



MICRO-435
Quantum and
Nanocomputing

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MIT FABRICATION: PART 2

MOLECULAR TRANSISTOR FABRICATION

OBJECTIVES

- a) EM PHYSICAL MODELING
- b) FAVOURING "GOOD" EM
- c) N.G. ARRAY BASED ON EM

2) EM PHYSICAL MODELING

EH EXPLANATION

STARTING POINT: } THERMODYNAMICS OF IRREVERSIBLE PROCESSES

3 FLUXES

J_e ELECTRON FLUX

J_m METAL FLUX
LOW

J_w ENERGY FLUX

INDUCED BY

3 FORCES

OR POTENTIAL GRADIENTS

X_k

$k: e, m, u$

LET'S STOP A MOMENT TO SEE THE FORCES AND PARAMETERS INVOLVED

for k
 e, m

$$X_k = -\nabla \mu_{ec}^k$$

POTENTIAL
GRADIENT

ELECTROCHEMICAL
POTENTIAL

$$\mu_{ec}^k = \mu + z \cdot e \cdot \varphi$$

φ \rightarrow electrostatic potential

μ \rightarrow chemical potential

z \rightarrow particle charge $\rightarrow -1 \text{ el.}$
 $\searrow +1 \text{ ions}$

for X_u local temperature gradient

GENERAL DESCRIPTION OF THE SYSTEM

$$\begin{cases} J_m = -L_{m,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{m,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{m,u} \left(\frac{\nabla T}{T^2} \right) \\ J_e = -L_{e,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{e,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{e,u} \left(\frac{\nabla T}{T^2} \right) \\ J_u = -L_{u,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{u,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{u,u} \left(\frac{\nabla T}{T^2} \right) \end{cases}$$

$L_{i,j}$ phenomenological constant.

RELATE EACH FLUX TO ALL FORCES

DESCRIBES :

ELECTROMIGRATION

THERMO-ELECTRIC EFFECT

THERMODIFFUSION

DEPENDING ON THE

CASE SUBPARTS ARE

MORE INVOLVED,

OTHER NEGLECTED

GENERAL DESCRIPTION OF THE SYSTEM

$$\begin{cases} \mathbf{J}_m = -L_{m,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{m,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{m,u} \left(\frac{\nabla T}{T^2} \right) \\ \mathbf{J}_e = -L_{e,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{e,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{e,u} \left(\frac{\nabla T}{T^2} \right) \\ \mathbf{J}_u = -L_{u,m} \nabla \left(\frac{\mu_{ec}^m}{T} \right) - L_{u,e} \nabla \left(\frac{\mu_{ec}^e}{T} \right) - L_{u,u} \left(\frac{\nabla T}{T^2} \right) \end{cases}$$

$L_{i,j}$ phenomenological constant.
RELATE EACH FLUX TO ALL FORCES

↓ ASSUMPTIONS TO SIMPLIFY IN CASE OF ET IN NCAP

↖ VERY DIFFICULT TO SOLVE IN 3D

1) $\nabla_{\mu} e$ ELECTROCHEMICAL POTENTIAL CAN BE
NEGLECTED FOR HIGH CONDUCTIVITY MAT.

2) THERMODIFFUSION FOR GOLD ELECTRODES IGNORED

3) IN METAL WIRE FLUX OF CHARGES IS DUE
ONLY TO ELECTRONS

electrostatic
potential

$$-\nabla \varphi = \rho \cdot j$$

ρ RESISTIVITY

j CURRENT
DENSITY

$$J_m = -L_{m,m} \left(\frac{\nabla_{\mu}^m - z^* e \rho j}{T} \right)$$

$$Z^* = Z - \frac{L_{me}}{L_{m,m}} \leftarrow \begin{array}{l} \text{CORRECTIVE TERM} \\ \text{DUE TO TRANSFER} \\ \text{OF MOMENTUM FROM} \\ \text{e TO IONS} \end{array}$$

EFFECTIVE CHARGE

$\frac{L_{me}}{L_{m,m}} \gg Z$ generally in EM

$\Rightarrow Z^*$ IS NEGATIVE

the NET FORCE acting on cold atom is in direction of electric force

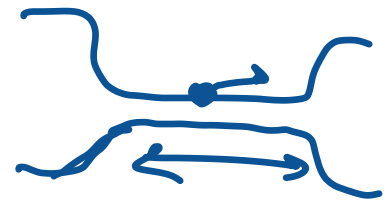
IN PRACTICE METAL ATOMS BEHAVES AS THEY HAD A CHARGE Z^* DUE TO MOMENTUM TRANSFER OF IONS

4) $\nabla_{\mu}^m = \int \frac{\Delta \sigma}{dx}$ FORCE due to stress σ

WIRE of L, $\Delta \sigma$ STRESS OVER LENGTH L

$$J_m = -L_{m,m} \left(\frac{-\int \frac{\Delta \sigma}{L} - z^* e_j}{T} \right)$$

AS LONG AS $\Delta \sigma < \Delta \sigma_{MAX}$



EM FORCE is BALANCED BY

STRESS GRADIENT

$$J_m = 0$$

When $j \rightarrow j_{min} = \frac{\Omega A \sigma_{max}}{z^* \rho e L}$



MINIMUM CURRENT DENSITY
NEEDED TO OVERCOME
STRESS AND SEE A POTENTIAL
MIGRATION OF IONS

IN NORMAL CIRCUIT GIVEN A j OF A SIGNAL,
A L EXISTS BELOW WHICH
NO EM CAN HAPPEN

Blech's
LENGTH

5) introducing $D = L_{m,m}^* \frac{d\mu}{dc}$

EINSTEIN
RELATION

DIFFUSION CONSTANT

The goal is
to introduce
the dependency
on T of
the ION FLOW

C CONCENTRATION OF METAL IONS

μ CHEMICAL POTENTIAL

$$\mu = kT \cdot \ln C$$



$$D = L_{m,m}^* \cdot \frac{kT}{C}$$

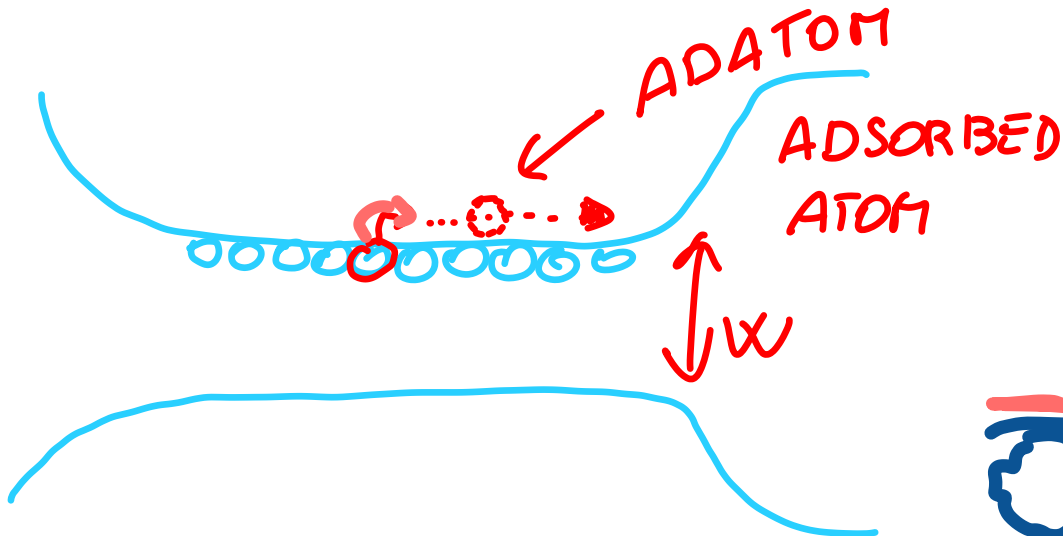
D CAN BE WRITTEN ALSO AS

$$D = D_0 \cdot e^{-\frac{E_a}{kT}}$$

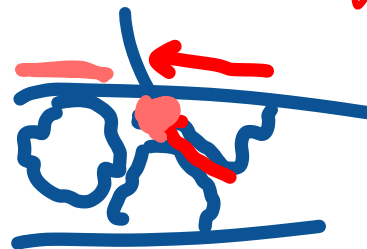
$$\frac{E_a}{kT}$$

ACTIVATION ENERGY OF DIFFUSION OF IONS ON SURFACE
gold 0.12 eV

IN SMALL WIRES FOR NG MIGRATION HAPPENS ONLY AT THE SURFACE



NOT IN GRAIN BOUNDARIES IF
 $w \sim$ GRAIN SIZE



$$D = D_0 e^{-\frac{E_0}{kT}}$$

$$D = L_{min}^* \frac{kT}{c}$$

$$\Rightarrow L_{m,m} = \frac{D_0}{kT} \cdot c \cdot e^{-\frac{E_0}{kT}}$$

J_m WAS

$$J_m = -L_{min}^* \left(\frac{\Omega \frac{\Delta\sigma}{2} - z^* e f_D}{T} \right)$$

WE OBTAINED
 J_{min} FOR
MECH. STRESS

\Rightarrow REWRITE $J_m \rightarrow f(J_{min}, T)$

$$J_m = \frac{\alpha}{T} (J - J_{min}) e^{-\frac{E_a}{kT}}$$

$$\alpha = c D_0 z^* e p / k$$

STRESS

•

$$\text{IF } J < J_{min} \Rightarrow J_m = 0$$

NO MIGRATION IF STRESS

IS NOT REACHED

• TEMPERATURE

$$J_m = \frac{\alpha}{T} (j - j_{min}) \cdot e^{-\frac{\bar{E}_a}{kT}}$$

$$j_m = \frac{\Sigma \Delta \sigma_{max}}{z^* e \rho L} \quad \alpha = c D_0 z^* e \frac{\rho}{k}$$

- FLUX OF IONS DEPENDS ON T
- GAP FORMATION STARTS AS SOON AS THE MASS FLUX IS LARGE ENOUGH
- WHEN IT HAPPENS THE LOCAL TEMPERATURE IN GAP ZONE IS INFLUENCED BY JOULE HEATING!

