Homework #1

September 23, 2021

Exercise 1:

Given the single cubit quantum gate:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

and the following qubits:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$
$$|\Psi_3\rangle = \frac{1}{4}|0\rangle + \frac{3}{4}|1\rangle$$

a) Verify if the three cubits are normalized and if not, normalize them.

b) Show that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are orthogonal.

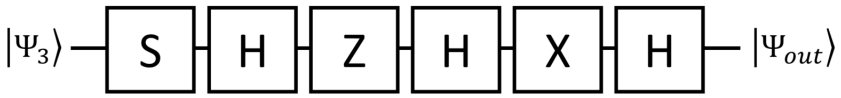
c) What is the probability that if we measured $|\Psi_3\rangle$ in the computational basis $\{|0\rangle, |1\rangle\}$ we would get $|0\rangle$?

d) What is the probability that if we measured $|\Psi_3\rangle$ in the $\{|\Psi_1\rangle, |\Psi_2\rangle\}$ basis we would get $|\Psi_2\rangle$?

e) How can we prepare $|\Psi_1\rangle$ and $|\Psi_2\rangle$ starting from $|0\rangle$ using only the X, Y, Z and H gates?

f) Show that S is unitary.

g) What is the output $|\Psi_{out}\rangle$ of the following quantum circuit?



h) Can you simplify the circuit from g)?