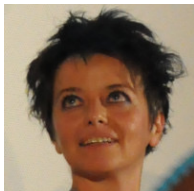


MICRO-435

Quantum and Nanocomputing

Edoardo Charbon
Mariagrazia Graziano



Prof. Mariagrazia Graziano
Politecnico di Torino
Torino, Italy

Email: mariagrazia.graziano@polito.it

URL: https://www.swas.polito.it/rubrica/scheda_pers.asp?matricola=004318

Office hours: by appointment



Prof. Edoardo Charbon
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Office hours: by appointment

Vladimir Pešić
EPFL

Email: vladimir.pesic@epfl.ch

URL: aqua.epfl.ch

Office hours: by appointment

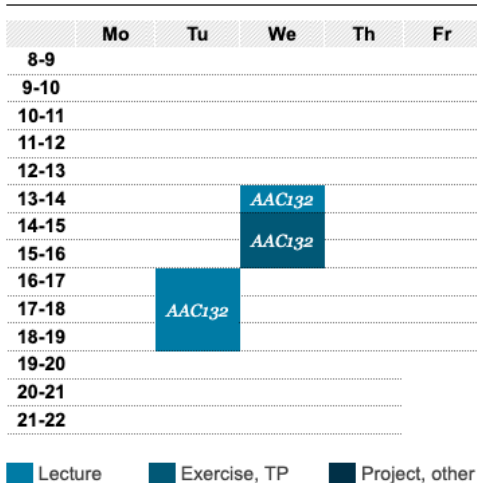
- Quantum computing (Week 1-7)
- Nano computing (Week 8-14)

- Prerequisites
 - Basic mathematics/physics

- Recommended courses
 - Basic quantum mechanics (e.g. Giovanni Boero, Pierre-André Besse)
 - Solid-state devices
 - CMOS circuit design

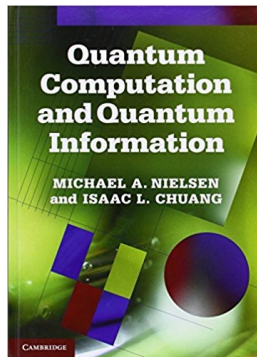
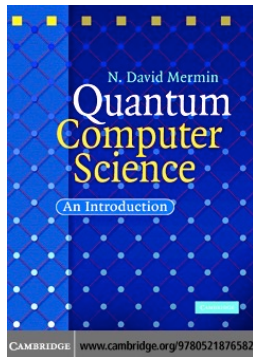
- Tuesday: 3h lectures with breaks
- Wednesday: 1h lecture/testimonial
2h homework/questions
- Evaluation
 - Weekly homework (50%)
 - Final written exam (50%)

REFERENCE WEEK



Recommended textbooks

- N.D. Mermin, Quantum Computer Science: An Introduction, Cambridge University Press, 5th printing, 2016. ISBN 978-0-521-87658-2
- M.A. Nielsen, I.L. Chuang, "Quantum Computation and Quantum Information", Cambridge University Press, 3rd printing, 2017. ISBN 978-1-107-00217-3



Quantum Computing Syllabus (Week 1-7)

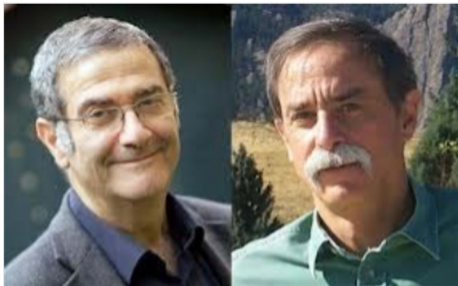
- **Fundamentals of quantum computing**
- Qubit realization & control
- Cryo-CMOS components
- Scalable quantum computers
- Quantum communication, sensing, and metrology

A computation is a physical process. It may be performed by a piece of electronics or on an abacus, or in your brain, but it is a process that takes place in nature and as such it is subject to the laws of physics. Quantum computers are machines that rely on characteristically quantum phenomena, such as quantum interference and quantum entanglement in order to perform computation.

- Artur Ekert

Overarching goal

*Solve intractable problems with massive speedup in computation...
...using the superposition and entanglement, two of the cornerstones
quantum mechanics*



Serge Haroche

David Wineland



2012 Physics Nobel Prize

Both Laureates work in the field of quantum optics studying the fundamental interaction between light and matter, a field which has seen considerable progress since the mid-1980s. Their ground-breaking methods have enabled this field of research to take the very first steps towards building a new type of super fast computer based on quantum physics. Perhaps the quantum computer will change our everyday lives in this century in the same radical way as the classical computer did in the last century.

—Announcement 2012 Nobel Prize

The Quantum Hype

Google

IBM Shows Off a Quantum Computing Chip

A new superconducting chip made by IBM demonstrates a technique crucial to the development of quantum computers.

By Tom Simonite on April 29, 2015

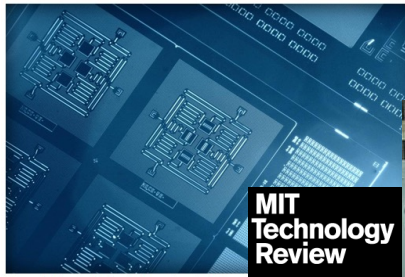
TC News Startups Mobile Gadgets Enterprise Social Trending Apple Google Face

Google AI quantum computing

Google Partners With UCSB To Build Quantum Processors For Artificial Intelligence

Posted Sep 2, 2014 by Frederic Lardinois (@fredric)

2,888 SHARES



TU Delft



 Microsoft

QuArC at Microsoft Research
Station Q @ UCSB

The Great Quantum Computing Race

1.1k
Shares

326



285



453



Companies and countries are pouring tens of billions of dollars into different qubit technologies, but it's still too early to predict a winner.

JULY 26TH, 2021 - BY: **MARK LAPEDUS**



Quantum computing is heating up, as a growing number of entities race to benchmark, stabilize, and ultimately commercialize this technology.

As of July 2021, a group from China appears to have taken the lead in terms of raw performance, but Google, IBM, Intel and other quantum computer developers aren't far behind. All of that could change overnight, though. At this point, it's too early to declare a winner in [quantum computing](#), a technology that promises to outperform today's conventional supercomputers.

Investments in Quantum Computing

- Big players and startups
- \$140M in Canada alone (2018)
- Extrapolated to €1T worldwide for the next decade



Microsoft



Zurich
Instruments



QUANTUM
MOTION

The screenshot shows the top portion of the D-Wave website. At the top left is the D-Wave logo with the tagline "The Quantum Computing Company™". To the right is a navigation menu with items: COMPANY, D-WAVE TECHNOLOGY (highlighted in blue), COMPUTING, RESOURCES, and NEWS. Below this is a secondary navigation bar with D-WAVE 2000Q, SOFTWARE, and SERVICES. The main content area features a large white heading: "The D-Wave 2000Q™ System" and a sub-heading: "The most advanced quantum computer in the world". The background of the main content area is a dark, stylized image of the quantum computer hardware.

D:wave
The Quantum Computing Company™

COMPANY ▾ D-WAVE TECHNOLOGY ▾ COMPUTING ▾ RESOURCES ▾ NEWS ▾

D-WAVE 2000Q SOFTWARE SERVICES

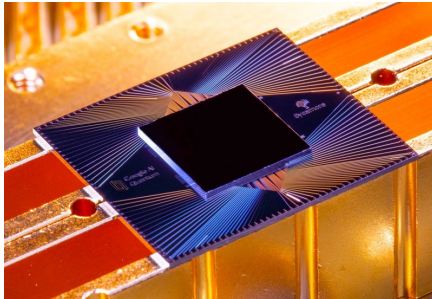
The D-Wave 2000Q™ System

The most advanced quantum computer in the world

Impact on (Scientific) Society?

