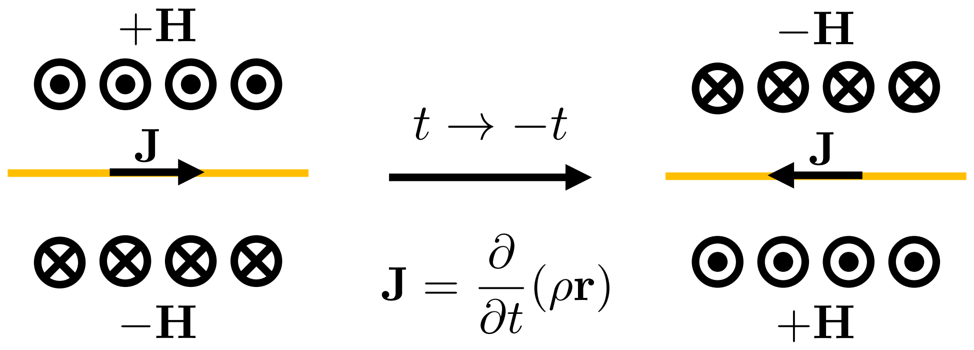
$$T : t \rightarrow -t$$



http://dx.doi.org/10.1103/PhysRevApplied.10.047001

Time-Reversal and Maxwell Equations





$$\mathbf{J} = \frac{\partial}{\partial t}(\rho \mathbf{r})$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

$$\nabla \cdot \mathbf{D} = q$$

$$\nabla \cdot \mathbf{B} = q_{\mathrm{m}}$$

even $\quad T\{\mathbf{E}(t,\mathbf{r})\} = +\mathbf{E}(-t,\mathbf{r})$

odd $\quad T\{\mathbf{H}(t,\mathbf{r})\} = -\mathbf{H}(-t,\mathbf{r})$

Maxwell equations are
even under time reversal

How even an odd
quantities combine

	even	odd
even	even	odd
odd	odd	even

Time-Reversal in the Frequency Domain

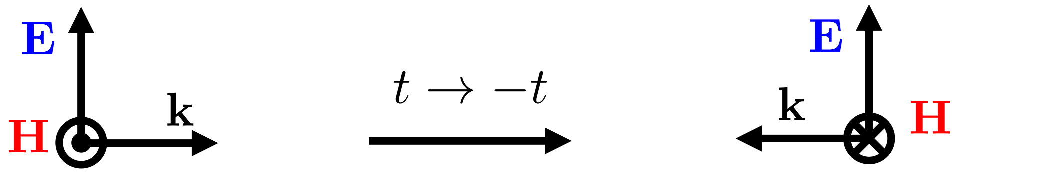
In the time-domain, we have:
$$\begin{cases} \text{even} & T\{\mathbf{E}(t,\mathbf{r})\} = +\mathbf{E}(-t,\mathbf{r}) \\ \text{odd} & T\{\mathbf{H}(t,\mathbf{r})\} = -\mathbf{H}(-t,\mathbf{r}) \end{cases}$$

Fourier transform

$$\mathbf{E}(\omega,\mathbf{r}) = \int_{-\infty}^{+\infty} \tilde{\mathbf{E}}(t,\mathbf{r}) e^{-j\omega t} dt \longrightarrow T\{\mathbf{E}(\omega,\mathbf{r})\} = \int_{-\infty}^{+\infty} \tilde{\mathbf{E}}(-t,\mathbf{r}) e^{+j\omega t} dt = \mathbf{E}(-\omega,\mathbf{r}) = \mathbf{E}^*(\omega,\mathbf{r})$$

In the frequency-domain, we have:
$$\begin{cases} \text{even} & T\{\mathbf{E}(\omega,\mathbf{r})\} = +\mathbf{E}(-\omega,\mathbf{r}) = +\mathbf{E}^*(\omega,\mathbf{r}) \\ \text{odd} & T\{\mathbf{H}(\omega,\mathbf{r})\} = -\mathbf{H}(-\omega,\mathbf{r}) = -\mathbf{H}^*(\omega,\mathbf{r}) \end{cases}$$

It follows that the time-reversed of $\mathbf{E}(z,\omega) = \mathbf{E}(\omega) e^{-jkz}$ is given by $T\{\mathbf{E}(z,\omega)\} = \mathbf{E}^*(\omega) e^{jk^* z}$



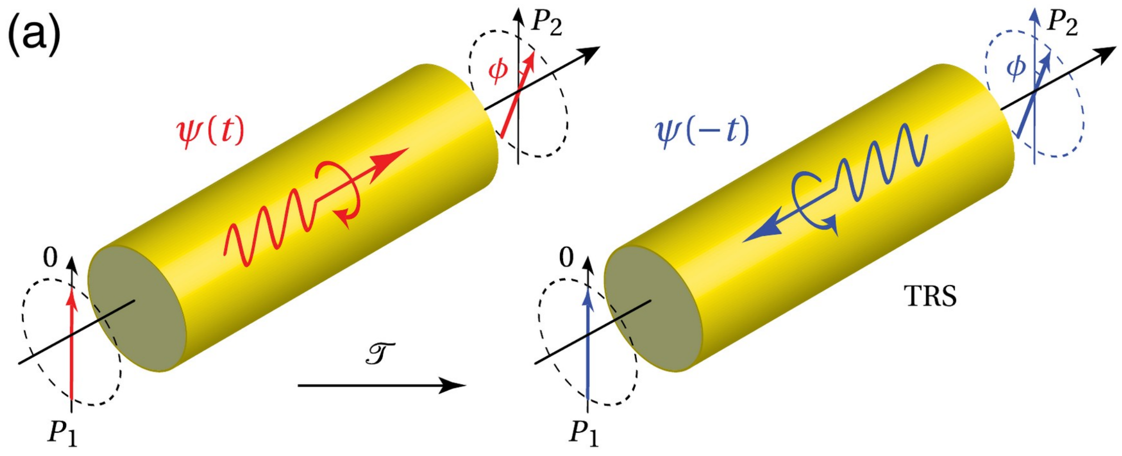
Direction of propagation is flipped!

Time Reversal Properties of Physical Quantities

Physical Quantity		Time Reversal
I. *Mechanical*		
Coordinate	$\mathbf{x}$	Even
Velocity	$\mathbf{v}$	Odd
Momentum	$\mathbf{p}$	Odd
Angular momentum	$\mathbf{L} = \mathbf{x} \times \mathbf{p}$	Odd
Force	$\mathbf{F}$	Even
Torque	$\mathbf{N} = \mathbf{x} \times \mathbf{F}$	Even
Kinetic energy	$p^2/2m$	Even
Potential energy	$U(\mathbf{x})$	Even
II. *Electromagnetic*		
Charge density	ρ	Even
Current density	$\mathbf{J}$	Odd
Electric field	$\mathbf{E}$	Even
Polarization	$\mathbf{P}$	Even
Displacement	$\mathbf{D}$	Even
Magnetic induction	$\mathbf{B}$	Odd
Magnetization	$\mathbf{M}$	Odd
Magnetic field	$\mathbf{H}$	Odd
Poynting vector	$\mathbf{S} = \mathbf{E} \times \mathbf{H}$	Odd
Maxwell stress tensor	$T_{\alpha\beta}$	Even

← and also $\mathbf{k} \parallel \mathbf{D} \times \mathbf{B}$

Time-Reversal Symmetry in a Lossless System



Perfect match between time-reversed case and case where time is not flipped but only the direction of wave propagation is reversed.

http://dx.doi.org/10.1103/PhysRevApplied.10.047001

Time-Reversal Symmetry in a Lossy System

$$\mathbf{D}(\omega) = \epsilon(\omega)\mathbf{E}(\omega)$$

$$\mathbf{B}(\omega) = \mu(\omega)\mathbf{H}(\omega)$$

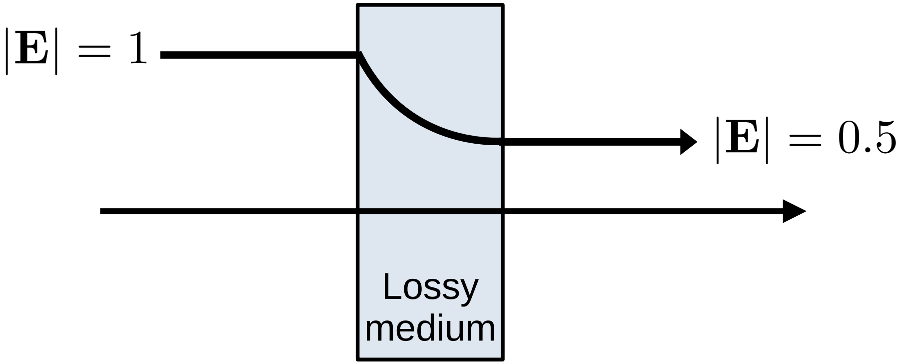
$$T\{\epsilon(\omega)\} = \epsilon(\omega)^*$$

