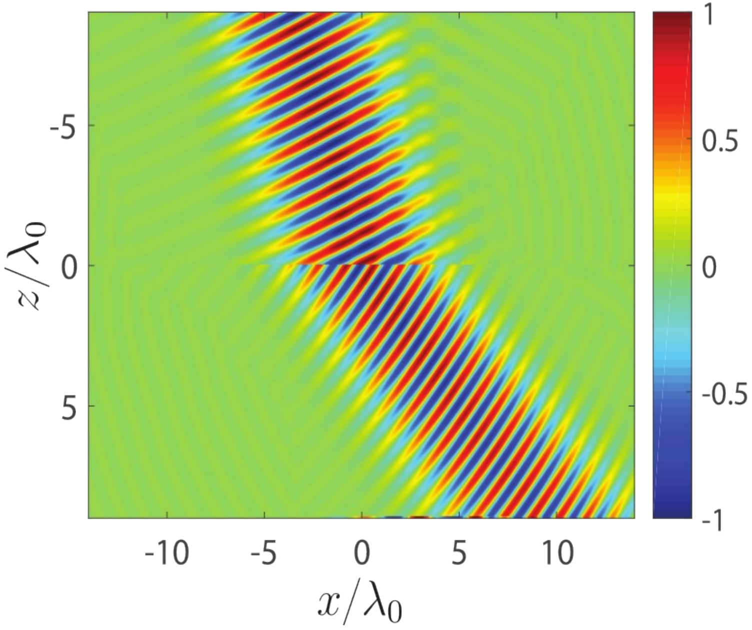
Lecture 14

Wave Control Strategies

Controlling the Phase is Important

Most metasurface wave transformations are based on controlling the phase

Refraction



Transfer function

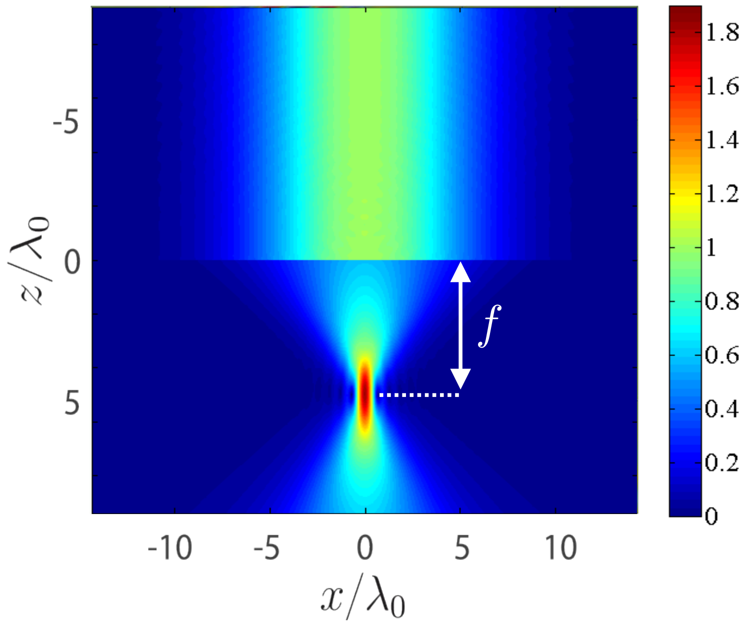
$$T = e^{-j(k_{x,\mathrm{t}} - k_{x,\mathrm{i}})x}$$

phase of scattered wave

$$T = \frac{\Phi_\mathrm{s}}{\Phi_\mathrm{i}}$$

phase of incident wave

Collimation



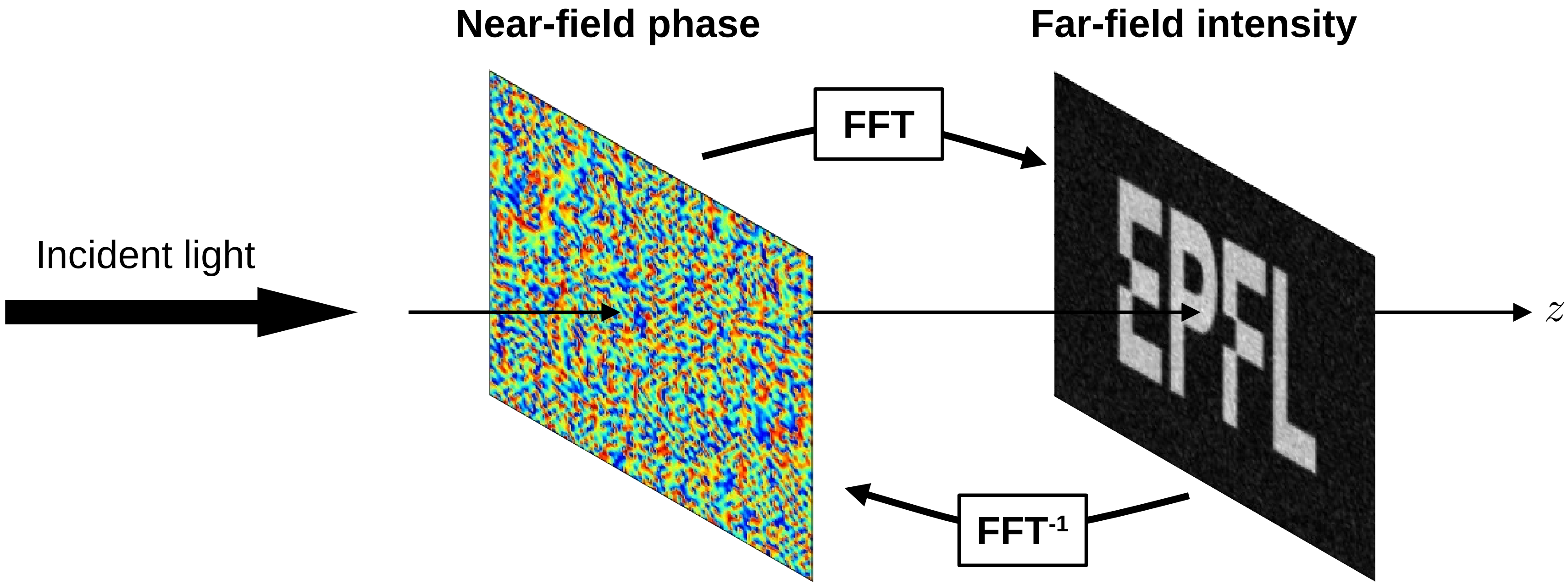
Transfer function

$$T = e^{-jk(\sqrt{x^2+y^2+f^2}-f)}$$

While approximate, this approach gives in practice decently good results.

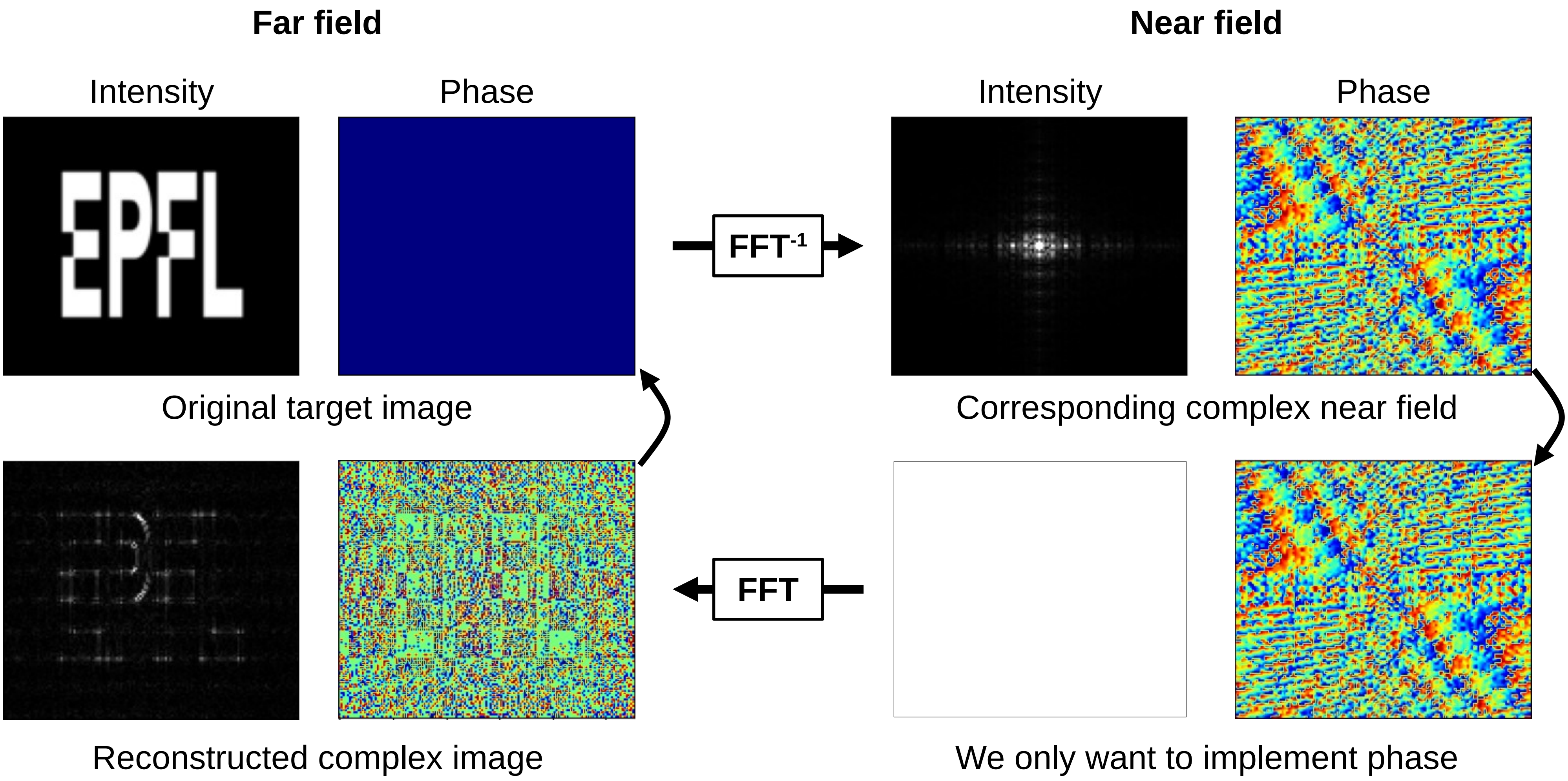
Holography

Holography Concept

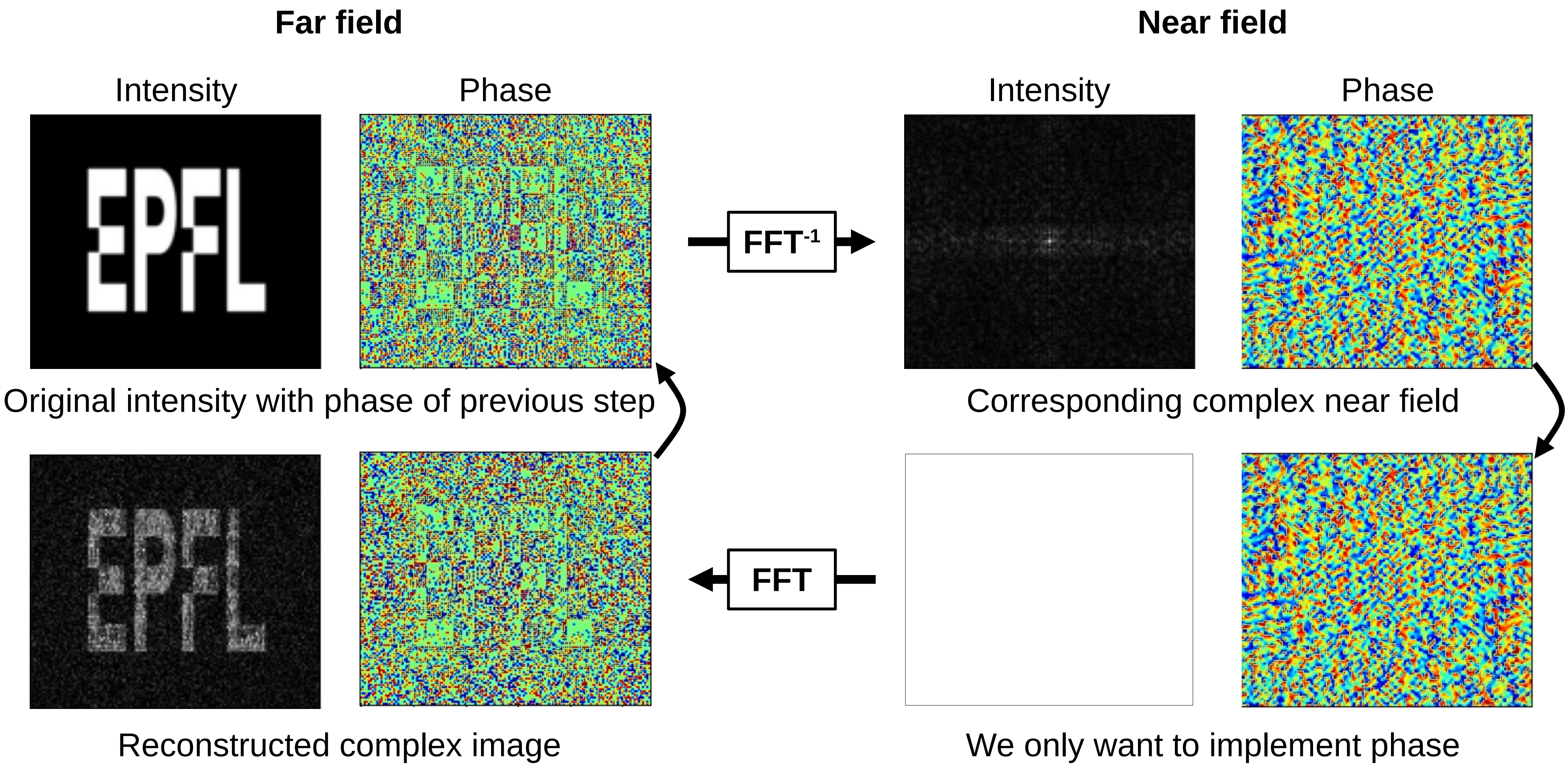


We know that the far-field of an aperture is proportional to the Fourier transform of the aperture field. We can therefore specify a target far-field intensity and determine the required near-field phase by using an inverse Fourier transform.

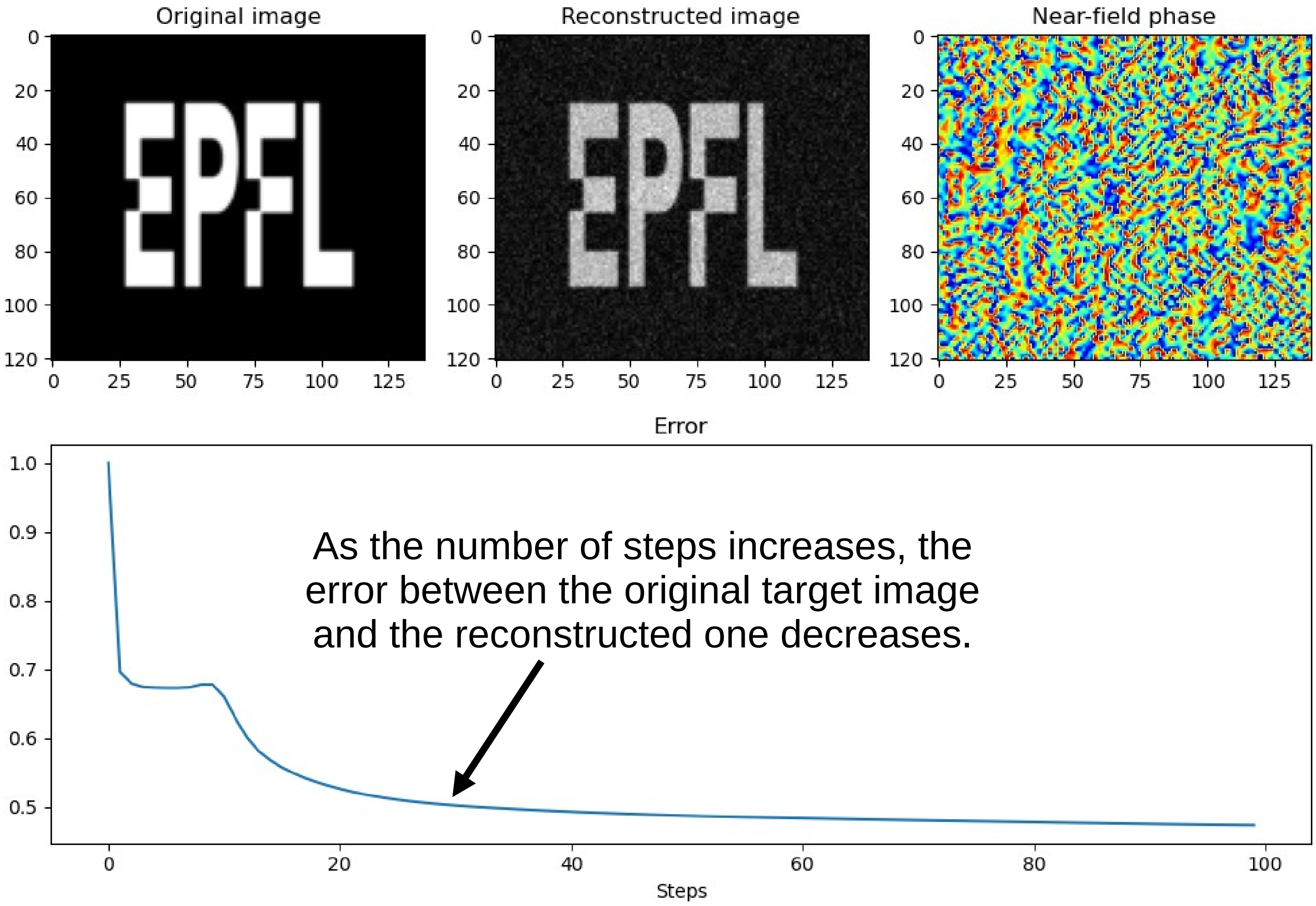
Gerchberg-Saxton Algorithm – First Step



Gerchberg-Saxton Algorithm – Second Step



Gerchberg-Saxton Algorithm – 100th Step



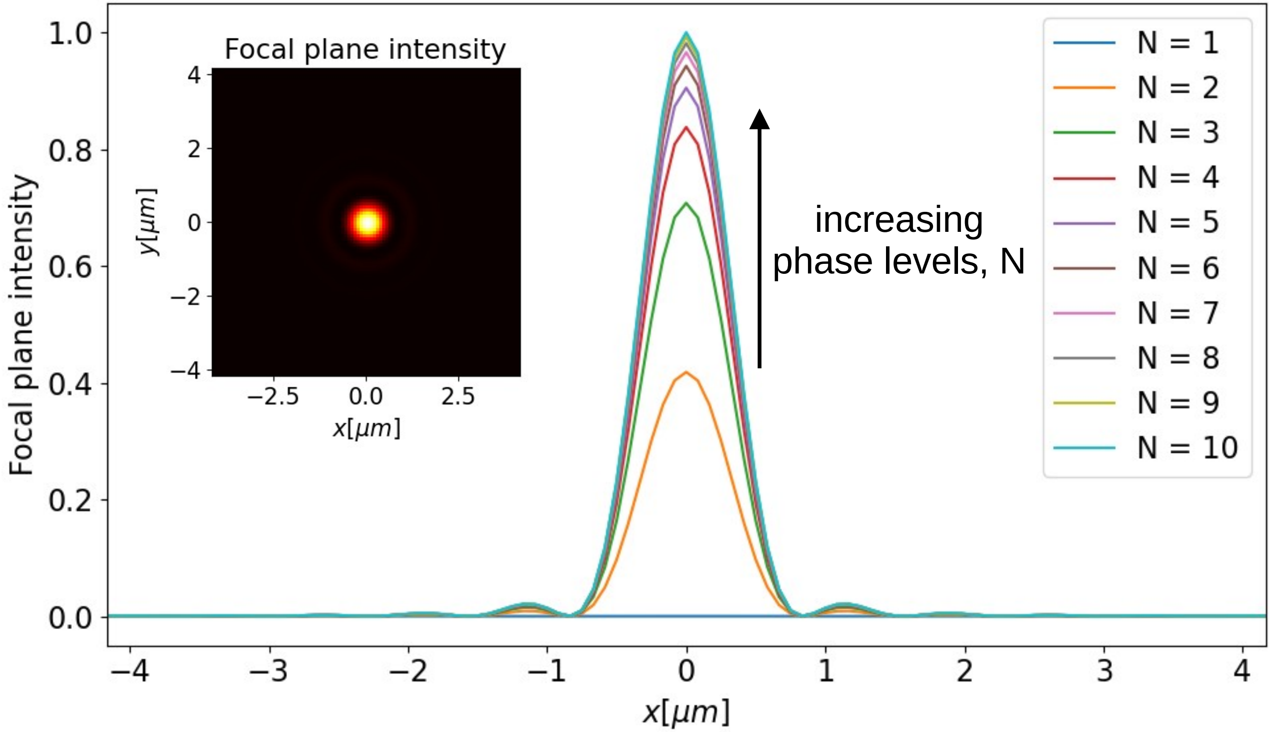
Python Implementation of the Gerchberg-Saxton Algorithm

```
from numpy import *
from matplotlib.pyplot import *
from PIL import Image

Nstep = 100
filename = 'EPFL.png'

Im = asarray(Image.open(filename).convert('L'))
A = fft.fftshift(fft.ifft2(fft.fftshift(Im)))
err = zeros(Nstep)
for n in range(Nstep):
    B = exp(1j*angle(A))                              # near-field phase
    C = fft.fftshift(fft.fft2(fft.ifftshift(B)))   # far-field image
    D = abs(Im)*exp(1j*angle(C))                      # update far-field phase
    A = fft.fftshift(fft.ifft2(fft.ifftshift(D)))  # new near field
    err[n] = sum(abs(abs(C) - abs(Im)))

figure(figsize=(10,7))
subplot(231)
imshow(Im,cmap='gray')
title('Original image')
subplot(232)
imshow(abs(C),cmap='gray')
title('Reconstructed image')
subplot(233)
imshow(angle(A),cmap='jet')
title('Near-field phase')
subplot(212)
plot(err/max(err))
xlabel('Steps')
title('Error')
tight_layout()
show()
```

