

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

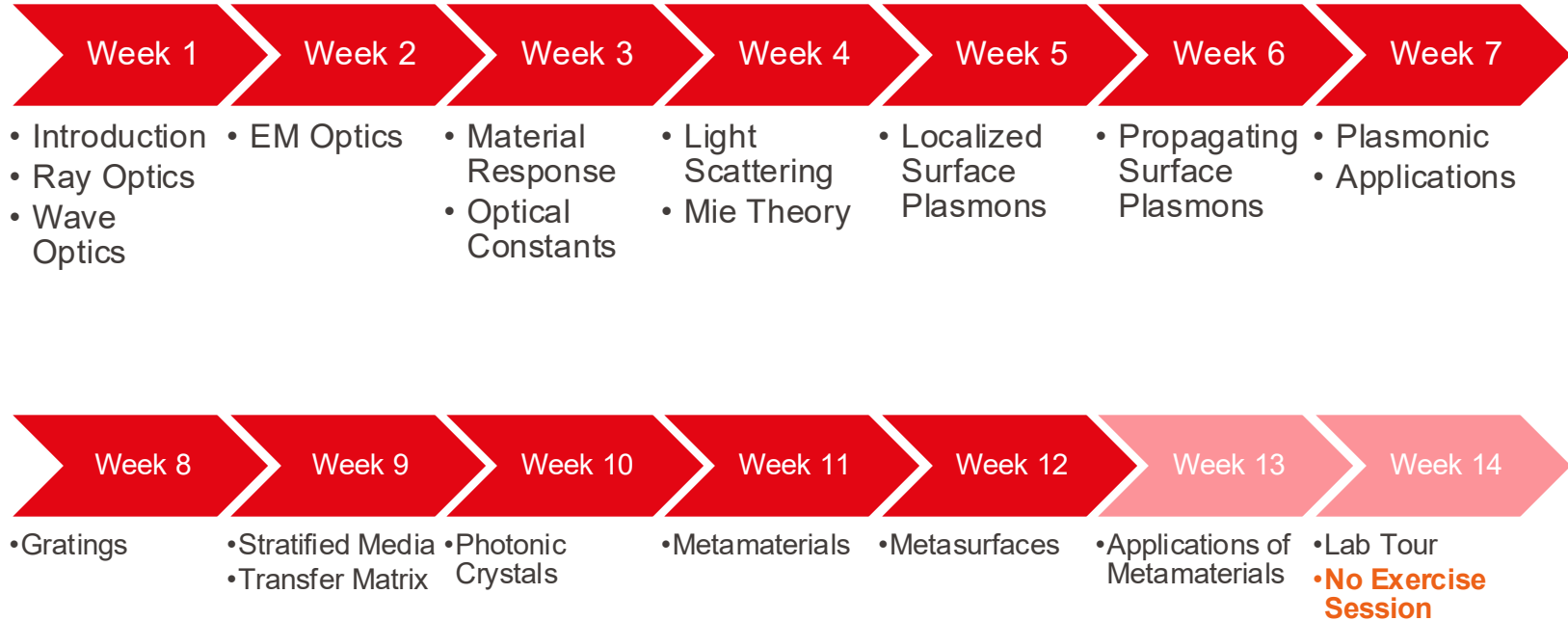
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Week 12