$$T\{\mu(\omega)\} = \mu(\omega)^*$$

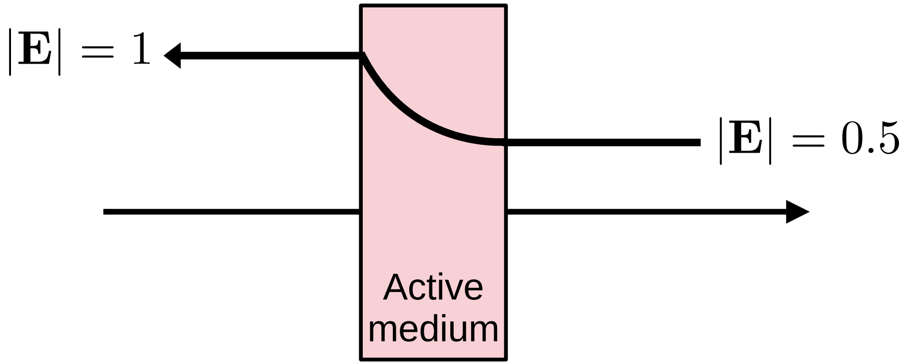
even

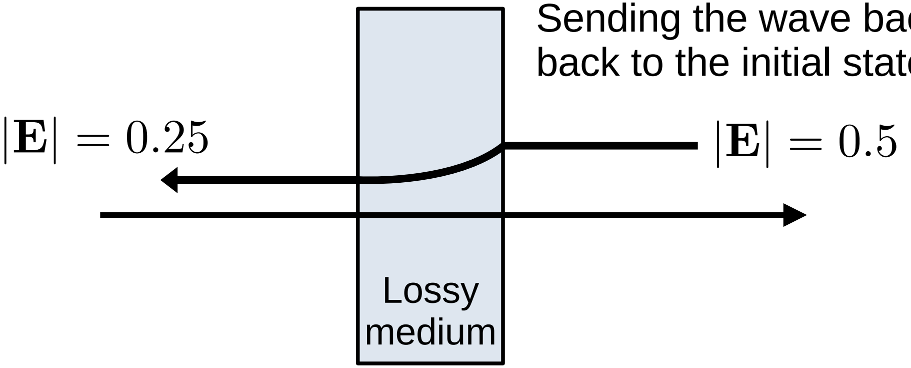
$$\epsilon(\omega) = \epsilon' - j\epsilon'' \xrightarrow{t \to -t} \epsilon(\omega) = \epsilon' + j\epsilon''$$

Loss becomes gain !!



$t \to -t$





Sending the wave backward does not get us back to the initial state, yet it is reciprocal.

$$\overline{\overline{\mathrm{S}}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} = \overline{\overline{\mathrm{S}}}^T$$

What Have We Learned So Far….

- Time-reversal symmetry may be used to investigate reciprocity.
- Time-reversal (TR) is simply defined as the transformation: $t \rightarrow -t$
- Electric quantities are even under TR meaning, while magnetic quantities are odd.
- Applying TR on a plane wave makes it propagate backward in space (phase conjugation). This is why TR symmetry is often used to assess the reciprocity of a system as it mimics exiting the system with a wave propagating in the backward direction.
- Since applying TR is equivalent (in Fourier space) to taking the complex conjugate, it follows that a lossy material becomes active under TR.
- It is generally better to use scattering parameters than TR to assess the reciprocity of a system with loss.

Onsager-Casimir Relations

Onsager-Casimir Relations Derivation

$$\langle a, b \rangle = \langle b, a \rangle \longrightarrow \int \mathbf{E}_{\mathrm{a}} \cdot \mathbf{J}_{\mathrm{b}} dV = \int \mathbf{E}_{\mathrm{b}} \cdot \mathbf{J}_{\mathrm{a}} dV$$

Instead of switching position, we flip time

$$\int \mathbf{E} \cdot T\{\mathbf{J}\} dV = \int T\{\mathbf{E}\} \cdot \mathbf{J} dV$$

$\downarrow$ Since both **J** and **E** are time-even quantities

$$-\int \mathbf{E} \cdot \mathbf{J}^* dV = \int \mathbf{E}^* \cdot \mathbf{J} dV$$

Defining the Time-Reversed Material Parameters

$$-\int \mathbf{E}\cdot\mathbf{J}^* dV = \int \mathbf{E}^*\cdot\mathbf{J} dV$$

Following the same procedure as for the Lorentz theorem →

$$\int \left(\mathbf{E}^*\cdot\mathbf{D} - \mathbf{E}\cdot\mathbf{D}^* + \mathbf{H}\cdot\mathbf{B}^* - \mathbf{H}^*\cdot\mathbf{B}\right) dV = 0$$

Bianisotropic constitutive relations

even $$\mathbf{D} = \overline{\overline{\epsilon}}(\mathbf{F}_0)\cdot\mathbf{E} + \overline{\overline{\xi}}(\mathbf{F}_0)\cdot\mathbf{H}$$

odd $$\mathbf{B} = \overline{\overline{\zeta}}(\mathbf{F}_0)\cdot\mathbf{E} + \overline{\overline{\mu}}(\mathbf{F}_0)\cdot\mathbf{H}$$

Where $\mathbf{F}_0$ is a time-odd bias

$t \rightarrow -t$ →

$$\mathbf{D}' = \overline{\overline{\epsilon}}'(-\mathbf{F}_0)\cdot\mathbf{E}' + \overline{\overline{\xi}}'(-\mathbf{F}_0)\cdot\mathbf{H}'$$

$$\mathbf{B}' = \overline{\overline{\zeta}}'(-\mathbf{F}_0)\cdot\mathbf{E}' + \overline{\overline{\mu}}'(-\mathbf{F}_0)\cdot\mathbf{H}'$$

↓ Alternatively

$$\mathbf{D}^* = \overline{\overline{\epsilon}}'(-\mathbf{F}_0)\cdot\mathbf{E}^* - \overline{\overline{\xi}}'(-\mathbf{F}_0)\cdot\mathbf{H}^*$$

$$\mathbf{B}^* = -\overline{\overline{\zeta}}'(-\mathbf{F}_0)\cdot\mathbf{E}^* + \overline{\overline{\mu}}'(-\mathbf{F}_0)\cdot\mathbf{H}^*$$

Primes indicate time-reversed quantities

Restricted Time-Reversed Material Parameters

$$\mathbf{D} = \overline{\overline{\epsilon}}(\mathbf{F}_0) \cdot \mathbf{E} + \overline{\overline{\xi}}(\mathbf{F}_0) \cdot \mathbf{H}$$

$$\mathbf{B} = \overline{\overline{\zeta}}(\mathbf{F}_0) \cdot \mathbf{E} + \overline{\overline{\mu}}(\mathbf{F}_0) \cdot \mathbf{H}$$

$t \rightarrow -t$

$$\mathbf{D}^* = \overline{\overline{\epsilon}}'(-\mathbf{F}_0) \cdot \mathbf{E}^* - \overline{\overline{\xi}}'(-\mathbf{F}_0) \cdot \mathbf{H}^*$$

$$\mathbf{B}^* = -\overline{\overline{\zeta}}'(-\mathbf{F}_0) \cdot \mathbf{E}^* + \overline{\overline{\mu}}'(-\mathbf{F}_0) \cdot \mathbf{H}^*$$

Taking the complex conjugate

$$\mathbf{D} = \overline{\overline{\epsilon}}'^*(-\mathbf{F}_0) \cdot \mathbf{E} - \overline{\overline{\xi}}'^*(-\mathbf{F}_0) \cdot \mathbf{H}$$

$$\mathbf{B} = -\overline{\overline{\zeta}}'^*(-\mathbf{F}_0) \cdot \mathbf{E} + \overline{\overline{\mu}}'^*(-\mathbf{F}_0) \cdot \mathbf{H}$$

Comparing the two

$$\overline{\overline{\epsilon}}'^*(-\mathbf{F}_0) = \overline{\overline{\epsilon}}(\mathbf{F}_0)$$

$$\overline{\overline{\mu}}'^*(-\mathbf{F}_0) = \overline{\overline{\mu}}(\mathbf{F}_0)$$

$$\overline{\overline{\xi}}'^*(-\mathbf{F}_0) = -\overline{\overline{\xi}}(\mathbf{F}_0)$$

$$\overline{\overline{\zeta}}'^*(-\mathbf{F}_0) = -\overline{\overline{\zeta}}(\mathbf{F}_0)$$

$$\overline{\overline{\epsilon}}'(-\mathbf{F}_0) = \overline{\overline{\epsilon}}(\mathbf{F}_0)$$

$$\overline{\overline{\mu}}'(-\mathbf{F}_0) = \overline{\overline{\mu}}(\mathbf{F}_0)$$

$$\overline{\overline{\xi}}'(-\mathbf{F}_0) = -\overline{\overline{\xi}}(\mathbf{F}_0)$$

$$\overline{\overline{\zeta}}'(-\mathbf{F}_0) = -\overline{\overline{\zeta}}(\mathbf{F}_0)$$

Here, we assume "restricted time-reversal". Everything is reversed except dissipation, so loss does not become gain. So we remove the '*'

Onsager-Casimir vs Lorentz Reciprocity Conditions

$$\int (\mathbf{E}^* \cdot \mathbf{D} - \mathbf{E} \cdot \mathbf{D}^* + \mathbf{H} \cdot \mathbf{B}^* - \mathbf{H}^* \cdot \mathbf{B})\, dV = 0$$