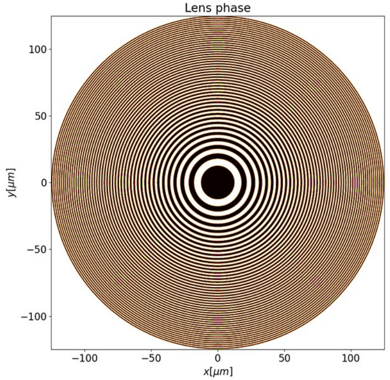
Implementation of Phase Controlling Structures

Phase Discretization

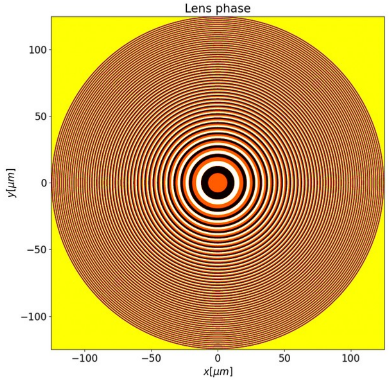
Wavelength: 632 nm, lens diameter: 250 µm, focal length: 250 µm



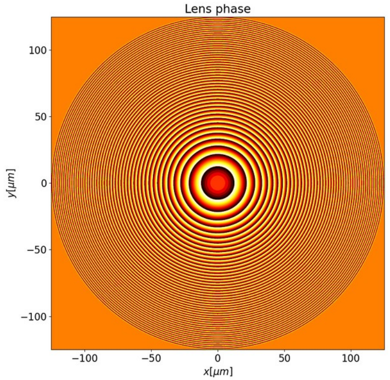
$N = 2$



$N = 3$



$N = 10$

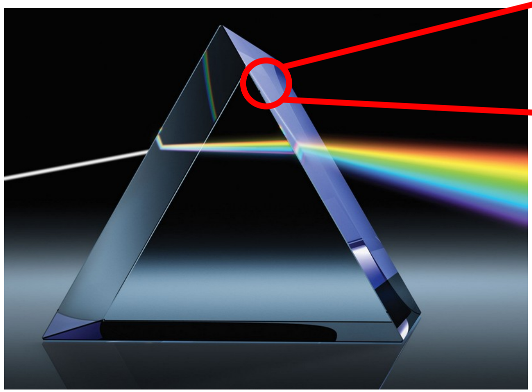


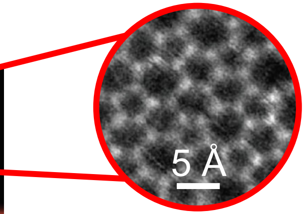
We need at least 5 phase levels to achieve 90% of maximum efficiency

Resolution limit: $R \approx 1.22\lambda\frac{f}{D}$

Metasurface Electromagnetic Properties

Conventional Material



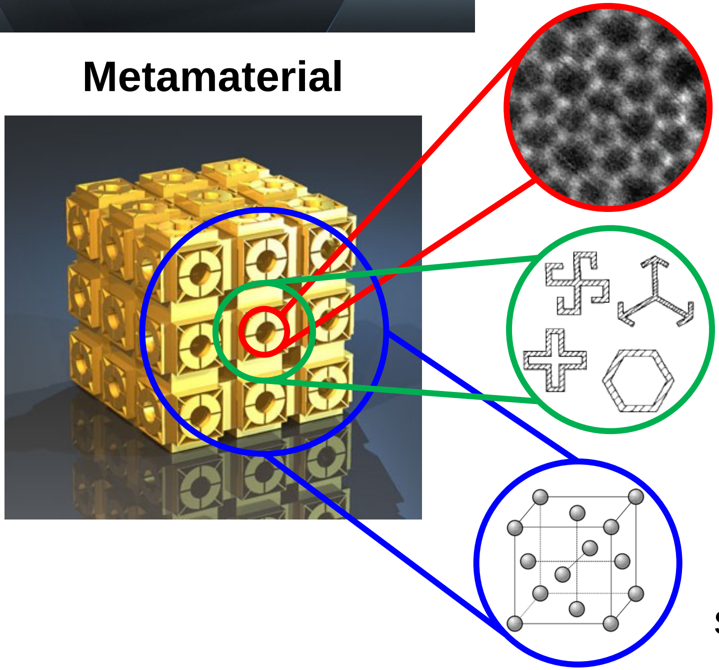


Amorphous silica (TEM)

Electromagnetic properties
due to chemical composition

Electromagnetic properties due to chemical composition,
shape of scatters and lattice arrangement

Metamaterial



Material

size < λ/1000

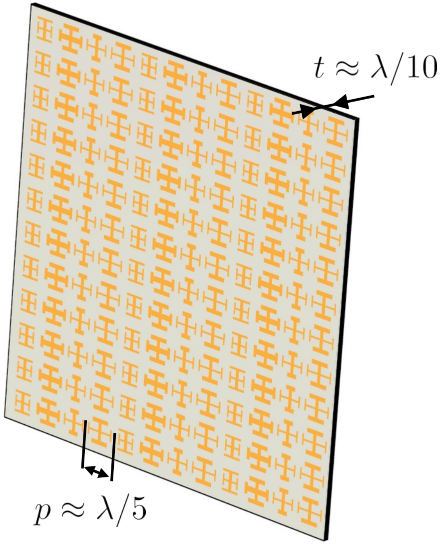
Shape

size < λ/5

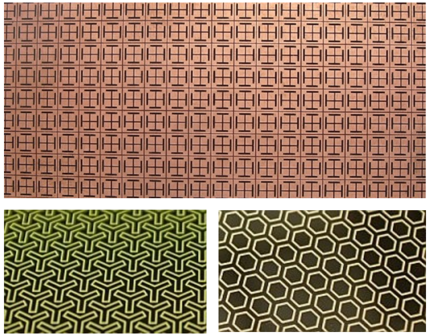
Lattice

size < λ/2

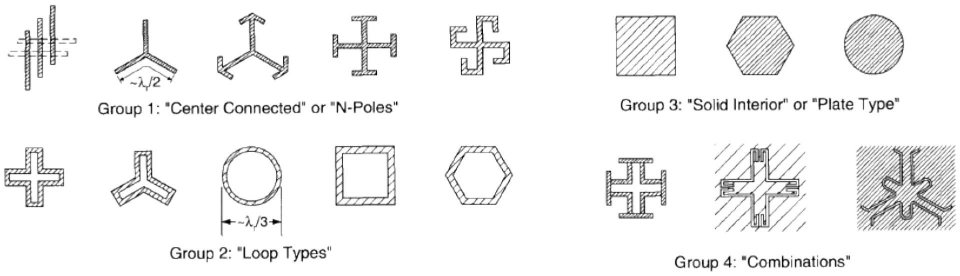
Metasurface



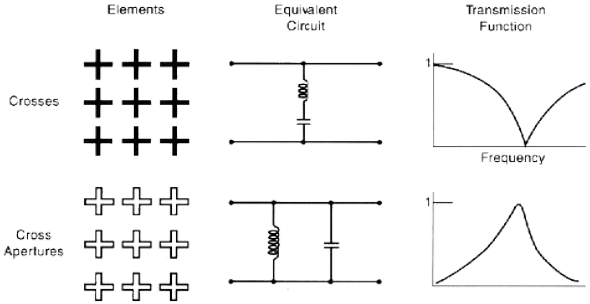
Controlling the Phase is Important



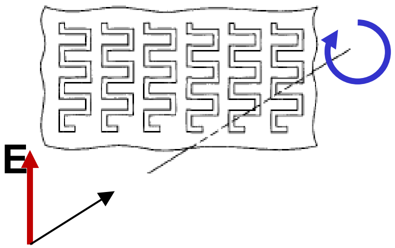
Scatterer Types



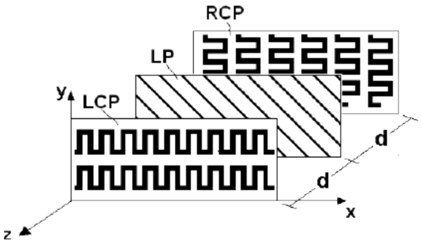
Spatial Filtering of Temporal Frequencies



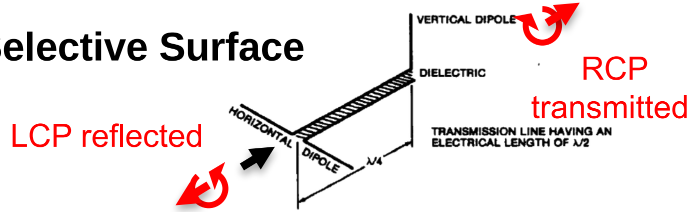
Polarization Transformers



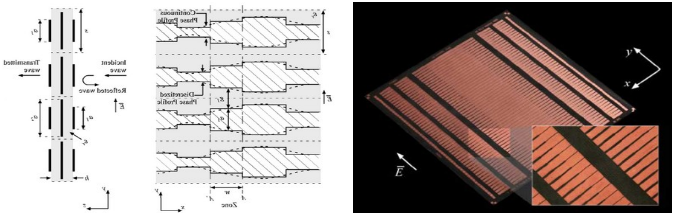
Multilayer BW Enhancement



CP Selective Surface



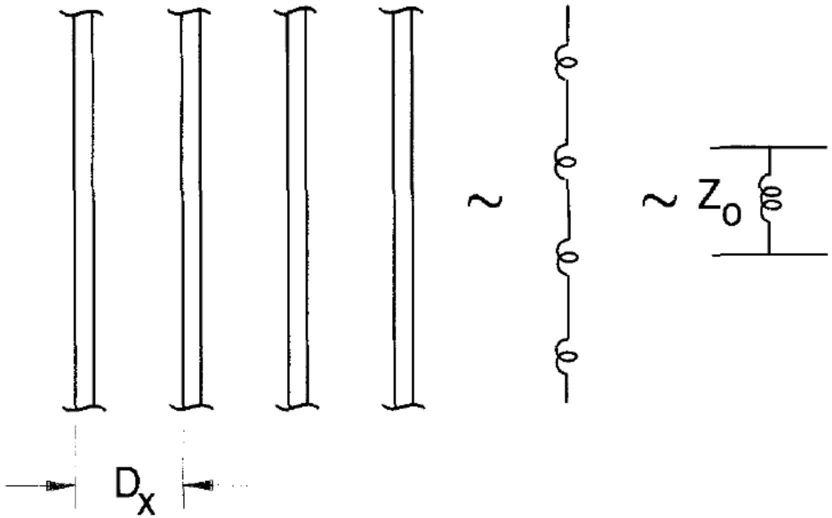
Beamforming Transmit Array



B. A. Munk, Frequency Selective Surfaces: Theory and Design, Wiley-Interscience, 2000.

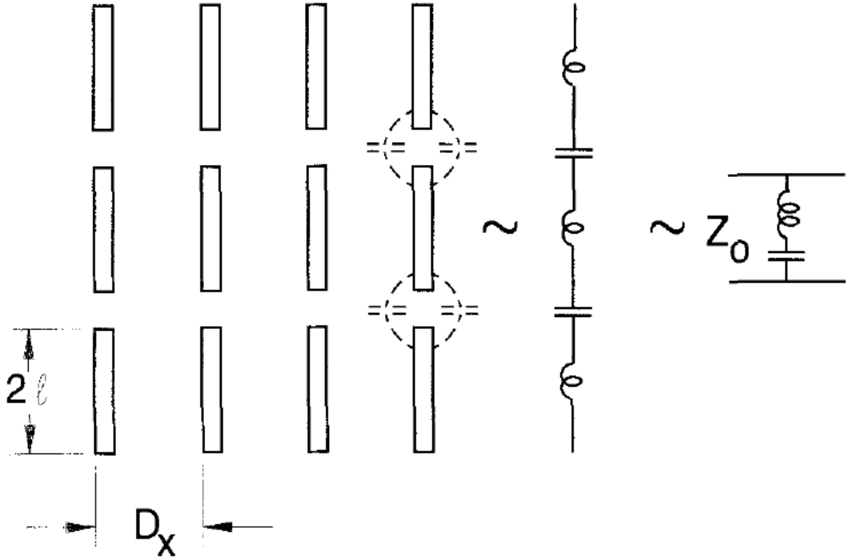
A Note About Resonance

Array of long metallic strips



Modeled as an effective
inductance => does not resonate

Array of metallic patches



Modeled as an effective
LC circuit => resonates

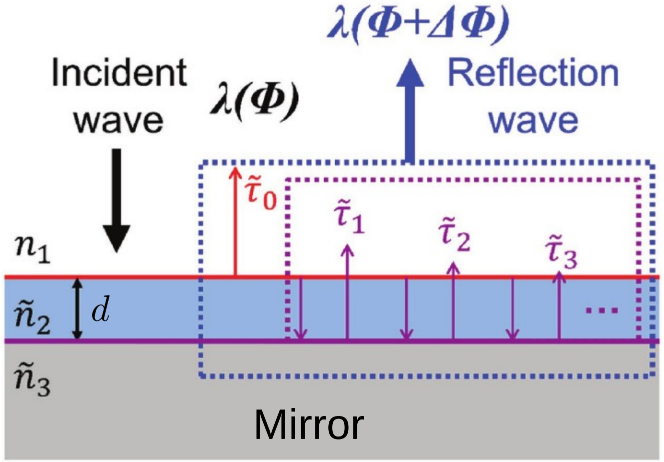
B. A. Munk, Frequency Selective Surfaces: Theory and Design, Wiley-Interscience, 2000.

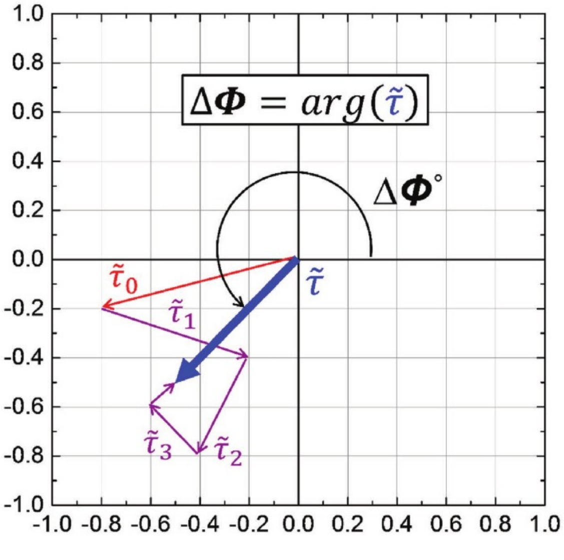
What Have We Learned So Far….

- Most metasurfaces are designed to control the phase of the light.
- Indeed, most metasurface applications consist in implementing transfer functions based on two-dimensional phase shifts.
- One of the most well-known applications is that of holography that can realized with the Gerchberg-Saxton algorithm.
- To implement a metasurface, one must discretize its transfer function. The more discretization levels, the smaller the spatial variations of the transfer function that can be implemented and the higher the efficiency. However, the smaller the unit-cells, the less they interact with the fields and the harder it is to realize them.
- To implement a unit-cell, there are three levels of control: 1) material, 2) shape and 3) lattice.
- Disconnected unit-cells resonate like LC resonators, whereas connected unit-cells typically do no resonate.
- Resonating unit-cells tend to be smaller than $\lambda/2$ due to the capacitive coupling in the array.

Gires-Tournois Effect

Gires-Tournois Effect





reflection coefficient

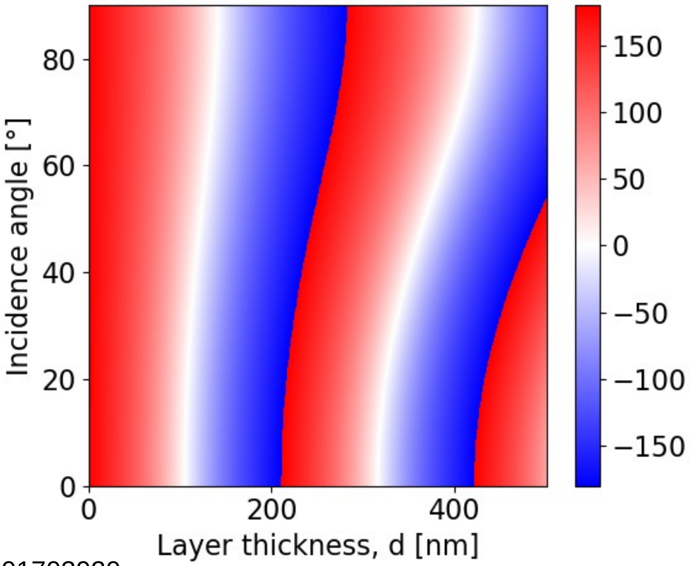
$$r = \frac{r_{12} + r_{23}e^{-j2\Psi}}{1 + r_{12}r_{23}e^{-j2\Psi}}$$

$$\Psi = k_0 n_2 d \cos\theta_2$$

θ_2: refraction angle into n_2

<u>Example</u>: $n_2 = 1.5$, $r_{23} = -1$

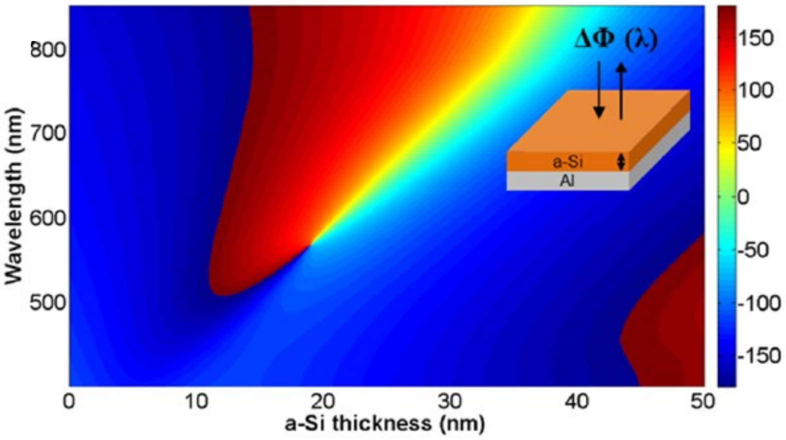
Reflection phase



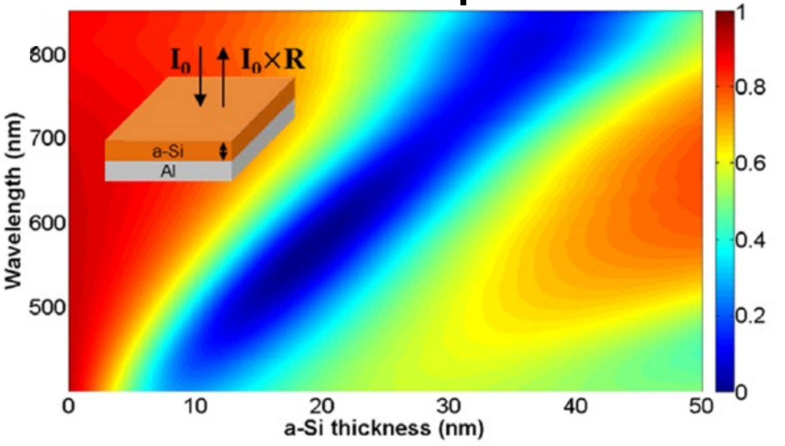
https://doi.org/10.1002/smll.201703920

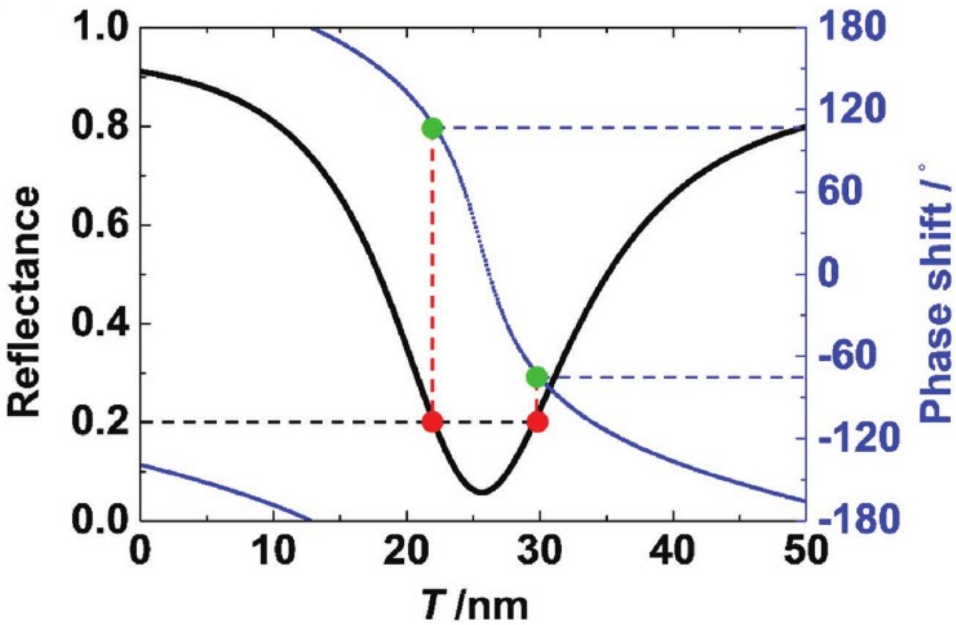
Gires-Tournois Effect with Subwavelength Thickness

