Homework #1

solutions

September 23, 2021

Exercise 1:

Given the single cubit quantum gate:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

and the following qubits:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$
$$|\Psi_3\rangle = \frac{1}{4}|0\rangle + \frac{3}{4}|1\rangle$$

a) Verify if the three cubits are normalized and if not, normalize them.

$\langle\Psi_1|\Psi_1\rangle = 1$ - *normalized*

$\langle\Psi_2|\Psi_2\rangle = 1$ - *normalized*

$\langle\Psi_3|\Psi_3\rangle = \frac{10}{16}$ - *not normalized;* $|\Psi_3\rangle = \frac{1}{\sqrt{10}}|0\rangle + \frac{3}{\sqrt{10}}|1\rangle$

b) Show that $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are orthogonal.

$\langle\Psi_1|\Psi_2\rangle = 0$ - *orthogonal*

c) What is the probability that if we measured $|\Psi_3\rangle$ in the computational basis $\{|0\rangle, |1\rangle\}$ we would get $|0\rangle$?

after normalization, $|\Psi_3\rangle = \frac{1}{\sqrt{10}}|0\rangle + \frac{3}{\sqrt{10}}|1\rangle;\quad p_{|0\rangle} = \left|\frac{1}{\sqrt{10}}\right|^2 = \frac{1}{10}$

or

after normalization, $p_{|0\rangle} = |\langle 0|\Psi_3\rangle|^2 = \frac{1}{10}$

d) What is the probability that if we measured $|\Psi_3\rangle$ in the $\{|\Psi_1\rangle, |\Psi_2\rangle\}$ basis we would get $|\Psi_2\rangle$?

after normalization, $|\Psi_3\rangle = \frac{2}{\sqrt{5}}|\Psi_1\rangle - \frac{1}{\sqrt{5}}|\Psi_2\rangle;\quad p_{|\Psi_2\rangle} = \left|-\frac{1}{\sqrt{5}}\right|^2 = \frac{1}{5}$

or

after normalization, $p_{|\Psi_2\rangle} = |\langle\Psi_2|\Psi_3\rangle|^2 = \frac{1}{5}$

e) How can we prepare $|\Psi_1\rangle$ and $|\Psi_2\rangle$ starting from $|0\rangle$ using only the X, Y, Z and H gates?

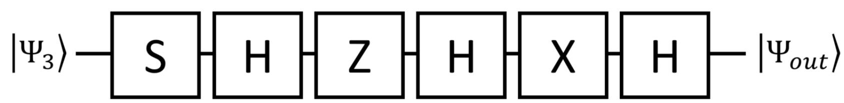
$|\Psi_1\rangle = H|0\rangle$

$|\Psi_1\rangle = ZH|0\rangle$

f) Show that S is unitary.

$SS^\dagger = I$

g) What is the output $|\Psi_{out}\rangle$ of the following quantum circuit?



after normalization, $|\Psi_{out}\rangle = HXHZHS\,|\Psi_3\rangle = \frac{1+3i}{\sqrt{20}}\,|0\rangle + \frac{1-3i}{\sqrt{20}}\,|1\rangle$

h) Can you simplify the circuit from g)?

Yes, $|\Psi_{out}\rangle = HXHZHS\,|\Psi_3\rangle = HS\,|\Psi_3\rangle$