EPFL

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Week 11
(Metamaterials)

Stavros Athanasiou

Lausanne, 25 Nov 2025

Course Timeline

Week 1
- Introduction
- Ray Optics
- Wave Optics

Week 2
- EM Optics

Week 3
- Material Response
- Optical Constants

Week 4
- Light Scattering
- Mie Theory

Week 5
- Localized Surface Plasmons

Week 6
- Propagating Surface Plasmons

Week 7
- Plasmonic
- Applications

Week 8
- Gratings

Week 9
- Stratified Media
- Transfer Matrix

Week 10
- Photonic Crystals

Week 11
- Metamaterials

Week 12
- Metasurfaces

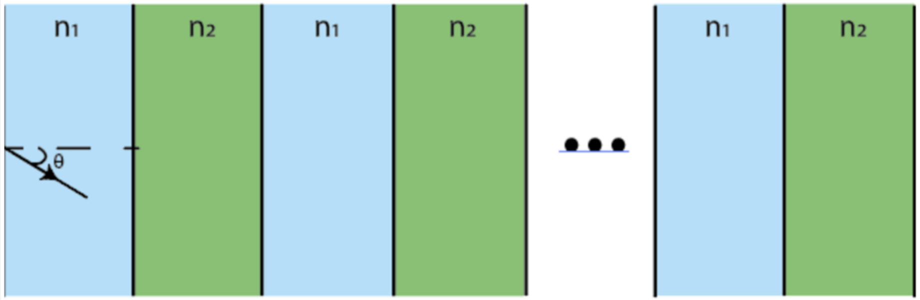
Week 13
- Applications of Metamaterials

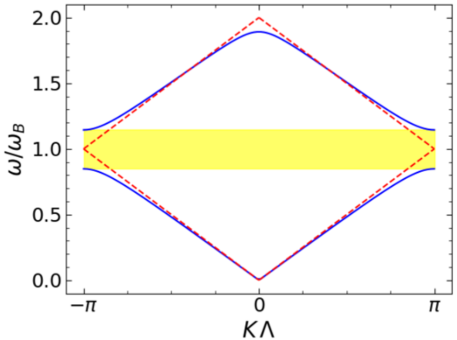
Week 14
- Lab Tour
- **No Exercise Session**