● TEMPERATURE

$$J_m = \frac{\alpha}{T} (J - J_{m,cr}) e^{-\frac{\bar{E}_a}{kT}}$$

$$J_m = \frac{\Sigma \Delta \sigma_{max}}{z^* e \rho L}$$

$$\alpha = c D_0 z^* e \frac{\rho}{k}$$

ONLY FOR T_{CRIT}
ADATONS ARE FORED!

when J reaches J_{CRIT} : T_{NARROW WIRE} → T_{CRIT} JOULE HEATING

$$T > T_{CRIT} \quad \rho > \rho_{CRIT}$$

$$\rho = \rho \cdot j^2 \quad \rho_{CRIT} = \rho \cdot j_{CRIT}^2$$

→ WE NEED TO REACH T_{crit}
FOR ION FLOWS AS ADATOMS
FRICTION

→ WE NEED TO REACH T_{CRIT}
FOR LOW FLOW AS ADATOMS
FRICTION

→ BUT... AS MIGRATION HAPPENS
THE WIRE REDUCES

$R \uparrow$ $P \uparrow$ $T \uparrow$

.....D AND WHAT

? DANGER OF
T.
RUNAWAY

b) FAVOURING A "GOOD" ETH

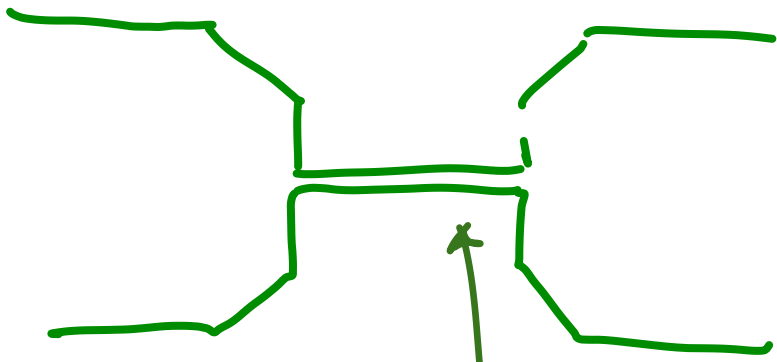
When T_{CRIT} MOBILITY OF IONS

INCREASES

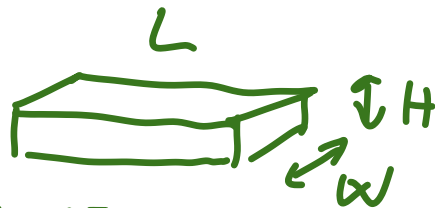
→ ADATOMS

→ VACANCIES

TERRACE TYPE CONNECTION



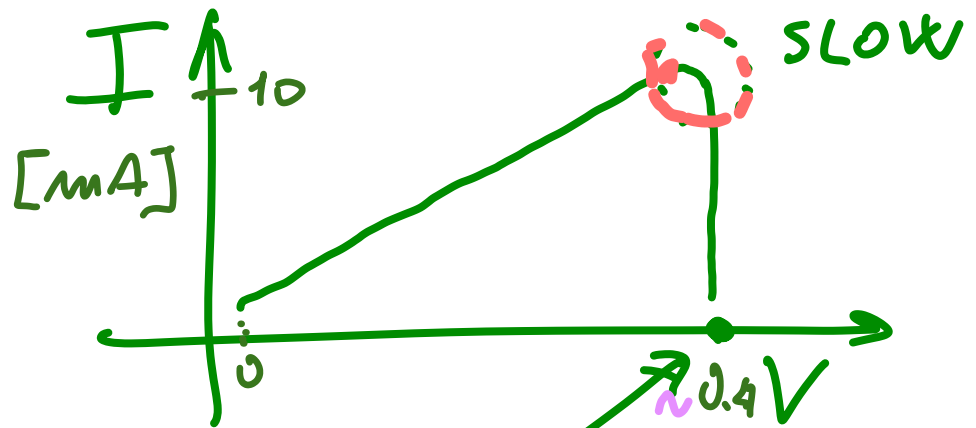
experiment ↑



$H = 15 \text{ nm}$
 $W = 100 \text{ nm}$

⊙ START

$L > 1 \mu\text{m}$

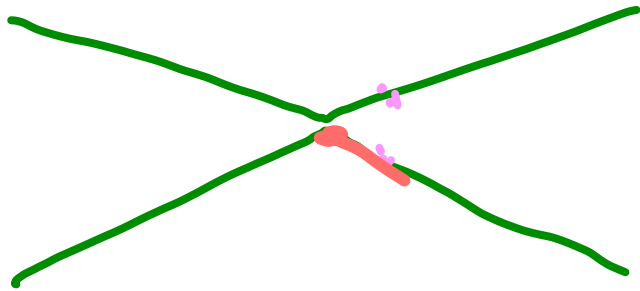


1 nm gap formed

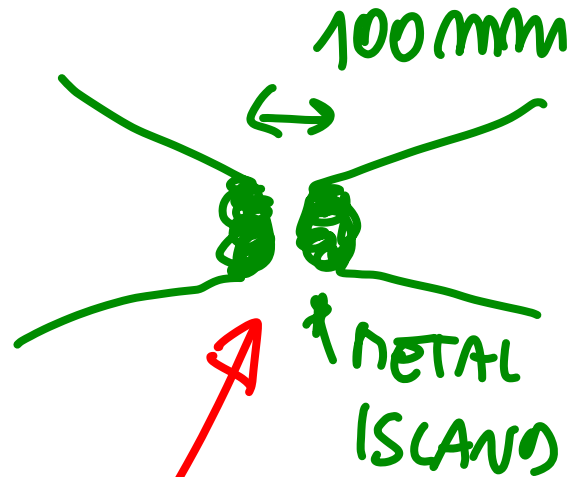
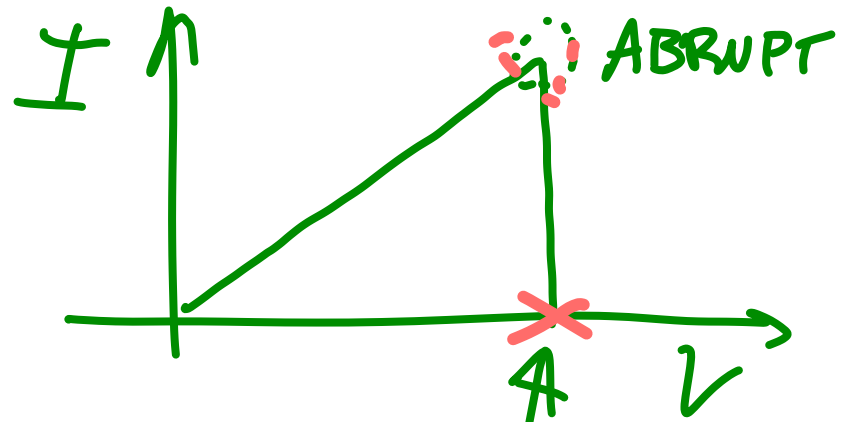
1 mA current
MEASURE
(TUNNEL)

