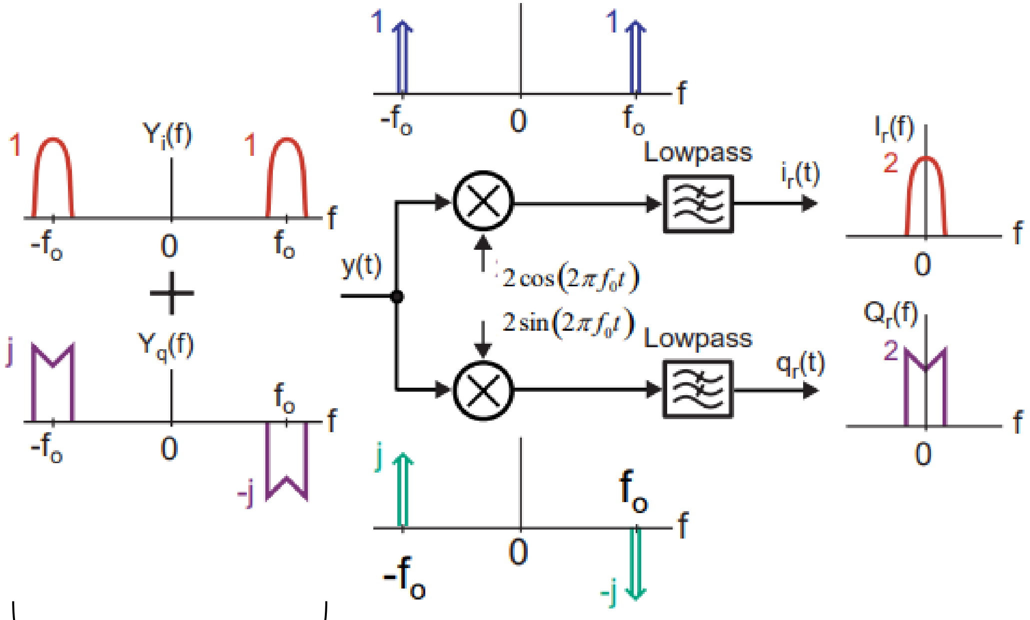
IQ Demodulation



$Acos(2\pi ft+\varphi)$

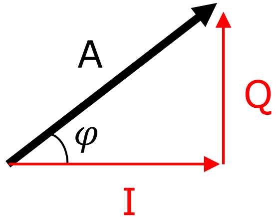
Important Trigonemetric Identities:

cos(a+b)=cos(a)cos(b)−sin(a)sin(b)
cos(a)cos(a) =1/2(cos(0)+cos(2a))
sin(a)cos(a) =1/2(sin(2a)+sin(0))
sin(a)sin(a) =1/2(cos(0)−cos(2a))

$Acos(2\pi ft+\varphi)$ is our signal that we want to reconstruct

$Acos(2\pi ft+\varphi) = Acos(2\pi ft)\cos(\varphi) - Asin(2\pi ft)\sin(\varphi)$

As known:
$Acos(\varphi) = I$
$Asin(\varphi) = Q$



$Acos(2\pi ft+\varphi) = Icos(2\pi ft) - Qsin(2\pi ft)$ ***is our signal***

Lets multiply our signal with **LO:** $2\times cos(2\pi ft)$ *and* $2\times sin(2\pi ft)$

$2\times Acos(2\pi ft+\varphi)\times cos(2\pi ft)$
$= 2\times (Icos(2\pi ft) - Qsin(2\pi ft))\times cos(2\pi ft)$
$= I\times (cos(0)+cos(4\pi ft)) - Q\times (sin(4\pi ft)+sin(0))$
$= \boldsymbol{I} + I\times \boldsymbol{cos(4\pi ft)} - Q\times \boldsymbol{sin(4\pi ft)}$