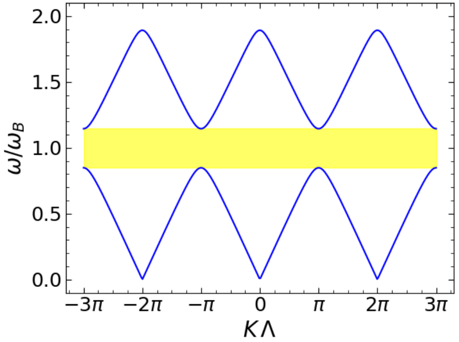
Last Week: Photonic Crystals

1. Dispersion Relation of a 1D Photonic Crystal

2. The large-N limit of Bragg Reflectors







DBRs and PCs

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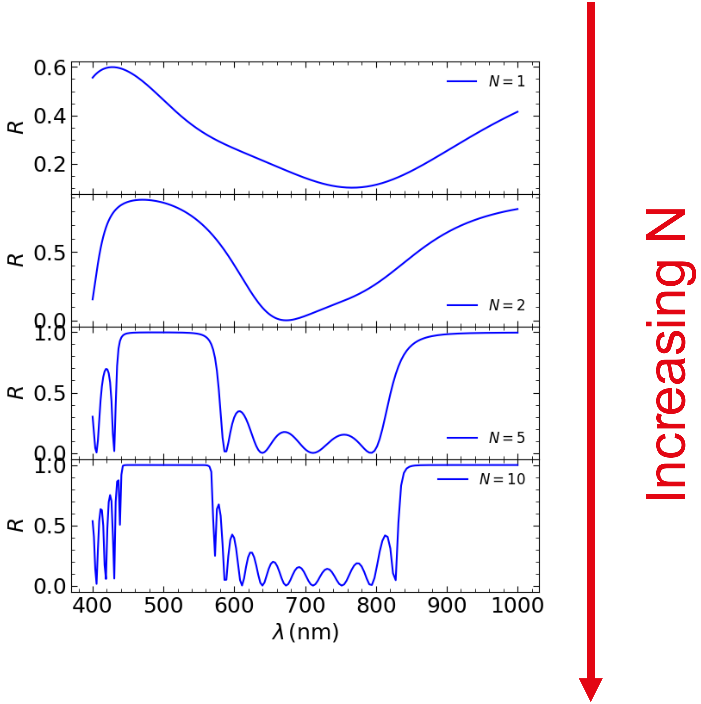
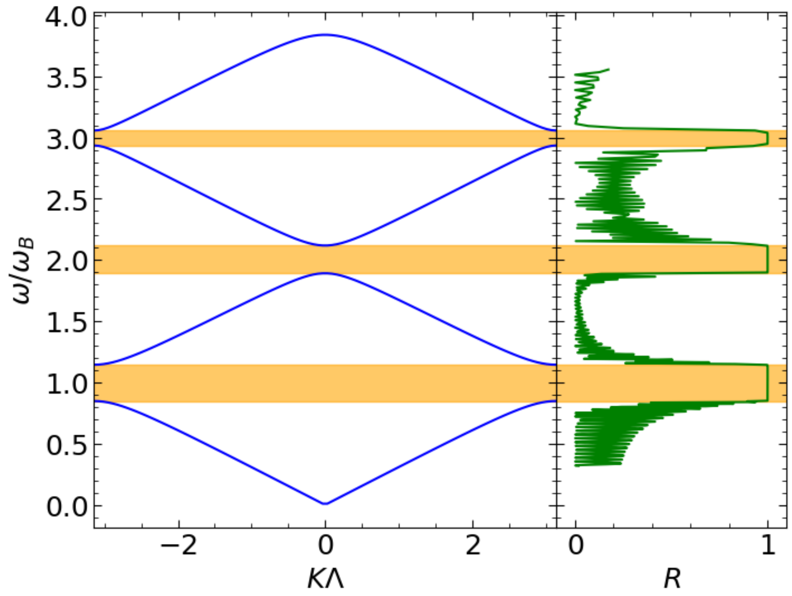
We obtain a 1D photonic crystal in the limit of large N in the DBR system.