Normal and complex-conjugated constitutive relations

$$\mathbf{D} = \overline{\overline{\epsilon}}(\mathbf{F}_0) \cdot \mathbf{E} + \overline{\overline{\xi}}(\mathbf{F}_0) \cdot \mathbf{H}$$

$$\mathbf{B} = \overline{\overline{\zeta}}(\mathbf{F}_0) \cdot \mathbf{E} + \overline{\overline{\mu}}(\mathbf{F}_0) \cdot \mathbf{H}$$

$$\mathbf{D}^* = \overline{\overline{\epsilon}}(-\mathbf{F}_0) \cdot \mathbf{E}^* + \overline{\overline{\xi}}(-\mathbf{F}_0) \cdot \mathbf{H}^*$$

$$\mathbf{B}^* = \overline{\overline{\zeta}}(-\mathbf{F}_0) \cdot \mathbf{E}^* + \overline{\overline{\mu}}(-\mathbf{F}_0) \cdot \mathbf{H}^*$$

Onsager-Casimir Relations

$$\overline{\overline{\epsilon}}(\mathbf{F}_0) = \overline{\overline{\epsilon}}^{\mathrm{T}}(-\mathbf{F}_0)$$

$$\overline{\overline{\mu}}(\mathbf{F}_0) = \overline{\overline{\mu}}^{\mathrm{T}}(-\mathbf{F}_0)$$

$$\overline{\overline{\xi}}(\mathbf{F}_0) = -\overline{\overline{\zeta}}^{\mathrm{T}}(-\mathbf{F}_0)$$

In the absence of external bias ($\mathbf{F}_0 = 0$), we retrieve the conventional Lorentz reciprocity conditions

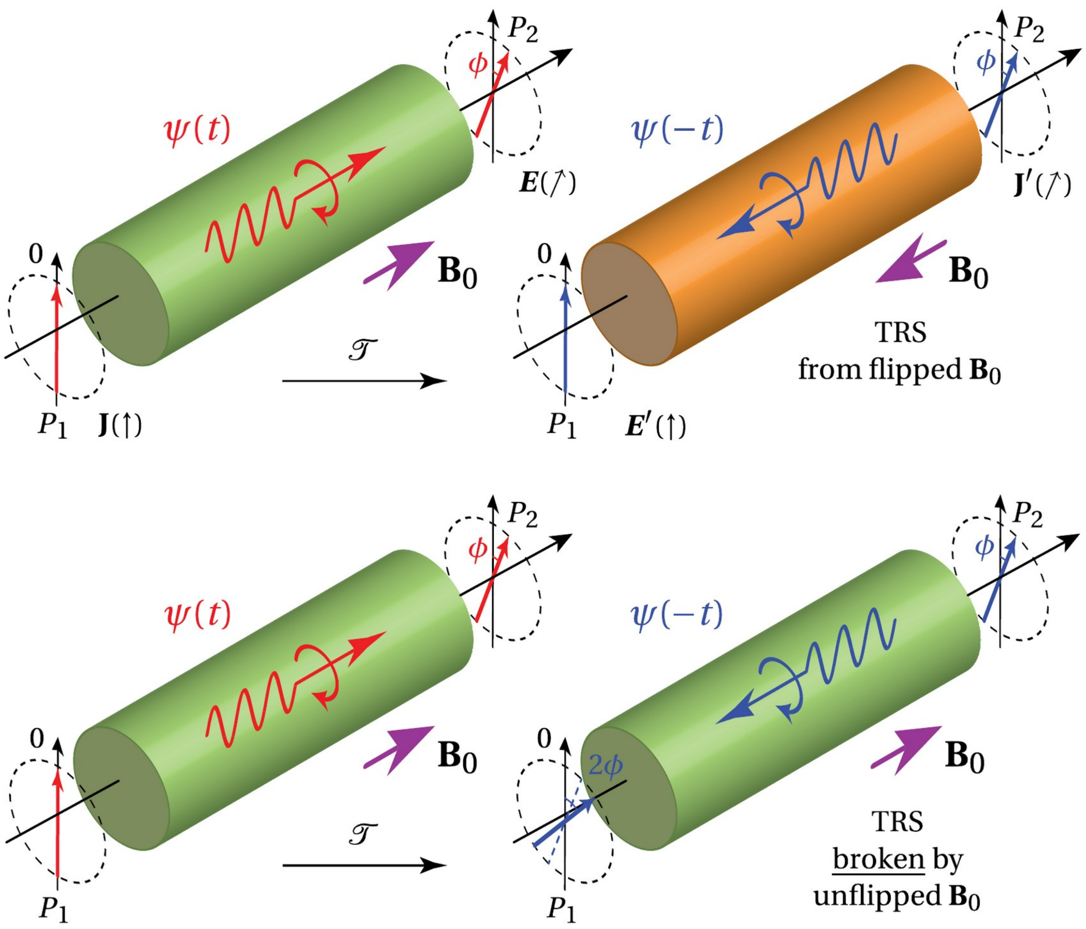
$$\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^{\mathrm{T}}$$

$$\overline{\overline{\mu}} = \overline{\overline{\mu}}^{\mathrm{T}}$$

$$\overline{\overline{\xi}} = -\overline{\overline{\zeta}}^{\mathrm{T}}$$

A time-odd bias must be flipped for the system to remain reciprocal

Time-Odd Bias Example



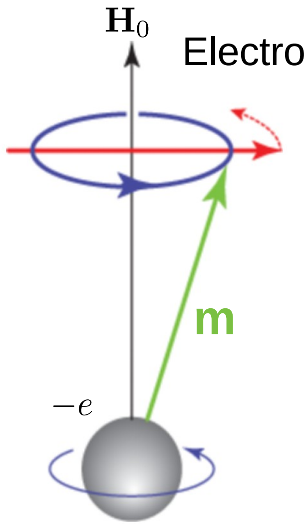
http://dx.doi.org/10.1103/PhysRevApplied.10.047001

What We Have Learned So Far

- **Breaking the spatial symmetry of an LTI system never breaks reciprocity.**
- **To achieve non-reciprocity, an LTI system must be biased with a time-odd quantity.**

Non-Reciprocal LTI Systems

The Faraday Effect



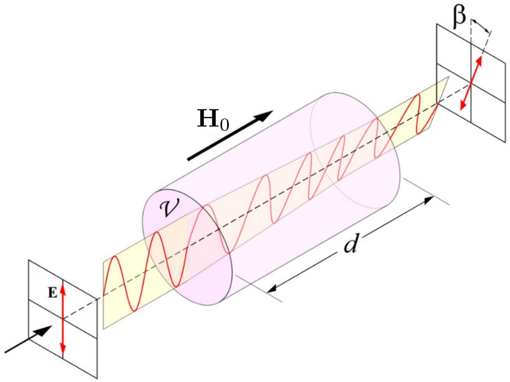
Electron spin precession

$$\overline{\overline{\mu}}(\omega, \mathbf{H}_0) = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

$$\mu = \mu_0 \left(1 + \frac{\omega_0 \omega_\mathrm{m}}{\omega_0^2 - \omega^2}\right) \qquad \kappa = \mu_0 \frac{\omega \omega_\mathrm{m}}{\omega_0^2 - \omega^2}$$

Lamor (precession) frequency: $\omega_0 = \mu_0 \gamma \mathbf{H}_0$

$\omega_\mathrm{m} = \mu_0 \gamma \mathbf{M}_\mathrm{s}$ ← DC saturation magnetization

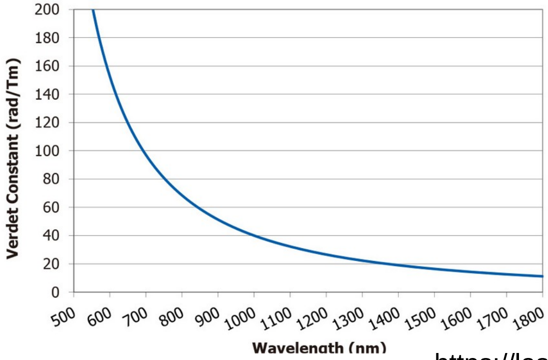


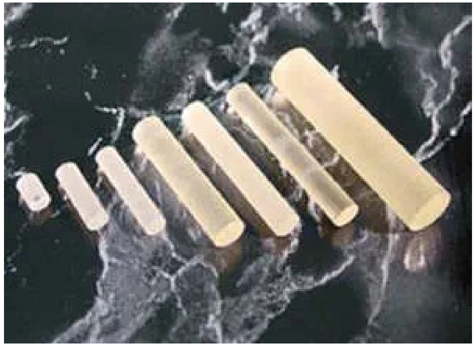
Rotation angle

$$\beta = \mu_0 \mathcal{V} d |\mathbf{H}_0|$$

$\mathcal{V}$: Verdet constant

Terbium gallium garnet (TGG)