Reflection phase



Reflection amplitude

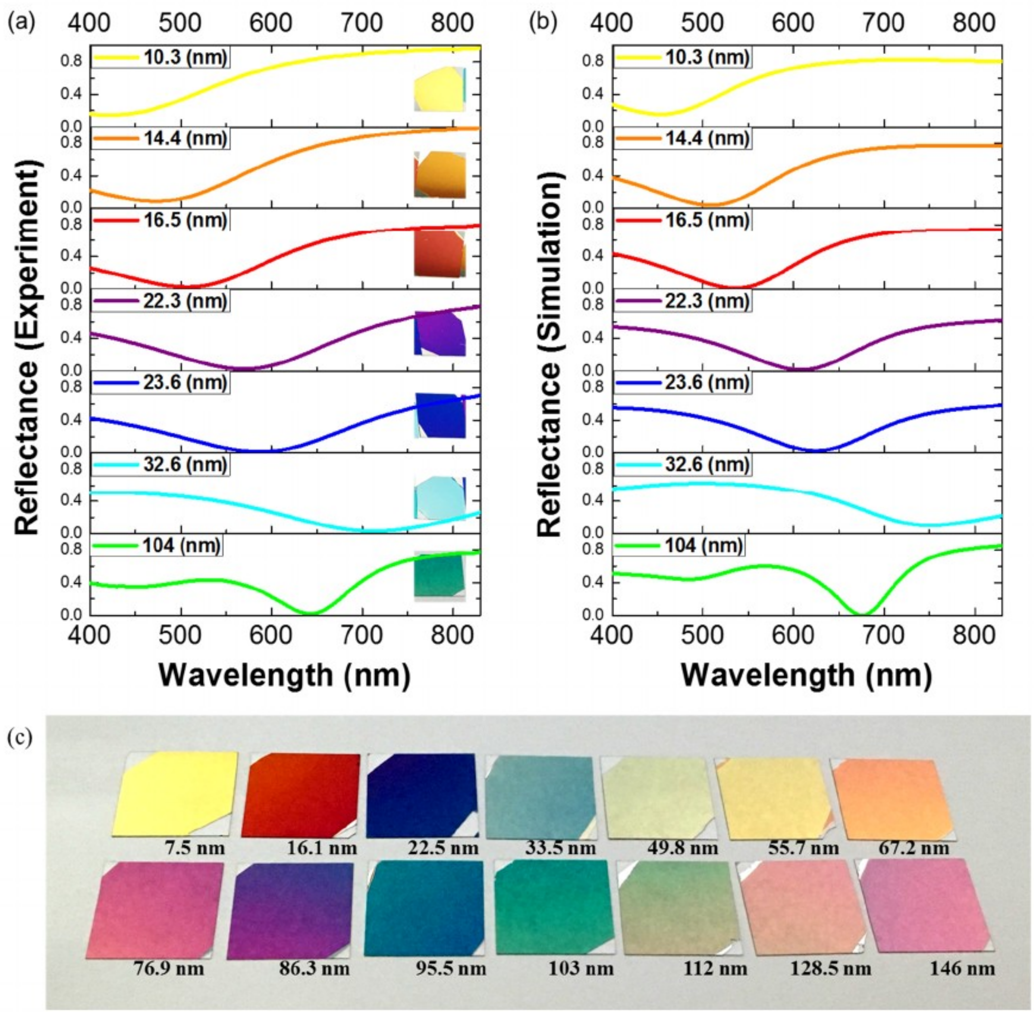




We select two thicknesses with the same reflectance and π-phase shift difference to create binary phase structures.

https://doi.org/10.1002/smll.201703920

Color Generation via Film Thickness



https://doi.org/10.1002/smll.201703920

Binary-Phase Reflective Lens

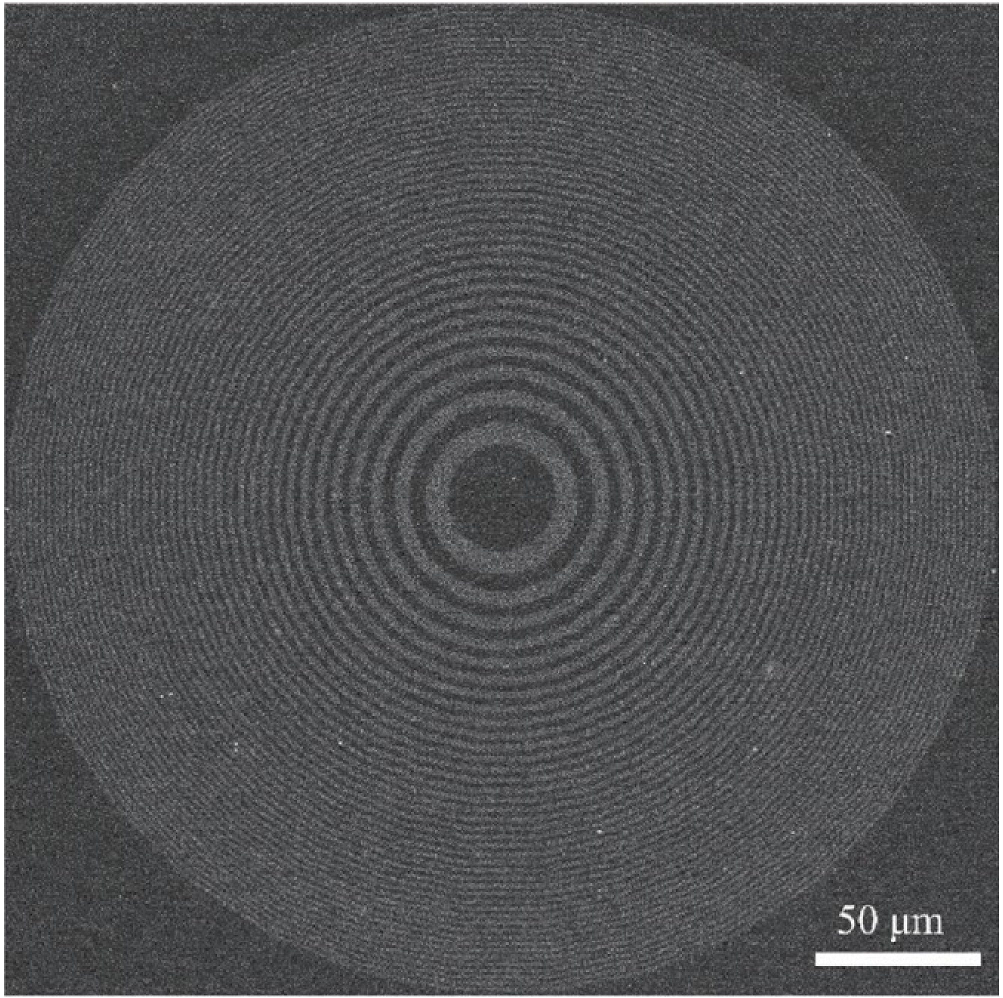
Transfer function

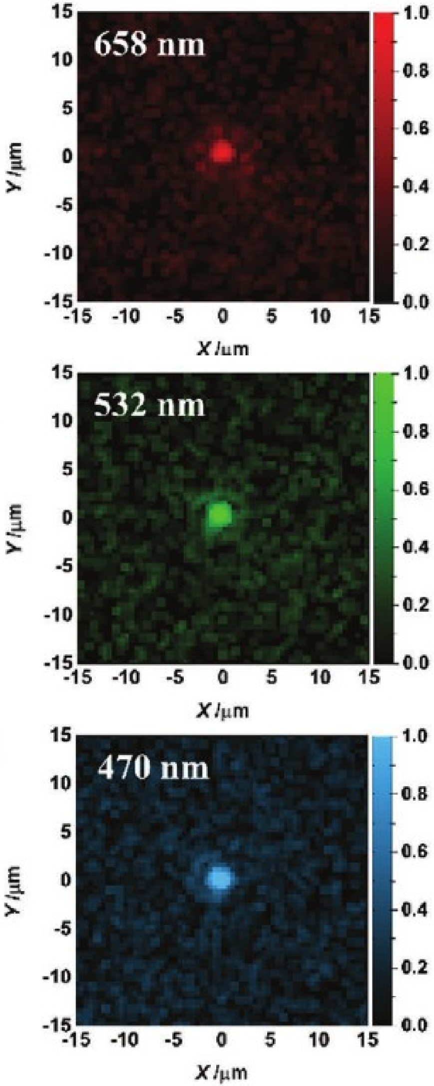
$$T = e^{-jk(\sqrt{x^2+y^2+f^2}-f)}$$

Designed wavelength = 658 nm

Diameter = 300 μm

Focal length = 400 μm

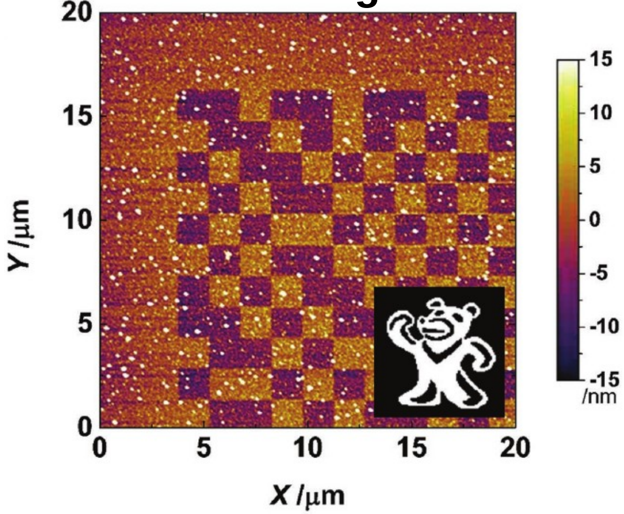


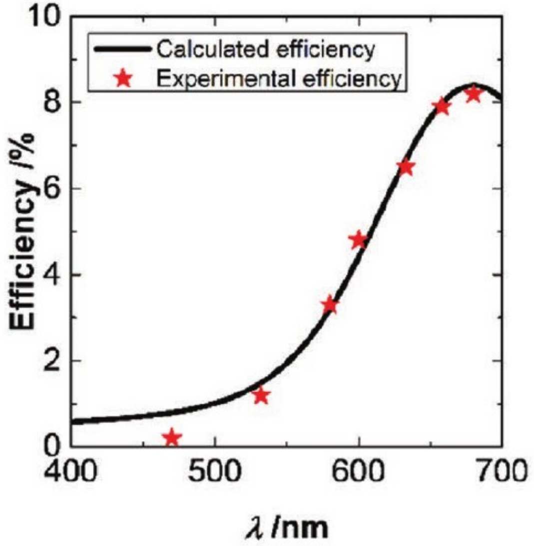


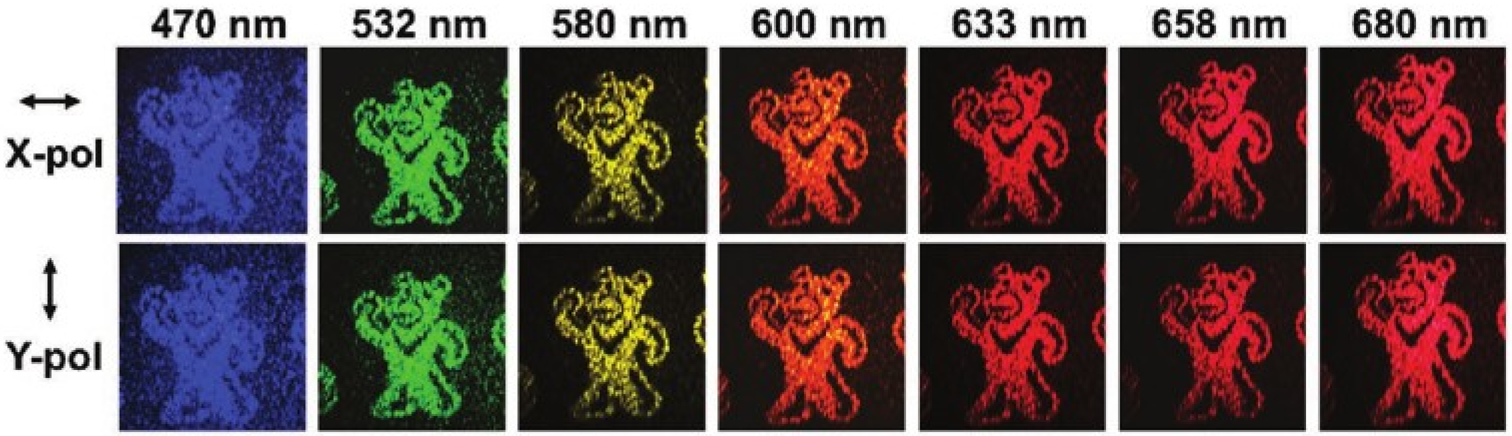
https://doi.org/10.1002/smll.201703920

Binary-Phase Hologram

AFM image







https://doi.org/10.1002/smll.201703920

This is not a New Concept

Reflectarray made of open-ended waveguides of various length in 1963

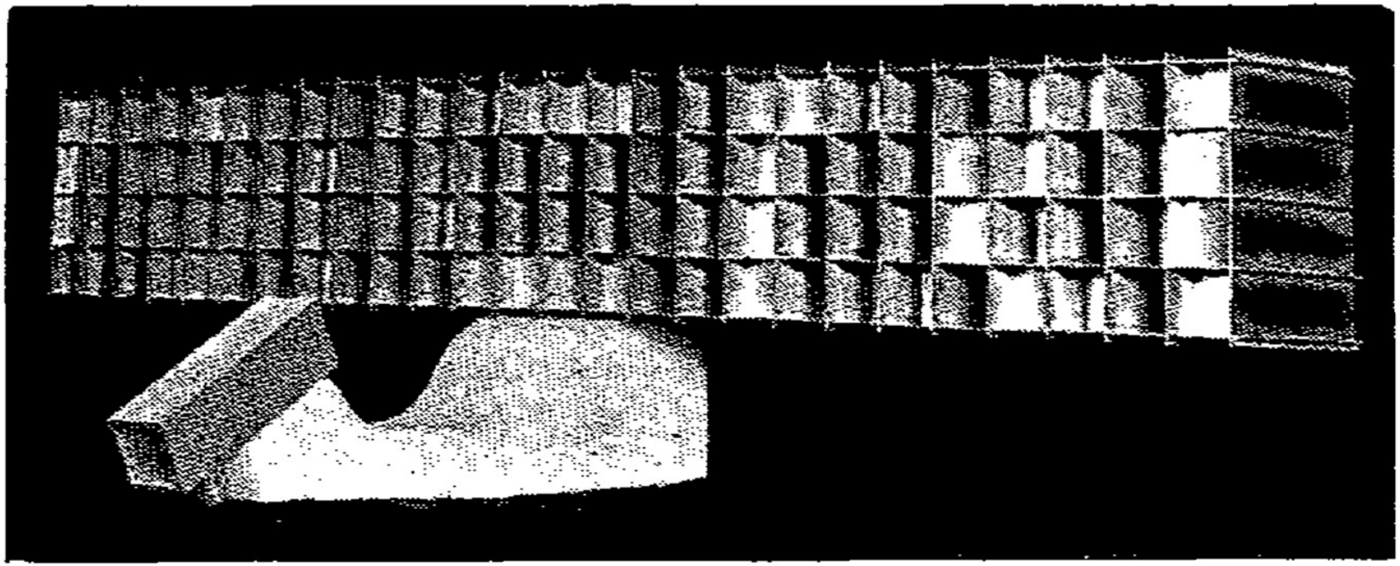
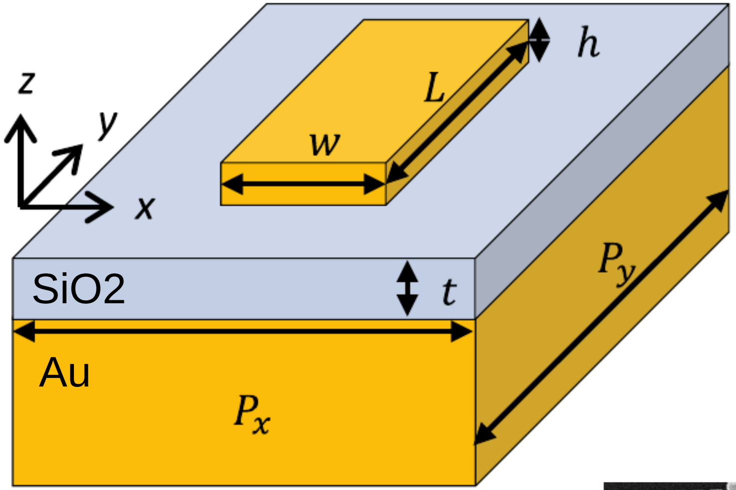


Fig. 3—Experimental model of the waveguide array type Reflectarray.

https://doi.org/10.1109/TAP.1963.1138112

Refractive Metasurface without Changing the Height

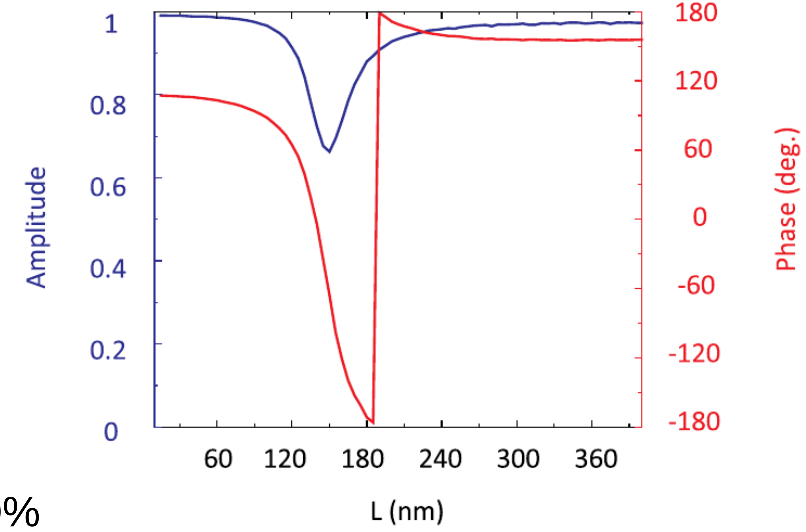
Tuning the size of the gold patch leads to an effect similar as tuning the thickness of the SiO_2 layer



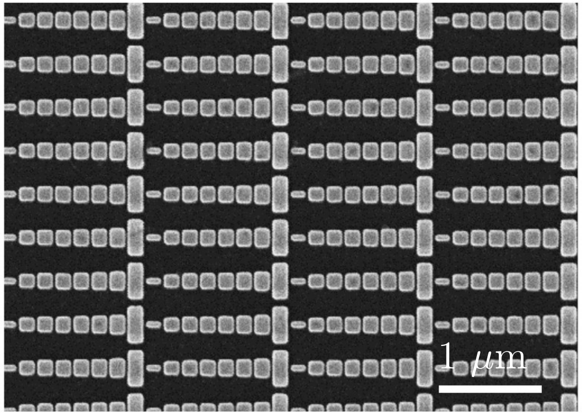
Parameters

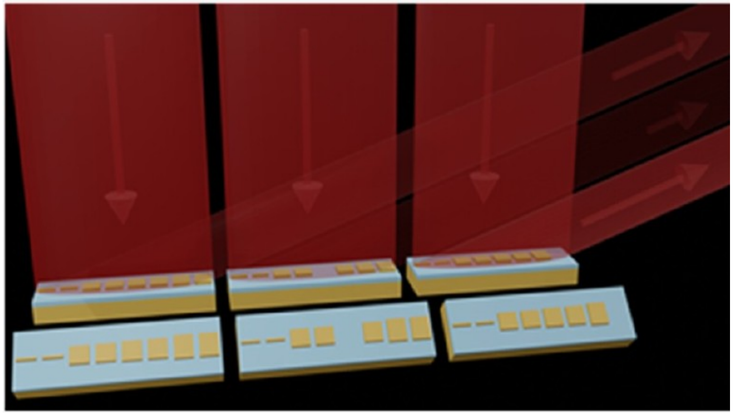
$$\lambda = 980 \text{ nm}$$
$$h = 30 \text{ nm}$$
$$t = 40 \text{ nm}$$
$$w = 140 \text{ nm}$$
$$P_x = 180 \text{ nm}$$
$$P_y = 430 \text{ nm}$$

Unit-cell reflection amplitude and phase



Efficiency ~60%





https://doi.org/10.1021/acsphotonics.2c00563

What Have We Learned So Far….