WELL FORMED GAP

BOWEN TIE TYPE



after the break

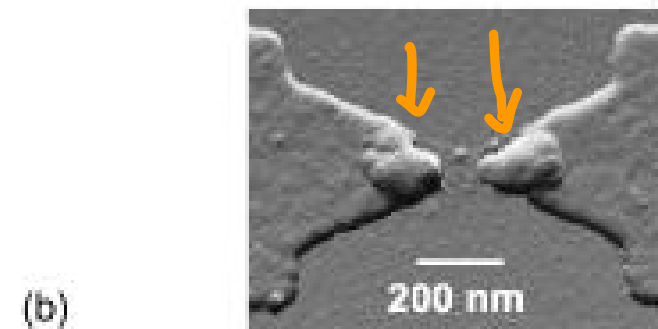
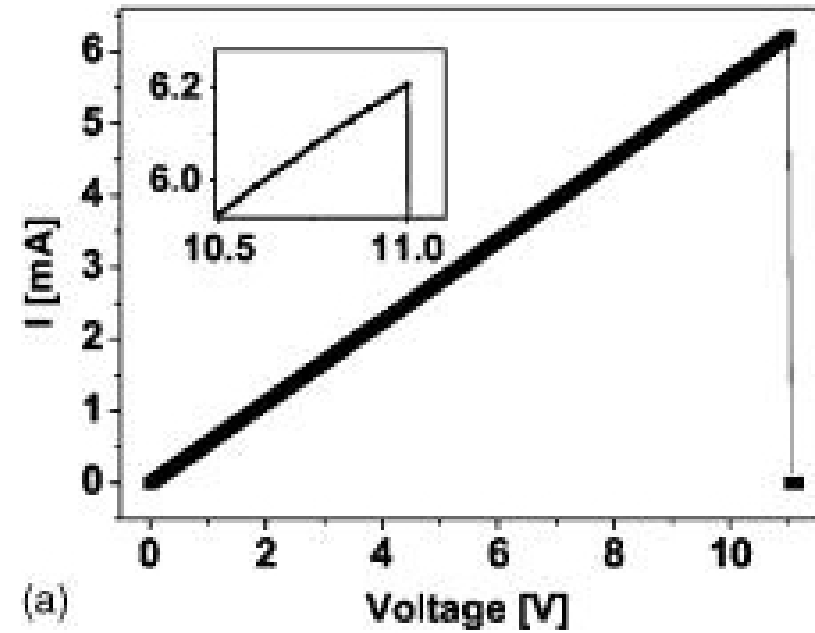
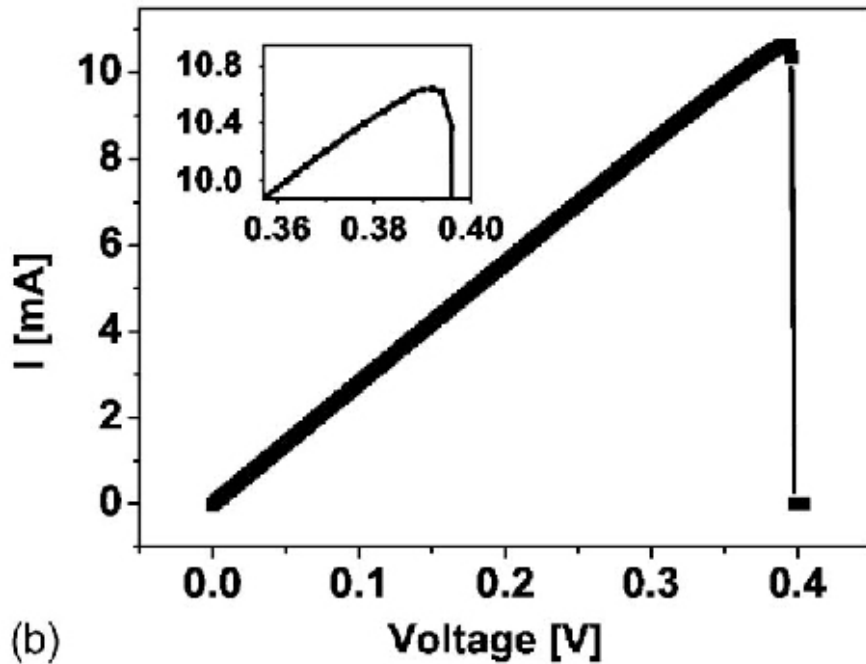
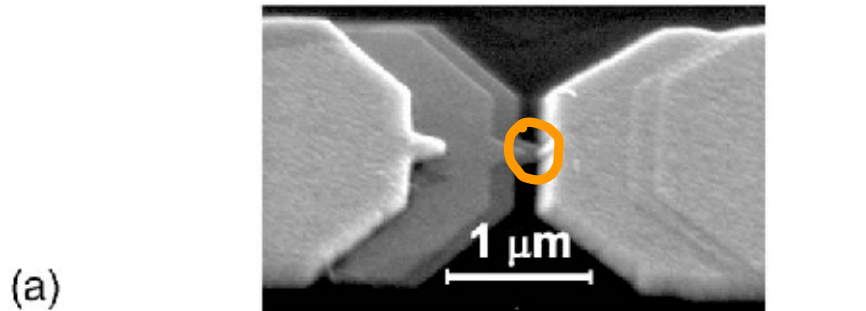


ACCUMULATION OF IONS ON GAP SIDES

IN THIS CASE THERMAL RUNAWAY
 $P \nearrow \nearrow T_{LOCAL} \nearrow \nearrow$ GOLD MELTING POINT!!

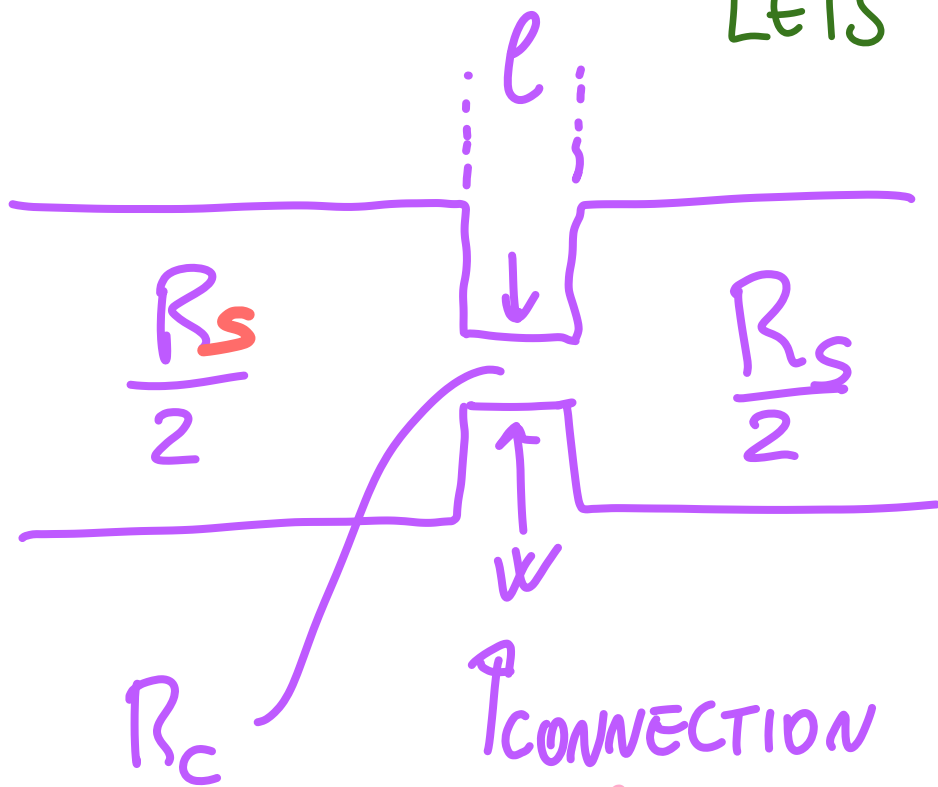
NO T.R.

T.R.