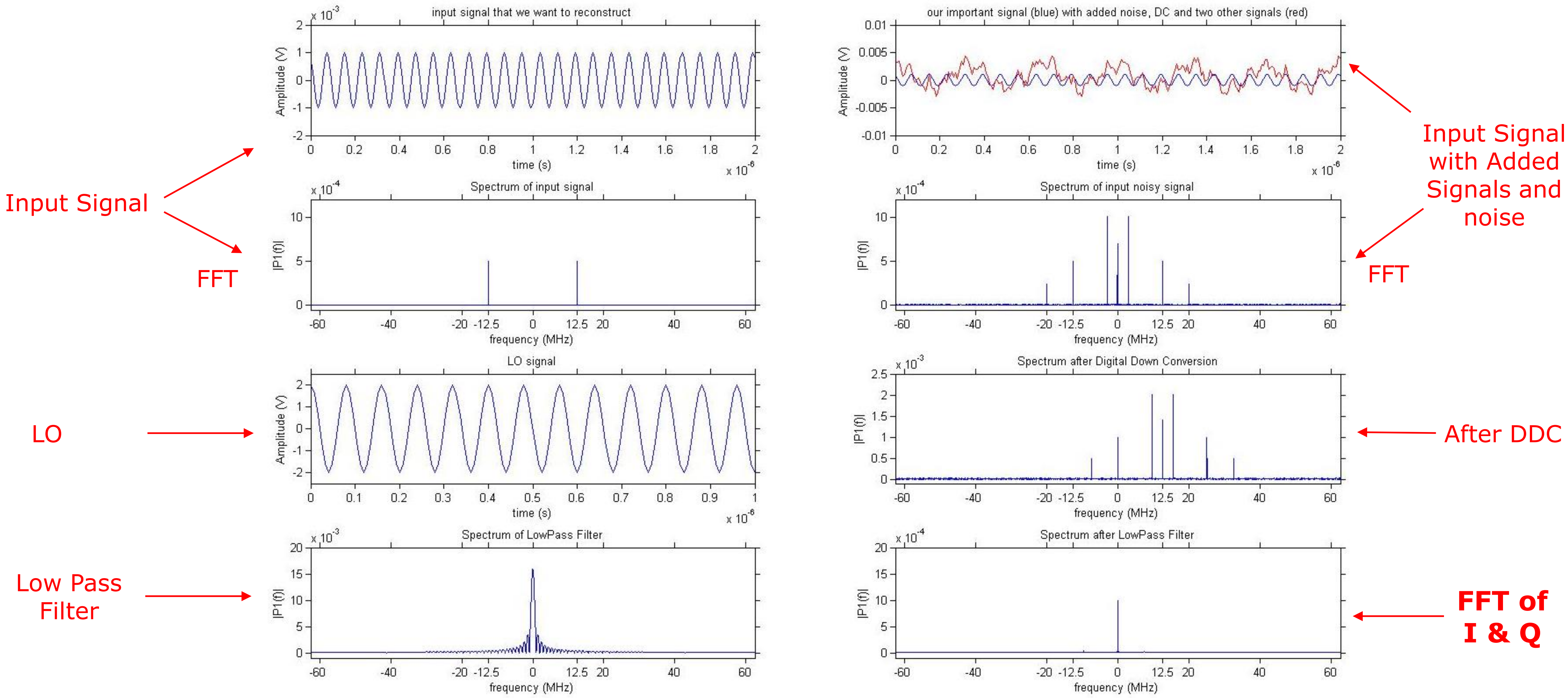
$2\times Acos(2\pi ft+\varphi)\times sin(2\pi ft)$
$= 2\times (Icos(2\pi ft) - Qsin(2\pi ft))\times sin(2\pi ft)$
$= I\times (sin(4\pi ft)+sin(0)) - Q\times (cos(0) - cos(4\pi ft))$
$= I\times \boldsymbol{sin(4\pi ft)} - \boldsymbol{Q} + Q\times \boldsymbol{cos(4\pi ft)}$

If you have I and Q, you can calculate A and φ, and reconstruct your signal

$A(t) = \sqrt{I(t)^2+Q(t)^2}$ $\varphi(t) = tan^{-1}\frac{Q(t)}{I(t)}$

$A(t)\,cos(2\pi ft+\varphi(t))$

Project Goal (Lockin_amplifier_solution.m)