- One of the simplest way to realize 2π phase shift is to use reflective layers of various heights.
- This approach is however not easy to implement in the optical regime due to the limitations of nano-fabrication techniques.
- In optics, this approach can be used to implement binary phase structures, which necessarily implies low efficiency.
- Besides realizing phase shifts, it is an interesting method for generating colors.
- Instead of changing the height of the layer, it is possible to tune its effective response by changing the lateral dimensions of a metallic patch on top of it. This allows creating more discretized phase levels.

Electric and Magnetic Phase Control

Achieving 2π Phase Shift

Scattering parameters

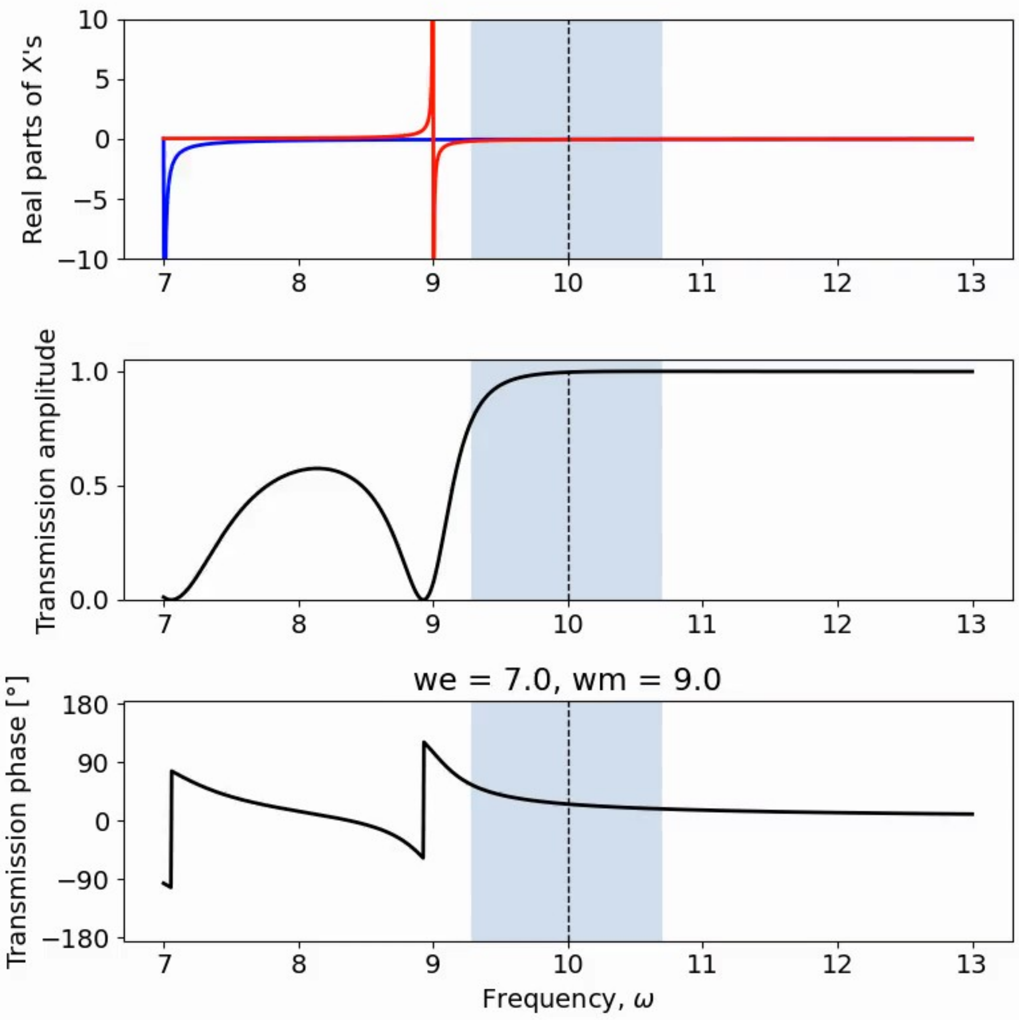
$$t = 1 - f_\mathrm{m} - f_\mathrm{e}$$

$$r = f_\mathrm{m} - f_\mathrm{e}$$

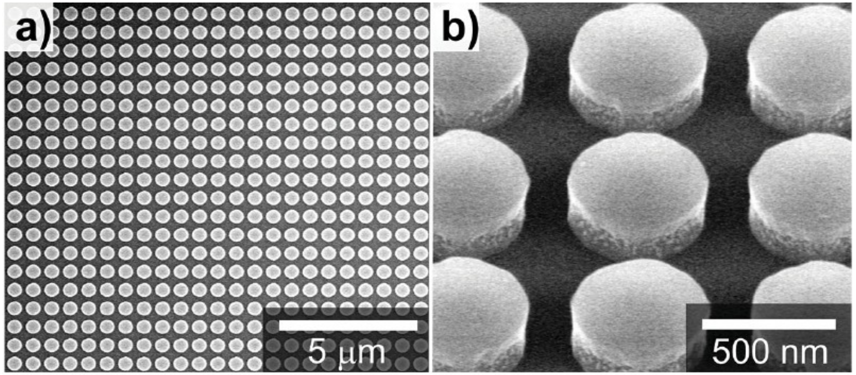
$$f_\mathrm{e} = \frac{A_\mathrm{e}k}{A_\mathrm{e}k - 2\gamma_\mathrm{e}\omega + 2j(\omega - \omega_\mathrm{e})(\omega + \omega_\mathrm{e})}$$

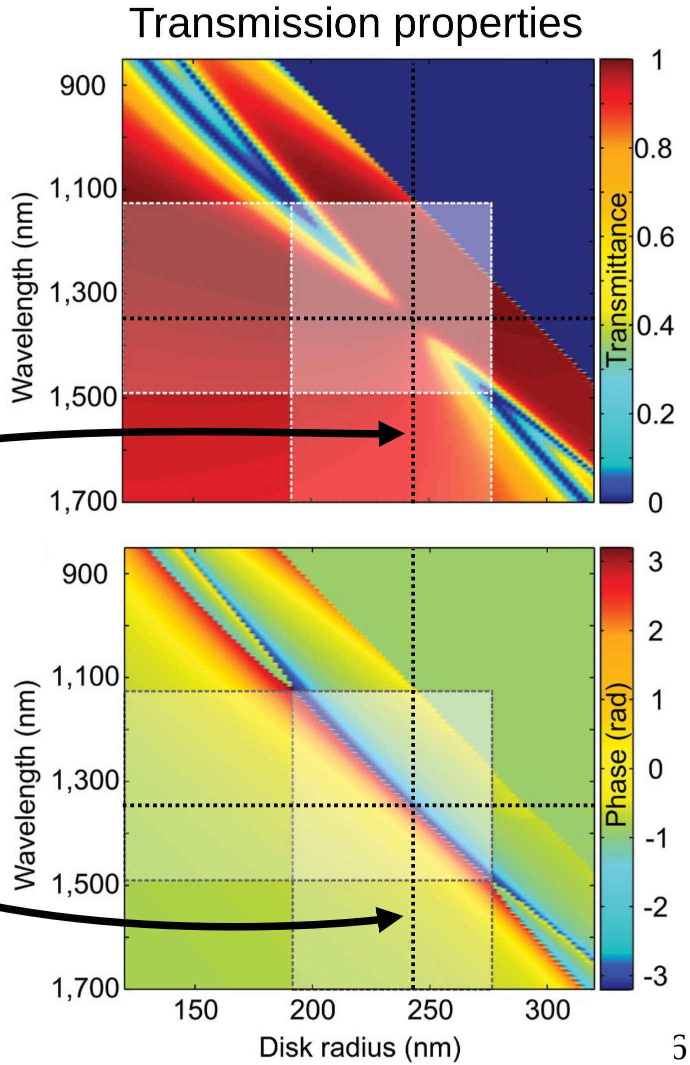
$$f_\mathrm{m} = \frac{A_\mathrm{m}k}{A_\mathrm{m}k - 2\gamma_\mathrm{m}\omega + 2j(\omega - \omega_\mathrm{m})(\omega + \omega_\mathrm{m})}$$

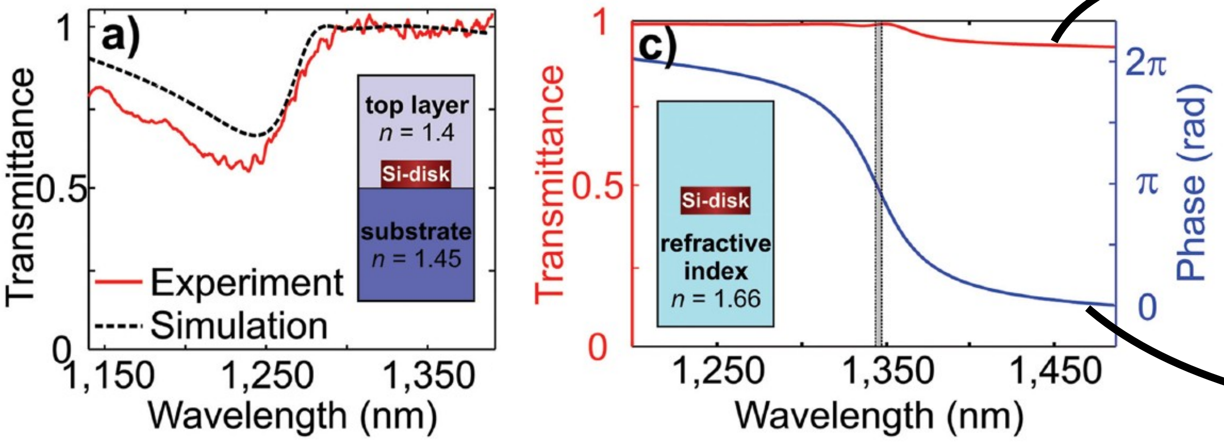
When the electric and magnetic resonances are close enough their interaction leads to a 2π phase shift



2π Phase Shift in Transmission with Dielectric Metasurfaces

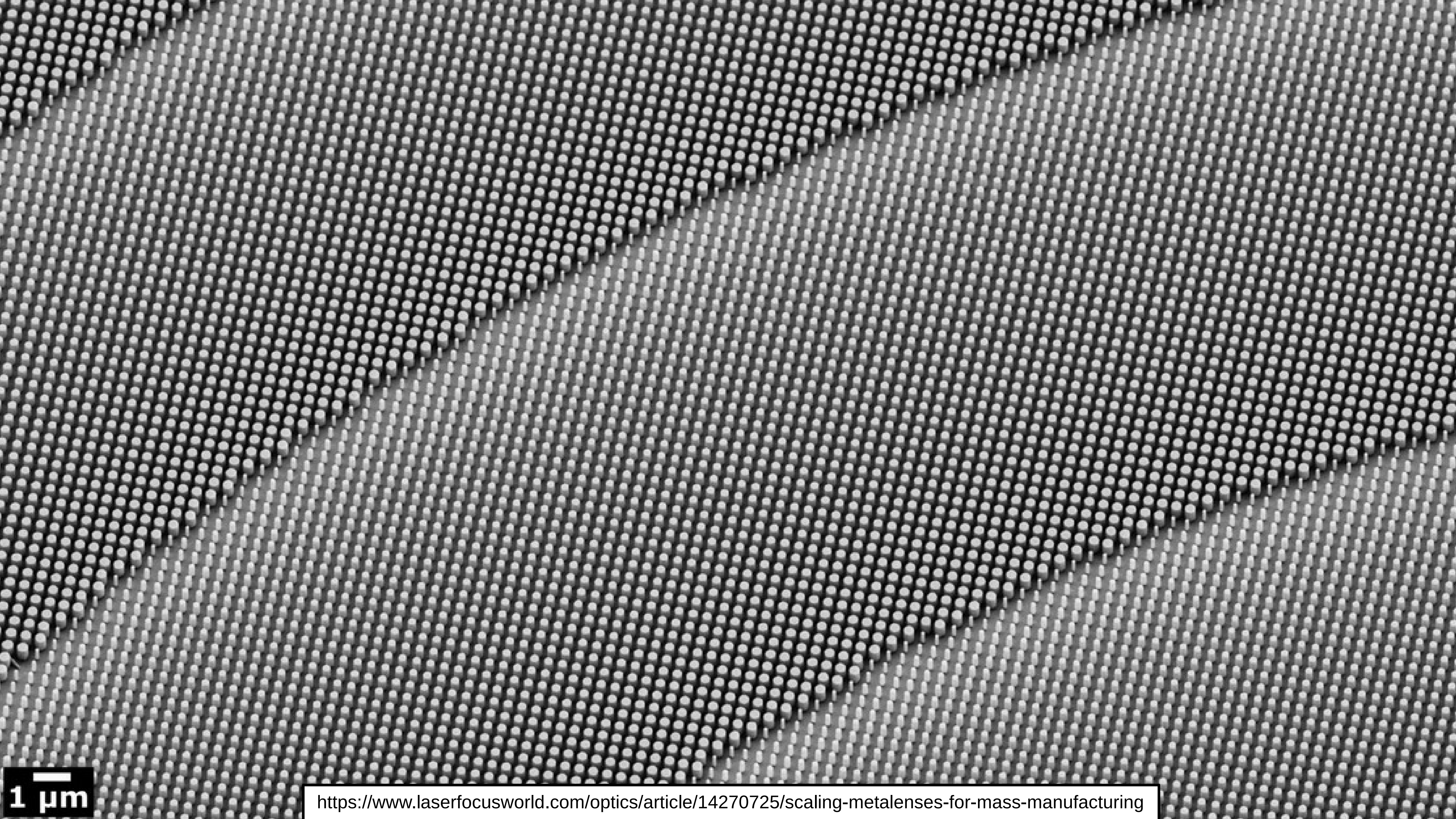






radius = 242 nm, height = 220 nm, period, period = 668 nm

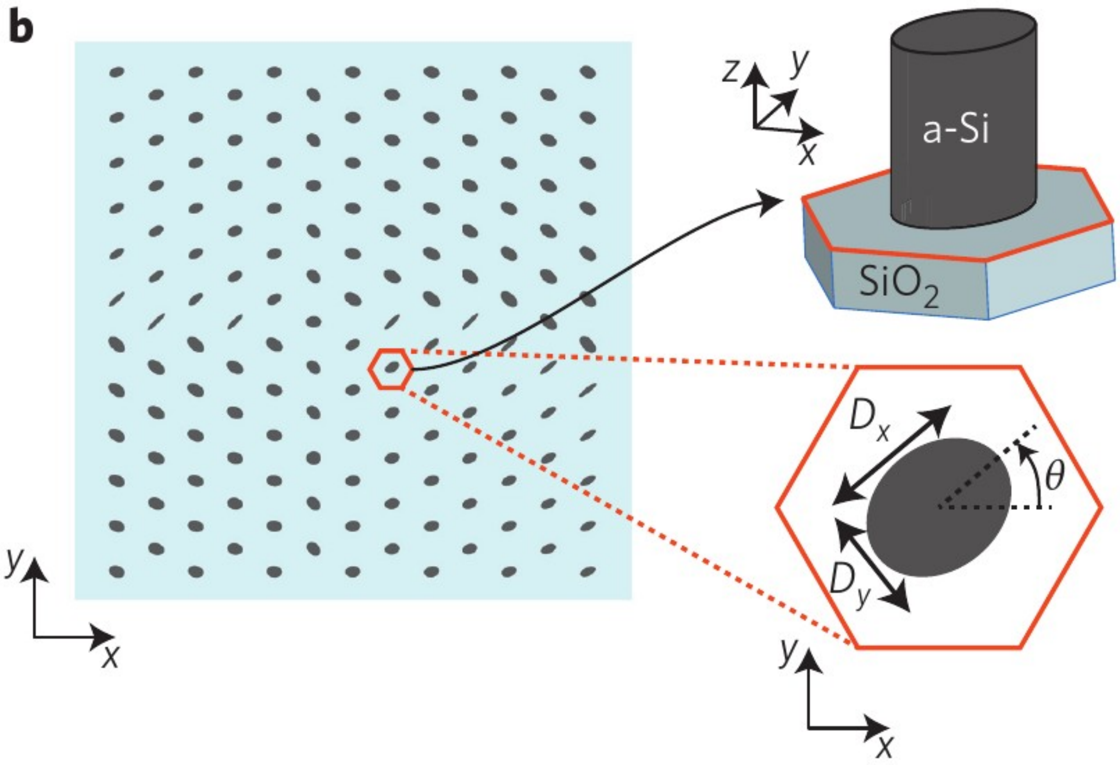
https://doi.org/10.1002/adom.201400584



https://www.laserfocusworld.com/optics/article/14270725/scaling-metalenses-for-mass-manufacturing

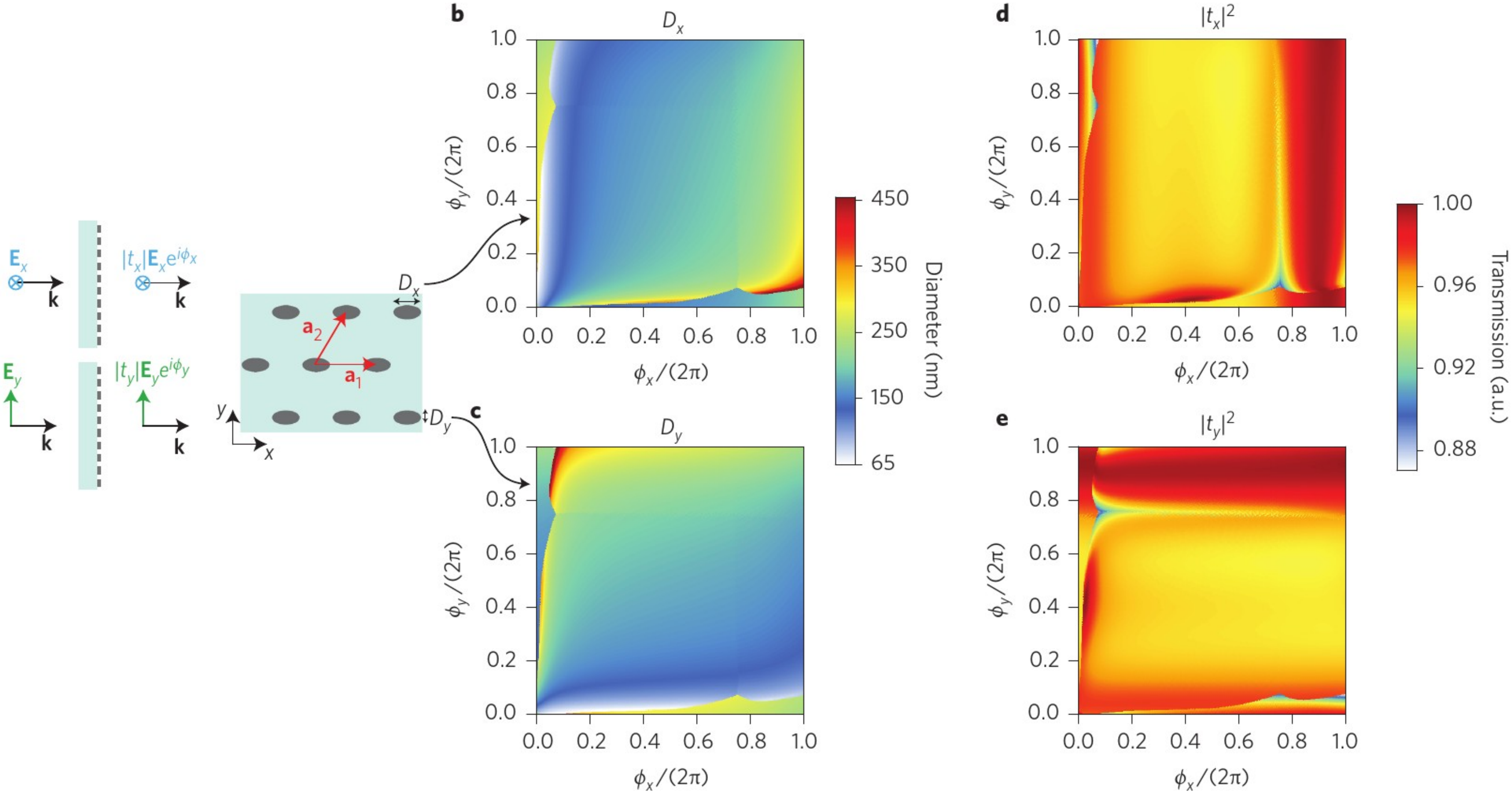
Birefringent Control with Elliptical Resonators