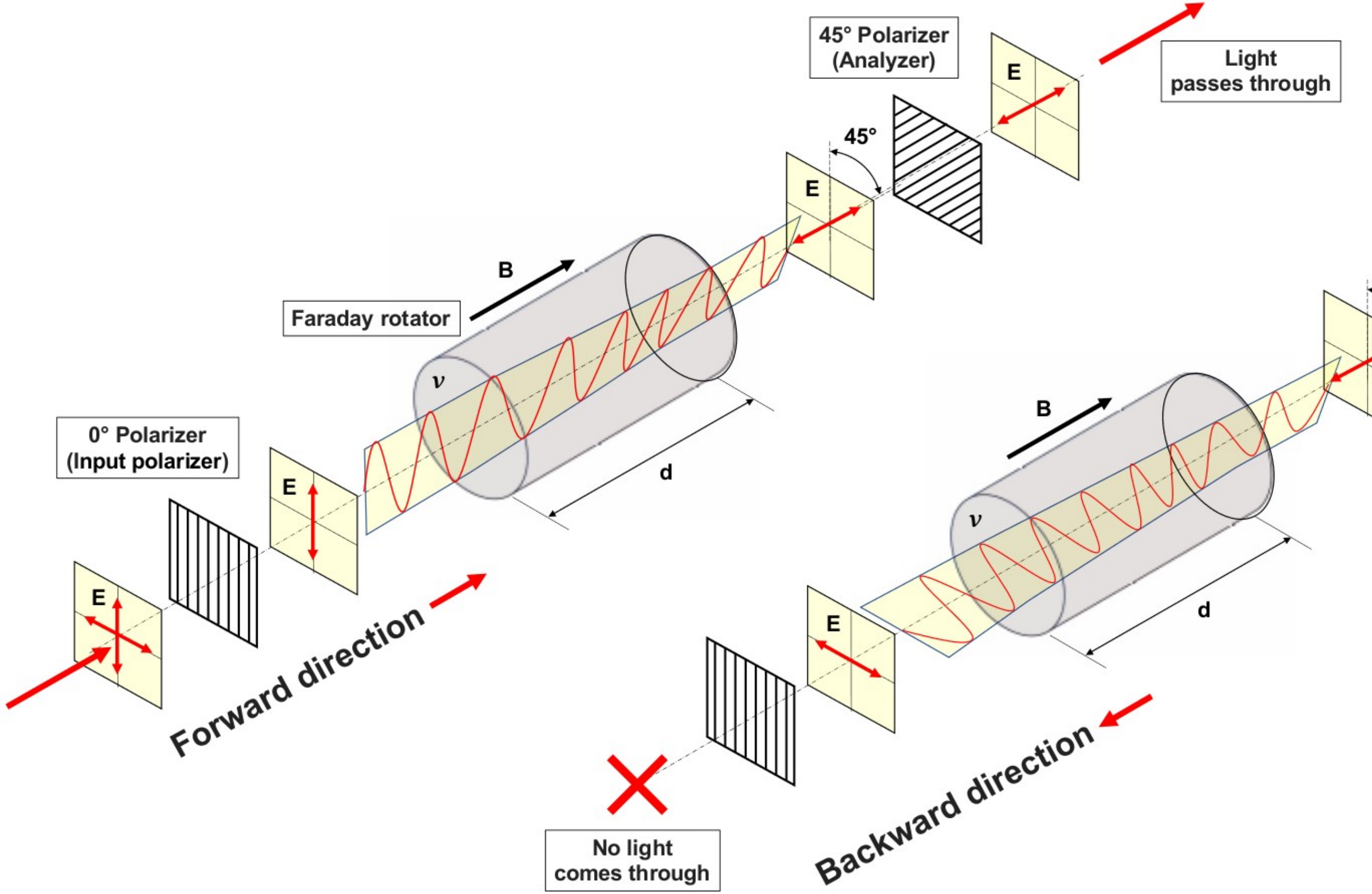


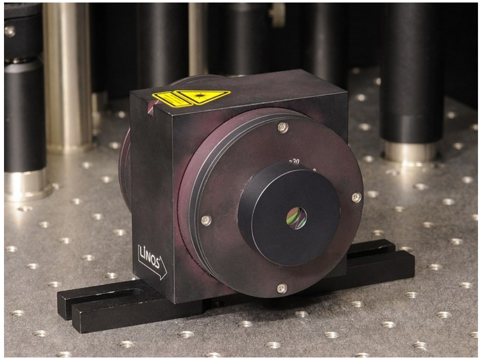


https://laserstates.com

Ferrite

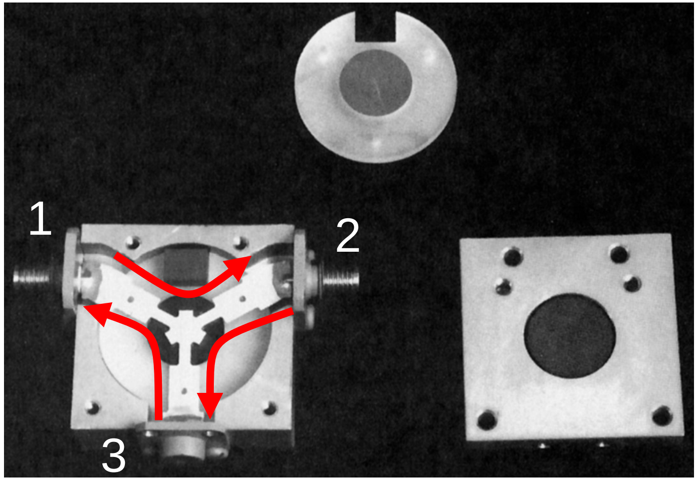
Optical Faraday Isolator





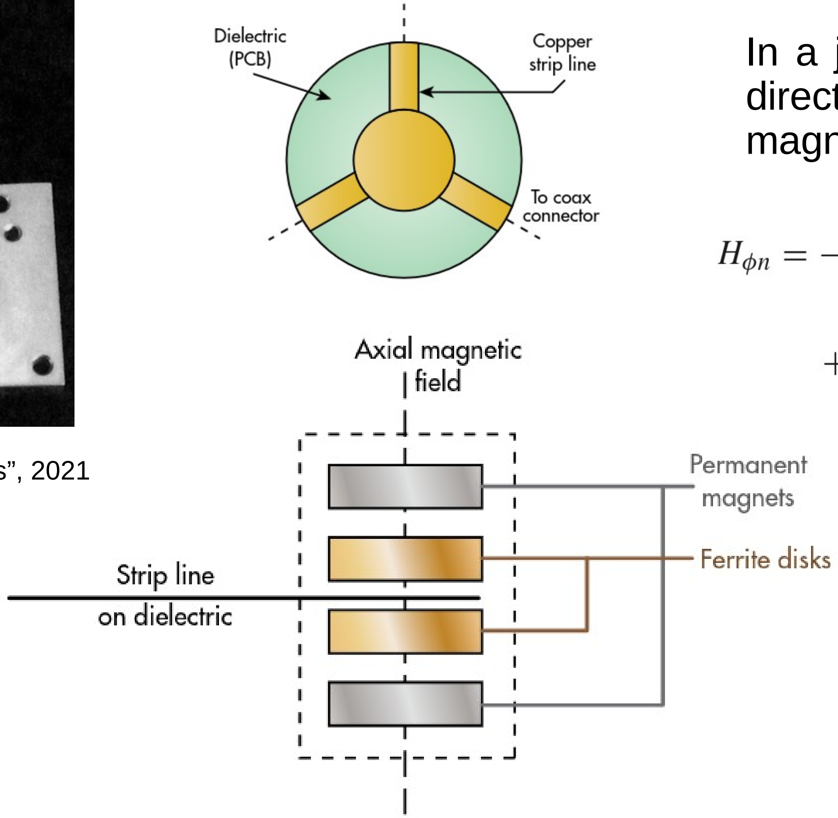
https://en.wikipedia.org/wiki/Optical_isolator

Microwave Circulator



Pozar,"Microwave engineering: theory and techniques", 2021

$$\bar{\bar{\mathrm{S}}} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



In a junction circulator, the field directly depends on the external magnetic field bias.

$$H_{\phi n} = -jY \left\{ A_{+n} e^{jn\phi} \left[J_n'(k\rho) + \frac{n\kappa}{k\rho\mu} J_n(k\rho) \right] + A_{-n} e^{-jn\phi} \left[J_n'(k\rho) - \frac{n\kappa}{k\rho\mu} J_n(k\rho) \right] \right\}.$$

where

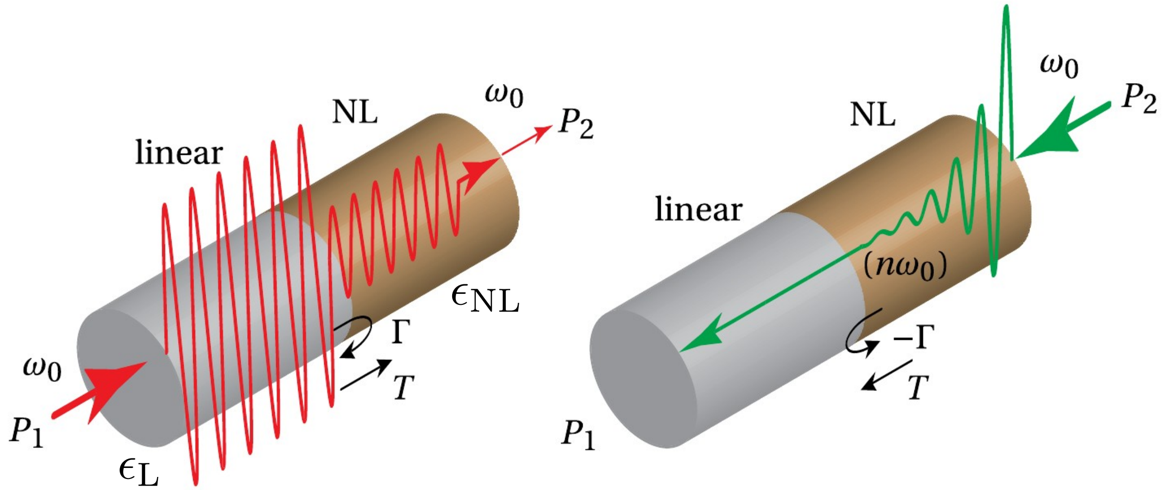
$$\kappa = \mu_0 \frac{\omega \omega_{\mathrm{m}}}{\omega_0^2 - \omega^2}$$

$$\omega_0 = \mu_0 \gamma \mathbf{H}_0$$

https://img.mwrf.com/files/base/ebm/mwrf/document/2019/03/mwrf_3420_21m_.pdf?dl=mwrf_3420_21m_.pdf