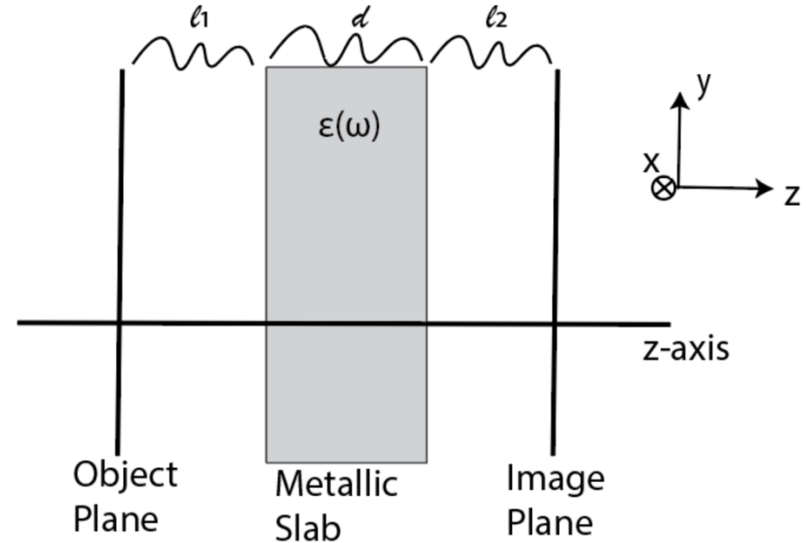
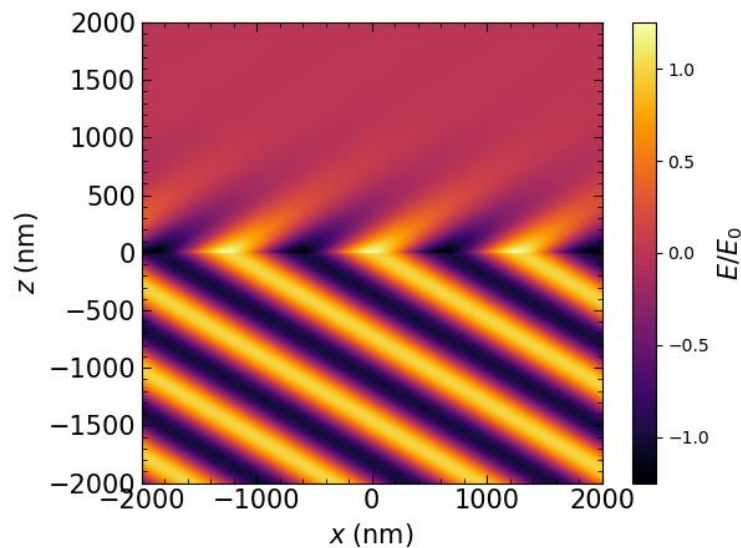
(Metasurfaces)

Stavros Athanasiou

Lausanne, 02 Dec 2025




1. Electromagnetic Properties and Refractive Index
2. Negative Refraction Makes a Perfect Lens



A note

Last week, we tried something like

$$\varepsilon = -1 - 0.5i, \quad \mu = -1 - 0.1i.$$

And there seemed to be a problem with the code. 

In fact, there is not!