- IBM allows anyone to program its quantum computer on line
- Google demonstrated quantum supremacy (quantum advantage) for the first time

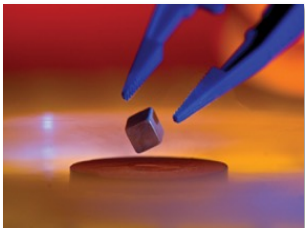
F. Arure et al., Nature **574**, 505-510 (2019)



Google's Sycamore chip
Credit: Erik Lucero

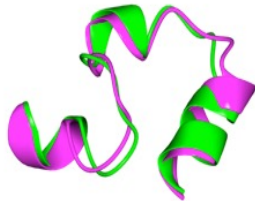


IBM Q System



Energy

Room-temperature
superconductivity



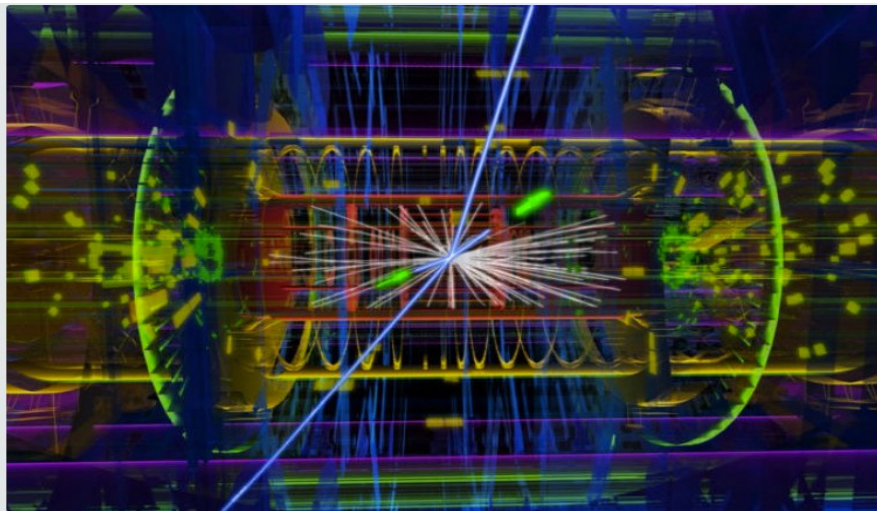
Health

Quantum chemistry



Internet Security

Source: L. Vandersypen, ISSCC 2017

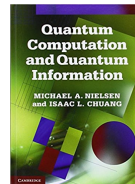


OCT 26, 2017

Ars Technica: Higgs Boson Uncovered By Quantum Algorithm On D-Wave Machine

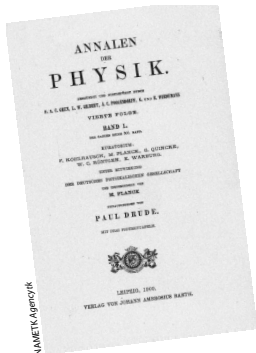
- Basic concepts
- Qubits & superposition
- 1-qubit measurement
- 1-qubit gates
- 2-qubit systems
- 2-qubit measurement

Chapter 1-4

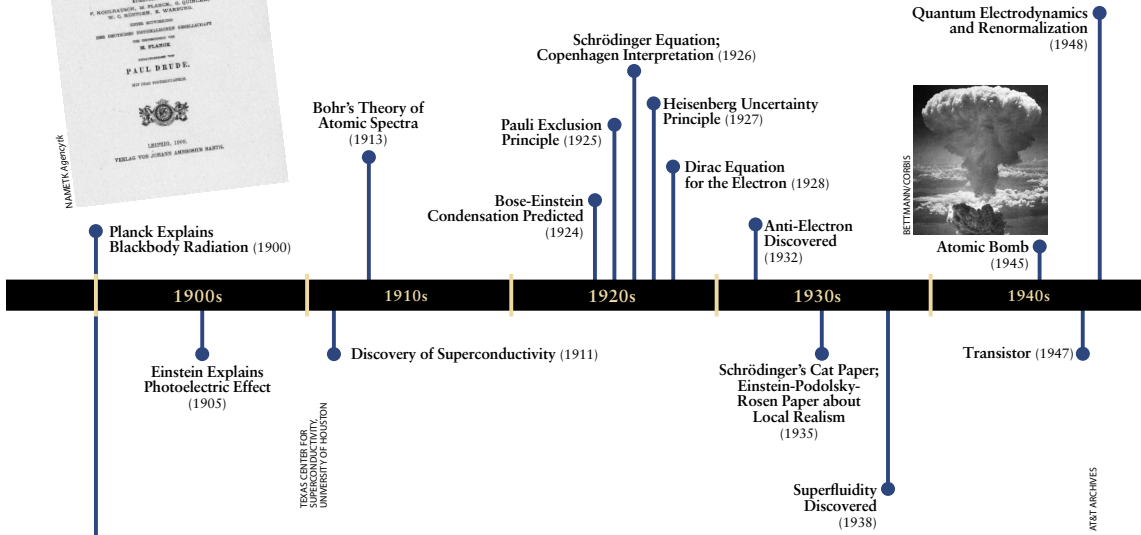


Basic Concepts

The First Quantum Revolution



M. Tegmark & J.A. Wheeler, 2001



NAMETK Agency

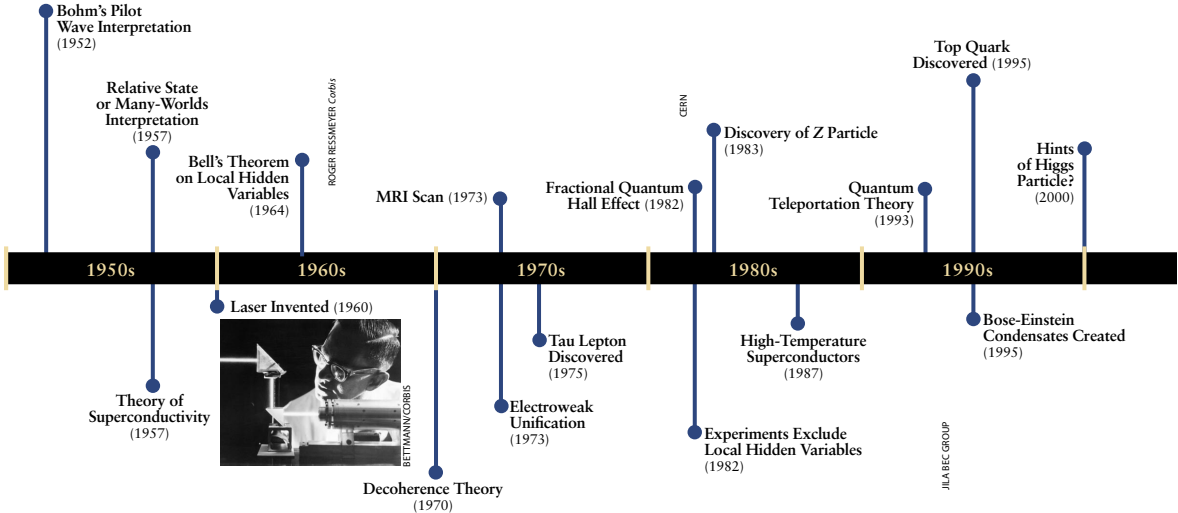
TEXAS CENTER FOR SUPERCONDUCTIVITY, UNIVERSITY OF HOUSTON

BETTMANN/CORBIS

AT&T ARCHIVES

The First Quantum Revolution

M. Tegmark & J.A. Wheeler, 2001



The Second Quantum Revolution

- Spearheaded by many, *in primis* Richard Feynman
- Proposal to use of **entanglement** and **superposition** for computation
- Fundamentals and theory developed in the 1980-2000s

There is plenty of space at the bottom

- Richard Feynman



- Enabling principles
 - Superposition & entanglement

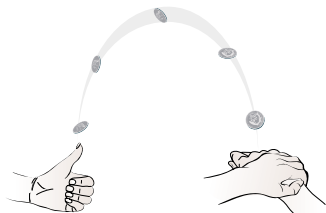
- Enabling methods/methodologies
 - Quantum algorithms
 - DiVincenzo criteria

- Enabling technologies
 - Solid-state qubits
 - Quantum-classical interfaces
 - Quantum-classical architectures

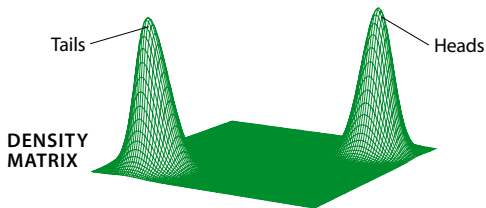
- *And...*
 - *Deep-cryogenic refrigerators*