Non-Reciprocity via Nonlinearity

Nonreciprocity via Nonlinear Absorption



Field-dependent permittivity: $\epsilon_{NL} = \epsilon_2 + [\epsilon'(E) - j\epsilon''(E)]$

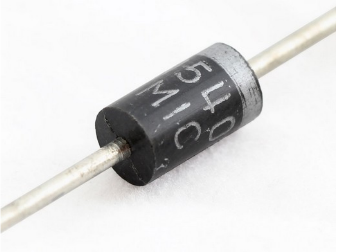
Reflection coefficient: $|\Gamma| = \left| \frac{\sqrt{\epsilon_{NL}} - \sqrt{\epsilon_L}}{\sqrt{\epsilon_{NL}} + \sqrt{\epsilon_L}} \right|$

Non-reciprocity achieved either through nonlinear absorption or reflection

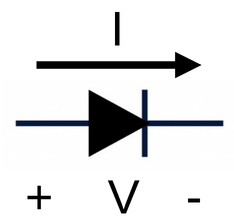
http://dx.doi.org/10.1103/PhysRevApplied.10.047001

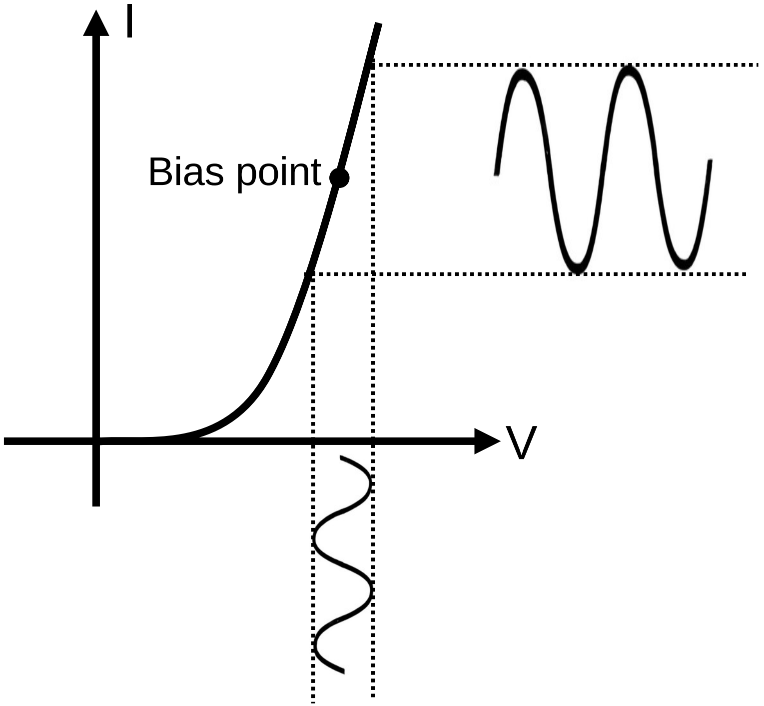
Electric Diodes and Reciprocity

Is an electric diode non-reciprocal?



Forward bias





When used as a “clipper”, a diode behaves as a non-reciprocal system.

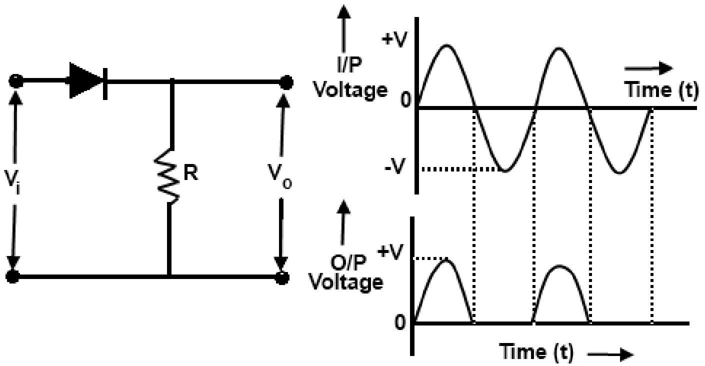
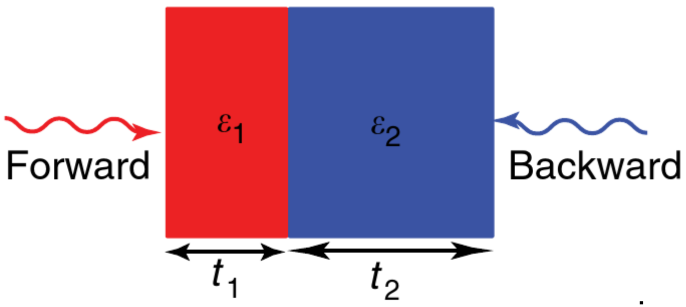


Figure 3: Series Negative Clipper

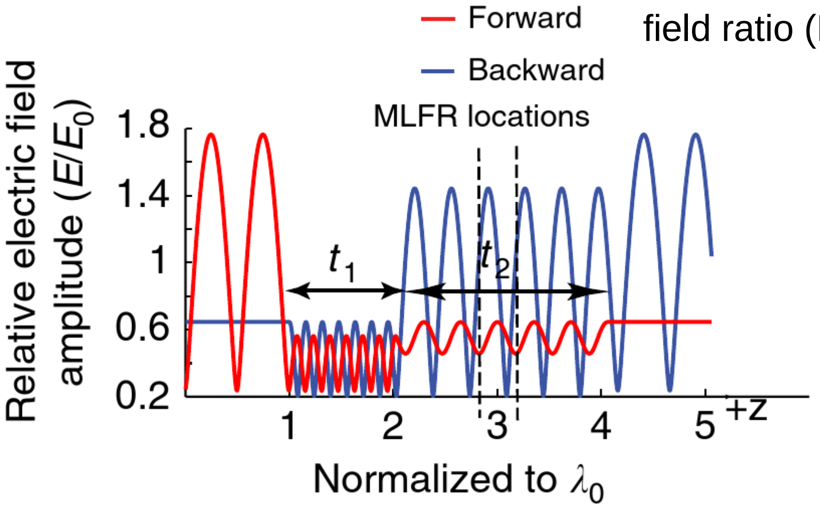
https://www.daenotes.com/electronics/digital-electronics/clipper-circuits

However, when considering its small signal behavior around a bias point, it is reciprocal!

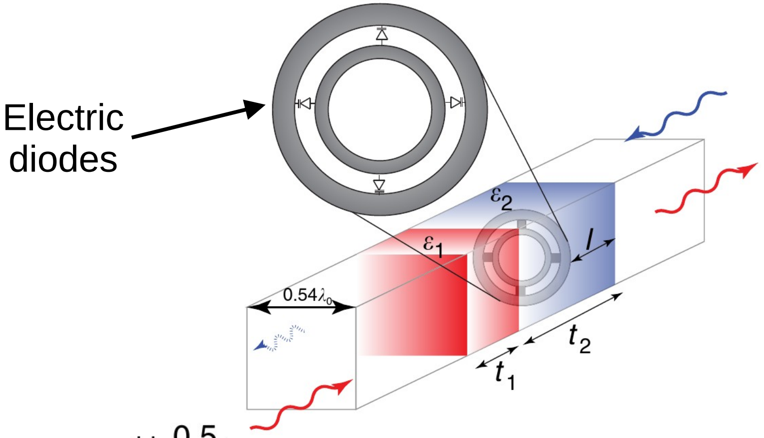
Non-Reciprocity via Nonlinear Absorption

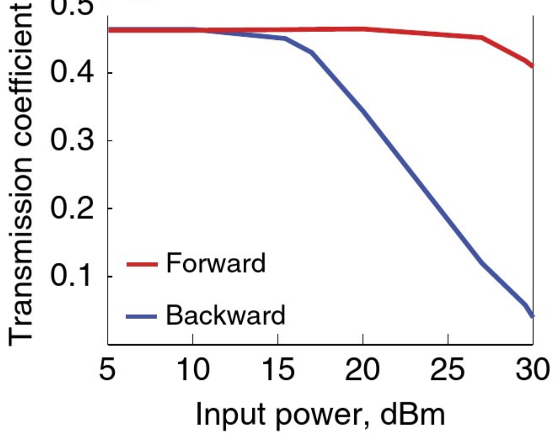


maximum local
field ratio (MLFR)