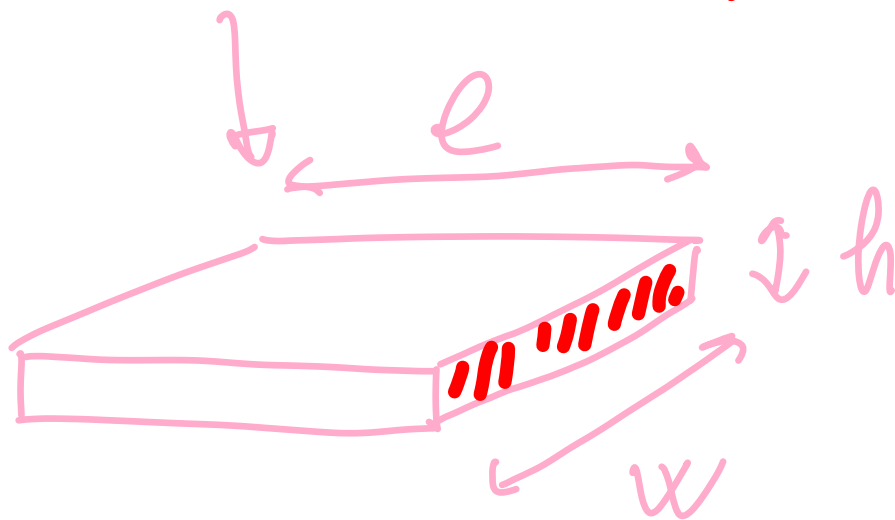
EFFECTS OF
THERMAL
RUNAWAY

LET'S MODEL



$$R_{TOT} = R_s + R_c(t)$$

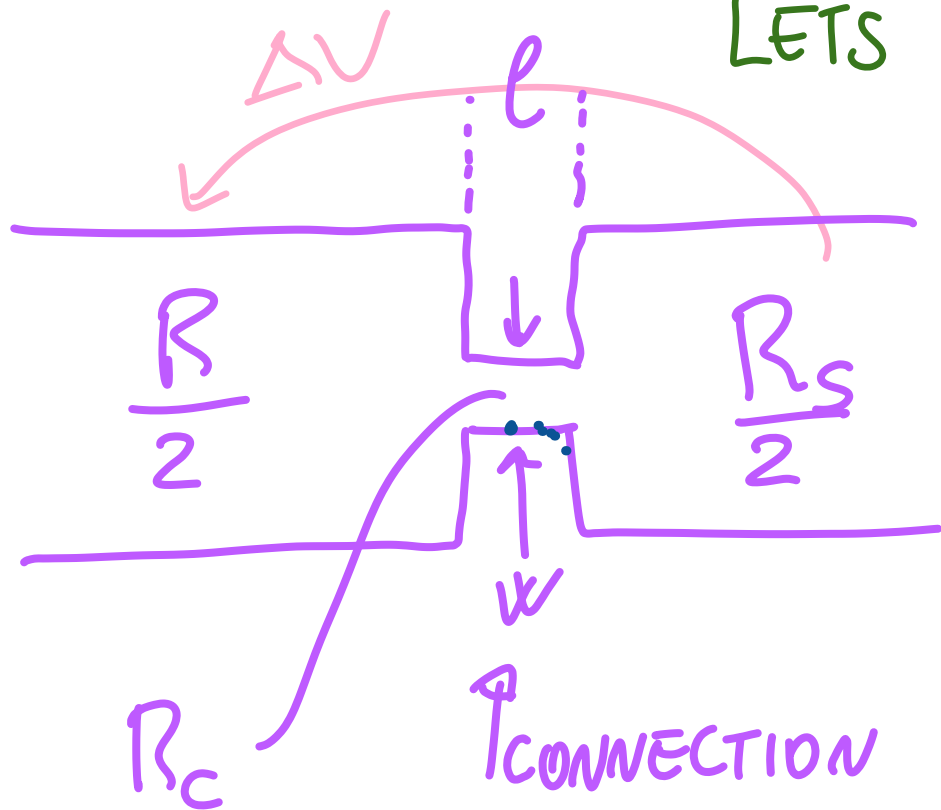
$$R_c(t) = \frac{\rho l}{h(t) w(t)}$$



When MIGRATION
HAPPENS

h and w
CHANGE IN
TIME

LET'S MODEL



$$R_{\text{TOT}} = R_s + R_c(t)$$

$$R_c(t) = \frac{\rho l}{h(t) w(t)}$$

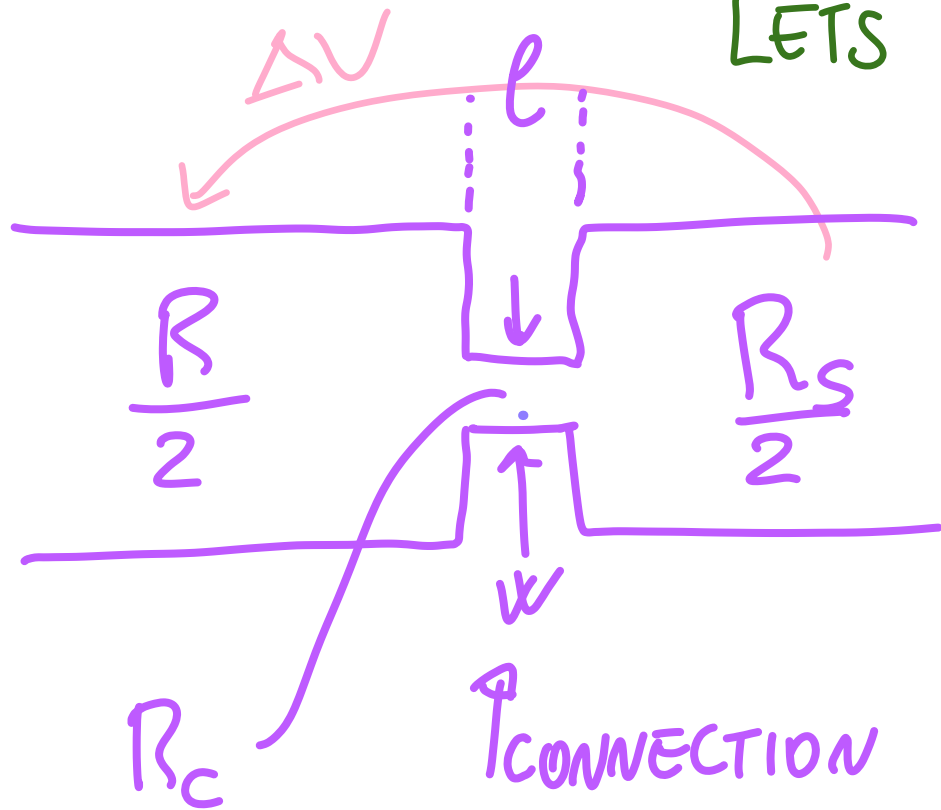
ΔV
APPLY



Time Reached
DUE TO
STRESS

FLUX NOT
NECESSARILY STARTS
HERE!

LET'S MODEL



$$R_{TOT} = R_S + R_C(t)$$

$$R_C(t) = \frac{\rho l}{h(t) w(t)}$$

ΔV
APPLY

J_{lim} Reached
DUE TO
STRESS

STARTS WHEN
FLUX, MOBILITY
REACHES A VALUE
DUE TO T

IN THE LOCAL POINT
OF THE CONNECTION

\dot{E}_H STARTS \rightarrow NARROW CUT (for example! 1 OR MORE)

$R_c \nearrow$

$p(t) \nearrow$



DESCRIBES INCREASE

of LOCAL DISSIPATION IN CONN.

$$p(t) = p_{\text{CRITIC}} \left(\frac{1 + \frac{R_s}{R_c(0)}}{1 + \frac{R_s}{R_c(t)}} \right)^2$$

WHEN \dot{E}_H BEGINS

$$p_{\text{CRIT}} = \frac{V_{\text{CRIT}}^2 \cdot \rho l^2}{\left(1 + \frac{R_s}{R_c(0)}\right)^2}$$

if $\frac{R_s}{R_c(t)} \ll 1$ SMALL
SERIES
RES $p(t) \sim p_{CRIT}$

$$p(t) = p_{CRITIC} \left(\frac{1 + \frac{I_s}{R_c(t)}}{1 + \frac{R_s}{R_c(t)}} \right)^2$$

→ NEGLIGIBLE

⇒ $p(t) \sim p_{CRIT} \Rightarrow T \sim T_{CRIT}$

GOOD FOR "NICE" MICRATION

OF AD ATOMS W/O T.R.

if $\frac{R_s}{R_c(0)} \gg 1$

**BIG
SERIES
RES**

$f(t) \uparrow \uparrow$
INCREASES
RAPIDLY

$()^2$

$T_{\text{CONNECTION POINT}}$

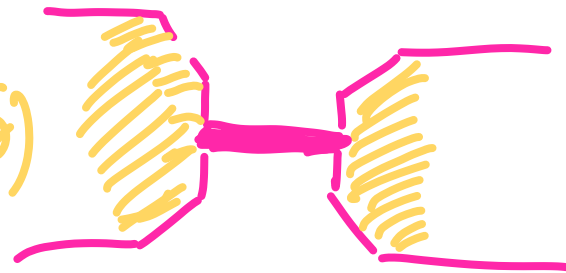
GROWS RAPIDLY TOWARD
MELTING POINT, IONS ACCUMULATE!