Project Goal

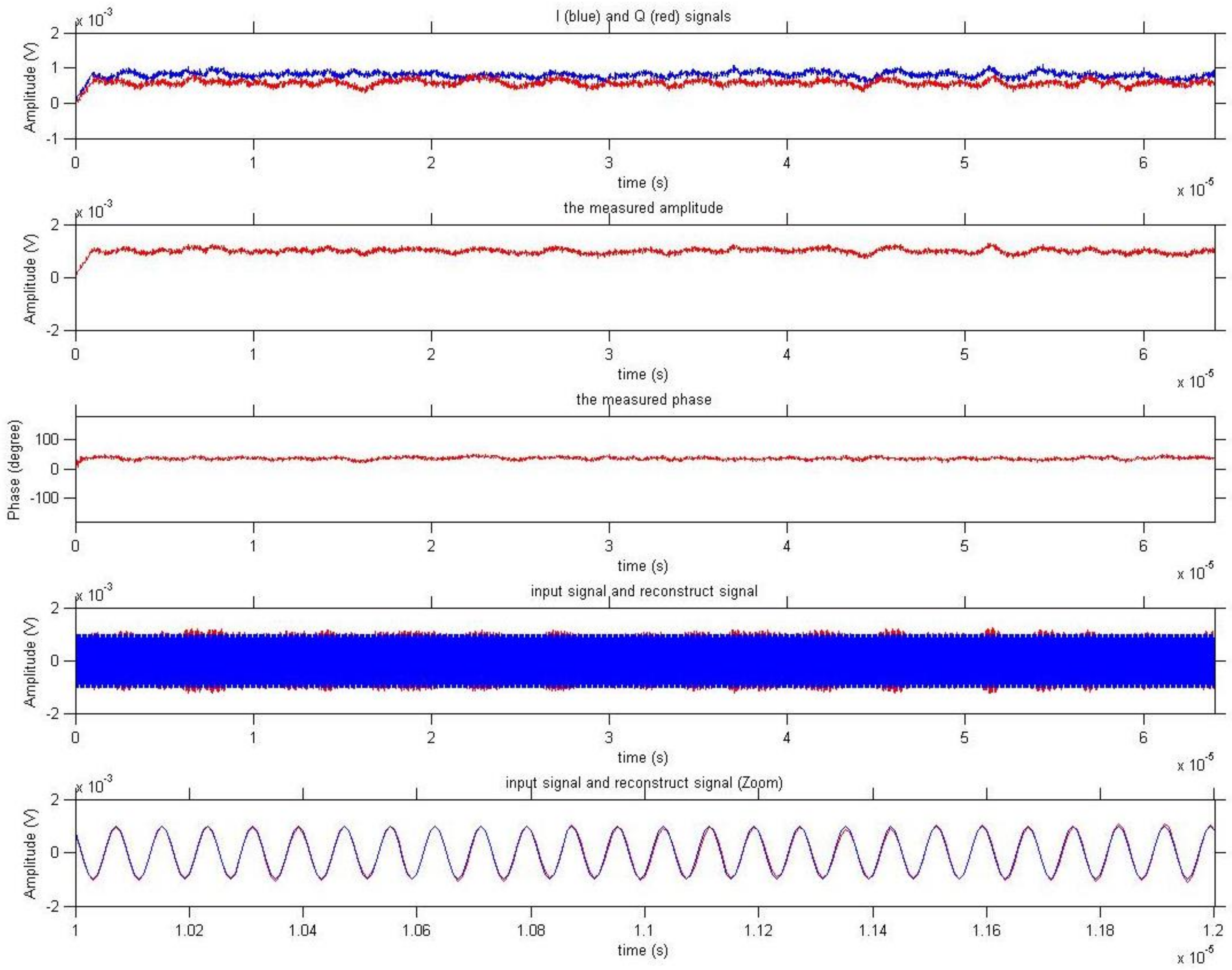
I & Q

Amplitude

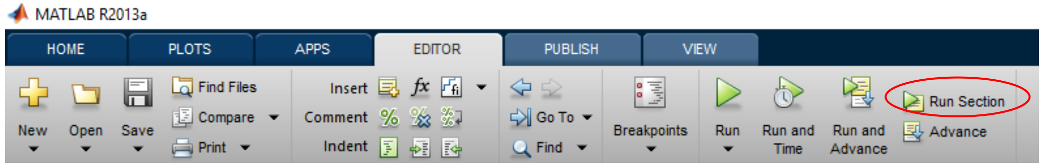
Phase

Reconstructed Signal

Reconstructed Signal (Zoomed)



Lab Session: IQ Demodulation to measure the phase and amplitude of a signal



Step by Step:

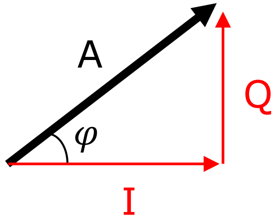
- Open lockin_amplifier.m file with matlab
- Search **xxxxx** in the file to see what you need to add or change.
- Check how our input signal signal_in is generated (lines 7-17). What is its phase (in degree and radian) ?
- Take the fft of signal_in (line 29). Run Section.
- Generate signal_s1 and signal_s2, which are added on top of signal_in. Use t_range to define time interval:
  signal_s1: 3 MHz cos signal with 0.002 V amplitude, 0 radian phase
  signal_s2: 20 MHz sin signal with 0.0005 V amplitude, pi/4 radian phase
- Take the fft of signal_in_noisy (line 64)
- Apply fftshift (line 65). Normalize fftshift value with number of samples (check line 30 as an example). fftshift puts 0 Hz to the middle, otherwise 0 Hz is in the left edge, which is not conventional for fft visualization. Run Section.
- Define your LO signals according to LO signal definitions at slide 16. (line 79-83)
- Multiply *signal_in_noisy* with LO signals (lines 85-86). Now you implemented digital down conversion (DDC).
- Use I_before_filter and Q_before_filter to define the signal with inphase and quandrature. Formula is here.
- Run Section. Check if the frequency components of *signal_in_noisy* are shifted after DDC.
- Check lines 113-120. It is a boxcar filter (averager of consecutive values) and the output is I and Q values. It filters out high frequency components. Check Slide 16. After boxcar filter for Q value will you get Q or –Q? According to that, multiply your filtered signals with 1 or -1 (Line 116-117).
- Run Section. Check the plots titled as "Spectrum after LowPass Filter" and "Spectrum of Low Pass Filter". Do you think is boxcar perfect as a low pass filter, why or why not ?

$$A\, e^{i\phi} = I + i\,Q$$

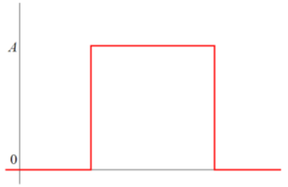
signal amplitude

Phase

In-phase and quadrature components



$$I = A\cos(\varphi)$$
$$Q = A\sin(\varphi)$$



Boxcar filter in time domain

Lab Session: IQ Demodulation to measure the phase and amplitude of a signal

Step by Step:

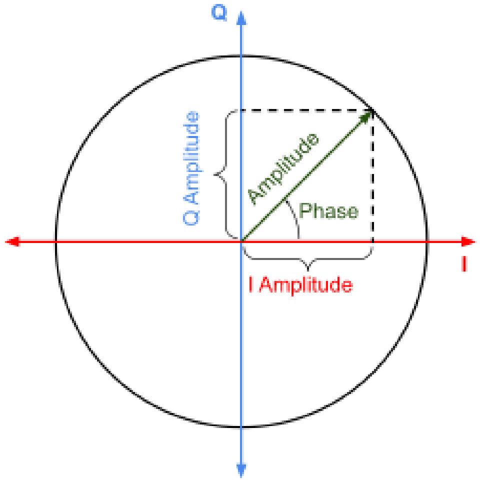
- Now we have I and Q components. We can extract amplitude and phase from I and Q. Then we can reconstruct our signal. *A_reconstruct* is the amplitude, *P_reconstruct* is the phase. Calculate them using the formulas below. Use *atan2(y,x)* function of matlab to calculate the arctangent. (Line 150-151)

$$A(t) = \sqrt{I(t)^2 + Q(t)^2} \qquad \varphi(t) = tan^{-1}\frac{Q(t)}{I(t)}$$

- Reconstruct the signal using the formula below. (Line 153)

$$A(t)\,cos(2\pi f t + \varphi(t))$$

- Run Section.
- Is the reconstruction successful ? Check the phase and amplitude plots. Is it matching with your input signal's amplitude and phase ?
- Change the phase of your input signal *signal_in* between 0 and 2π, and check if the reconstruction is correct.
- Check the plots of I and Q values and the Phase. Considering that I and Q values are pozitive or negative, phase can be at one of the four regions around the sphere (0 to 2π). Does it match in your plot (for example when both I and Q are negative, phase should be between π and 1.5π)?



Lab Session: IQ Demodulation to measure the phase and amplitude of a signal

Step by Step:

- By looking at I and Q components and their region at complex plane, we can distinguish two qubit states (*e* and *g*) after measurement.
- measurements.mat file includes 3 measurement signals. Type clear all in console. Click measurements.mat. You will see 3 data sequence in workspace. You expect to see your quantum signal at 12.5 MHz after analog down conversions.
- In line 17, type signal_in=0;
- Comment out *clear all* at line3 (to dont click measurements.mat file every time)
- In line 17, type: signal_in=0;
- In line 48, type: signal_in_noisy = measurement_1;
- Run all code, and repeat the runs by assigning measurement_2 and measurement_3 to signal_in_noisy.
- Answer the questions for each run, considering that you expect a response from your qubits as given in figures below:

  When do the measurements start and stop ?

  What is the qubit's state for signal 1 & 2 ?

  What happens at 2.8e-5 s in signal 3 ?