Resonators with different dimensions along x and y allow controlling the transmission phase of TE and TM waves independently



http://dx.doi.org/10.1038/nnano.2015.186

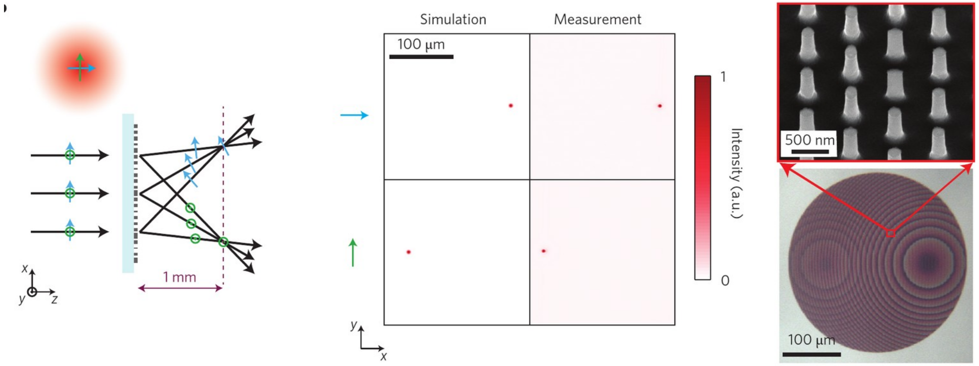
Birefringent Transmission Amplitude and Phase



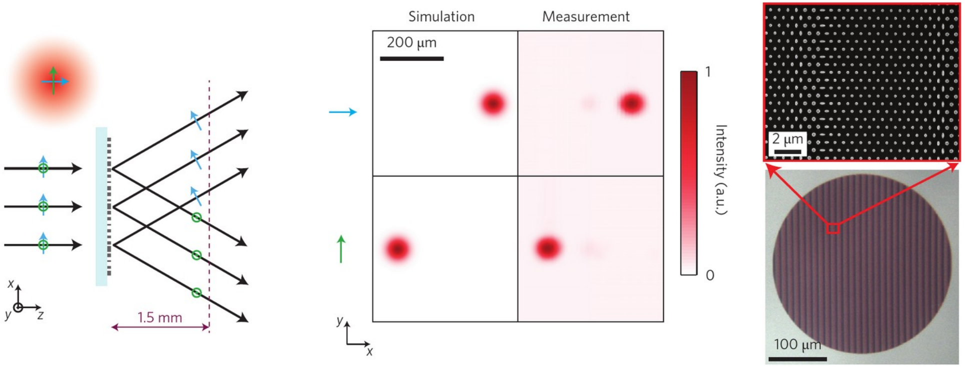
http://dx.doi.org/10.1038/nnano.2015.186

Birefringent Transformations

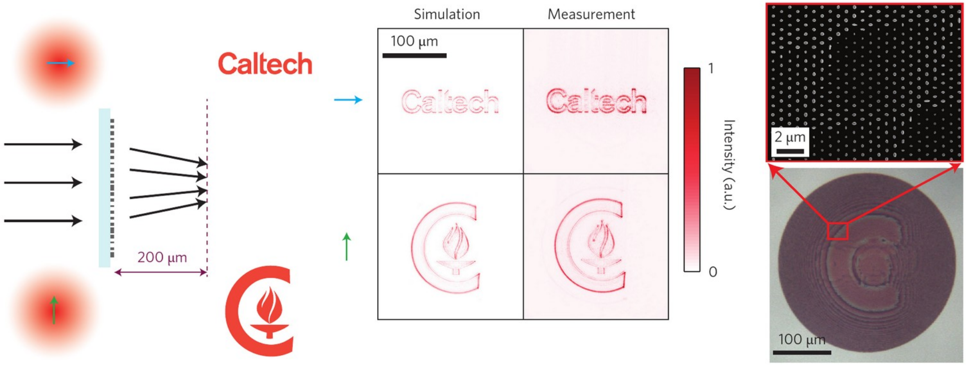
Different focus for TE and TM waves



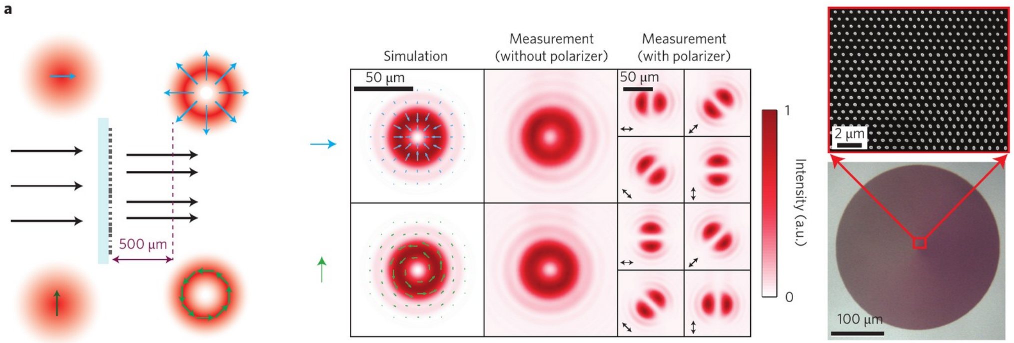
TE and TM beam splitting (± 5°)



Different holograms for TE and TM waves



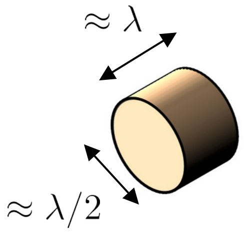
Vortex beam generation



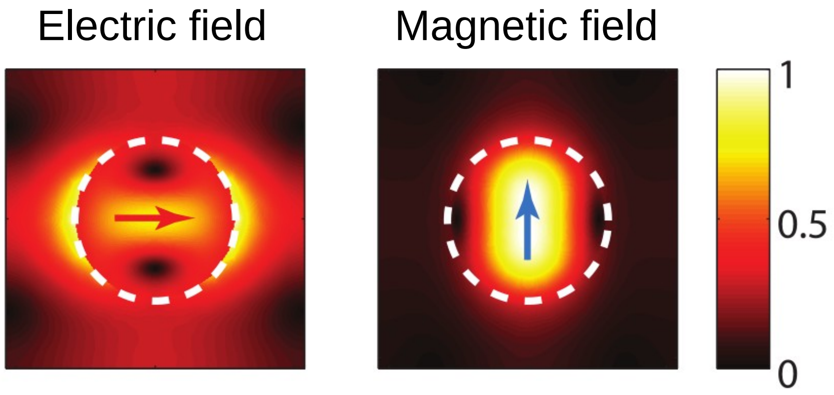
http://dx.doi.org/10.1038/nnano.2015.186

Controlling the Phase – Optical vs Microwave Regime

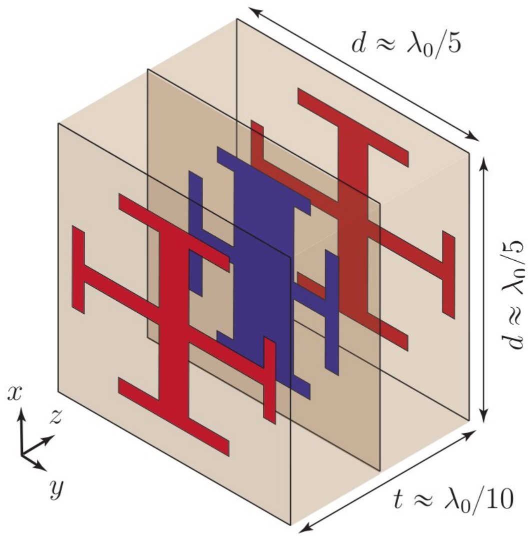
Dielectric resonators

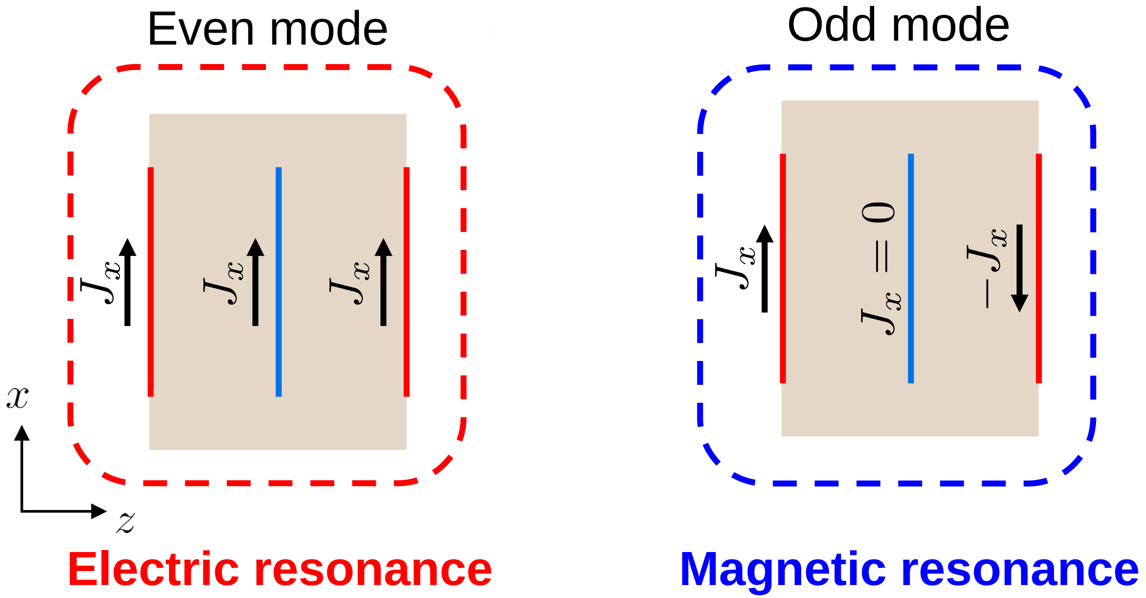


- Simple structure
- Limited control of fields
- Good at optical frequencies



Cascaded metallic layers

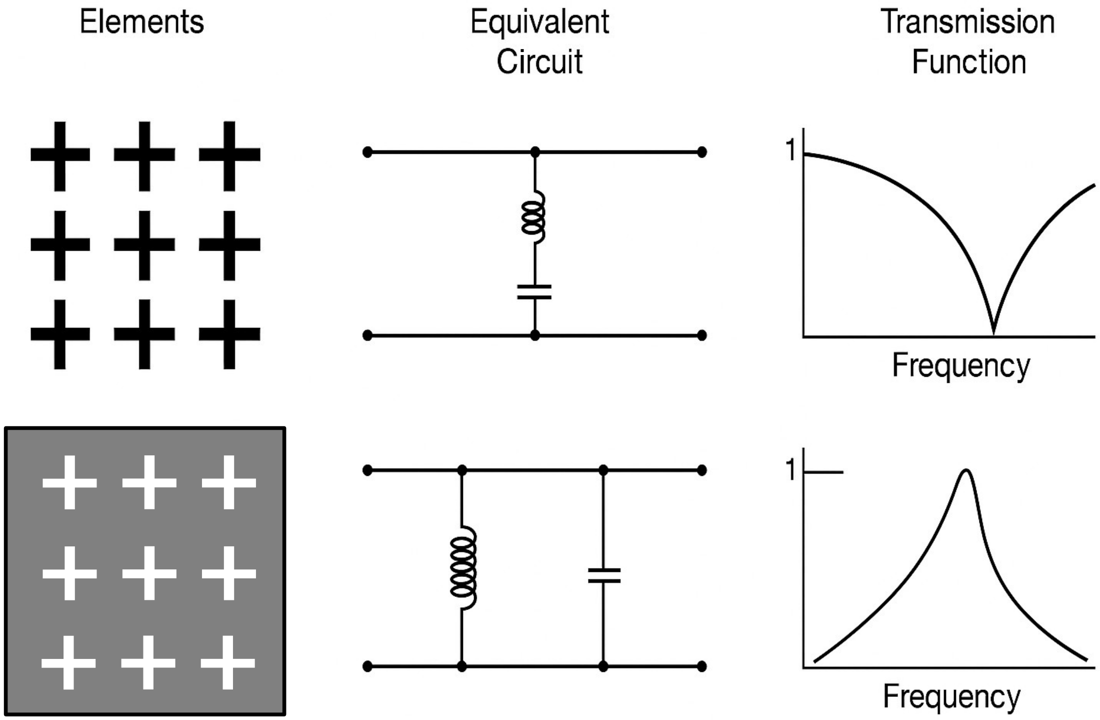




Screen vs Films

Crosses
(film)

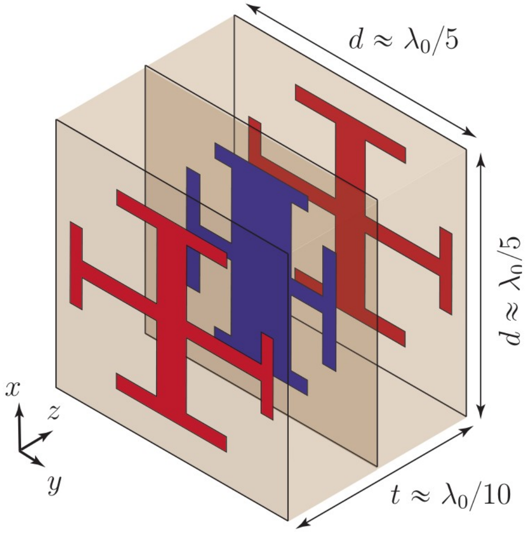
Aperture crosses
(screen)



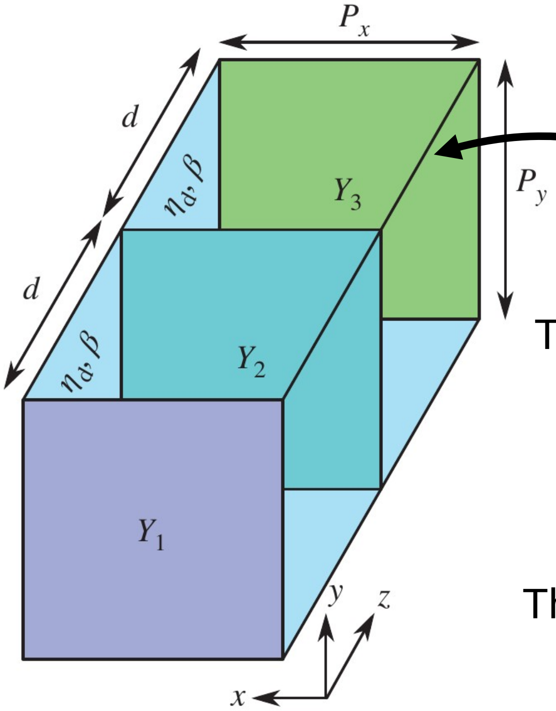
Band stop

Band pass

Modeling the Structure Response



We assume that the outer layers are identical ($Y_1 = Y_3$)



Each metallic sheet is model as an impedance layer

$$\overline{\overline{\mathrm{Y}}}_a = \begin{bmatrix} 1 & 0 \\ Y_a & 1 \end{bmatrix}$$

The dielectric spacers are modeled as

$$\overline{\overline{\mathrm{M}}} = \begin{bmatrix} \cos(\beta d) & j\eta_\mathrm{d}\sin(\beta d) \\ \frac{j\sin(\beta d)}{\eta_\mathrm{d}} & \cos(\beta d) \end{bmatrix}$$

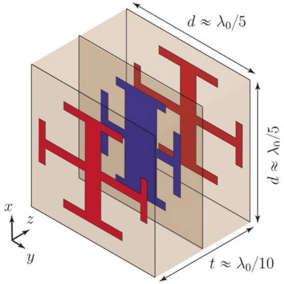
The ABCD matrix of the entire structure

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \overline{\overline{\mathrm{Y}}}_1 \cdot \overline{\overline{\mathrm{M}}} \cdot \overline{\overline{\mathrm{Y}}}_2 \cdot \overline{\overline{\mathrm{M}}} \cdot \overline{\overline{\mathrm{Y}}}_1$$

The ABCD matrix is mapped to the scattering parameters using

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{2A + B/\eta_0 + C\eta_0} \begin{bmatrix} B/\eta_0 - C\eta_0 & 2 \\ 2 & B/\eta_0 - C\eta_0 \end{bmatrix}$$

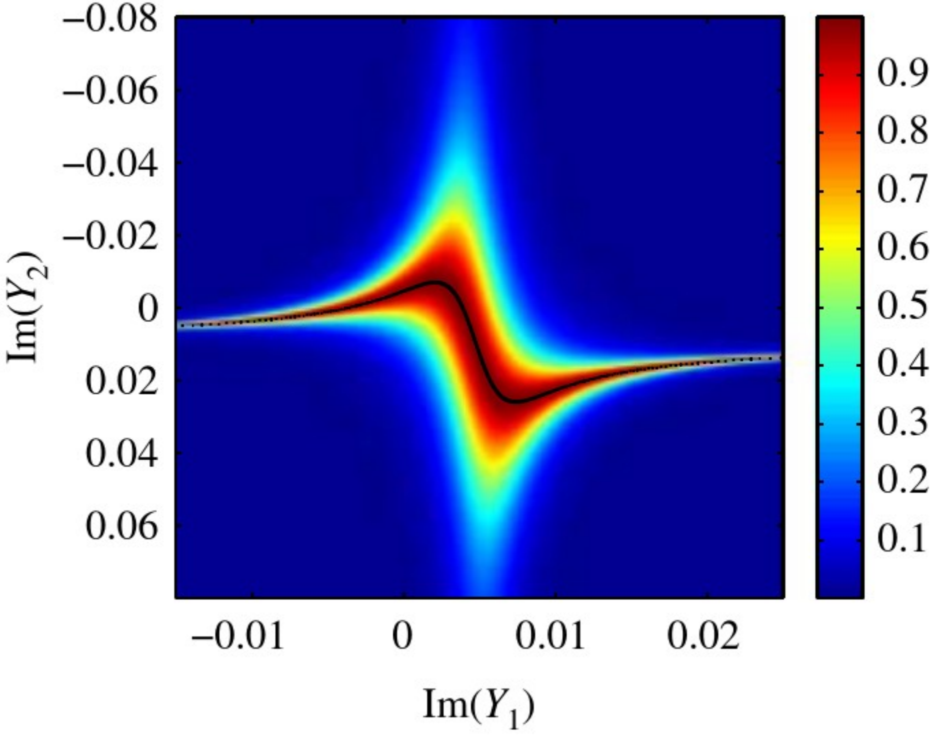
Pozar, “Microwave engineering: theory and techniques”, 2021