Why? This choice does not satisfy the physical constraints we imposed. (see last week presentation)

```
# plot field for oblique angle
n1 = 1.0 # refractive index of (incident) medium 1
epsilon2 = -1.0 - 0.5j # permittivity of medium 2
mu2 = -1.0 - 0.1j # permeability of medium 2

theta_inc = 30.0 # angle of incident in degrees
wavelength = 633 # wavelength of incident wave in nm

# compute Re and Im of epsilon * mu
print( "Imaginary part of epsilon * mu:\t", np.imag( epsilon2 * mu2 ) )

# show field
show_field_in_space(n1=n1, eps2=epsilon2, mu2=mu2, theta_inc=theta_inc,
                    wavelength=wavelength, only_incident=True)

⊗ 0.2s

Imaginary part of epsilon * mu: 0.6
Did not find a physical kz.

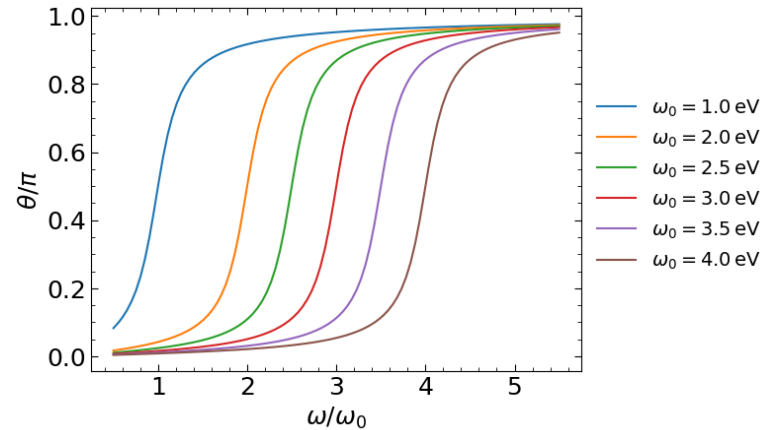
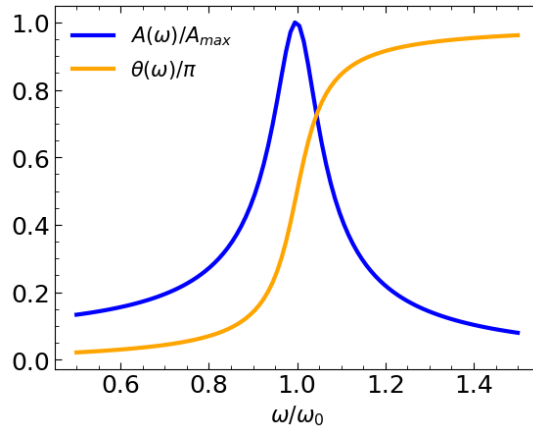
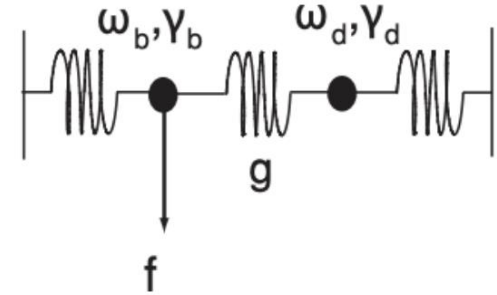
-----
TypeError                                Traceback (most recent call last)
Cell In[11], line 13
     10 print( "Imaginary part of epsilon * mu:\t", np.imag( epsilon2 * mu2 ) )
     12 # show field
--> 13 show_field_in_space(n1=n1, eps2=epsilon2, mu2=mu2, theta_inc=theta_inc,
     14                    wavelength=wavelength, only_incident=True)

Cell In[10], line 44
     42 z = np.linspace( zmin, z_interf, 100, endpoint=False)
     43 X, Z = np.meshgrid( x, z )
--> 44 E_d = efield(x=X, z=Z, theta=theta_inc, n1=n1, eps2=eps2, mu2=mu2,
     45                    wvl=wavelength, only_incident=only_incident)
     47 # upper plane
     48 x = np.linspace( xmin, xmax, 200 )

Cell In[9], line 43
     41 eps1 = n1**2.0
     42 # compute Fresnel coefficients
--> 43 rTE, tTE = fresnel_TE(eps1=eps1, mu1=1.0, eps2=eps2, mu2=mu2, wvl=wvl, theta=theta)
     45 if not upper_plane:
     46
     47     # compute kz vector component below the interface for incident
     48     kz1 = k0 * np.cos(theta) * n1
     ...
--> 42 rTE = (mu2 * kz1 - mu1 * kz2) / (mu2 * kz1 + mu1 * kz2)
     43 # compute transmission Fresnel coefficient
     44 tTE = (2 * mu2 * kz1) / (mu2 * kz1 + mu1 * kz2)

TypeError: unsupported operand type(s) for *: 'float' and 'NoneType'
Output is truncated. View as a scrollable element or open in a text editor. Adjust cell output settings...
```

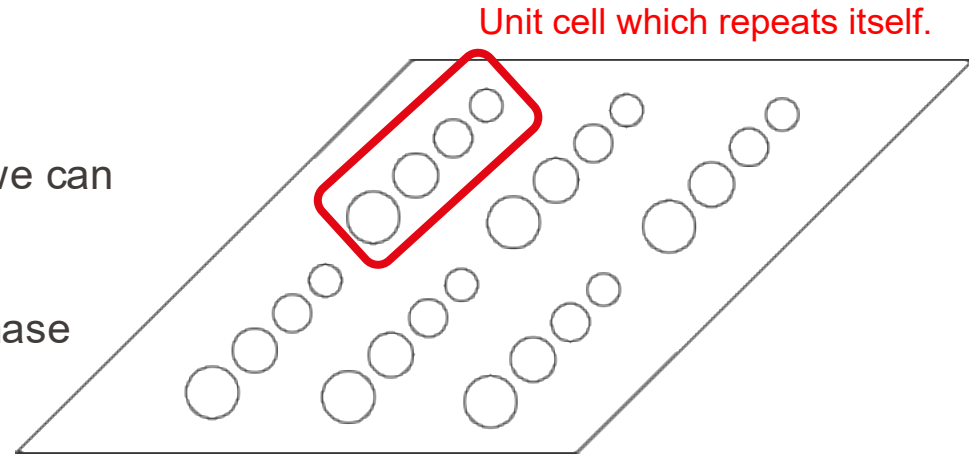
1. A Damped Oscillator in an External Field
2. Studying Phase Shifts with Simple Oscillators
3. Mechanical Analogue of Fano Resonance



A Simple Metasurface / 1

Consider the following metasurface.