IN TERRACE TYPE

POWER REMAINS (CONSTANT) LIMITED

BECAUSE
 $\frac{R_s}{R_c(0)} \ll 1$

$R_s \ll R_c(0)$



EXAMPLE $T_{CRIT.} \sim 345K$

IN AN EXP. TERRACE TYPE

$J_{MIN} \sim 4 \cdot 10^8 \frac{A}{cm^2}$
for gold

BUT

if

$T_{ENV.} 295K$

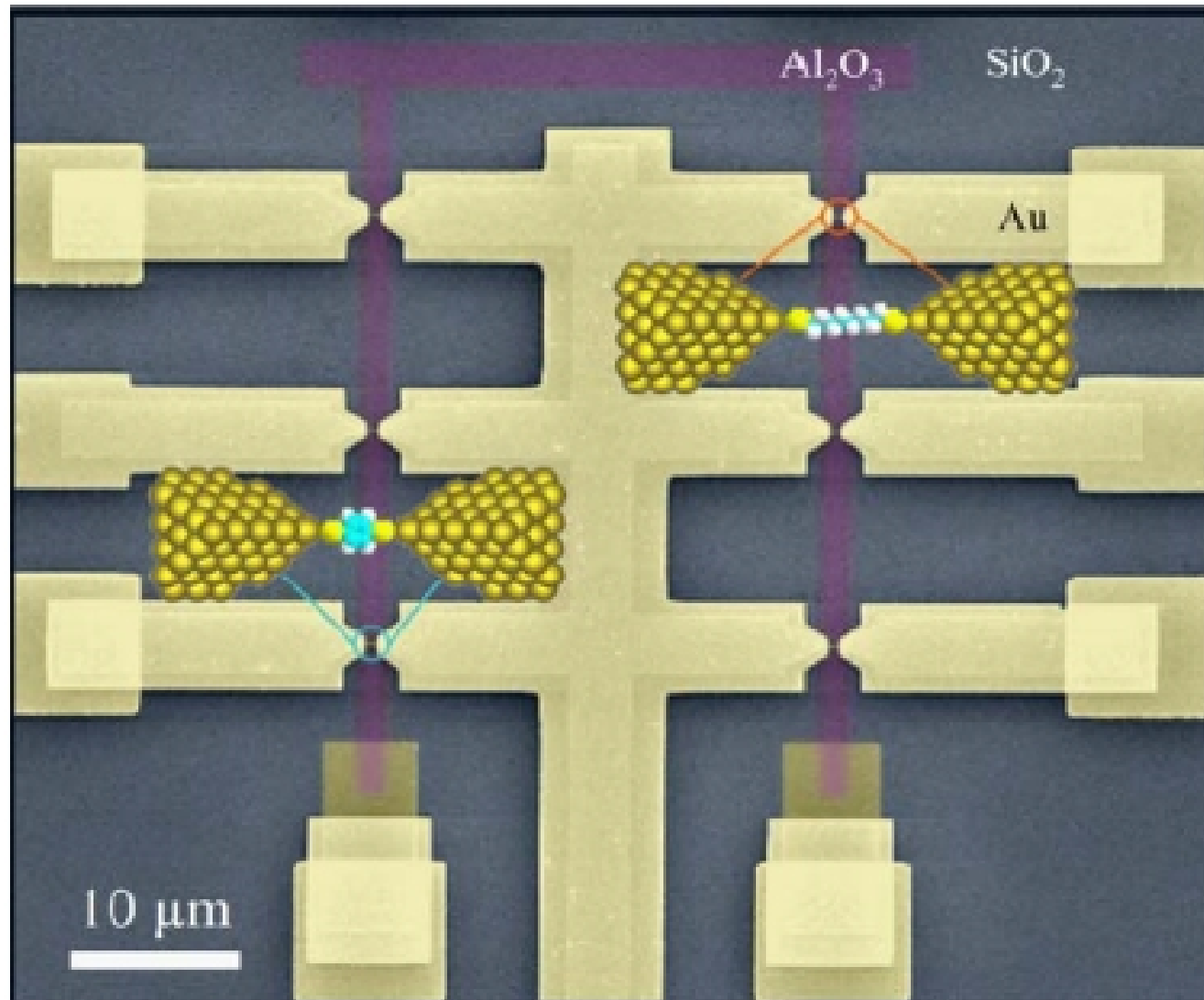
$J \rightarrow 4.2K$

$J_{CRITICAL} \nearrow 50\%$
HAS TO RISE TO

to reach $P_{CRIT.}$

T_{CRIT}

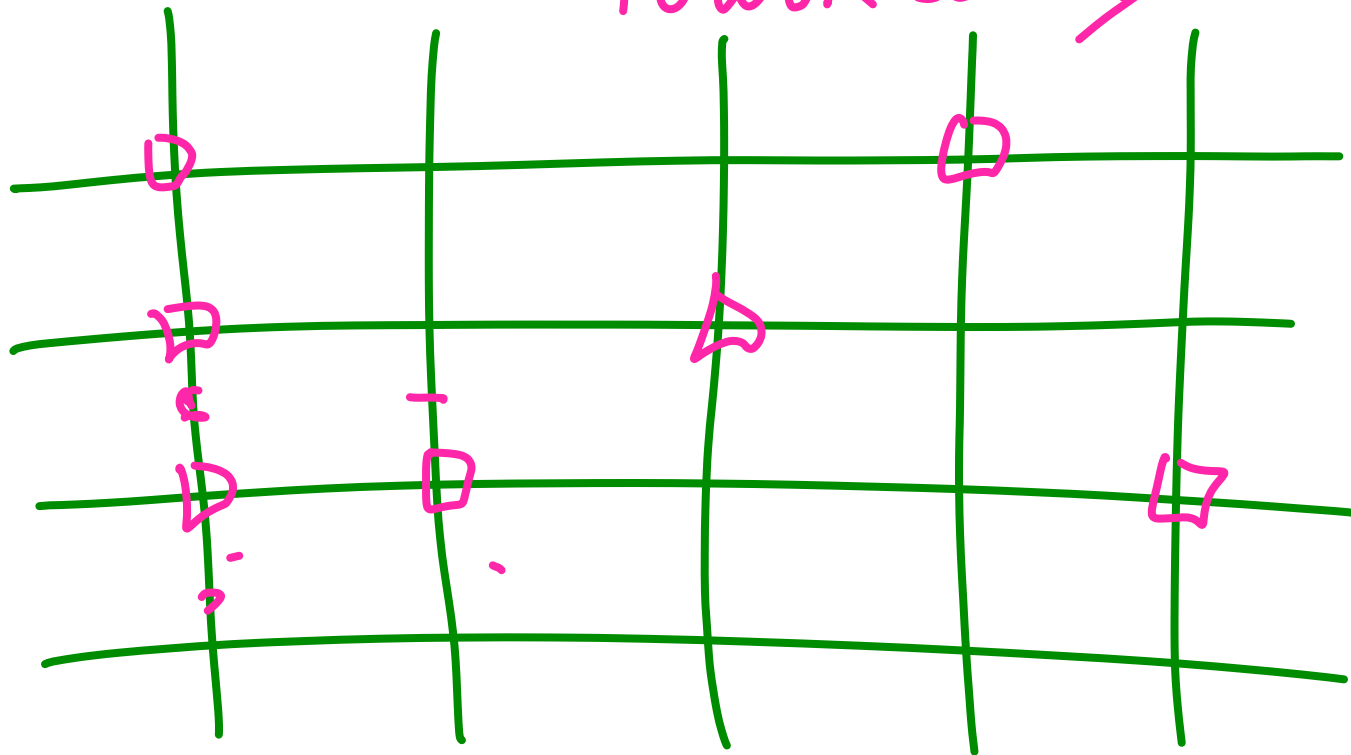
C) N.G. ARRAYS BASED ON N.G.



IN A SYSTEM WE EXPECT

POWER SUPPLY

INPUTS

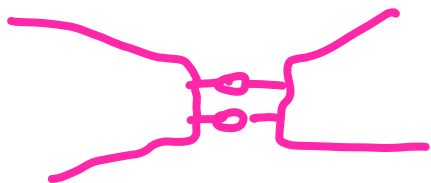


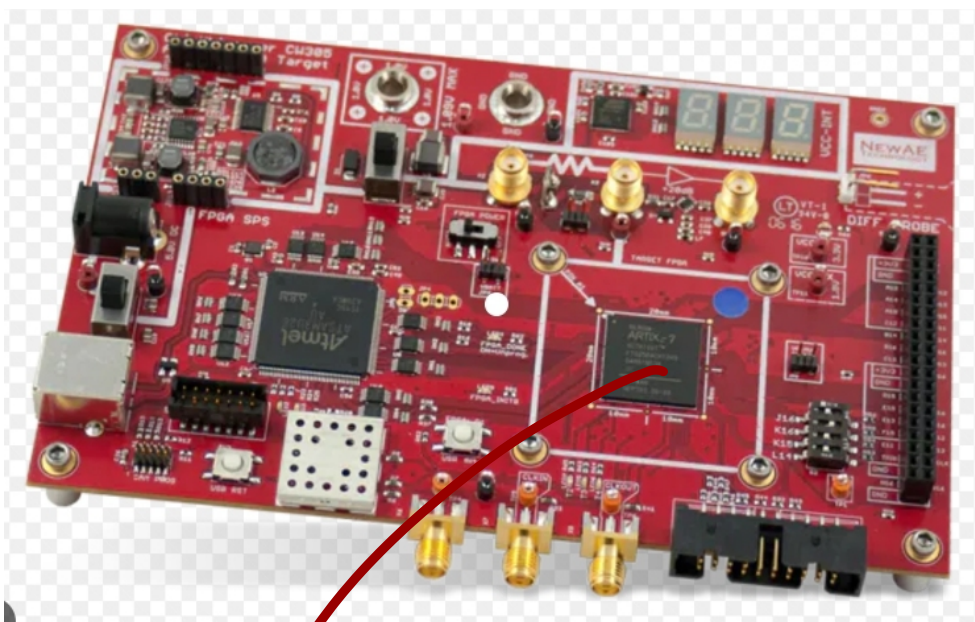
MATRIX
OF WIRES
OF P.T
CONNECTED
// OR SOME
→ LOGIC!
FUNCTION