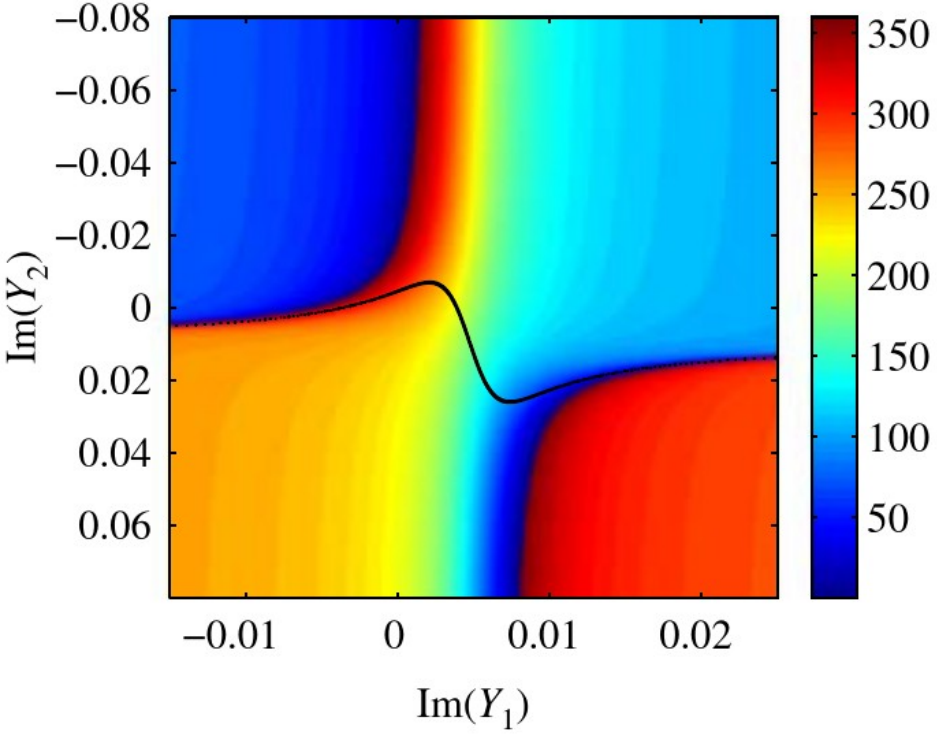
Scattering Response

Changing the shape (admittance Y_a) of the middle and outer layers allows achieving 2π phase shift with full transmission

Transmission amplitude



Transmission phase

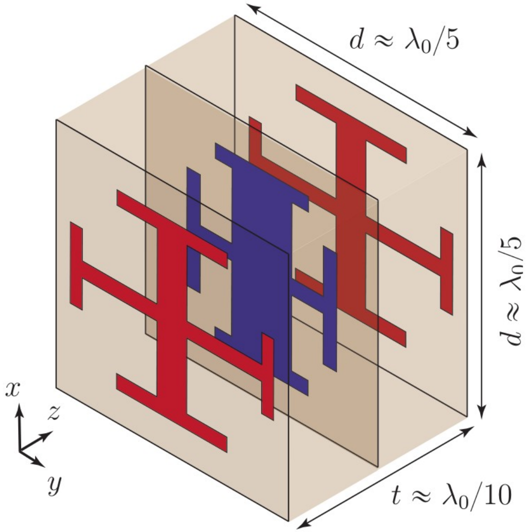


Note: lossless metallic sheets are modeled with imaginary admittances

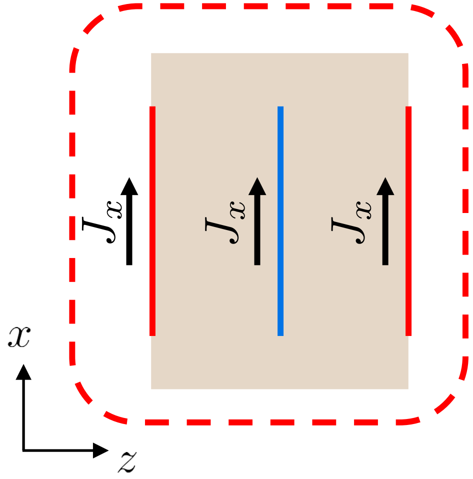
Achouri K, ”Electromagnetic metasurfaces: Theory and applications”, 2021

Controlling Electric and Magnetic Responses Independently

Cascaded metallic layers

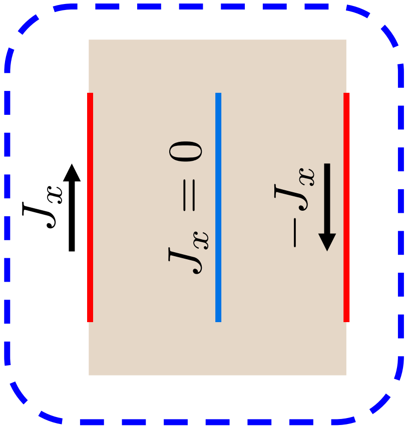


Even mode



Electric resonance

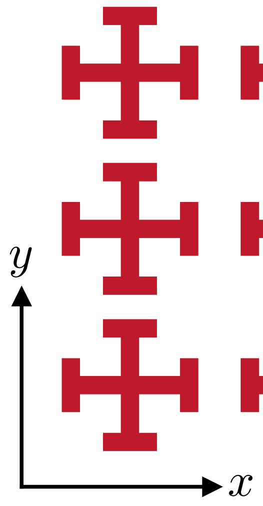
Odd mode



Magnetic resonance

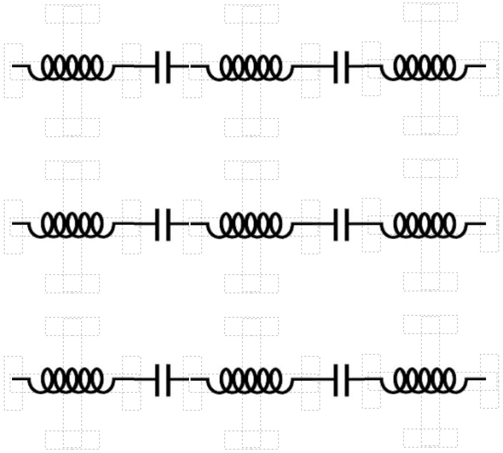
Since the current is zero on the middle layer for the magnetic resonance, the middle layer can be used to control the electric response with affecting the magnetic response. One can start by adjusting the outer layers to control the magnetic response and then adjust the middle layer to control the electric response.

Lump Element Model



For x polarization

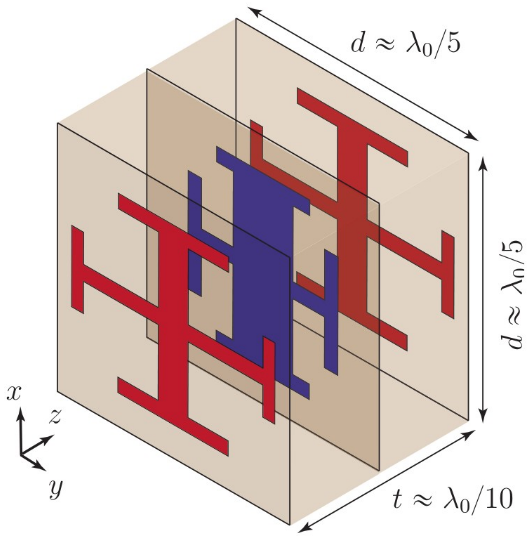
≈

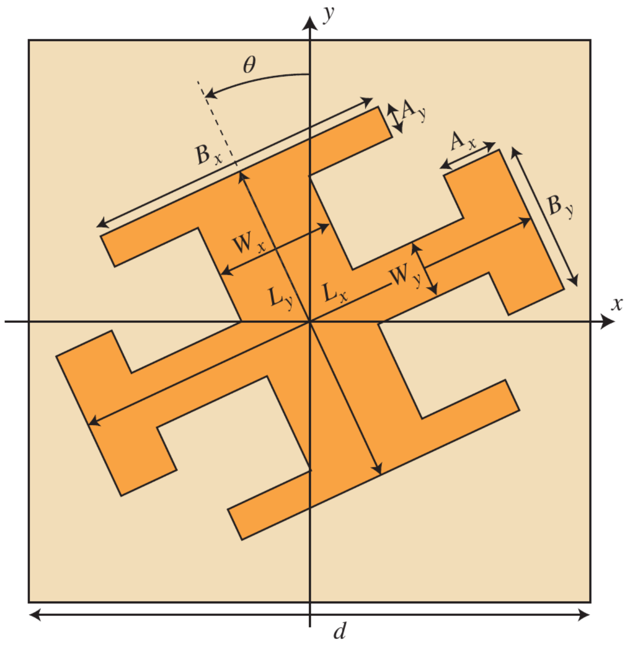


Increasing the length of the cross arms increases both the inductance and capacitance leading to a decrease of the resonance frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

Controlling Electric and Magnetic Responses Independently



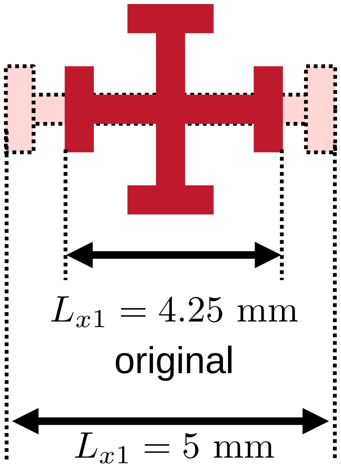


Layers dimensions in [mm] for θ = 0°

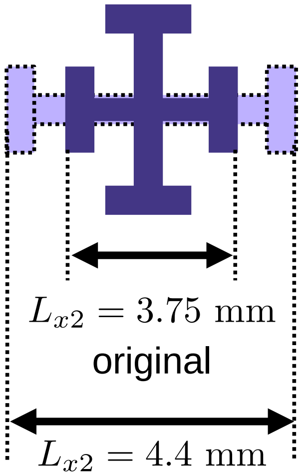
	L_x	L_y	W_x	W_y	A_x	A_y	B_x	B_y
Layer 1	4.25	4.75	0.625	0.25	0.5	0.5	4.25	3
Layer 2	3.75	5.25	0.5	0.375	0.5	0.5	2.25	4.5

Demonstration of Independent Control

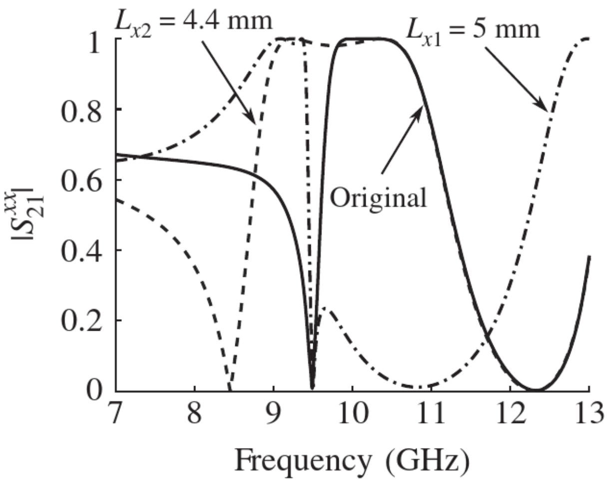
outer layer

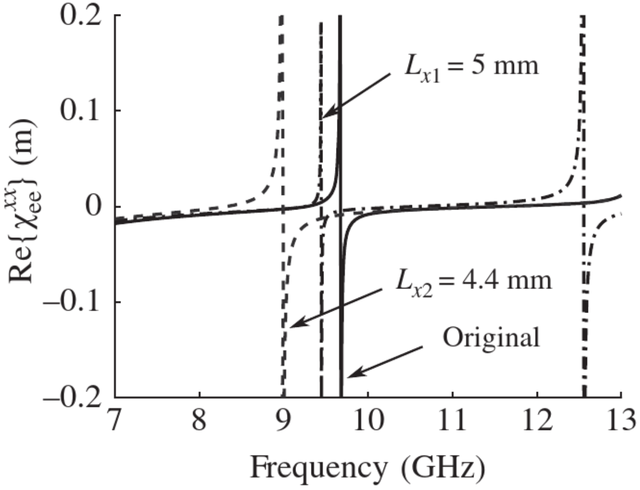


middle layer

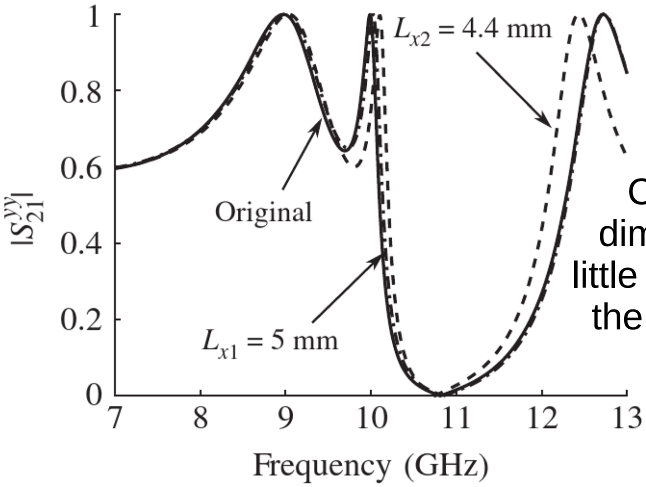


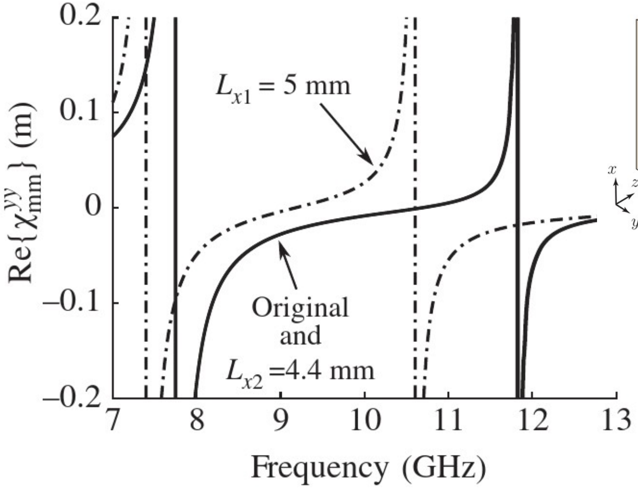
TM excitation



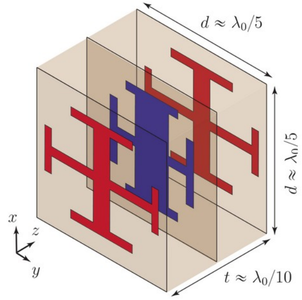


TE excitation





Changing x-dimensions has little to no effect on the TE response



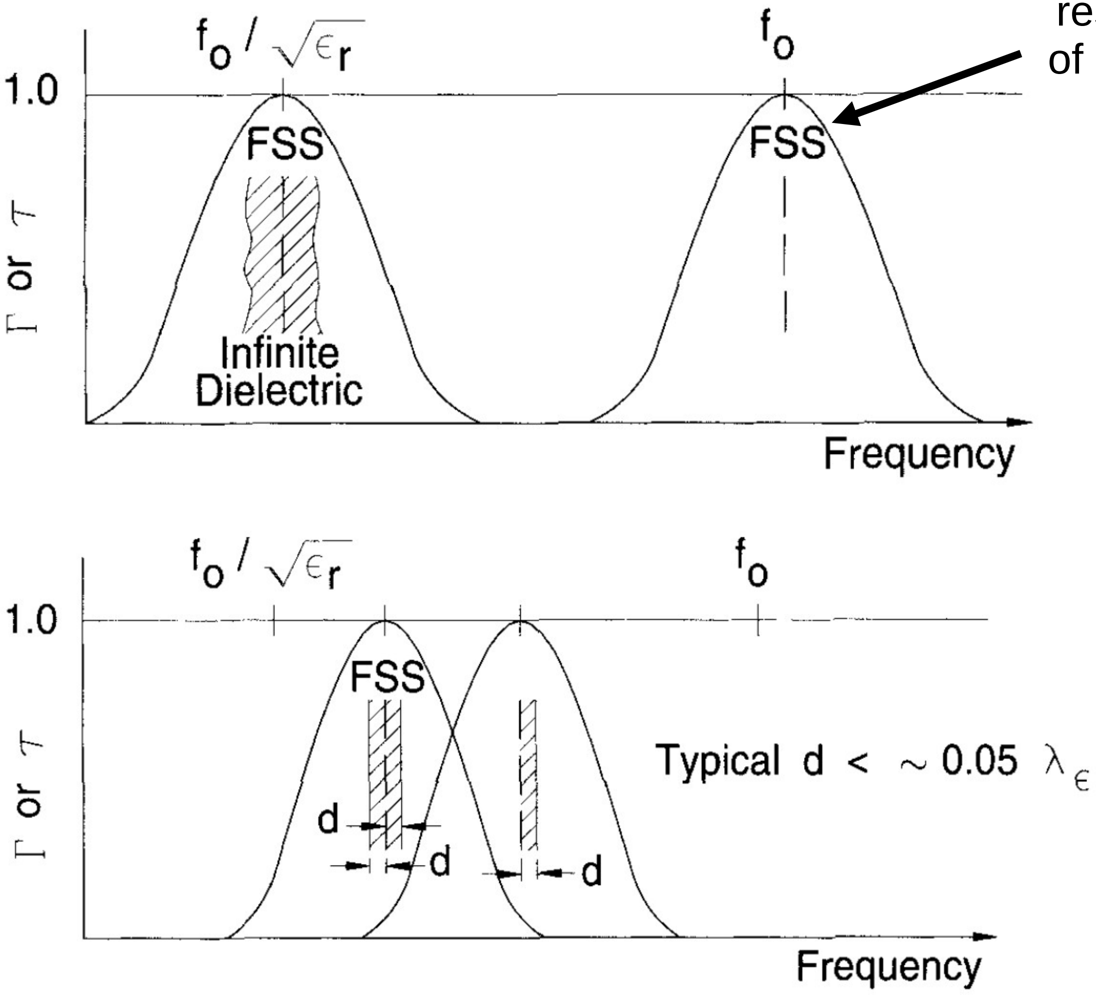
Achouri K, "Electromagnetic metasurfaces: Theory and applications", 2021

Effect of the Dielectric Spacers

The presence of a dielectric around the surface shifts the resonance to lower frequencies

The dielectric increases the stored electric energy (capacitance)

$$\omega = \frac{1}{\sqrt{LC}}$$



resonance frequency of single layer surface without dielectric

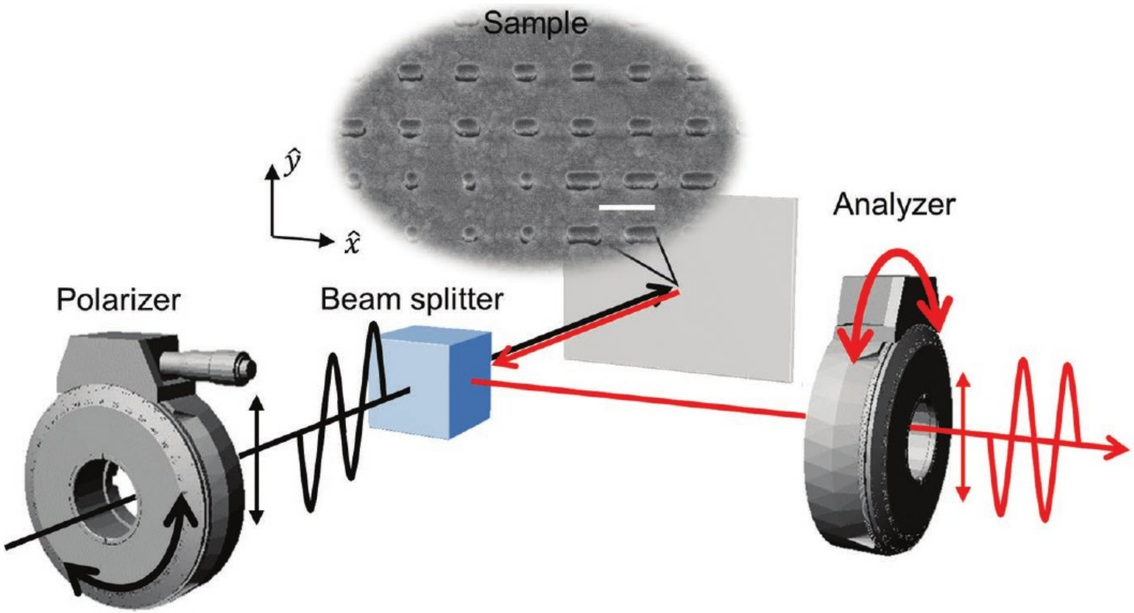
B. A. Munk, Frequency Selective Surfaces: Theory and Design, Wiley-Interscience, 2000.

What Have We Learned So Far….

- The scattering response of both metallic and dielectric metasurface may be expressed in terms of interactions of electric and magnetic responses. These interactions can not only lead to the Kerker effect (canceling of the reflectance and maximization of the transmittance) but also to a 2π phase shift.
- Many dielectric metasurfaces have been realized with this approach by using elliptical high-index resonators that have the advantage of allowing the control of both x- and y-polarized waves.
- In the microwave regime, it is more common to use multilayer metallic structures typically having the shape of crosses for near independent birefringent control.
- A three-layer metallic structure with identical outer layers can be shown to exhibit full transmittance with full 2π phase coverage.
- Another advantage of this design is that it offers a very easy control of its electric and magnetic responses. The two outer layers can be used to control the magnetic response and the middle layer, which barely affects the magnetic response, may be used to control its electric response.