CLASSICAL UNCERTAINTY

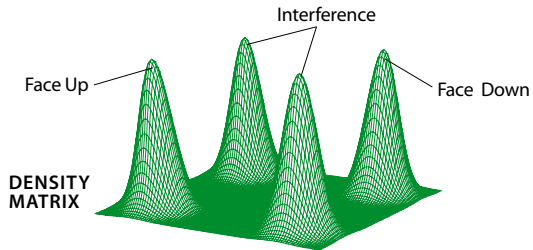


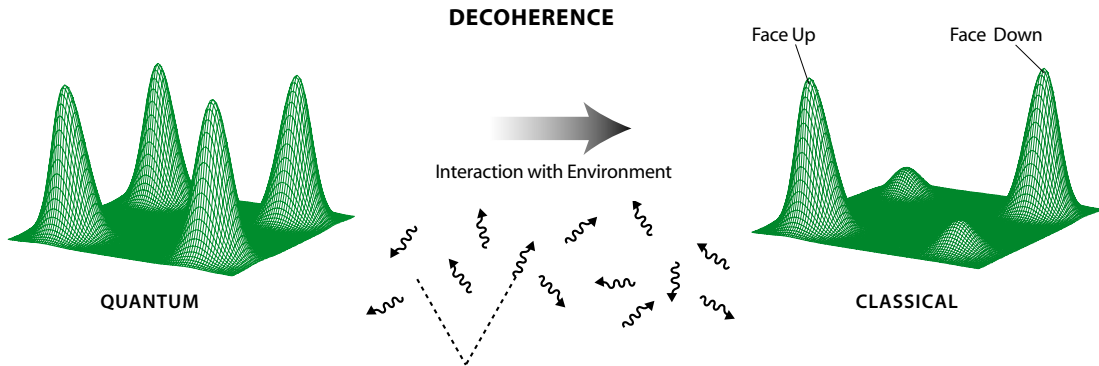
COIN TOSS



QUANTUM UNCERTAINTY

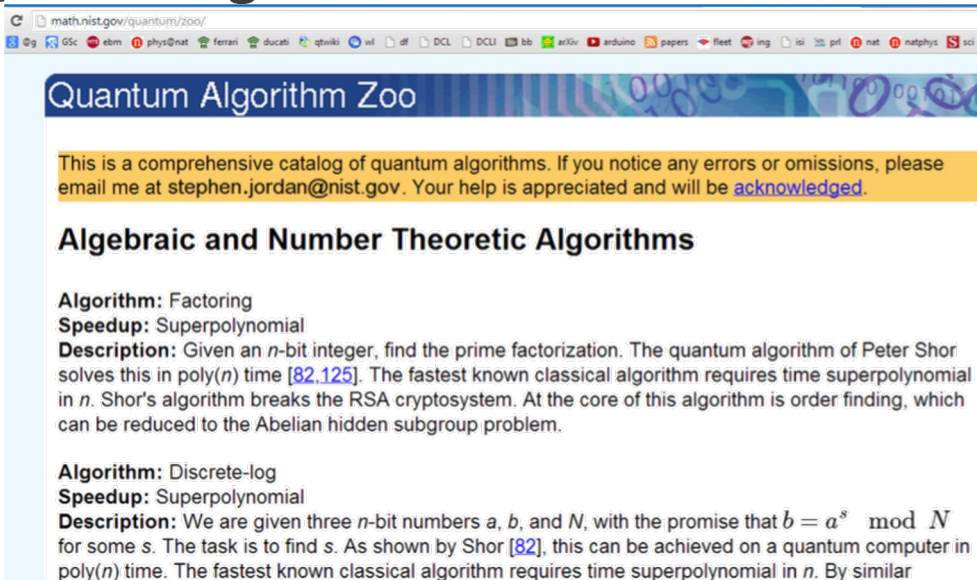
COHERENT SUPERPOSITION





Definition: two particles are entangled if the quantum state of one particle cannot be described independently from the quantum state of the other particle.

Intuition: measuring the quantum state of one particle implies knowledge of the quantum state (e.g. momentum, spin, polarization, etc.) of the other entangled particle using the same projection.



The image shows a browser window displaying the Quantum Algorithm Zoo website. The address bar shows the URL `math.nist.gov/quantum/zoo/`. The page title is "Quantum Algorithm Zoo". A yellow banner contains the text: "This is a comprehensive catalog of quantum algorithms. If you notice any errors or omissions, please email me at stephen.jordan@nist.gov. Your help is appreciated and will be [acknowledged](#)." Below this, the section "Algebraic and Number Theoretic Algorithms" is highlighted. It lists two algorithms: Factoring and Discrete-log, each with its speedup and a description.

math.nist.gov/quantum/zoo/

Quantum Algorithm Zoo

This is a comprehensive catalog of quantum algorithms. If you notice any errors or omissions, please email me at stephen.jordan@nist.gov. Your help is appreciated and will be [acknowledged](#).

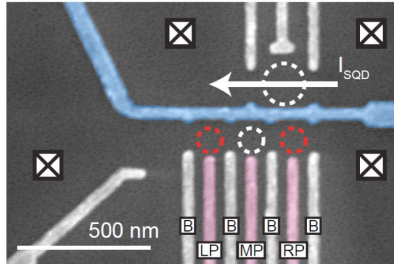
Algebraic and Number Theoretic Algorithms

Algorithm: Factoring
Speedup: Superpolynomial
Description: Given an n -bit integer, find the prime factorization. The quantum algorithm of Peter Shor solves this in $\text{poly}(n)$ time [82,125]. The fastest known classical algorithm requires time superpolynomial in n . Shor's algorithm breaks the RSA cryptosystem. At the core of this algorithm is order finding, which can be reduced to the Abelian hidden subgroup problem.

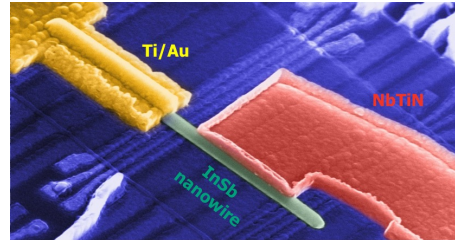
Algorithm: Discrete-log
Speedup: Superpolynomial
Description: We are given three n -bit numbers a , b , and N , with the promise that $b = a^s \pmod N$ for some s . The task is to find s . As shown by Shor [82], this can be achieved on a quantum computer in $\text{poly}(n)$ time. The fastest known classical algorithm requires time superpolynomial in n . By similar

~50 algorithms with quantum speedup, but most people know 2.

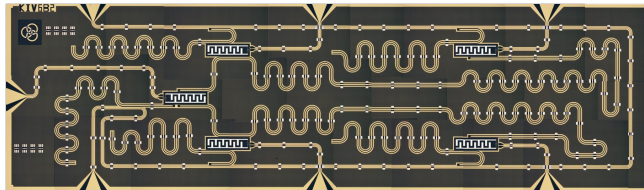
1. A scalable physical system with well characterized qubits.
2. The ability to initialize the state of the qubits to a simple fiducial state.
3. Long relevant decoherence times.
4. A “universal” set of quantum gates.
5. A qubit-specific measurement capability.
6. The ability to interconvert stationary and flying qubits.
7. The ability to faithfully transmit flying qubits between specified locations.