P.L.A → FPGA

IN CONVENTIONAL
TECH

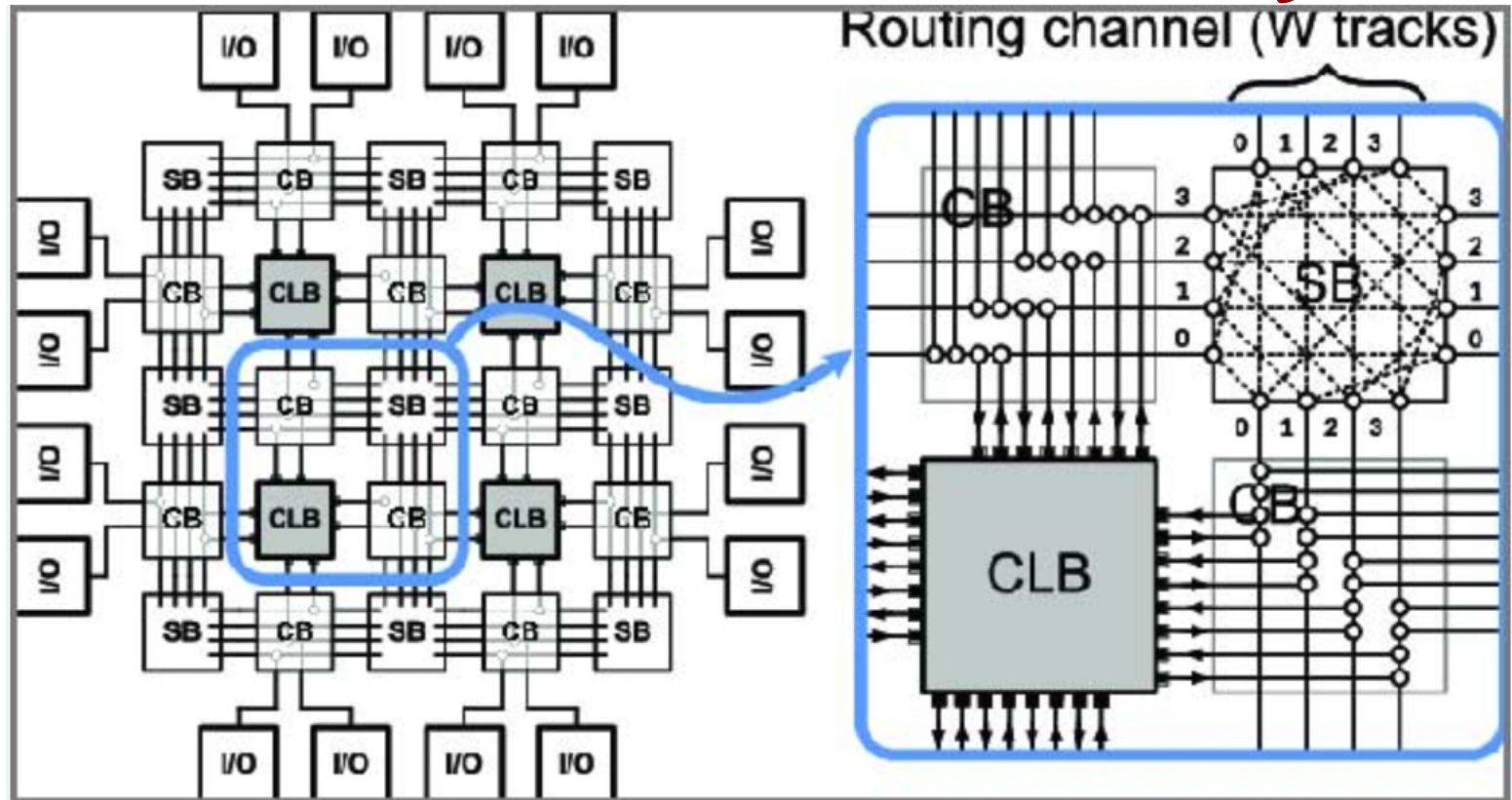
FABRICATION
OF ARRAY OF
NGAPS IS
NEEDED!