Different standing wave amplitude in both slabs

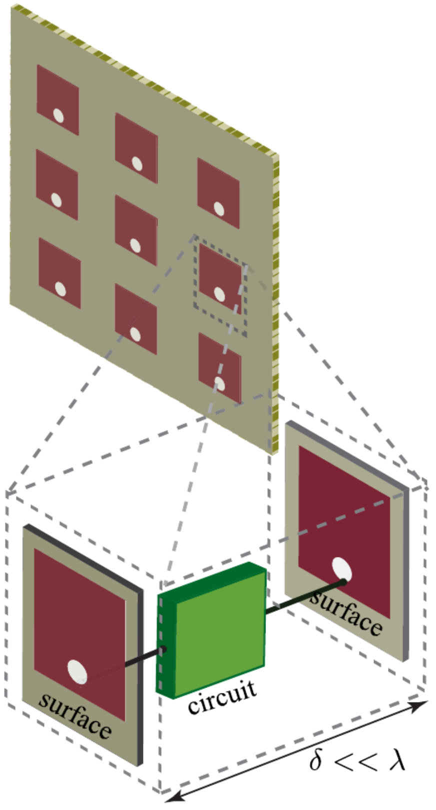


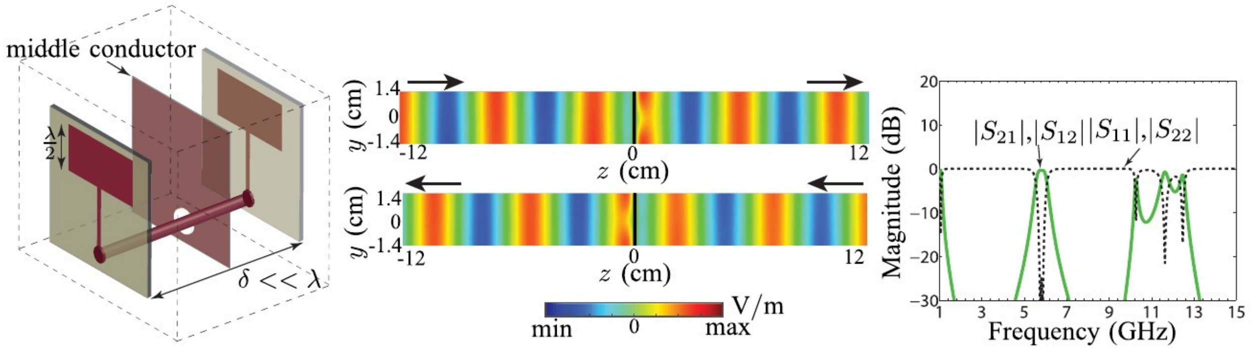


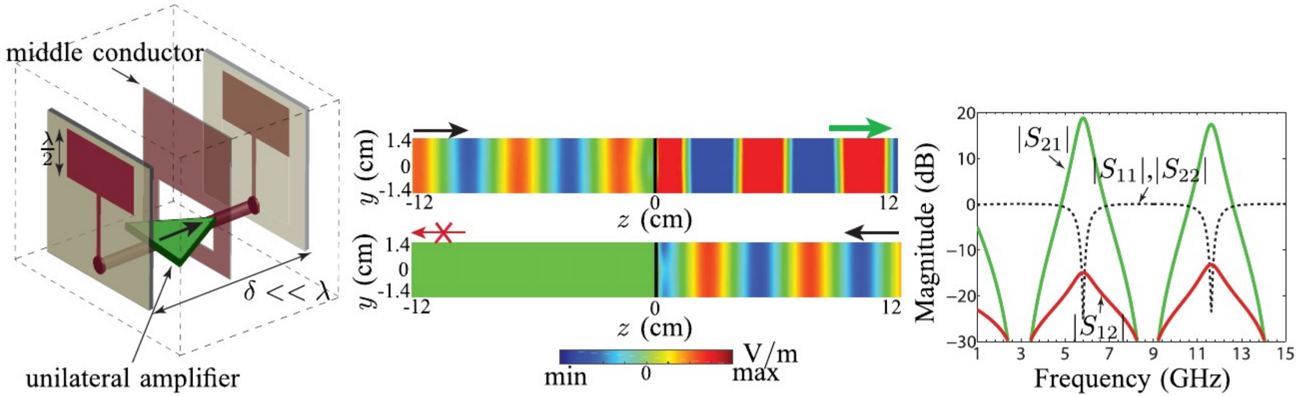
The double-ring system exhibits different effective capacitance based on which direction the wave propagates leading to different transmission coefficients.

https://doi.org/10.1038/ncomms9359

Non-Reciprocity via Nonlinear Absorption

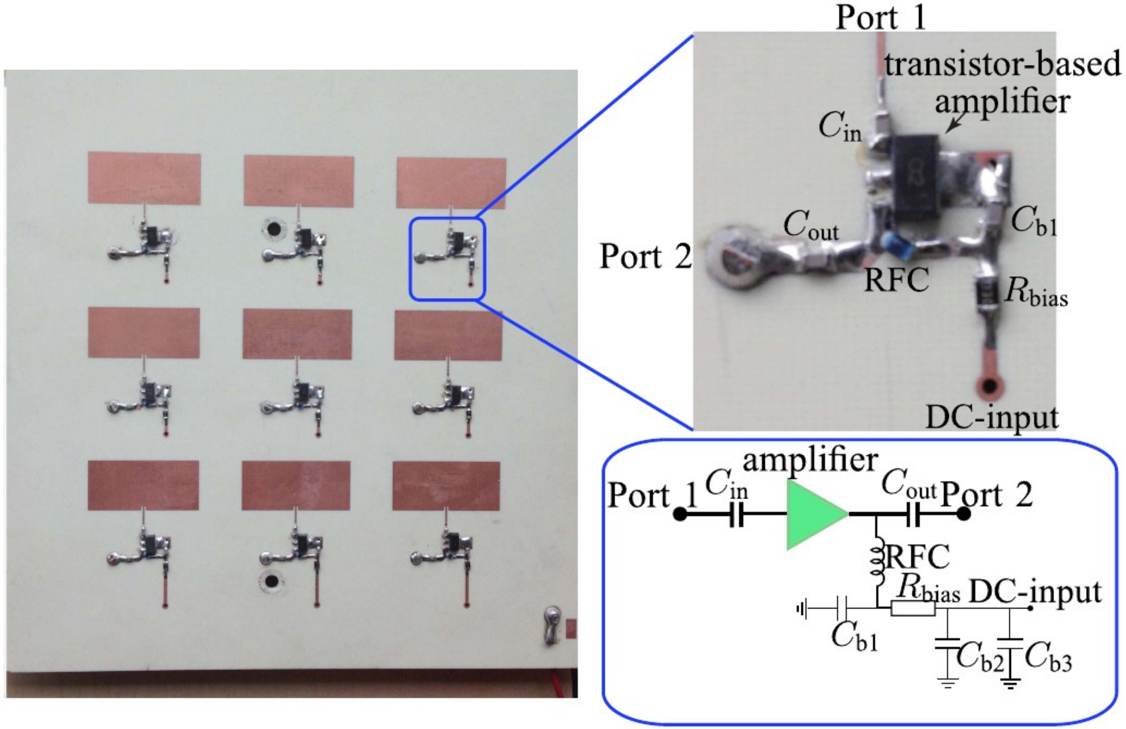


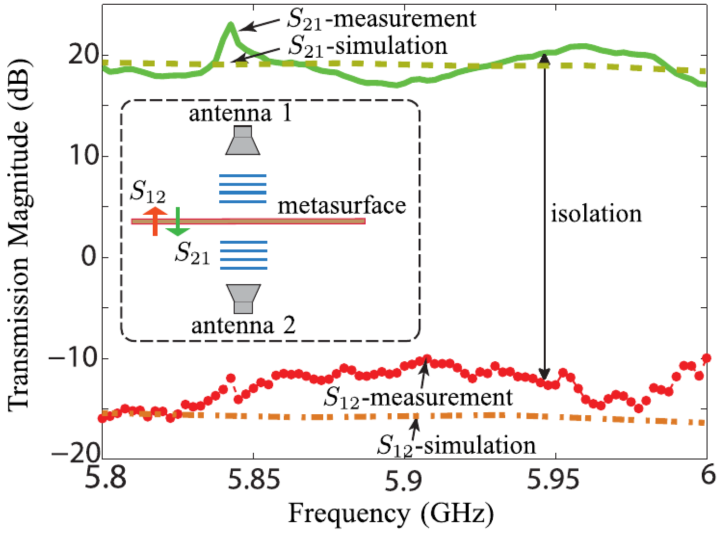




https://doi.org/10.1109/TAP.2017.2702712

Non-Reciprocity via Nonlinear Absorption





https://doi.org/10.1109/TAP.2017.2702712

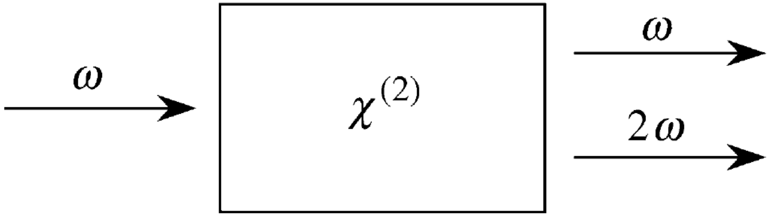
Nonlinear Second-Harmonic Generation

Medium polarization assuming nonlinear response

$$P(t) = \epsilon_0 \left[\chi^{(1)} E(t) + \chi^{(2)} E^2(t) + \dots\right] = P^{(1)}(t) + P^{(2)}(t) + \dots$$

Incident wave field

$$E(t) = E_0 e^{j\omega t} + \text{c.c.}$$

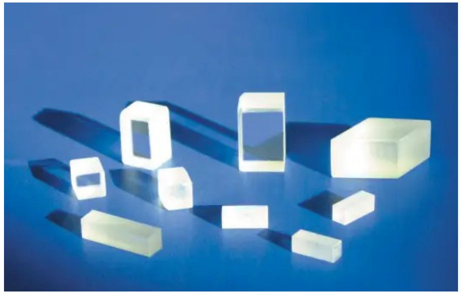


Resulting second-order polarization

$$P^{(2)}(t) = \epsilon_0 \chi^{(2)} \left[E_0 E_0^* + E_0^2 e^{j2\omega t} + \text{c.c.}\right]$$

Second-harmonic wave is produced

Example: Barium borate (BBO)



Boyd, "Nonlinear optics", 2008

Is Second-Harmonic Generation Non-Reciprocal ?