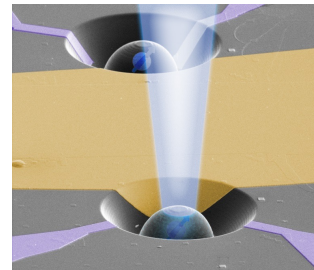
Semiconductor quantum dots



Semiconductor-superconductor hybrids



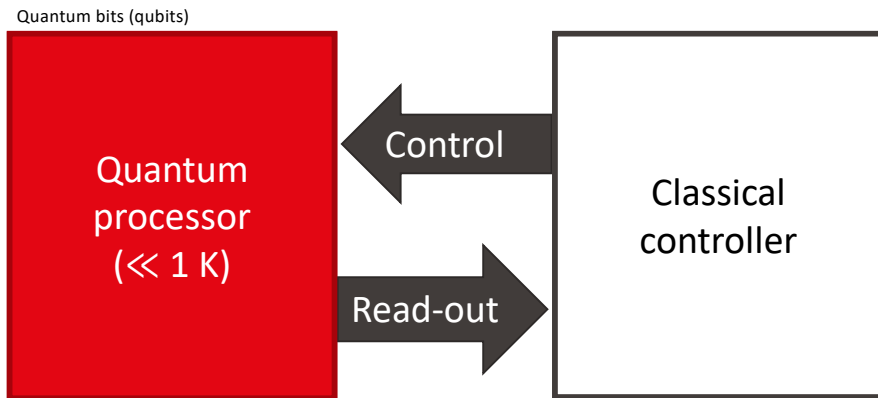
Superconducting circuits



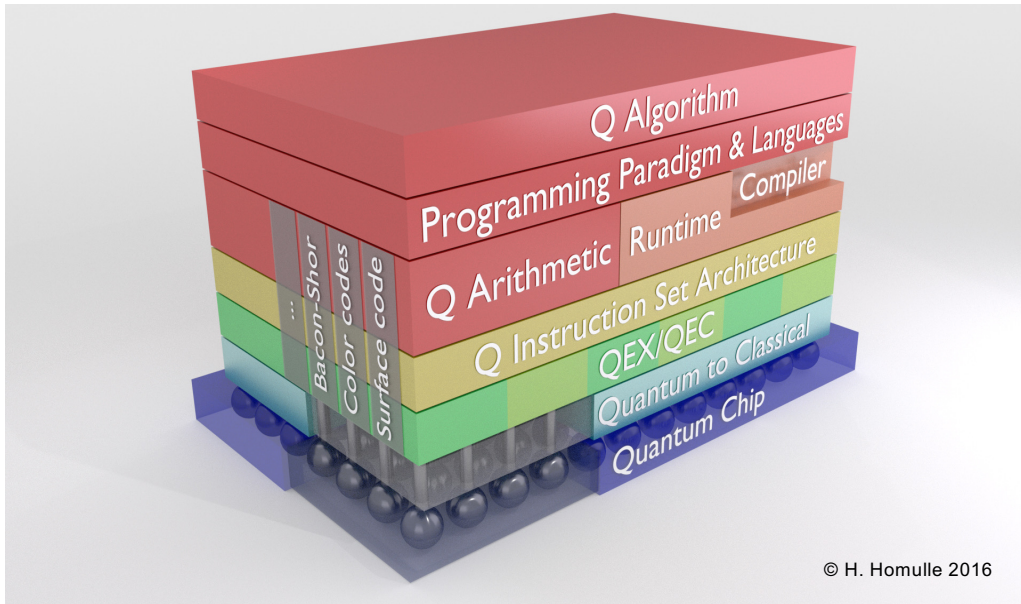
Impurities in diamond

Image source: L. Vandersypen, 2017

Quantum-Classical Interfaces



- Carrier frequency: 100 MHz – 15 GHz, 70 GHz
- Pulses: 10 – 100 ns



72 Qubit system with ~50%
of room temperature cabling

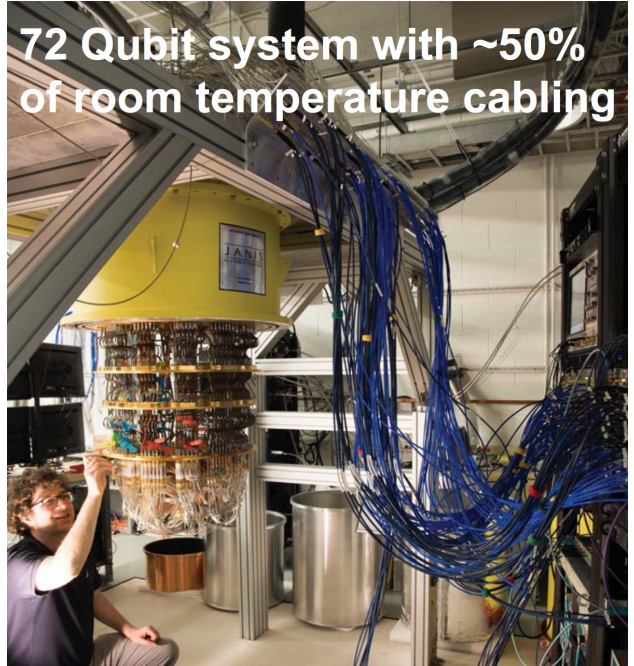


Image: Google Bristlecone. Taken from: J.C. Bardin et al.,
"An Introduction to Quantum Computing for RFIC
Engineers", RFIC Symposium 2019

Qubits & Superposition

Quantum Bit (Qubit)

- A quantum bit or qubit is a quantum system in which the Boolean states 0 and 1 are represented by a pair of mutually orthogonal quantum states labeled as $|0\rangle$ and $|1\rangle$.

Dirac bra-ket notation

- A vector notation is also used

$$|0\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Superposition of states is represented as follows

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

where

$$\alpha_{0,1} \in \mathcal{C}$$

α_i is a probability amplitude

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

$|\alpha_i|^2$ is the probability of finding the qubit in state $|i\rangle$ when measured

$$|\psi\rangle \doteq \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

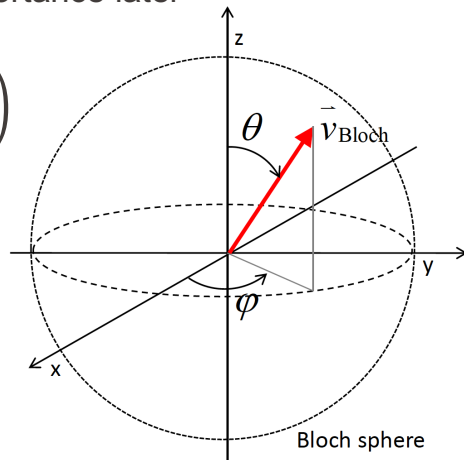
- A single qubit is represented in three dimensions
- X,Y,Z axes represent possible projections for qubit readout
- The x-y plane is important – see its importance later

$$|\psi\rangle = e^{i\delta} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle \right)$$

θ : polar angle

φ : azimuth

δ : global phase (ignored in the Bloch sphere)



- A quantum state $|\psi\rangle \doteq \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$ is normalized iff $\langle\psi|\psi\rangle = 1$,

$$\begin{aligned}\langle\psi|\psi\rangle &= (\alpha_0^* \quad \alpha_1^*) \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \alpha_0^* \alpha_0 + \alpha_1^* \alpha_1 \\ &= |\alpha_0|^2 + |\alpha_1|^2\end{aligned}$$

Note:

$\langle\psi|\psi\rangle$: self inner product or self overlap

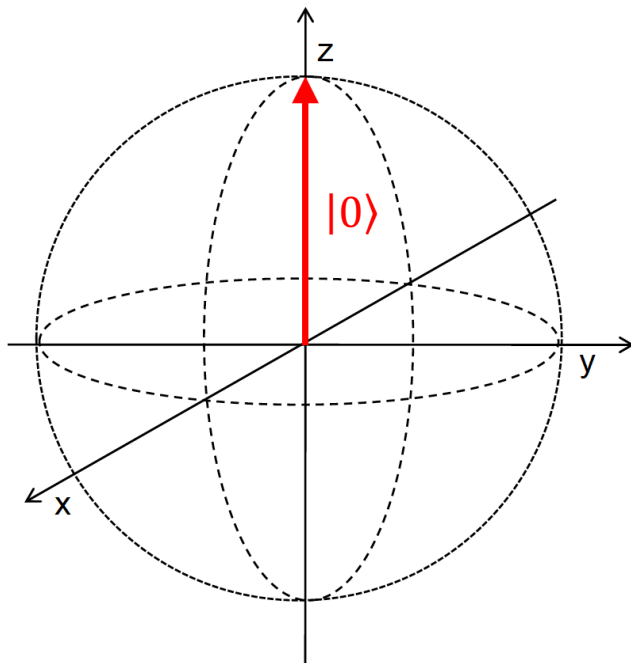
Orthogonality

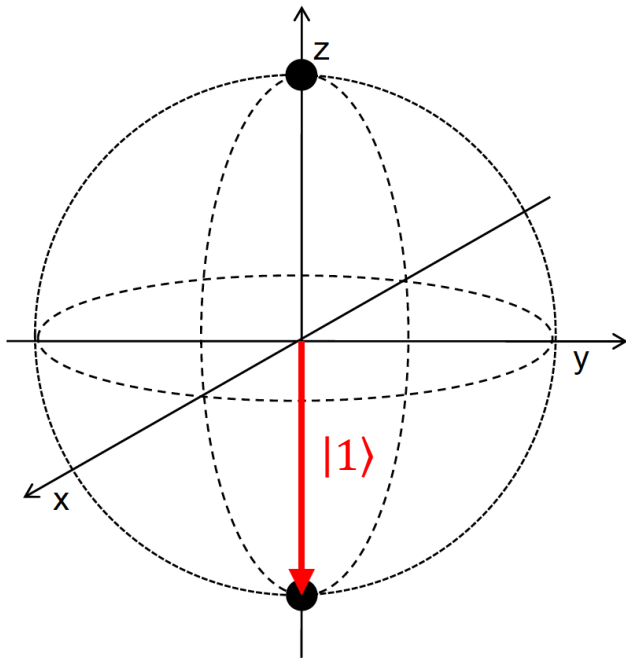
- Two quantum states $|\psi\rangle \doteq \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$ and $|\psi'\rangle \doteq \begin{pmatrix} \alpha'_0 \\ \alpha'_1 \end{pmatrix}$ are mutually orthogonal iff

$$\langle \psi | \psi' \rangle = (\alpha_0^* \quad \alpha_1^*) \begin{pmatrix} \alpha'_0 \\ \alpha'_1 \end{pmatrix} = \alpha_0^* \alpha'_0 + \alpha_1^* \alpha'_1 = 0$$