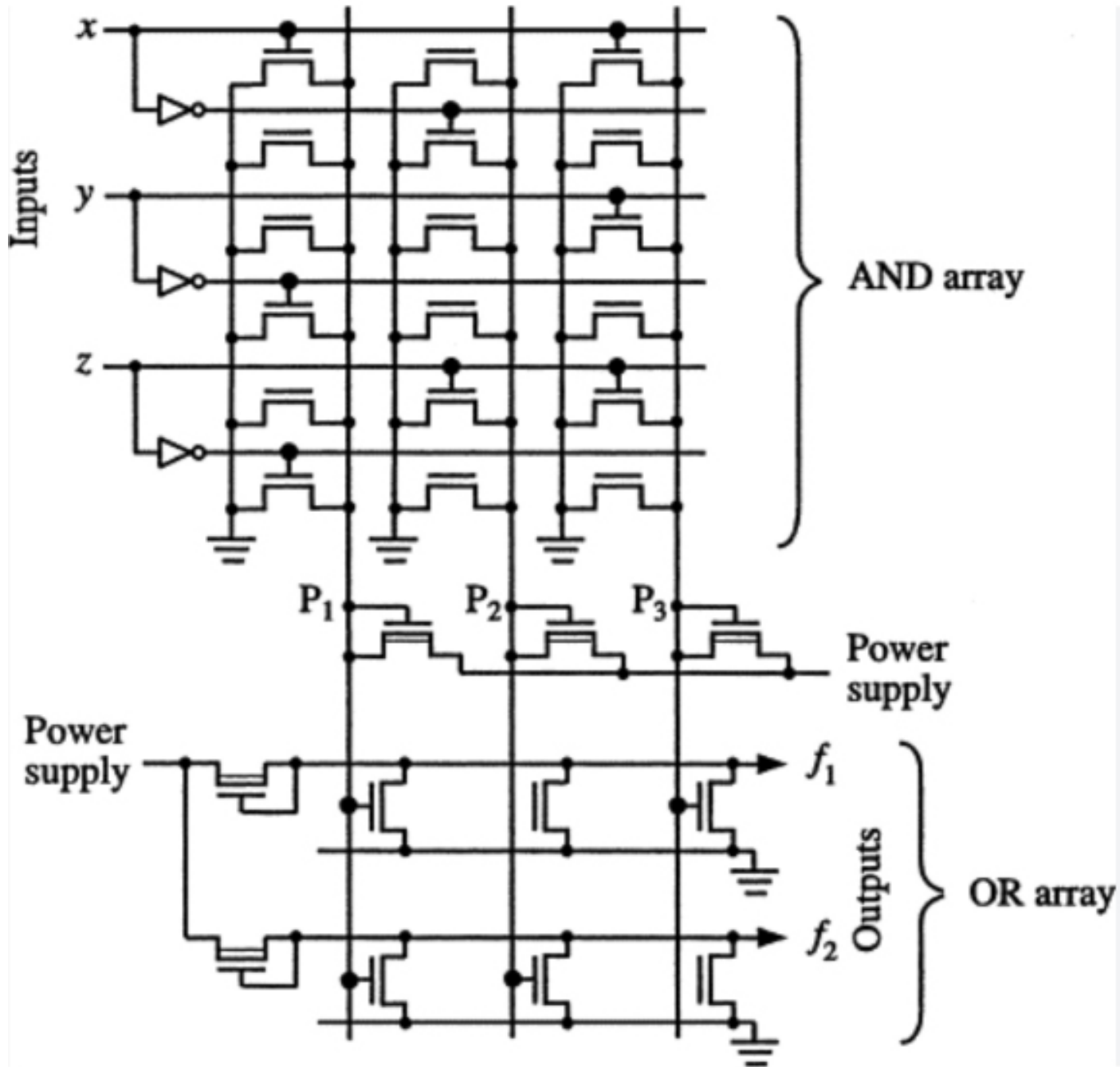




MODERN FPGA

FIELD PROGRAMMABLE
GATE ARRAY





FOR MER

PLA

PROGRAMMABLE

LOGIC

ARRAY

WHEN TECH

WAS LESS

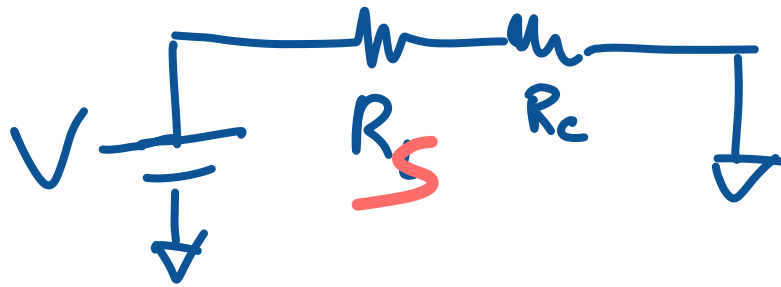
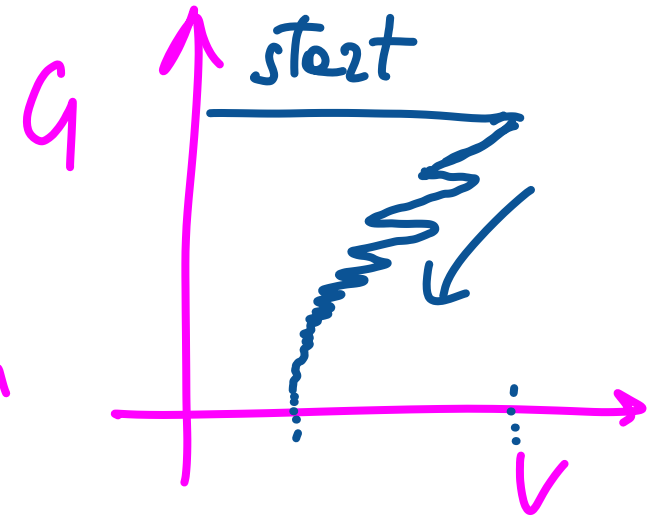
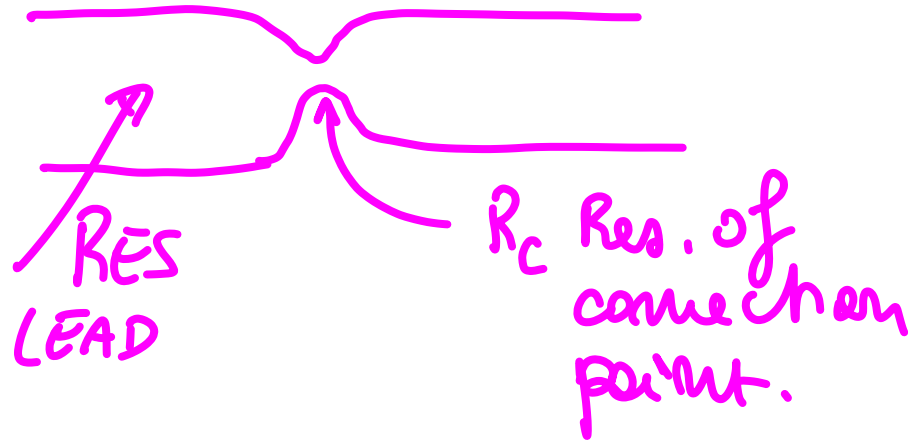
RELIABLE AND

EVOLVED

REGULARITY HELPS !!

PARALLEL FABRICATION OF NGAP V_i's FCE

CASE (A)
1 DEVICE



FCE ADJUSTS V
so that power
at junction

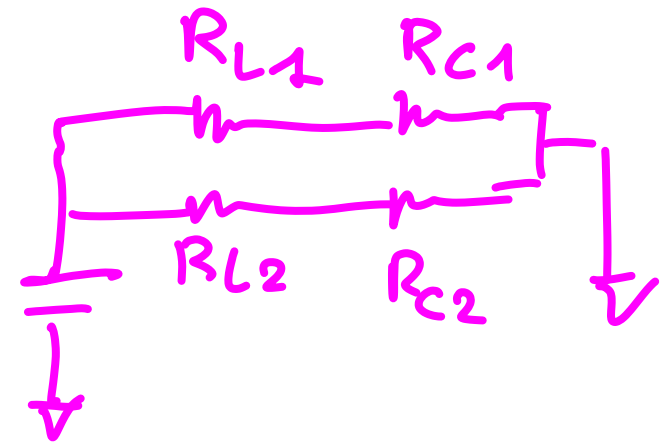
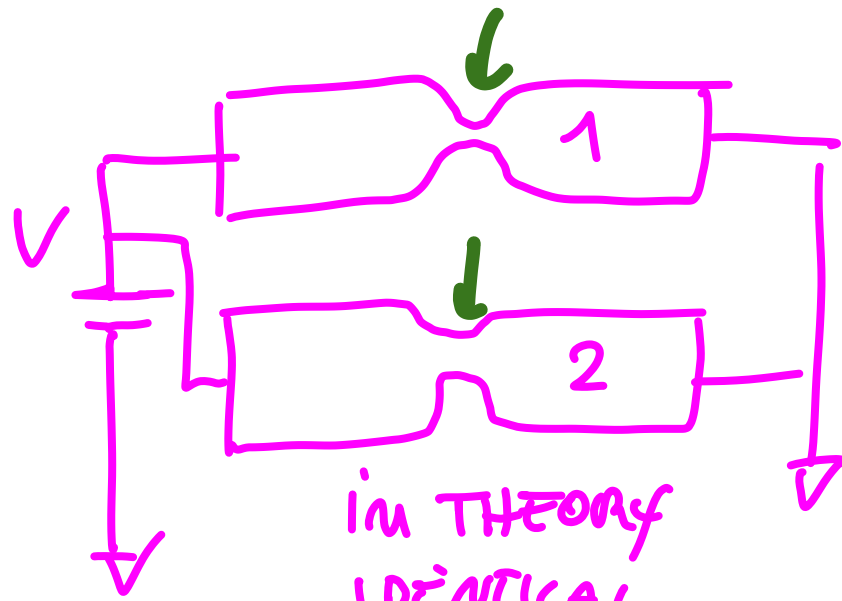
SEVERAL STEPS
UNTIL

$R_c \sim 4k\Omega$
FINAL LIMIT BEFORE
BREAK

$P > P_{CRITICAL}$
 $P < MELTING POINT$

CASE (3)

2 DEVICES
SEPARATED
CONTROLLED
in //



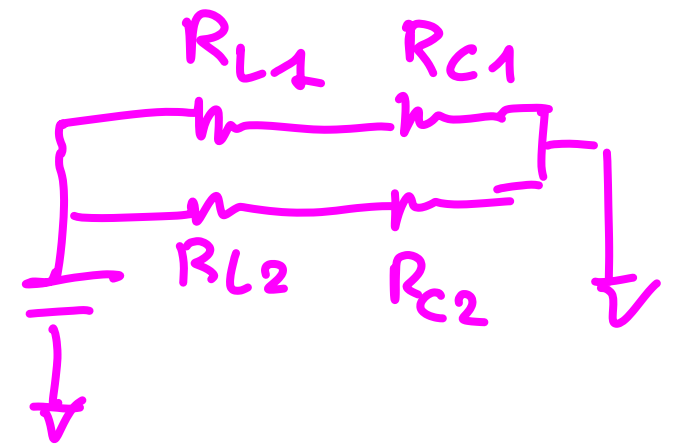
in THEORY
IDENTICAL

$$R_{L1} = R_{L2} \quad R_{C1} = R_{C2}$$

WE EXPECT

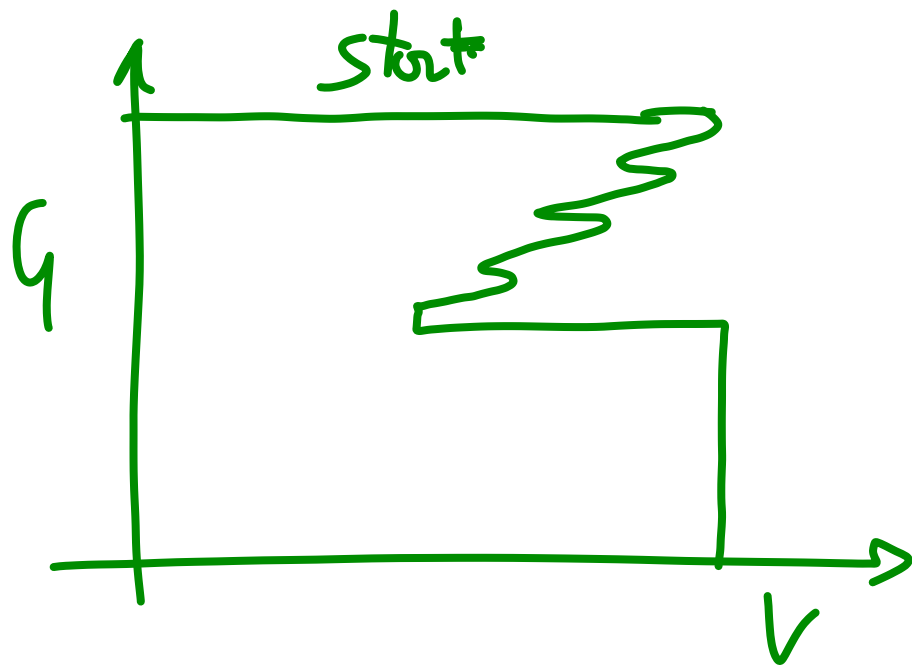
THEY BEHAVE IN THE SAME WAY

CASE (3)
 2 DEVICES
 SEPARATED
 CONTROLLED
 in //



WE EXPECT
 THEY BEHAVE

in THEORY
 IDENTICAL
 $R_{L1} = R_{L2}$ $R_{j1} = R_{j2}$
 IN THE SAME WAY



RESULT: 1 GAP IS OK

THE SECOND HAS

↗ BUMPS, BIGGER

THERMAL RUNAWAY THERMAL
 REACHED!