- Each unit cell contains a series of structures, called **meta-atoms**.
- Under the dipolar approximation, we can treat them as simple **classical oscillators**, with amplitude and phase



$$A = \frac{q|\mathbf{E}|}{m} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}}, \quad \theta = \arctan\left(\frac{\omega\gamma}{\omega_0^2 - \omega^2}\right).$$

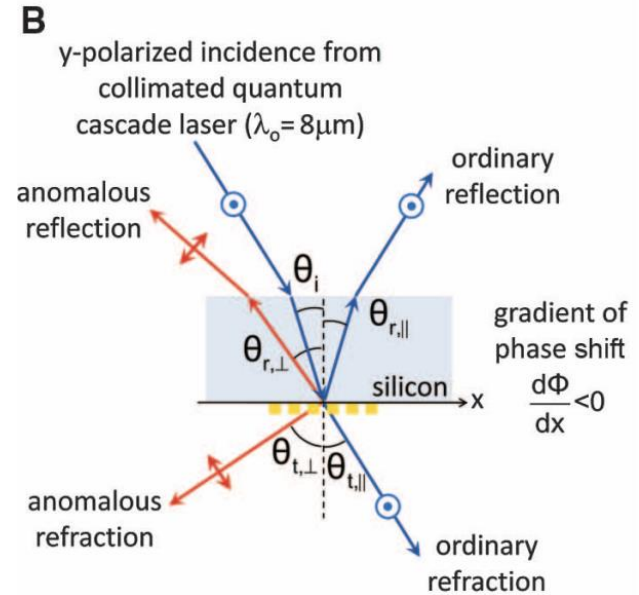
A Simple Metasurface / 2

Following the generalized Snell law

$$\sin \theta_r - \sin \theta_i = \frac{1}{n_1 k_0} \frac{d\Phi(x)}{dx},$$

we can realize anomalous reflection/refraction.

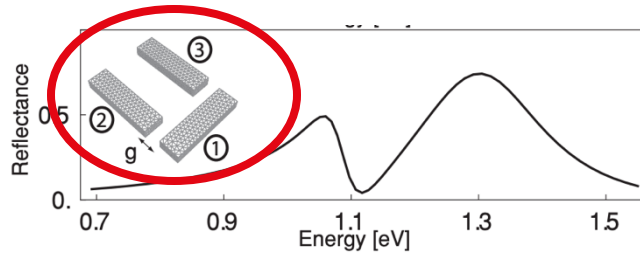
In our simple model of oscillators, each oscillator needs to oscillate at different phase and collectively generate a phase shift.



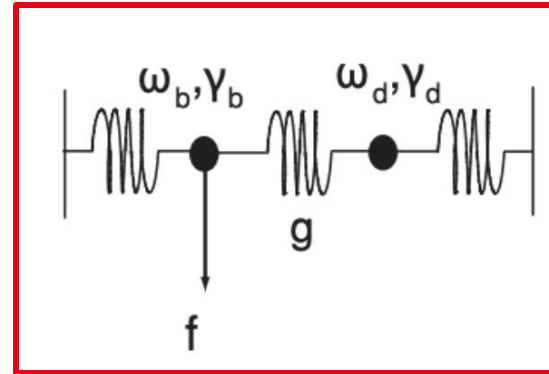
Nanfeng Yu et al. Science 334, 333 (2011)

Fano Resonance / 1

We can model the interference of a dark mode to a bright mode through a coupled oscillator model.