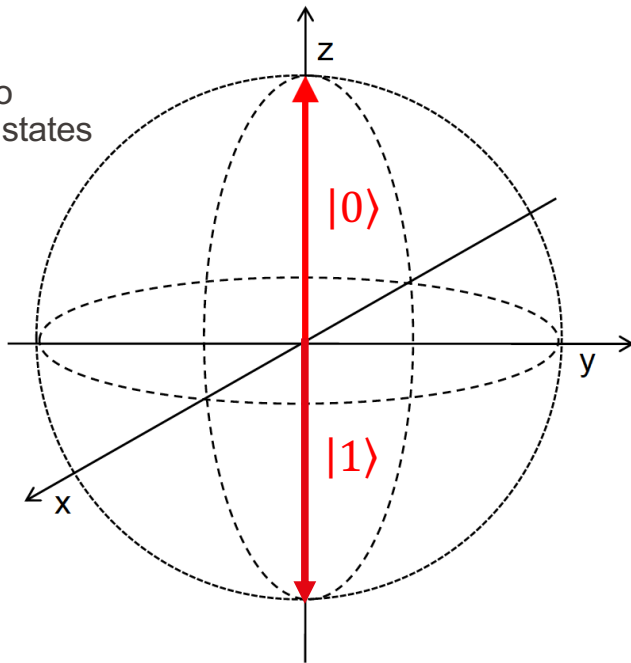
Note:

$\langle \psi | \psi' \rangle$: inner product or overlap

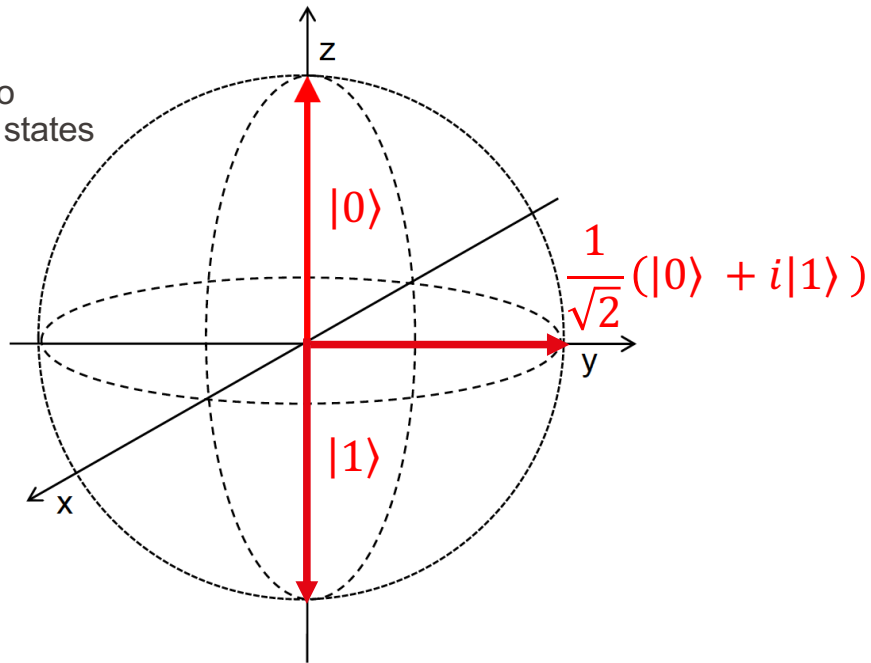




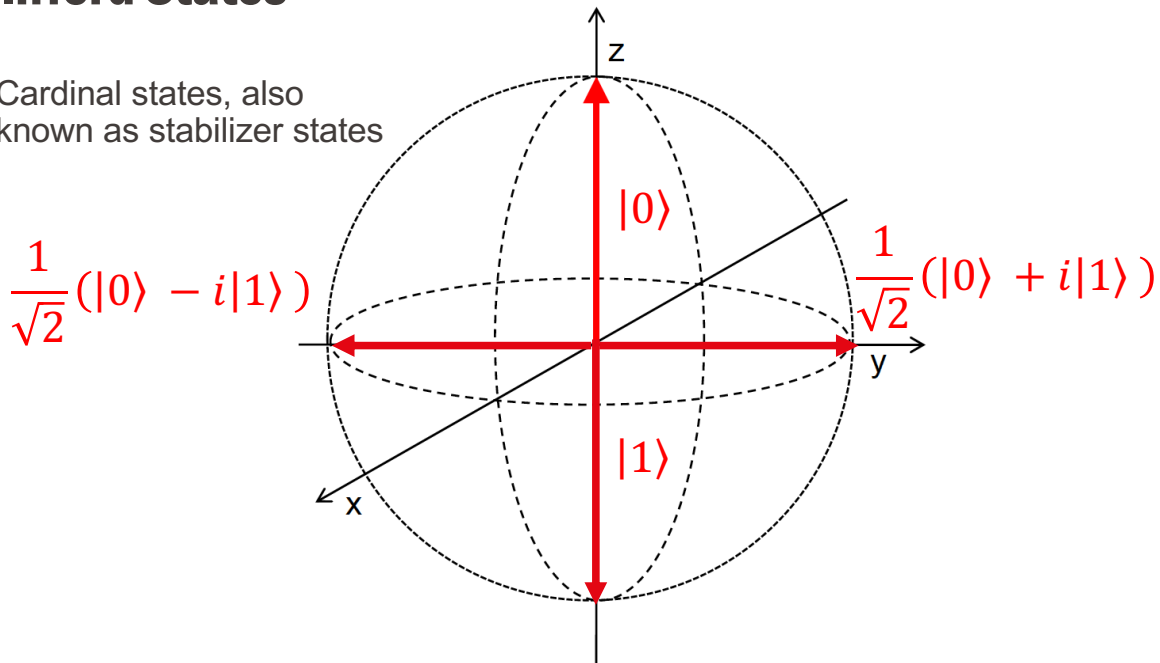
- Cardinal states, also known as stabilizer states



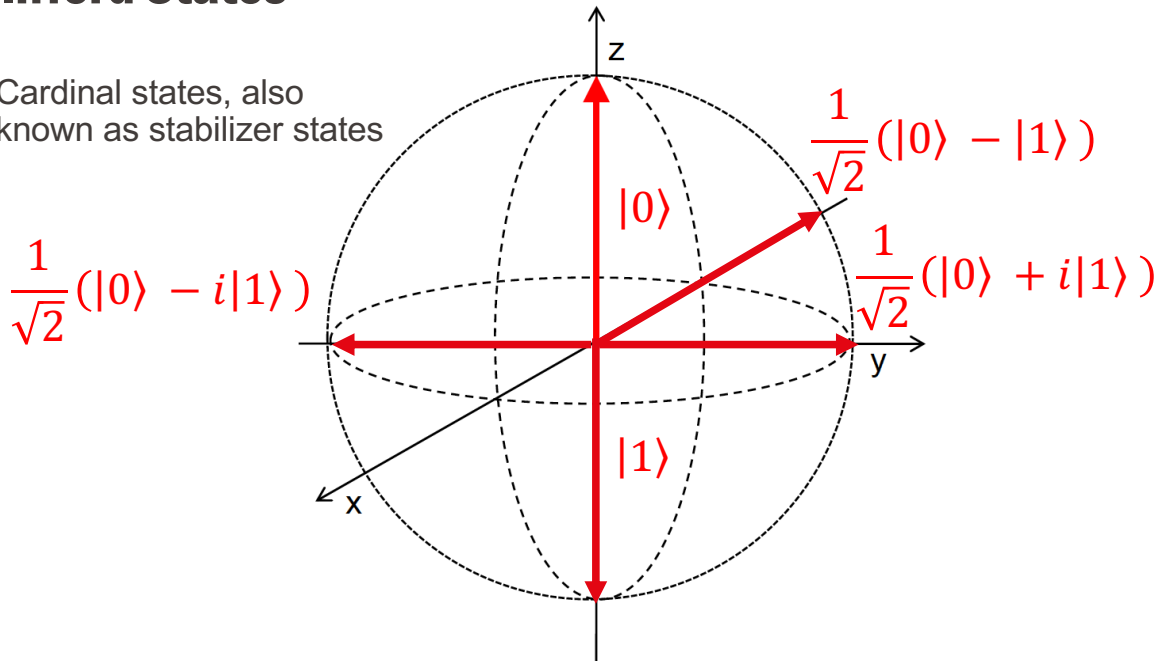
- Cardinal states, also known as stabilizer states