PROCESS VARIATIONS !

$$R_{c1} \neq R_{c2}$$

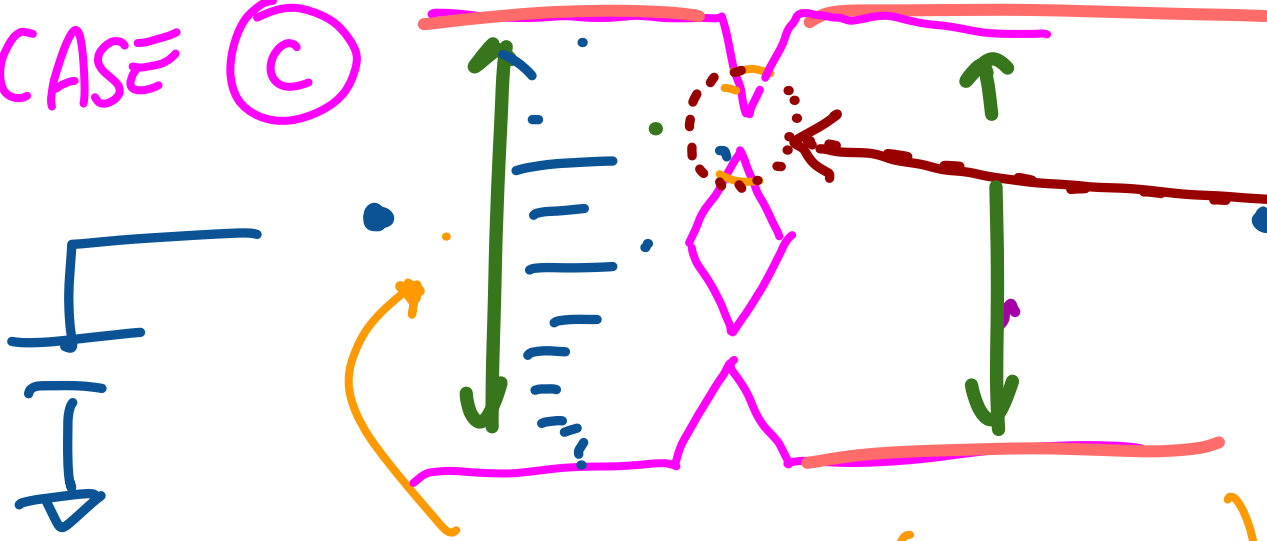
ex. $R_{c1} \sim 4.2 \Omega$ $R_{c2} \sim 9 \Omega$

AFTER A WHILE R_{c1} WILL BREAK AT FIRST
BECAUSE IT REFLECTS A SMALLER WIRE, SO IT
IS CONSUMED FIRST: AS SOON AS IT BREAKS

→ ALL CURRENT GOES SUDDENLY
IN THE SECOND CONNECTION

→ REFLECTING WILL
BE THE RESULT

CASE (C)



$$R_{TOT} = R_L + R_S + (R_{L1} // R_{L2})$$

TOTAL SAMPLE

R_S HAS TO BE WELL DESIGNED W.R.C. R_c

if $R_s < R_{c1}, R_{c2}$ then

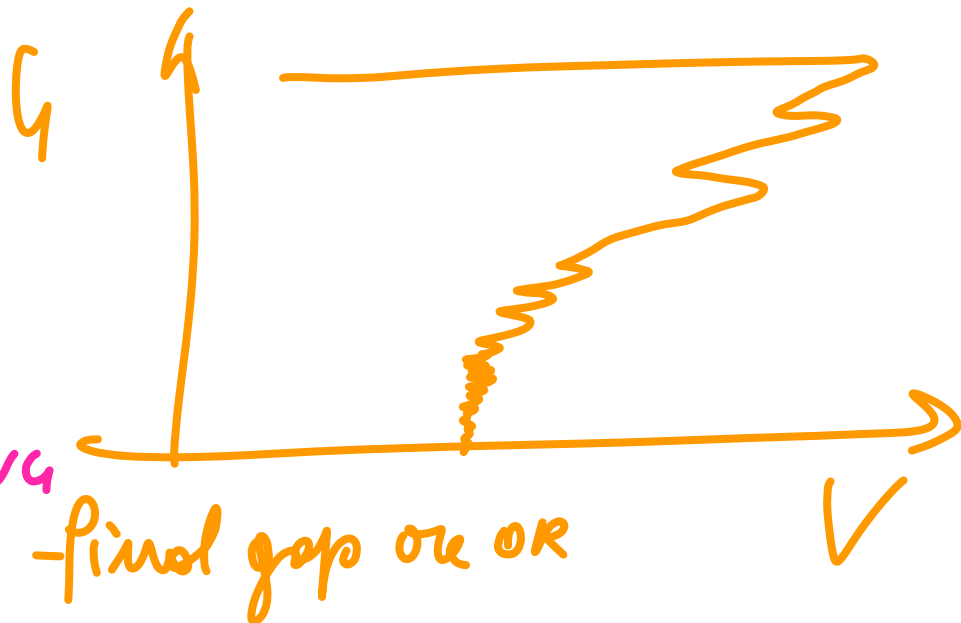
even if for ex. $R_{c1} > R_{c2}$ DUE TO P. VARIATION

$$\frac{\partial P_1}{\partial R_{c1}} < \frac{\partial P_2}{\partial R_2}$$

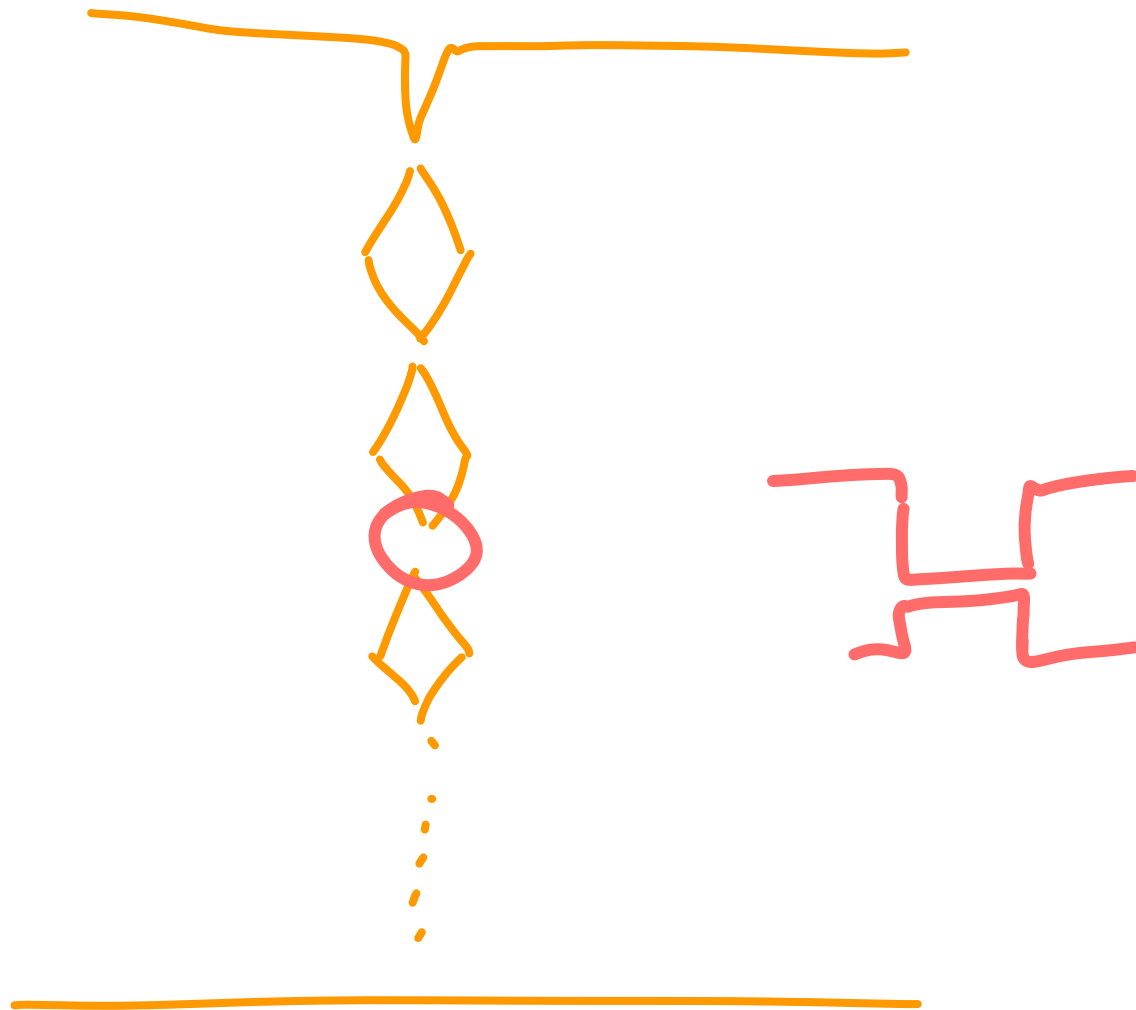
↓
CURRENT GOES TO
OTHER CONNECTION
 P_c INCREASE

GAPS ARE
GENERATED
ALMOST AT THE
SAME TIME

POWER DUE TO SELF HEATING
REMAINS N POINT EVEN
IF 1 CONN. BREAKS



SAME THING WITH SEVERAL CONNECTIONS
IN PARALLEL



⇒ IT WORKS
WELL UP
TO A CERTAIN
NUMBER OF
ELEMENT

⇒ REQUIRES
CAREFUL CONTROL
& WIRING

CRISTALLIZATION

- a) ET PHYSICAL MODELLING
- b) FOUOURING A GOOD ET
- c) ARRAY NG. WITH ET