Wave Plate Control

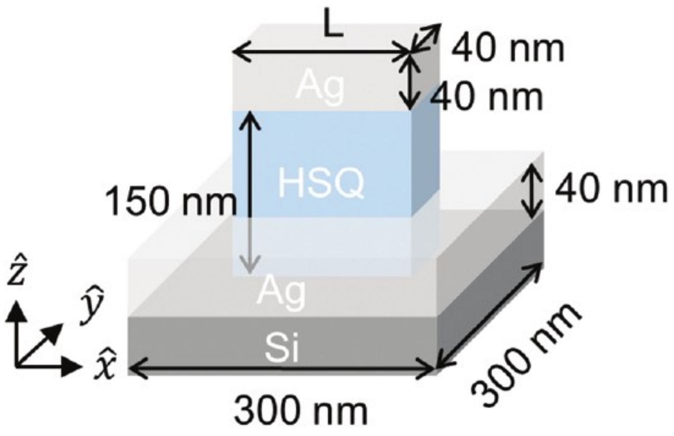
Exploiting Rotation of Polarization

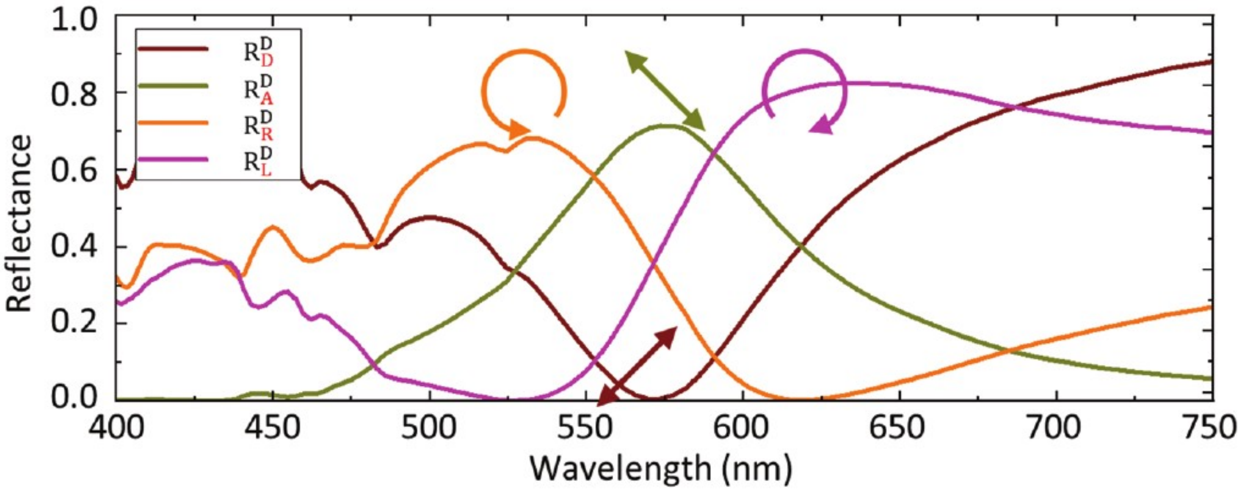


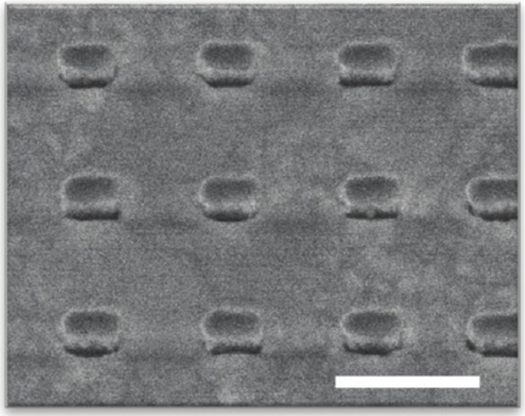
Black = illumination, **Red** = measurement

https://doi.org/10.1002/adom.202202165

Exploiting Rotation of Polarization

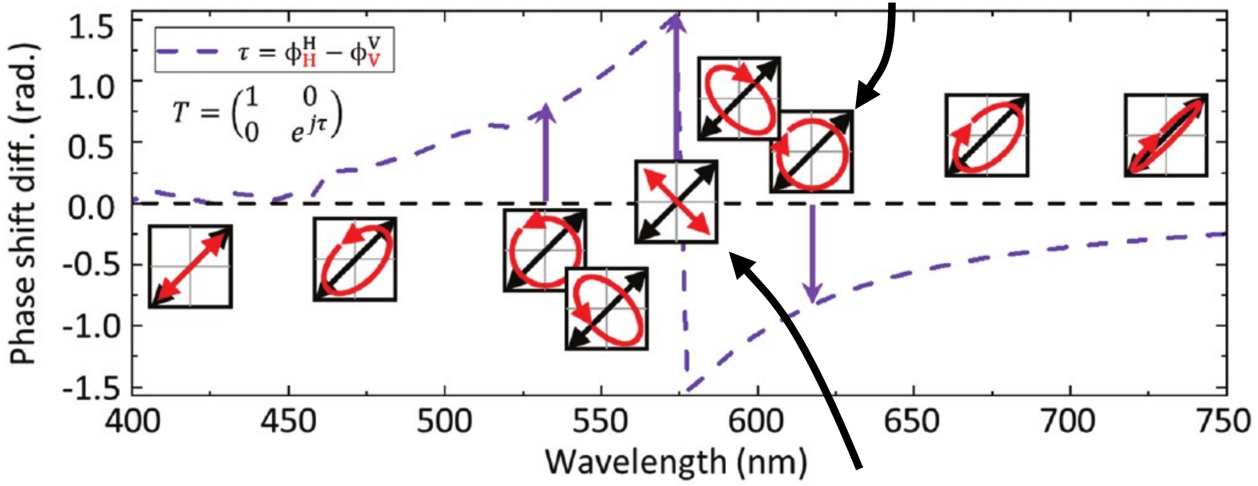






300 nm

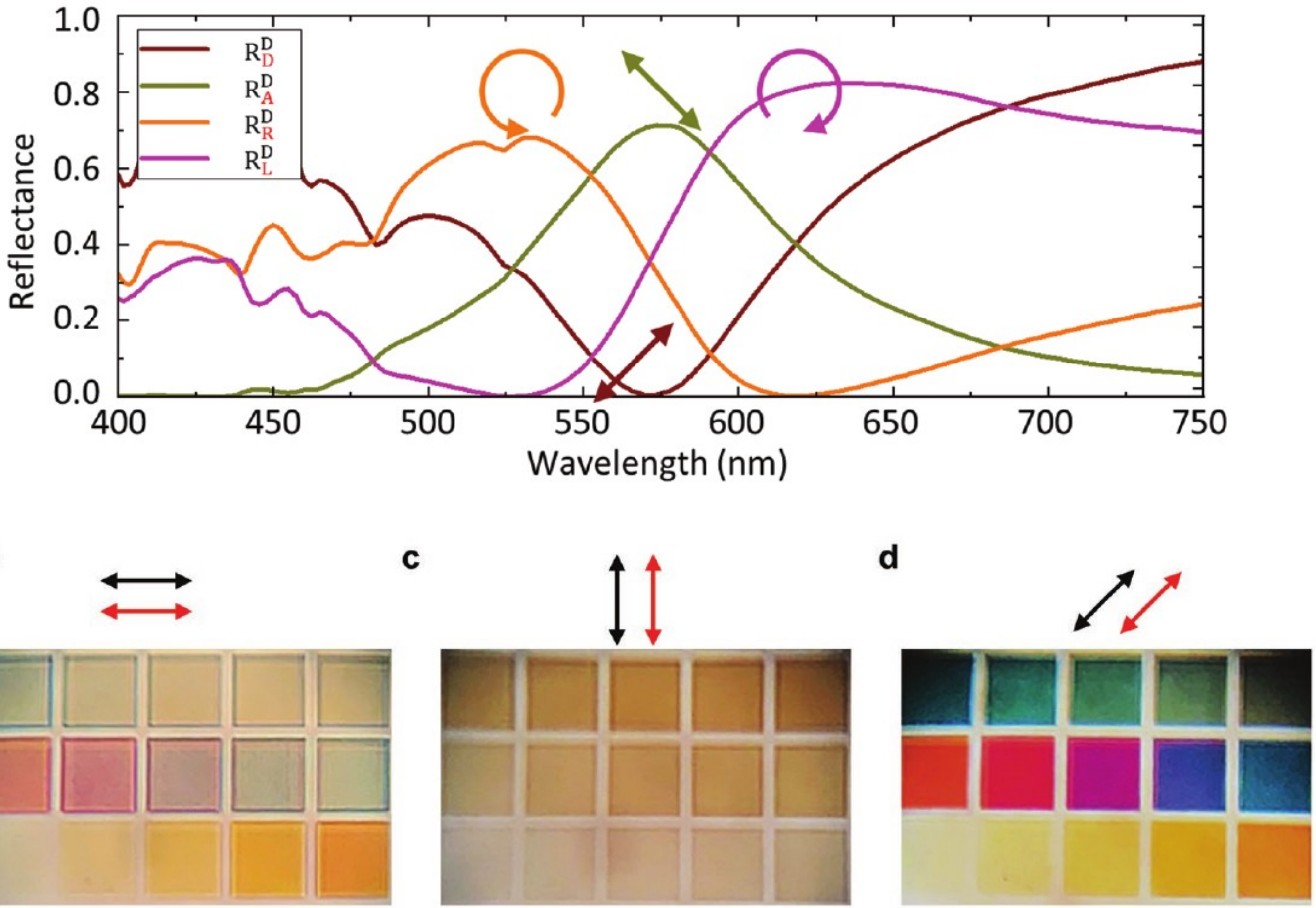
behaves as a quarter-wave plate



behaves as a half-wave plate

https://doi.org/10.1002/adom.202202165

Exploiting Rotation of Polarization



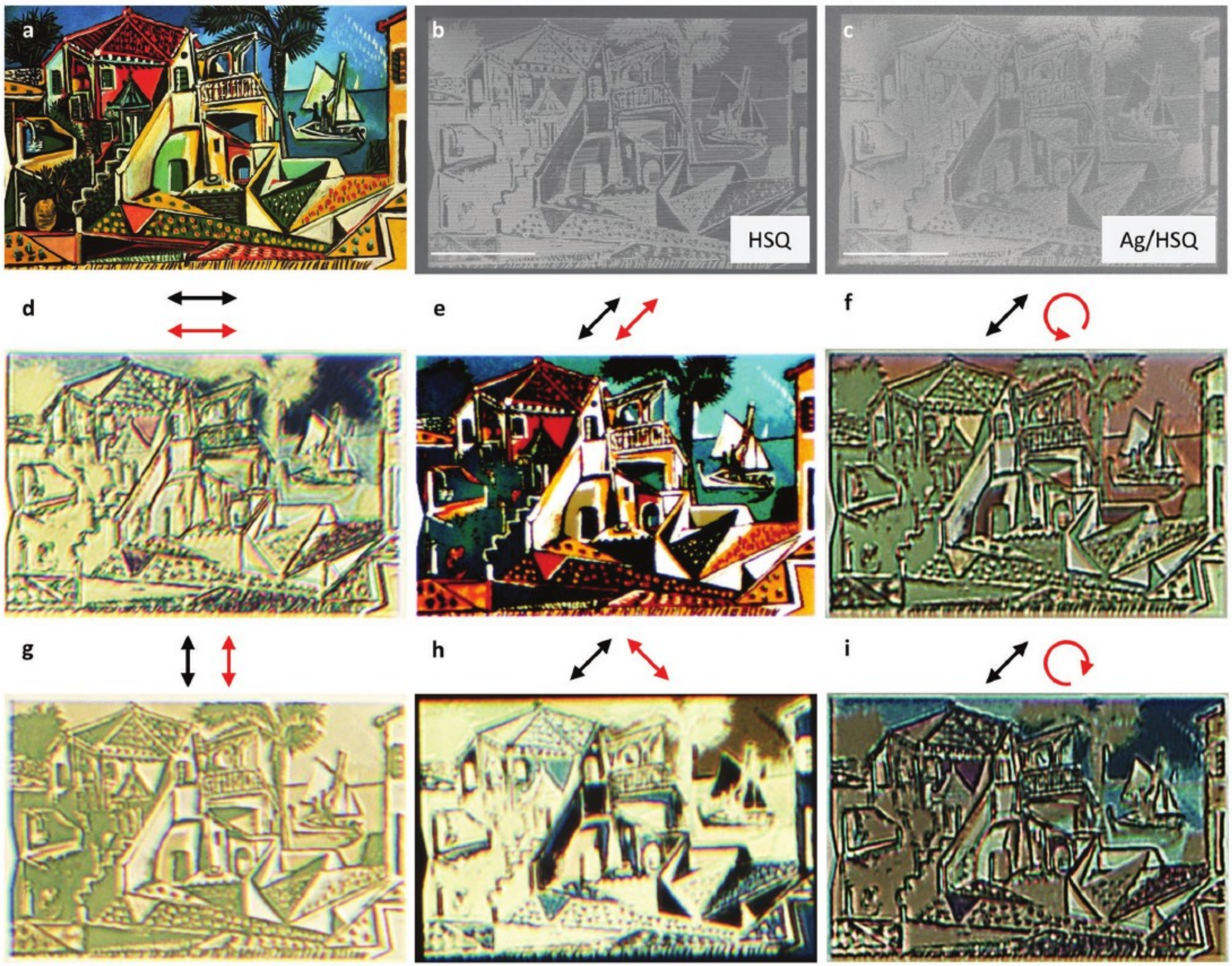
https://doi.org/10.1002/adom.202202165

Nano Painting

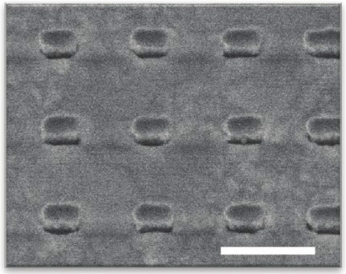
360 µm

Target image

235 µm



SEM images



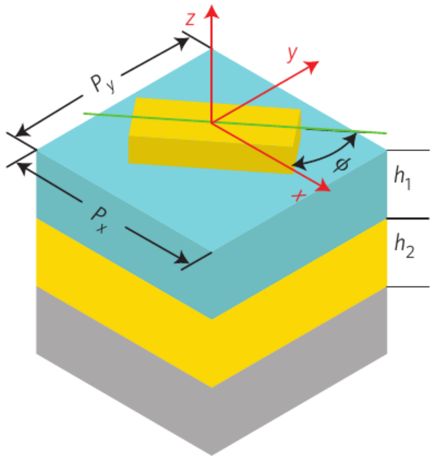
300 nm

Optical images

https://doi.org/10.1002/adom.202202165

Pancharatnam-Berry Effect

Pancharatnam-Berry Effect



Jones vector of the scattered wave

Jones vector of the incident wave

$$\mathbf{J}_\mathrm{s} = \overline{\overline{\mathrm{R}}}(\theta) \cdot \overline{\overline{\mathrm{M}}}_\mathrm{HWP} \cdot \overline{\overline{\mathrm{R}}}(-\theta) \cdot \mathbf{J}_\mathrm{i}$$

LCP or RCP incident wave

$$\mathbf{J}_\mathrm{i} = \begin{bmatrix} 1 \\ \pm j \end{bmatrix}$$

$$\mathbf{J}_\mathrm{s} = e^{j2\theta} \begin{bmatrix} 1 \\ \mp j \end{bmatrix}$$

The polarization of the scattered wave is flipped

Rotation matrix

$$\overline{\overline{\mathrm{R}}}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Jones matrix of a half-wave plate

$$\overline{\overline{\mathrm{M}}}_\mathrm{HWP} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Reflective Metasurface Based on PB Phase