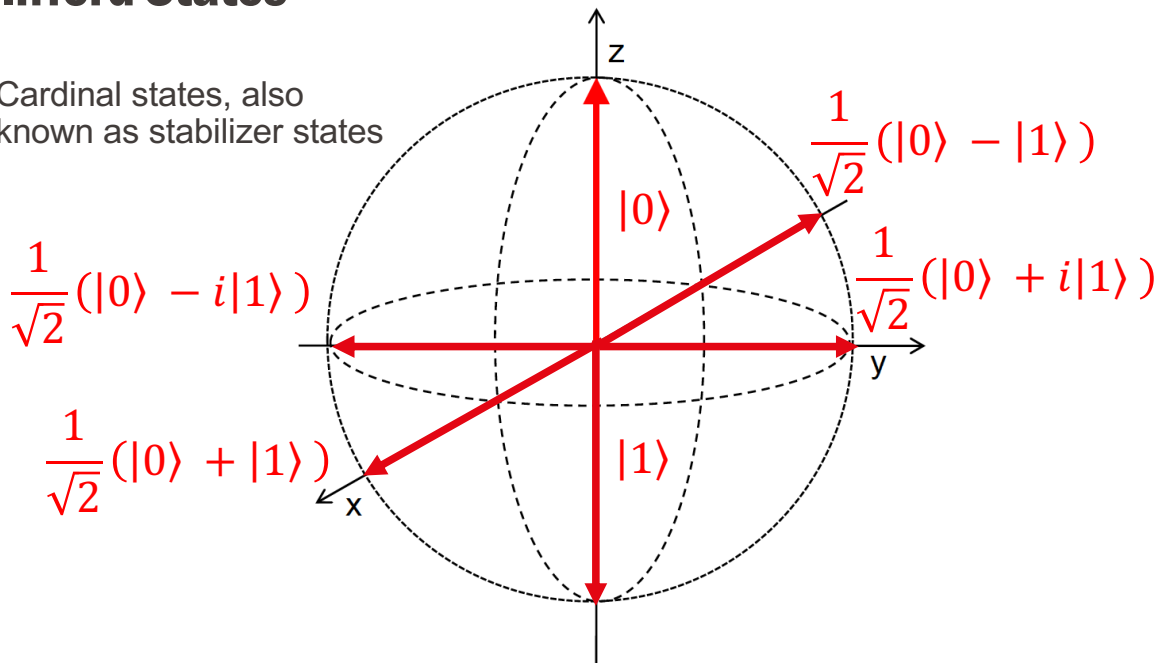
- Cardinal states, also known as stabilizer states



- Cardinal states, also known as stabilizer states



- Cardinal states, also known as stabilizer states



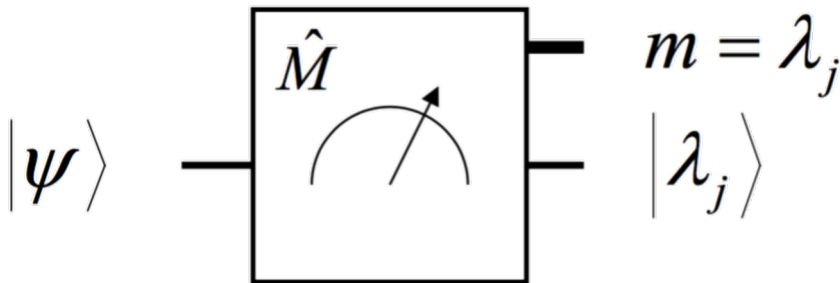
Altering a Quantum State

- By performing a measurement
- By using a gate

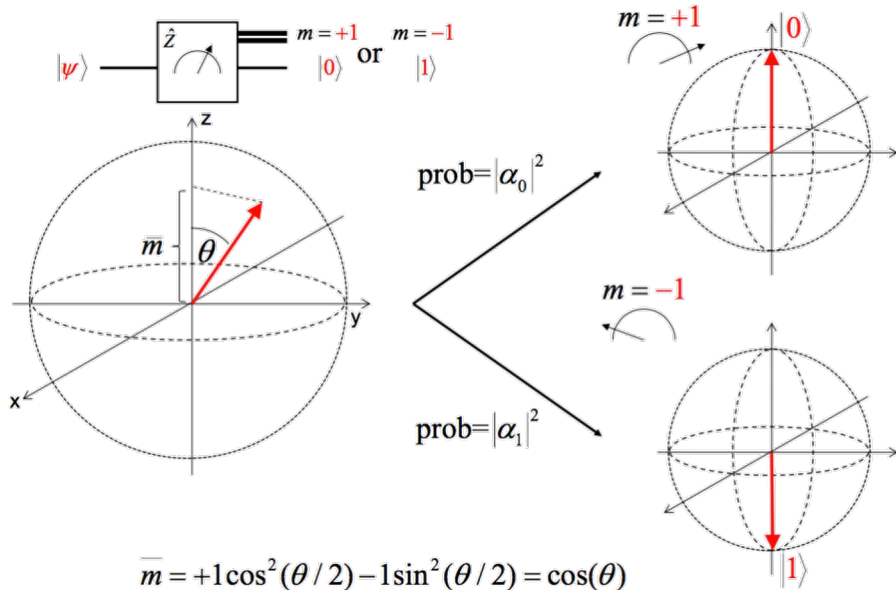
1-Qubit Measurement

1-qubit Measurement

- We need to understand how to measure a qubit
- When a qubit in the Bloch sphere is read its wavefunction is collapsed and a *probabilistic* measurement results from the measurement
- Note that up to measurement, the state is deterministic!



Example



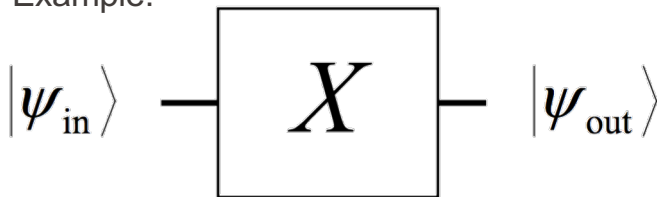
Source:
Leo DiCarlo

1-Qubit Gates

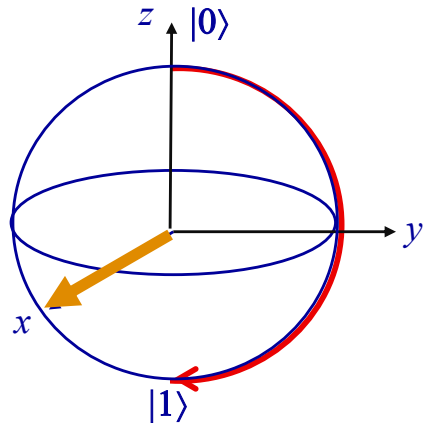
1-qubit Gate

- When a gate is used the qubit is transformed and the result is *deterministic*
- This remains the case until it is read out
- A 1-qubit gate will rotate the qubit in the Bloch sphere

- Example:

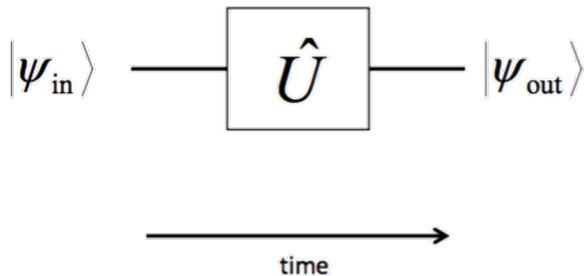


- This is a π -rotation wrt X axis



Unitary Transform

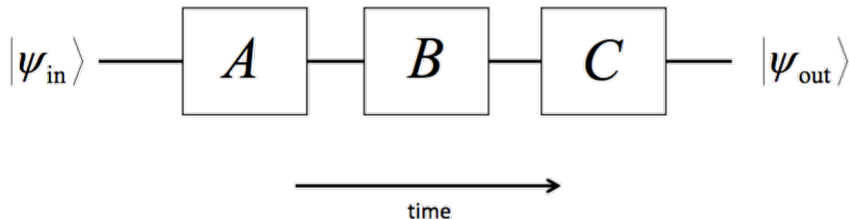
- A unitary transformation is a specific **rotation** on the Bloch sphere where condition (1) is satisfied
- Note that a transform requires **time** to be executed



(1) \hat{U} is a unitary operator when: $\hat{U}\hat{U}^\dagger = \hat{U}^\dagger\hat{U} = \hat{I}$
 † = conjugate and transpose

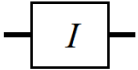
Chain Transformations

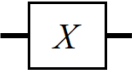
- A chain transformations is written from left to right but mathematically from right to left
- The input is on the right and the output on the left!