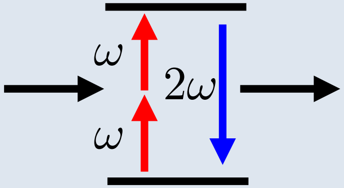
Generalized nonlinear scattering matrix

$$\bar{\bar{S}} = \begin{bmatrix} \bar{\bar{S}}^{\omega\to\omega} & \bar{\bar{S}}^{\omega\to 2\omega} \\ \bar{\bar{S}}^{2\omega\to\omega} & \bar{\bar{S}}^{2\omega\to 2\omega} \end{bmatrix}$$

$$S_{21}^{\omega\to 2\omega} \neq S_{12}^{2\omega\to\omega}$$

SHG

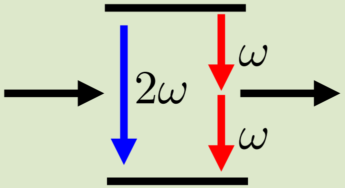
(Second-harmonic generation)



Classical

SPDC

(spontaneous down conversion)



Quantum

$$\eta_{\text{SHG}} \gg \eta_{\text{SPDC}}$$

Non-reciprocal !

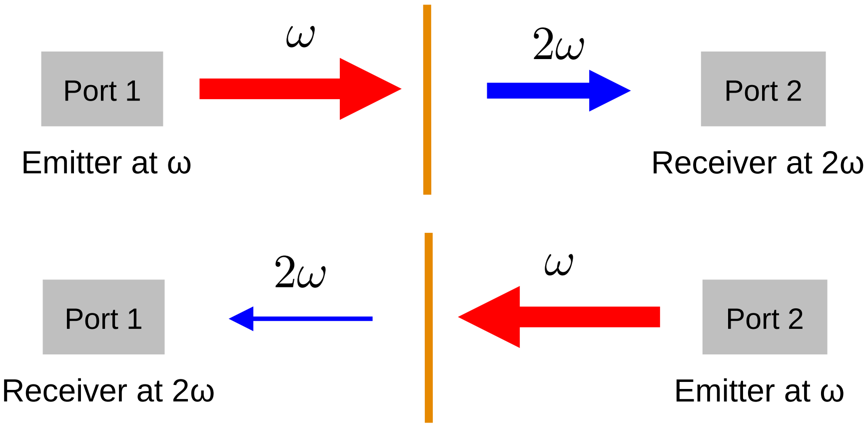
here the sub-matrices are of the form $\bar{\bar{S}}^{\omega\to\omega} = \begin{bmatrix} S_{11}^{\omega\to\omega} & S_{12}^{\omega\to\omega} \\ S_{21}^{\omega\to\omega} & S_{22}^{\omega\to\omega} \end{bmatrix}$

Is the reciprocity condition still valid?

$$\bar{\bar{S}} = \bar{\bar{S}}^{T}$$

Asymmetric SHG ≠ Non-reciprocity

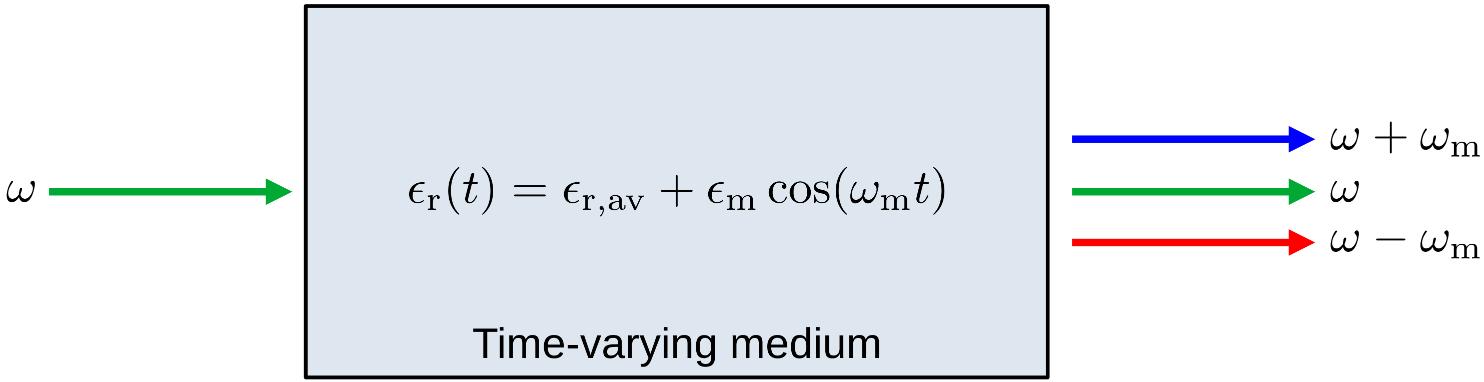
$$S_{21}^{\omega\to 2\omega} \neq S_{12}^{\omega\to 2\omega}$$



More details in https://doi.org/10.1109/TAP.2018.2863116

Non-Reciprocity via Time-Variations

Time-Varying Medium



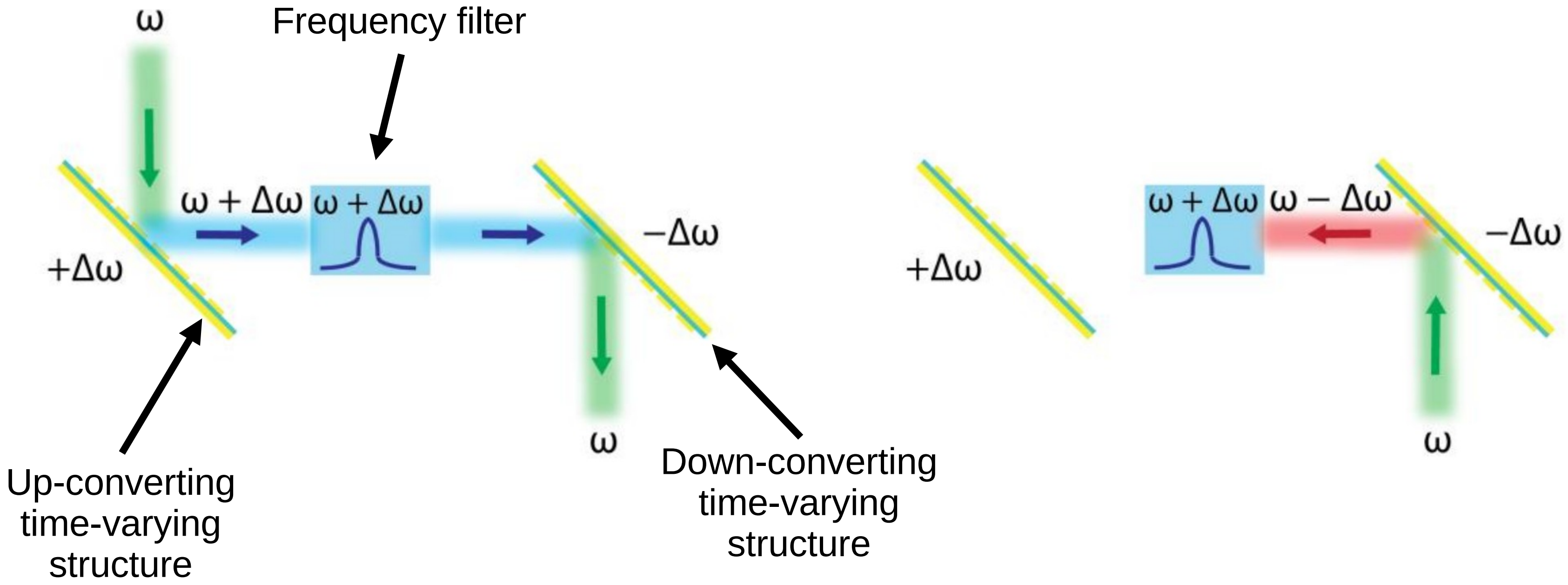
Scattered field contains all time harmonics

$$E(t) \propto \left[\epsilon_{\mathrm{r,av}} + \epsilon_{\mathrm{m}} \cos(\omega_{\mathrm{m}} t)\right] e^{j\omega t} = \epsilon_{\mathrm{r,av}} e^{j\omega t} + \frac{\epsilon_{\mathrm{m}}}{2} e^{j(\omega - \omega_{\mathrm{m}})t} + \frac{\epsilon_{\mathrm{m}}}{2} e^{j(\omega + \omega_{\mathrm{m}})t}$$