Parameters

$P_x = P_y = 300$ nm

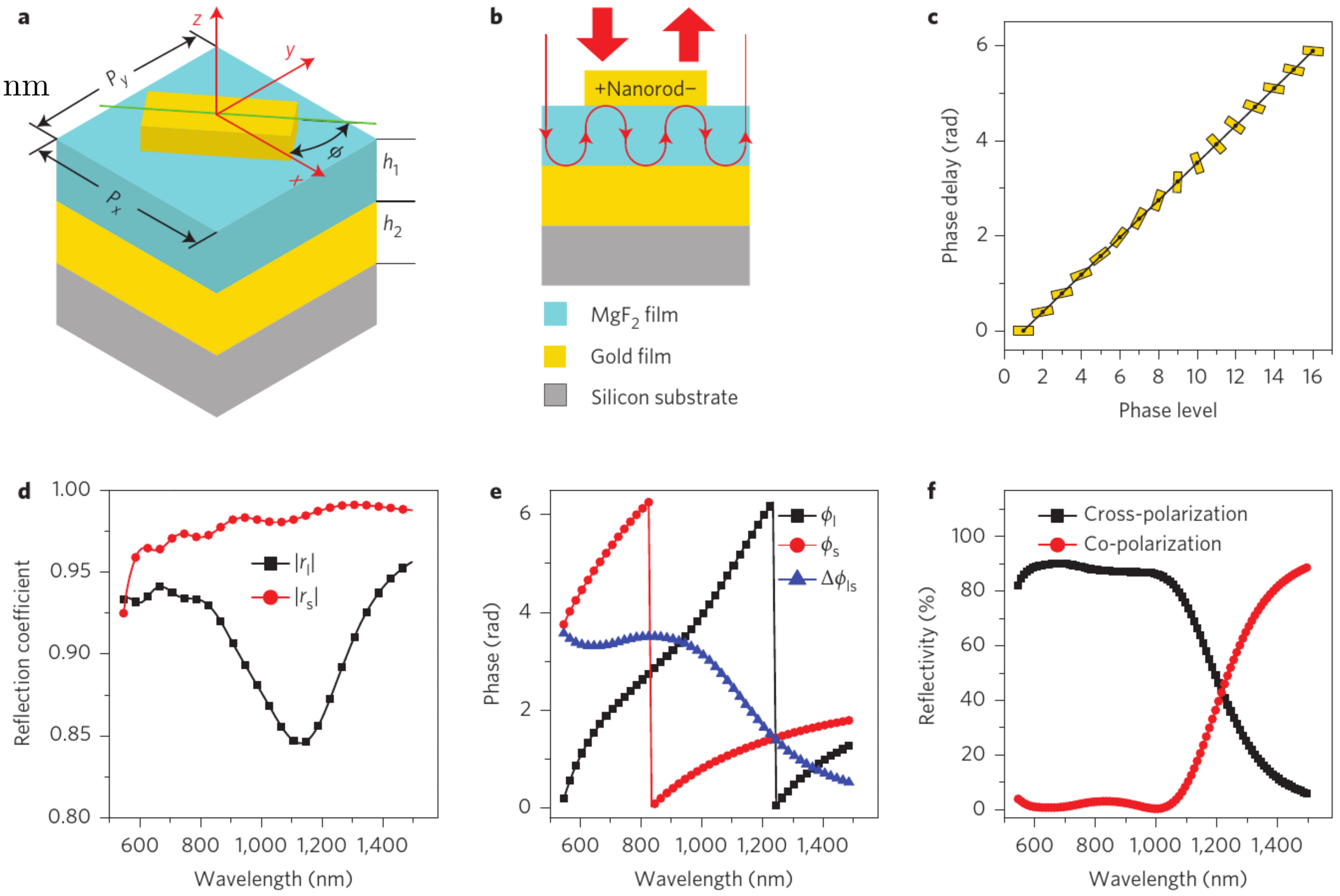
$h_1 = 90$ nm

$h_2 = 130$ nm

$L = 200$ nm

$W = 80$ nm

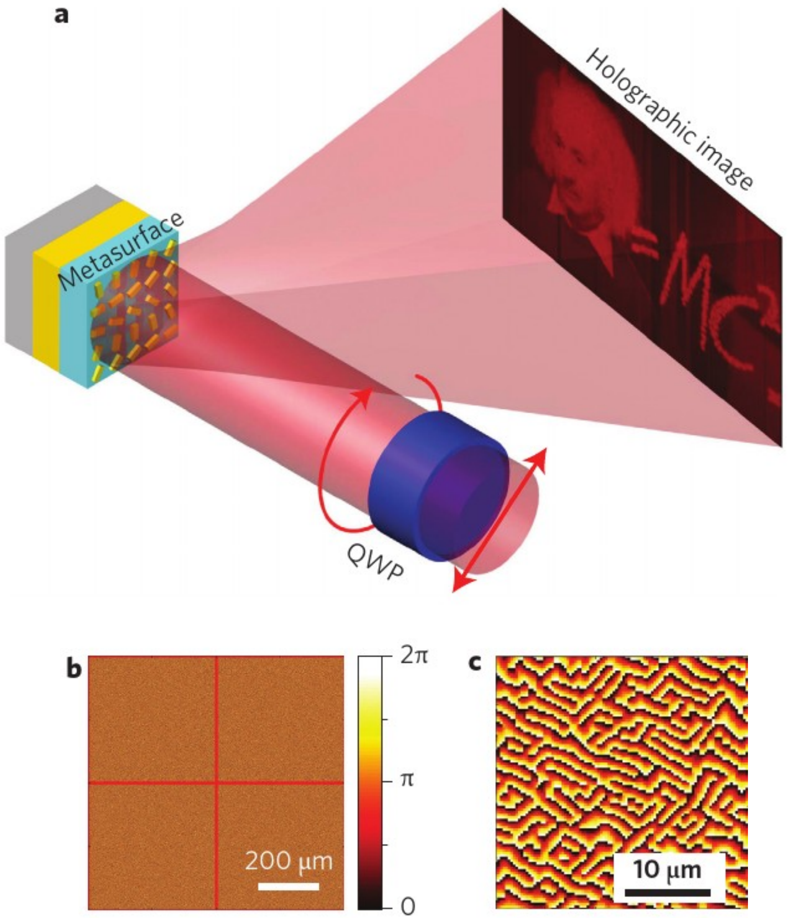
$H = 30$ nm

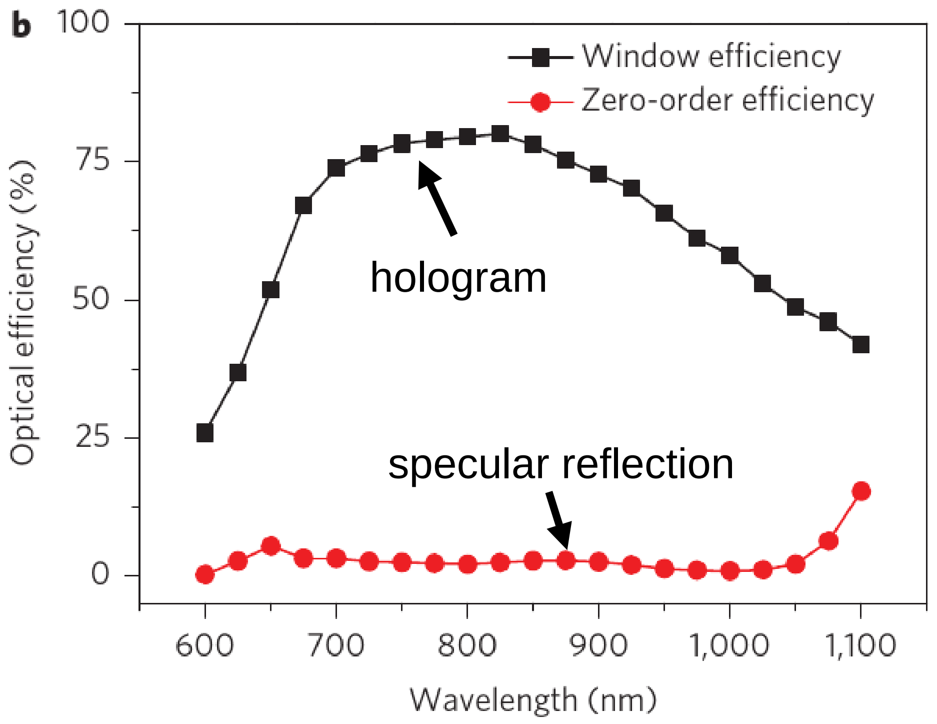


$$\mathbf{J}_t = e^{j2\theta}\begin{bmatrix}1 \\ \mp j\end{bmatrix}$$

http://doi.org/10.1038/nnano.2015.2

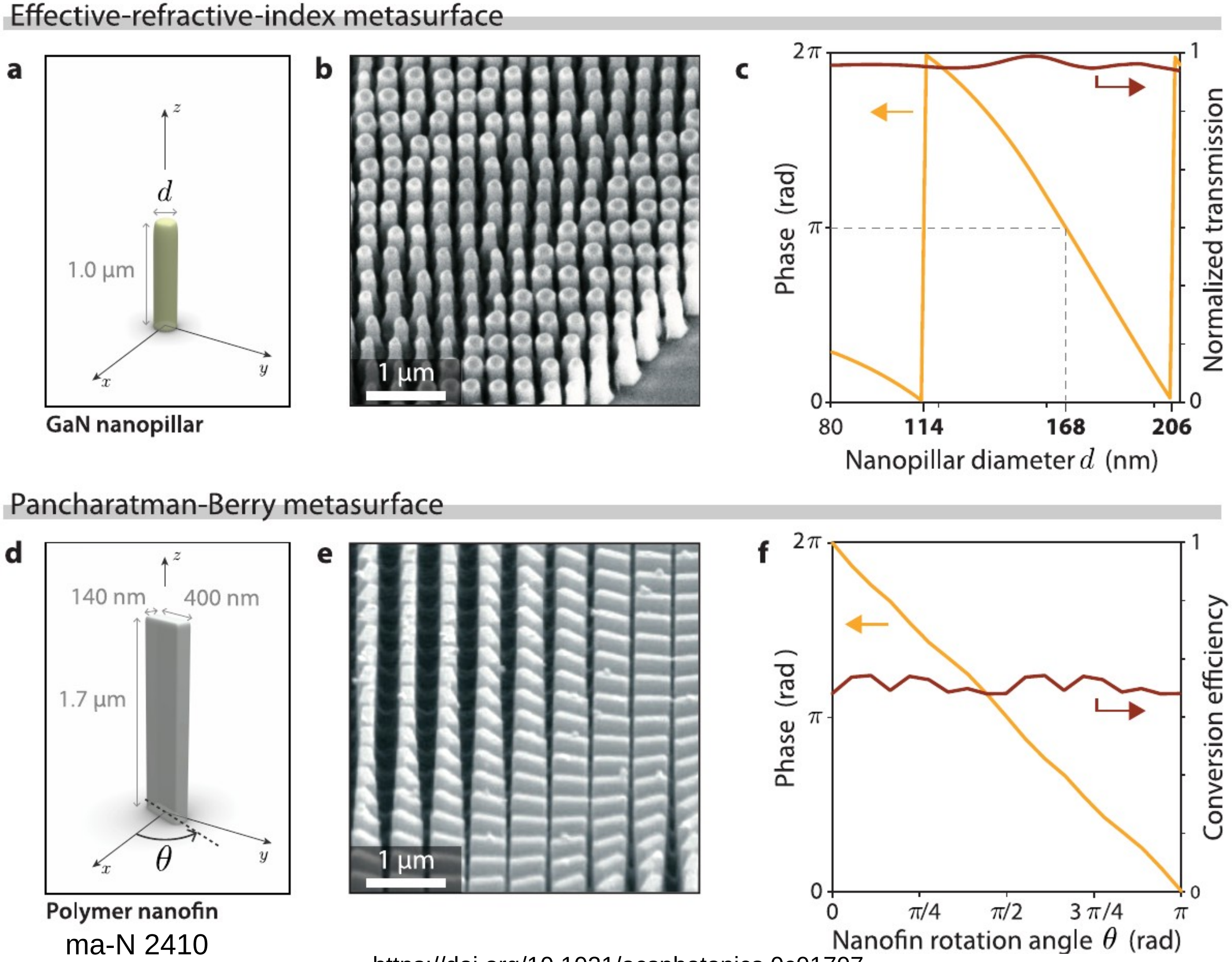
Hologram Efficiency Reaching 80%





http://doi.org/10.1038/nnano.2015.2

Example in Transmission

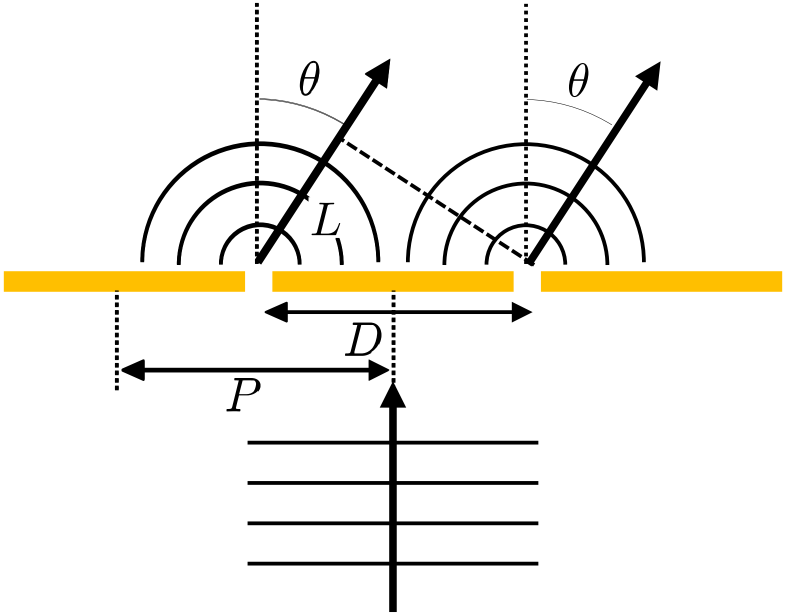


ma-N 2410

https://doi.org/10.1021/acsphotonics.0c01707

Detour Phase

Detour Phase



A diffraction grating of period P produces ±1 orders at angles

$$\sin\theta = \frac{\lambda}{P}$$

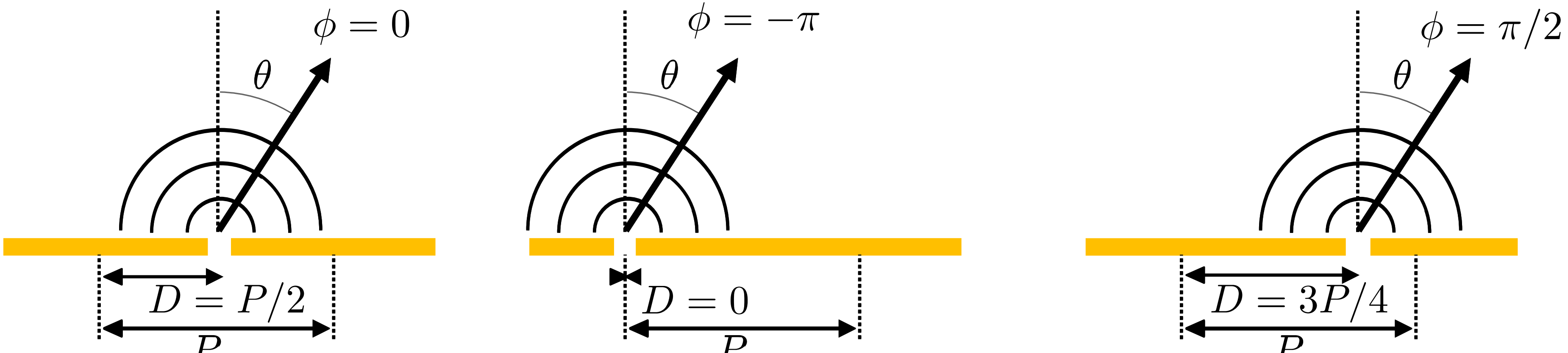
The path difference between the two beams emerging from slits separated by a distance D is

$$L = D\sin\theta$$

The phase shift between the two beams is

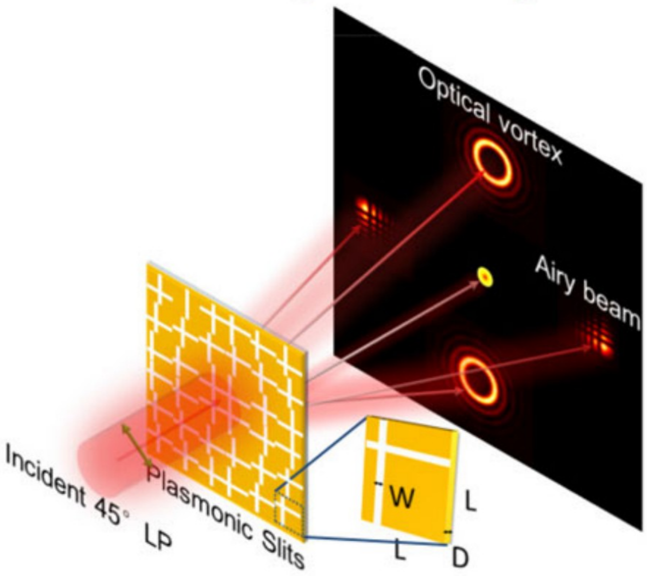
$$\Delta\phi = kL = \frac{2\pi}{\lambda} D\sin\theta$$

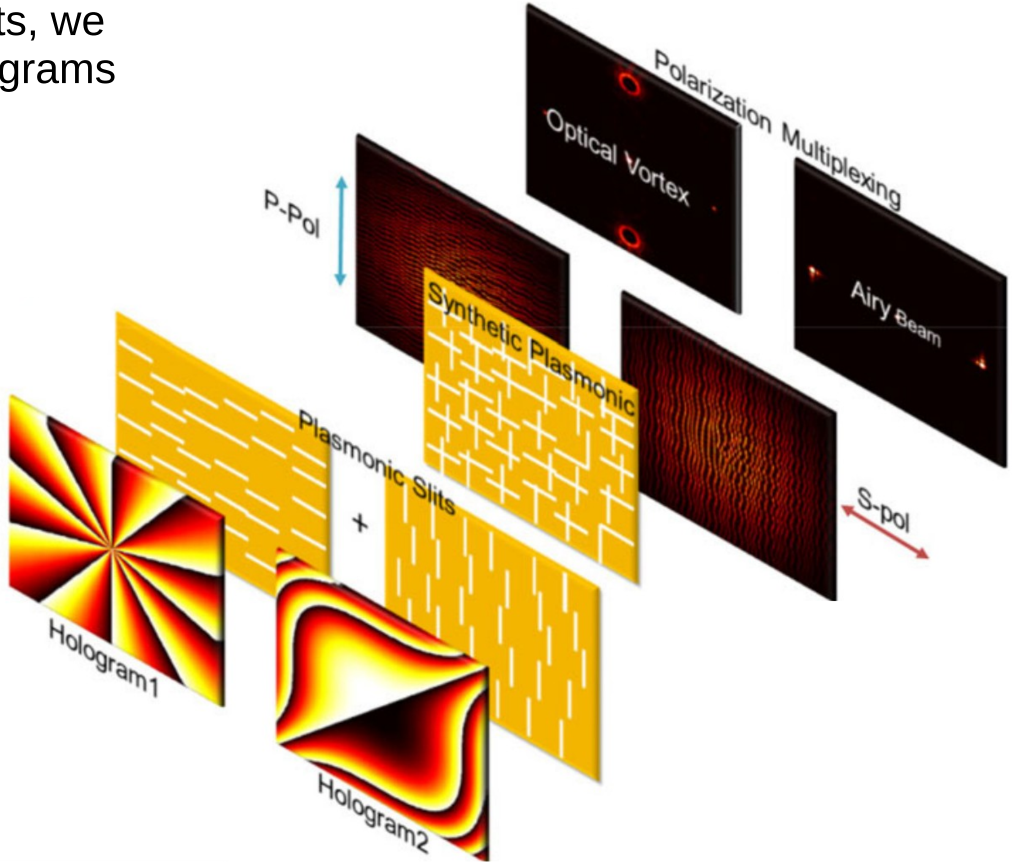
Choosing that the phase shift is zero for a slit in the middle of its period, we can cover the range ±π



Polarization Sensitive Holography Using Detour Phase

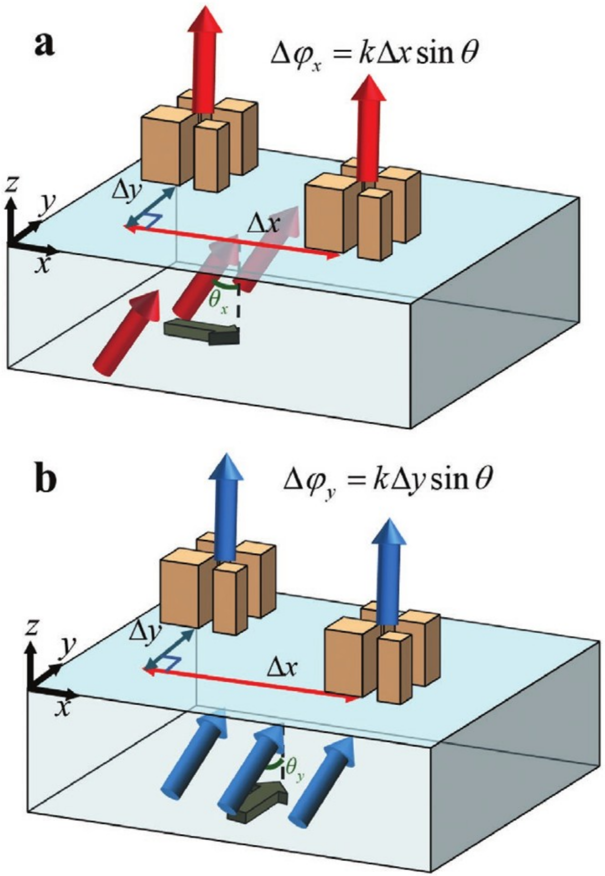
By combining horizontal and vertical slits, we can generate polarization sensitive holograms

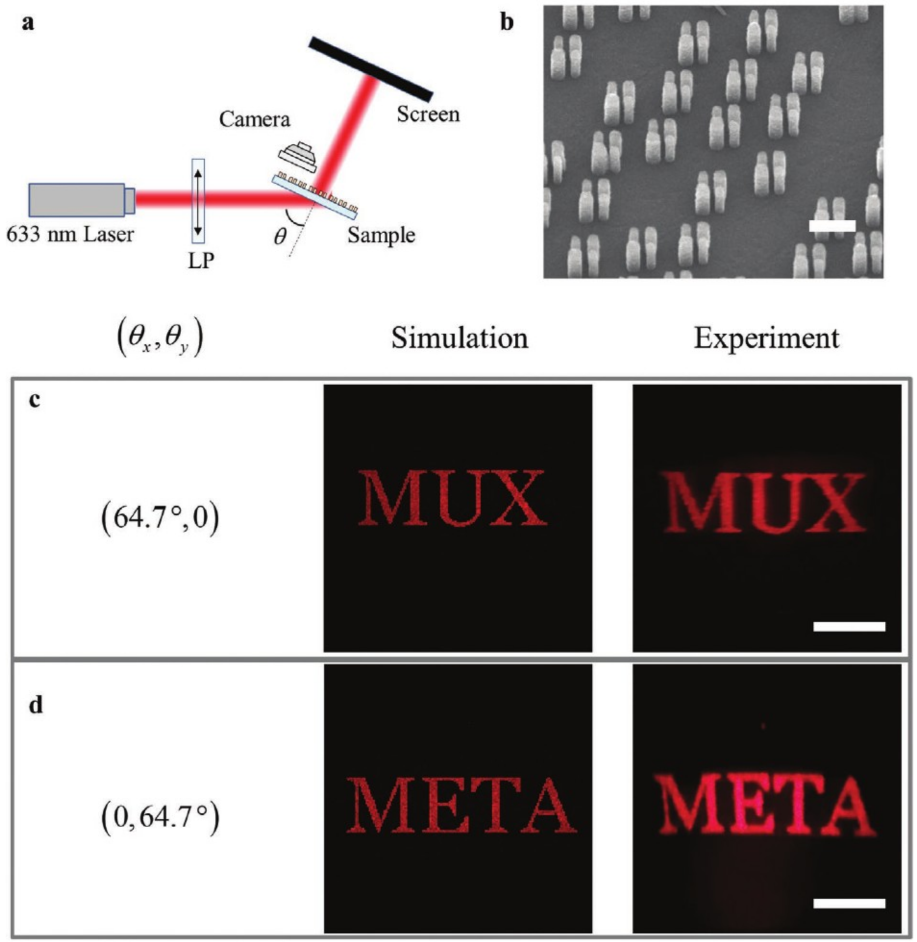




doi.org/10.1002/lpor.201600101

Example with Oblique Incidence and Normal Transmission





doi.org/10.1002/adom.202100678

What Have We Learned So Far….

- Birefringent metasurfaces maybe used to create very differently looking optical structures when the polarization of the incident and scattered light are tuned.
- A half-wave plate (HWP) changes the handedness of circularly polarized light.
- A rotation of the HWP in the metasurface plane leads to phase shift of the scattered light.
- This allows to easily create a large amount of phase discretization leading to high efficiency holograms.
- HWP metasurfaces can be used in both reflection and transmission. The technique is however limited to circularly polarized light.
- An other approach for controlling the phase is via the “detour phase” technique.
- The detour phase technique consists in shifting the lateral position of the scattering element within its unit-cell. Depending on the angle of scattering and the angle of incidence, this lateral shift leads to a phase shift of the scattered light.