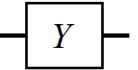
$$|\psi_{out}\rangle = CBA|\psi_{in}\rangle$$

Standard Transformations

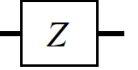
Identity  $\hat{I} \doteq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Pauli X  $\hat{X} \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Also known as σ_x

Pauli Y  $\hat{Y} \doteq \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

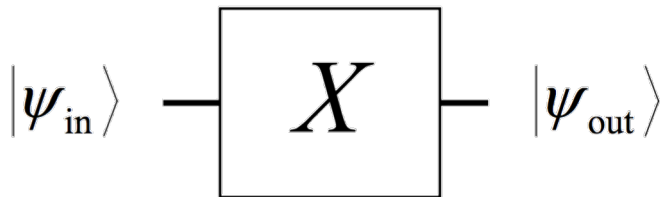
Also known as σ_y

Pauli Z  $\hat{Z} \doteq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Also known as σ_z

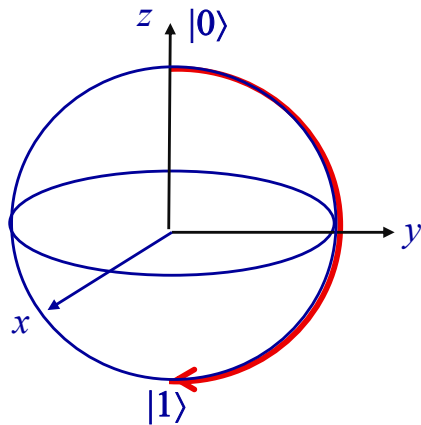
Source:
Leo DiCarlo

Example



$$|\psi_{in}\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi_{out}\rangle \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$\hat{X}\hat{X}^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \hat{I} \quad \text{Unitary operator}$$

$$\hat{X} = \hat{X}^\dagger \quad \text{hermitian}$$

$$\hat{X}\hat{X}^\dagger = \hat{Y}\hat{Y}^\dagger = \hat{Z}\hat{Z}^\dagger = \hat{I} \quad \text{unitary}$$

$$\hat{X}\hat{Y} = i\hat{Z} \quad \hat{Y}\hat{X} = -i\hat{Z}$$

$$[\hat{X}, \hat{Y}] = 2i\hat{Z}, \quad [\hat{Z}, \hat{X}] = 2i\hat{Y}, \quad [\hat{Y}, \hat{Z}] = 2i\hat{X}$$

$$\{\hat{X}, \hat{Y}\} = \{\hat{Z}, \hat{X}\} = \{\hat{Y}, \hat{Z}\} = 0$$

definition

$$[A, B] \equiv AB - BA$$

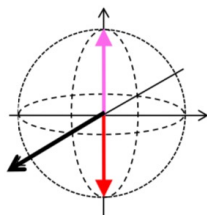
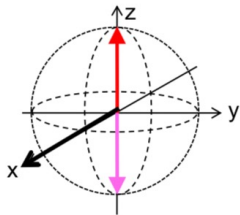
$$\{A, B\} \equiv AB + BA$$

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X-Rotation Examples

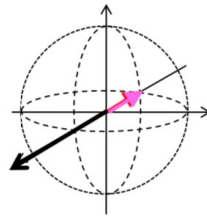
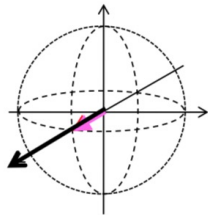
$$|\psi\rangle \xrightarrow{X} |\psi\rangle$$

$$|0\rangle \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} |1\rangle$$



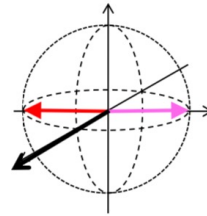
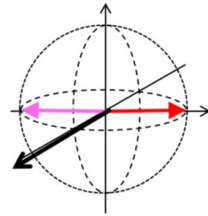
$$|0\rangle + |1\rangle \quad +1(|0\rangle + |1\rangle)$$

$$|0\rangle - |1\rangle \quad -1(|0\rangle - |1\rangle)$$



$$|0\rangle + i|1\rangle \quad i(|0\rangle - i|1\rangle)$$

$$|0\rangle - i|1\rangle \quad -i(|0\rangle + i|1\rangle)$$



What happened to normalization?
Well, usually we tend to forget it.

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The Hadamard Gate

$$|\psi\rangle \longrightarrow \boxed{H} \longrightarrow |\psi\rangle$$

$$|0\rangle \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad |0\rangle + |1\rangle$$

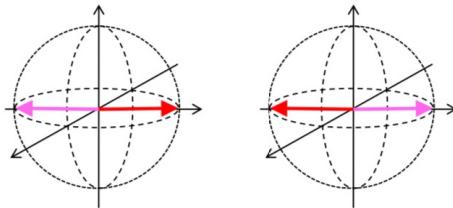
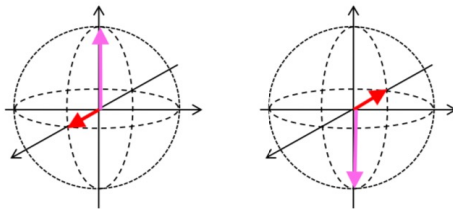
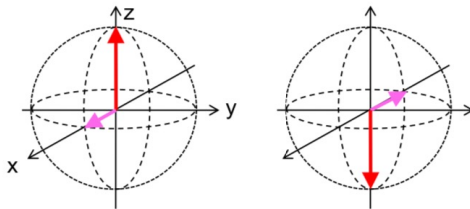
$$|1\rangle \quad |0\rangle - |1\rangle$$

$$|0\rangle + |1\rangle \quad |0\rangle$$

$$|0\rangle - |1\rangle \quad |1\rangle$$

$$|0\rangle + i|1\rangle \quad e^{i\pi/4} (|0\rangle - i|1\rangle)$$

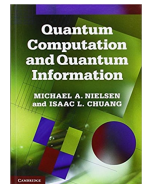
$$|0\rangle - i|1\rangle \quad e^{-i\pi/4} (|0\rangle - i|1\rangle)$$



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2-Qubit Systems

Chapter 4



- We go from a single qubit to a 2-qubit system
- The state is fully characterized by 6 real parameters

$$|\Psi\rangle \doteq \alpha_{11}|11\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{00}|00\rangle \doteq \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

$$\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11} \in \mathbb{C}$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

- Two qubits are entangled when their joint wavefunction cannot be separated into a product of individual qubit wavefunctions

$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle \neq |\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle + |\varphi'\rangle \otimes |\psi'\rangle$$

Unentangled = separable = product

Entangled = non-separable = non-product

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

- States that form an orthonormal basis for the 4-dimensional Hilbert space of 2 qubits. This basis is called the Bell basis.

$$\begin{aligned}
 |\Phi_+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
 |\Phi_-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)
 \end{aligned}
 \left. \vphantom{\begin{aligned} |\Phi_+\rangle \\ |\Phi_-\rangle \end{aligned}} \right\} \textit{The even parity Bell states.}$$

$$\begin{aligned}
 |\Psi_+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\
 |\Psi_-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)
 \end{aligned}
 \left. \vphantom{\begin{aligned} |\Psi_+\rangle \\ |\Psi_-\rangle \end{aligned}} \right\} \textit{The odd parity Bell states.}$$

The singlet.

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- Write the state of two qubits as follows:

$$|\Psi\rangle = \alpha|0\rangle \otimes |\psi\rangle + \beta|1\rangle \otimes |\psi'\rangle$$

Where

$$|\alpha|^2 + |\beta|^2 = 1 \quad \text{and} \quad |\psi\rangle, |\psi'\rangle \text{ are normalized}$$

If $|\psi\rangle = |\psi'\rangle$ then, they are separable

else, they are entangled

- Write the state of two qubits as follows:

$$\frac{1}{5} |00\rangle + \frac{2}{5} |01\rangle + \frac{2}{5} |10\rangle + \frac{4}{5} |11\rangle$$

$$= \frac{1}{\sqrt{5}} |0\rangle \otimes \frac{1}{\sqrt{5}} (|0\rangle + 2|1\rangle) + \frac{2}{\sqrt{5}} |1\rangle \otimes \frac{1}{\sqrt{5}} (|0\rangle + 2|1\rangle)$$

$$= \frac{1}{\sqrt{5}} (|0\rangle + 2|1\rangle) \otimes \frac{1}{\sqrt{5}} (|0\rangle + 2|1\rangle)$$

Not entangled!

- A collection of n qubits is called quantum register of size n .
- The information is stored in a register of 'conventional' binary form
- Example ($n=2$):

$$|0\rangle = |0\rangle \otimes |0\rangle = |00\rangle$$

$$|1\rangle = |0\rangle \otimes |1\rangle = |01\rangle$$

$$|2\rangle = |1\rangle \otimes |0\rangle = |10\rangle$$

$$|3\rangle = |1\rangle \otimes |1\rangle = |11\rangle$$

- Remember, the number of **Hilbert states** $N = 2^n$!!

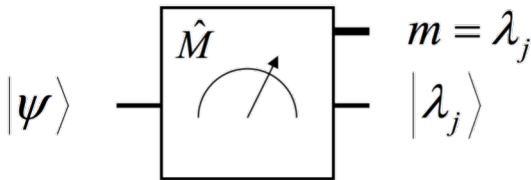
$$\frac{1}{\sqrt{2}} (|2\rangle + |3\rangle) = \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) = |1\rangle \otimes \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$\begin{aligned} \frac{1}{2} (|0\rangle + |1\rangle + |2\rangle + |3\rangle) &= \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\ &= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \end{aligned}$$

2-Qubit Measurement

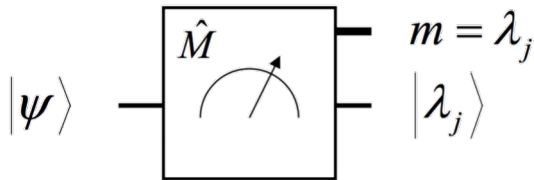
Reminder: 1-qubit Measurement

- We need to understand how to measure a qubit
- When a qubit in the Bloch sphere is read its wavefunction is collapsed and a *probabilistic* measurement results from the measurement
- Note that up to measurement, the state is deterministic!