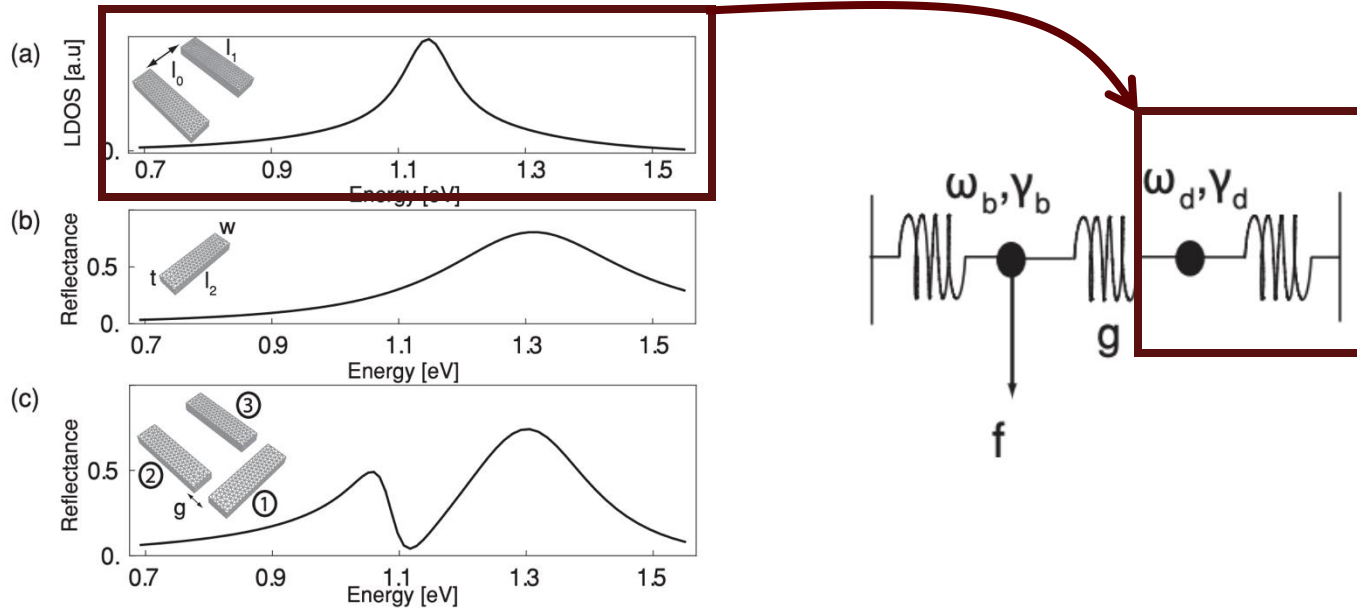
Electromagnetic Problem.



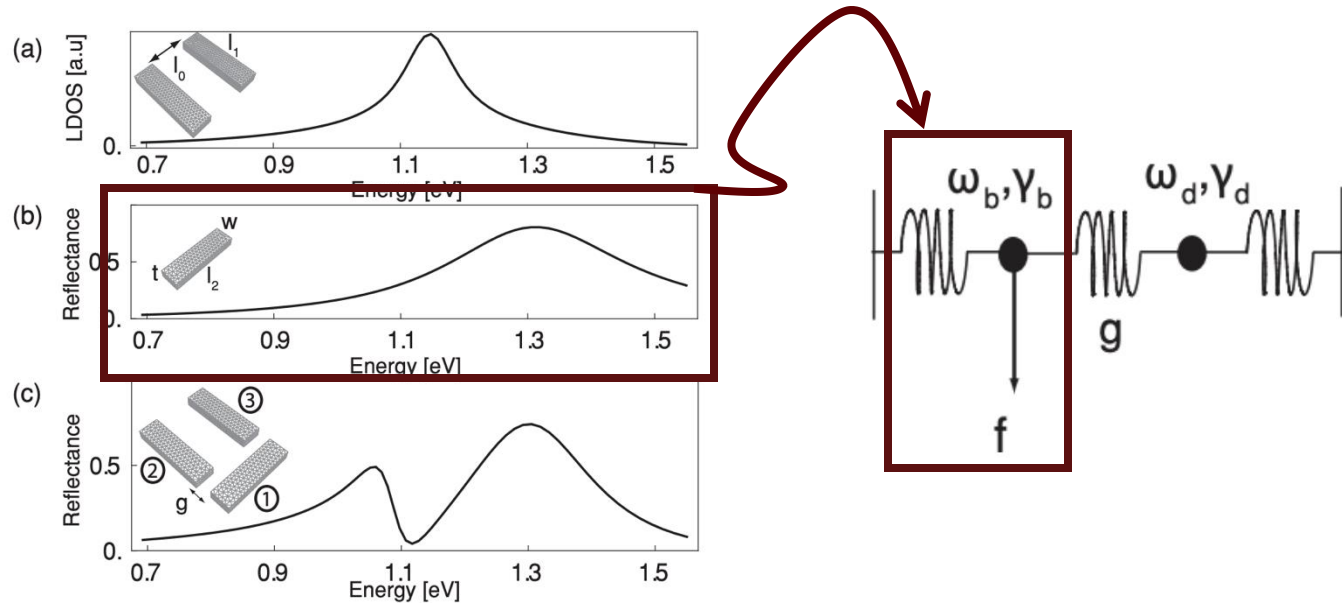
Coupled Oscillator Model

Fano Resonance / 2

We can model the interference of a dark mode to a bright mode through a coupled oscillator model.



We can model the interference of a dark mode to a bright mode through a coupled oscillator model.



Fano Resonance / 2

We can model the interference of a dark mode to a bright mode through a coupled oscillator model.