$$\vdots$$

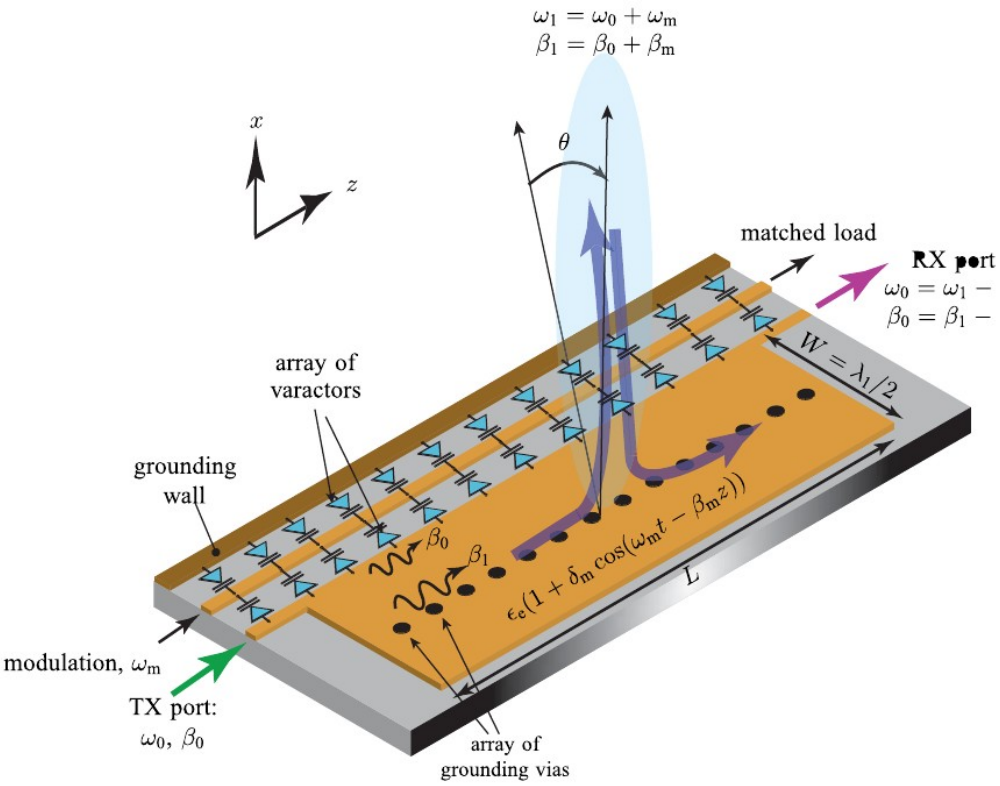
$$E(t) \propto \sum_{n} e^{j(\omega + n\omega_{\mathrm{m}})t}$$

Example of Non-Reciprocal Time-Varying System



https://doi.org/10.1364/OME.5.002459

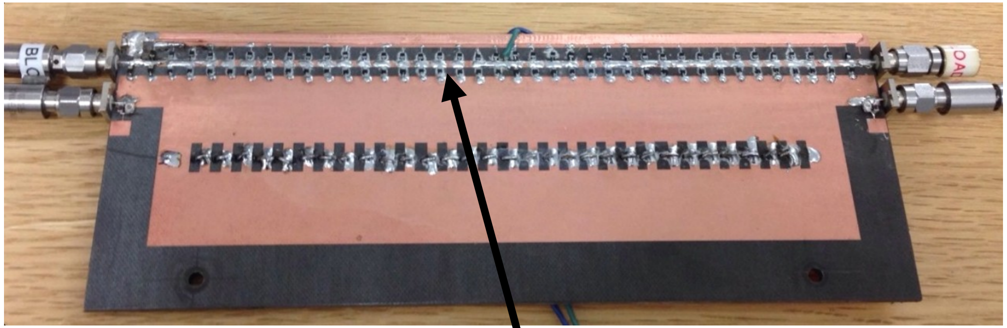
Example of a Time-Varying System



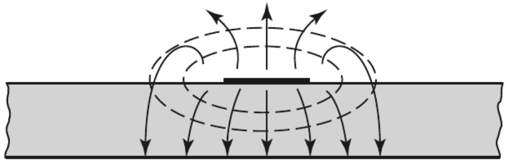
Varactors: voltage-dependent capacitance

https://doi.org/10.1109/TAP.2016.2632735

Space and time varying transmission line

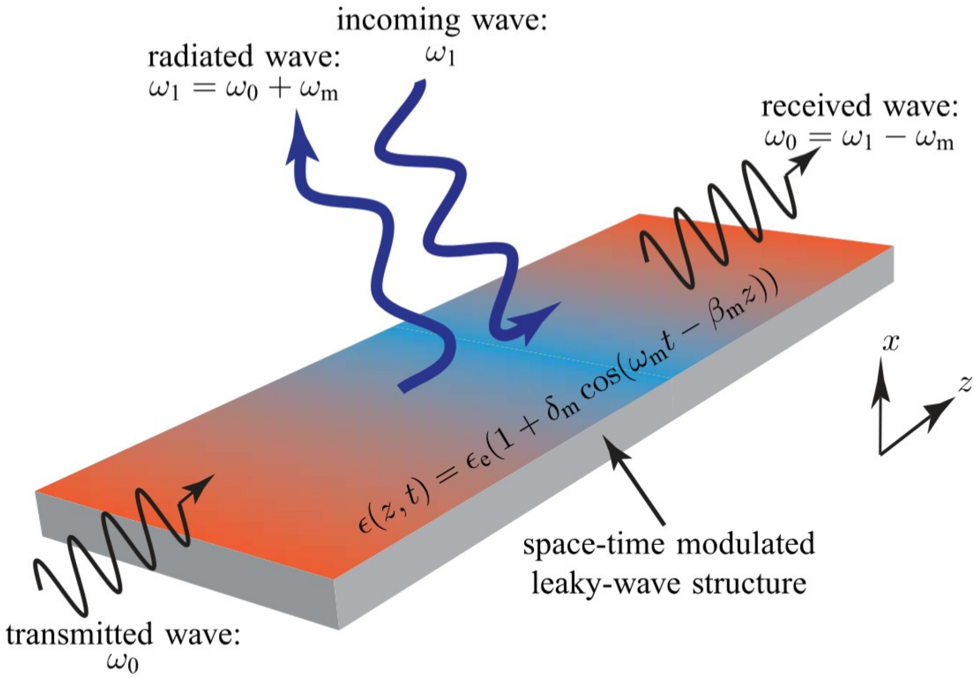


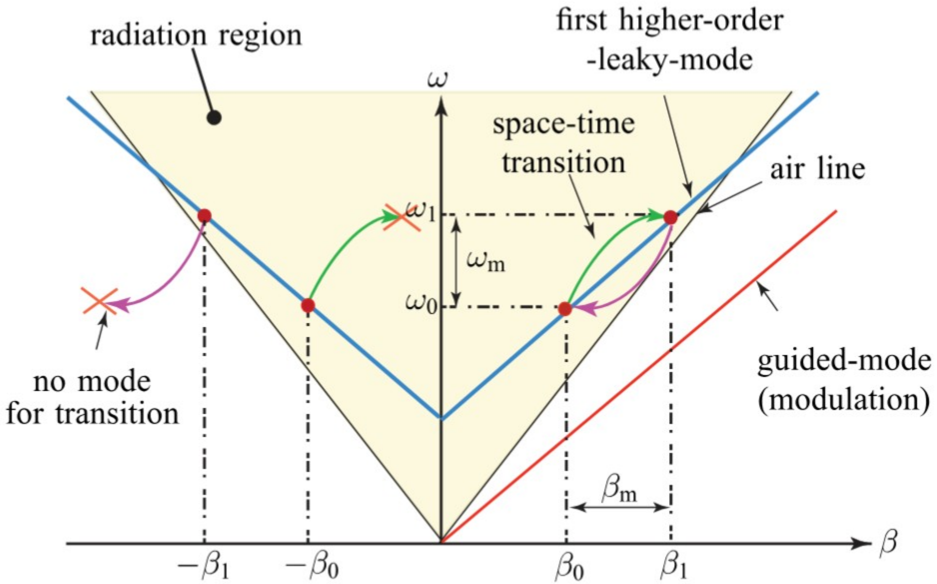
Microstrip transmission line



Pozar,"Microwave engineering: theory and techniques", 2021

Non-Reciprocal Leaky Wave Antenna





https://doi.org/10.1109/TAP.2016.2632735

What Have We Learned So Far….

- There are different ways to break reciprocity. In a LTI system, this can be achieved only by biasing the system with a time-odd quantity such as a magnetic field (e.g., Faraday isolator)
- Nonlinearity may be used to break reciprocity by creating different absorption/reflection at the interface between two media.
- In electronics, diodes may intuitively be seen as non-reciprocal devices but that depends on how they are used. Transistors are better non-reciprocal devices than diodes.
- In optics, second-harmonic generation (SHG) is intrinsically non-reciprocal. Asymmetric SHG should not be confused with non-reciprocal SHG.
- Time-modulations can also be exploited to create non-reciprocal devices. The principle works on a similar basis as harmonic generation in nonlinear structures.