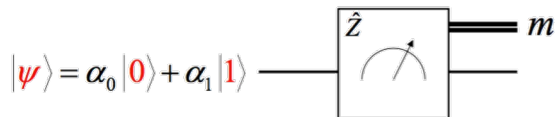
1-qubit Measurement Details

- Every measurement is associated with an operator \hat{M} called hermitian operator.
- The eigenvalues λ_i of the hermitian.
- Post-measurement state of the qubit is $|\lambda_i\rangle$ the eigenstate of the hermitian.
- The probability of the result being λ_i is computed as $|\langle\lambda_i|\psi\rangle|^2$ the squared overlap between input state and eigenstate.



with probability $|\langle\lambda_j|\psi\rangle|^2$

where $\hat{M}|\lambda_j\rangle = \lambda_j|\lambda_j\rangle$



$$\hat{Z} \doteq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle +1 | \psi \rangle = (1 \ 0) \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \alpha_0$$

$$\langle -1 | \psi \rangle = (0 \ 1) \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \alpha_1$$

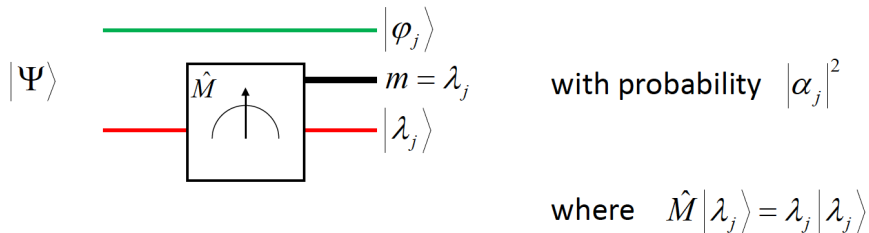
eigenvalues $+1,$ -1

eigenvectors $|+1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$ $| -1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

probabilities: $|\alpha_0|^2$ $|\alpha_1|^2$

average: $\bar{m} = +1|\alpha_0|^2 - 1|\alpha_1|^2$
 $= \cos^2(\theta/2) - \sin^2(\theta/2)$
 $= \cos(\theta)$

Source:
Leo DiCarlo



Without loss of generality, we can always write a 2-qubit state as

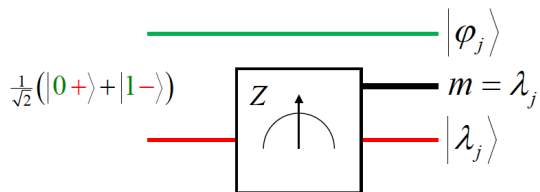
$$|\Psi\rangle = \sum_j \alpha_j |\varphi_j\rangle |\lambda_j\rangle$$

where $|\lambda_j\rangle$ form an orthonormal basis,

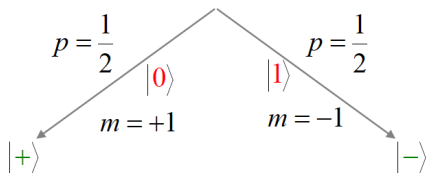
$|\varphi_j\rangle$ are normalized but not necessarily orthogonal

and $\sum_j |\alpha_j|^2 = 1$.

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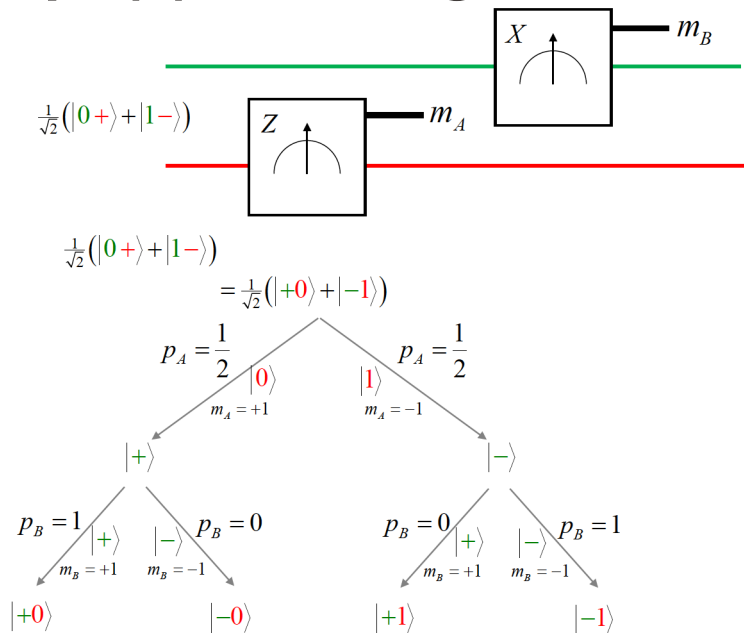


$$\begin{aligned}
 & \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle) \\
 &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}(|10\rangle - |11\rangle) \\
 &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) \\
 &= \frac{1}{\sqrt{2}}(|+0\rangle + |-1\rangle)
 \end{aligned}$$



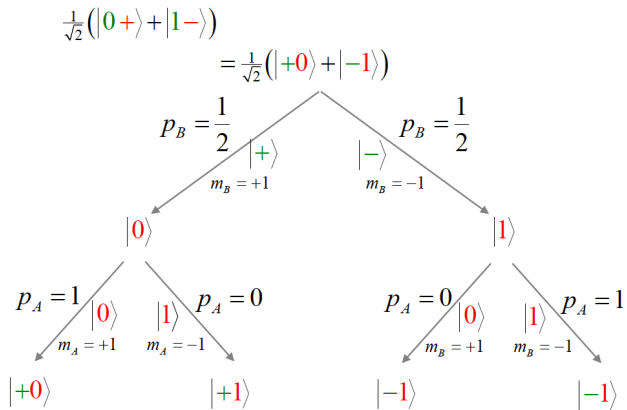
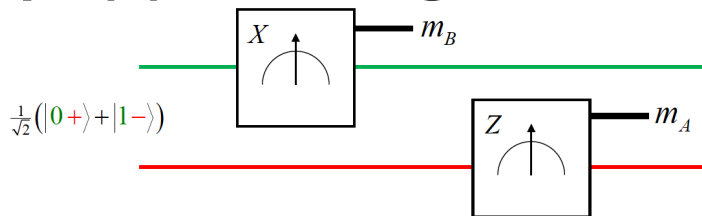
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Example (2) - Measuring both Qubits

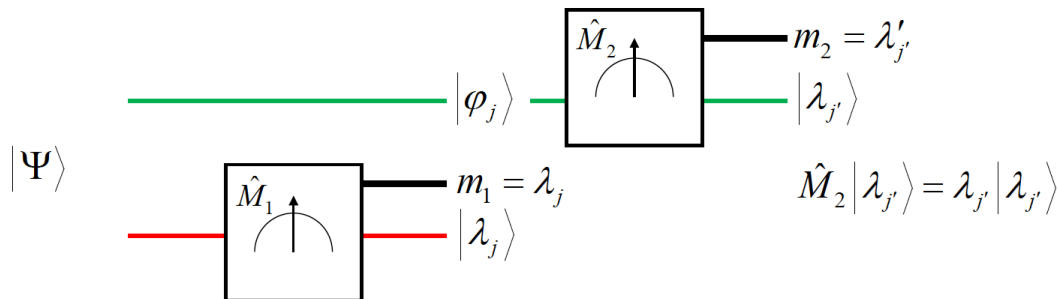


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Example (2) - Measuring both Qubits



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- We can show that the statistics of the readout outcomes do not depend on the order of the measurements.
- Interesting properties:

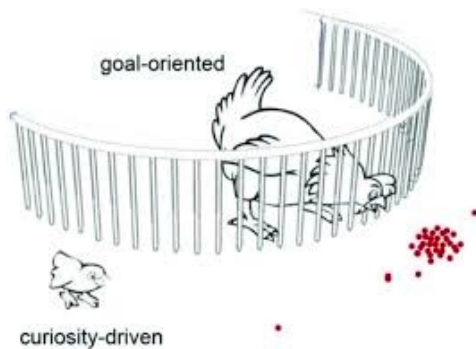
$$\bar{m}_1 = \langle \Psi | \hat{I} \hat{M}_1 | \Psi \rangle$$

$$\bar{m}_2 = \langle \Psi | \hat{M}_2 \hat{I} | \Psi \rangle$$

$$\overline{m_2 m_1} = \langle \Psi | \hat{M}_2 \hat{M}_1 | \Psi \rangle$$

Source:
Leo DiCarlo

Thank you



T. Haensch