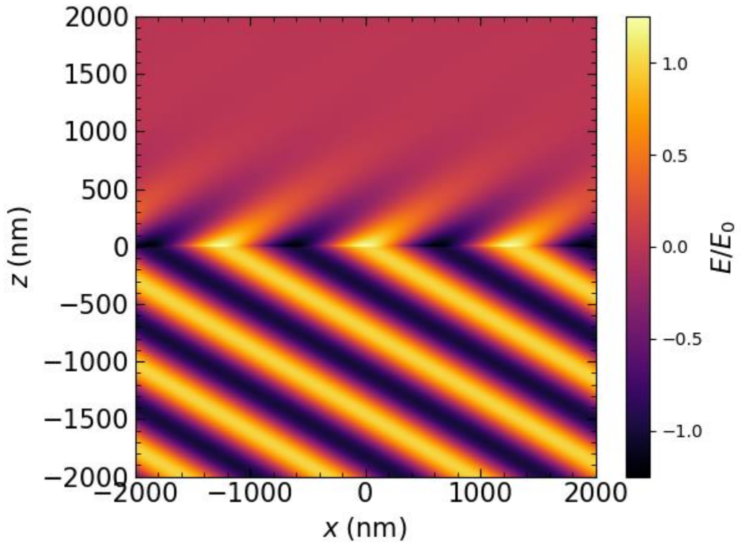


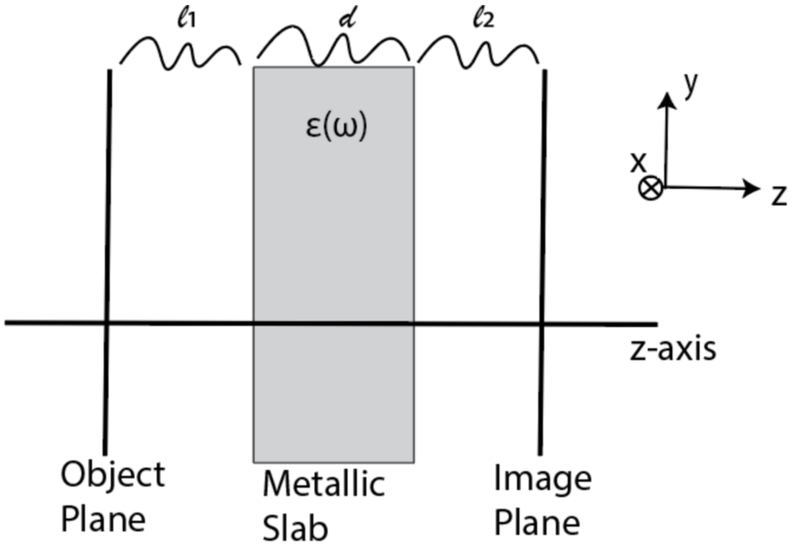


This Week: Metamaterials

1. Electromagnetic Properties and Refractive Index

2. Negative Refraction Makes a Perfect Lens





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EPFL

Choice of Sign

The exploration of substances of generic complex permittivity and permeability requires careful treatment.

$$k_z^2 = \varepsilon\mu\frac{\omega^2}{c^2} - k_x^2 \quad \Longrightarrow \quad k_z = \pm\sqrt{\varepsilon\mu\frac{\omega^2}{c^2} - k_x^2}$$

To choose the correct sign, we need to satisfy:

1. Causality, i.e. the imaginary part of k_z must be non-negative, and
2. Energy should flow away from the interface.

➔ This problem requires **complex analysis**.

Choice of Sign

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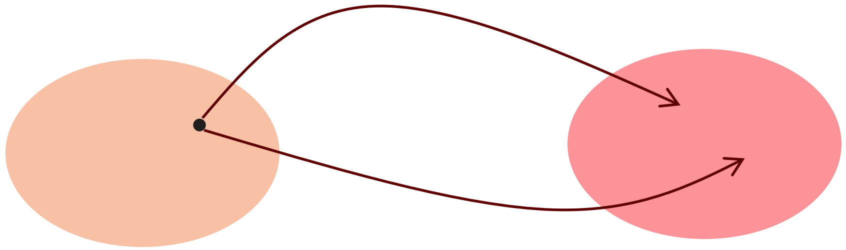
Taking the second power of a complex number z is a one-to-one map.

However, taking the square root of a complex number z is a one-to-two map.

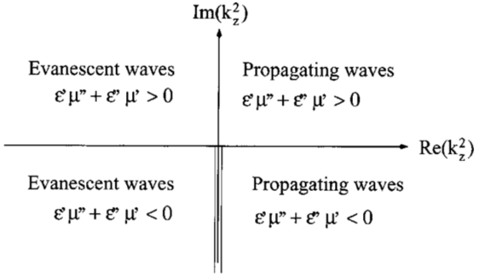


Result: A multi-valued function. Physical quantities **must be** single-valued.

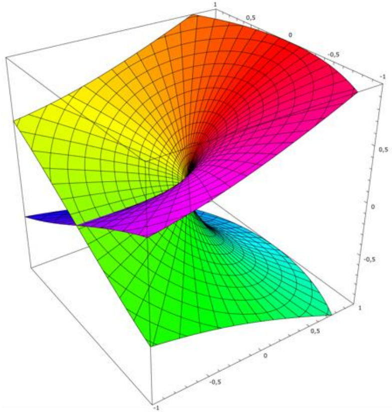
Choice of Sign

Options:

1. **Branch cut:** Analytical continuation from z to sqrt(z) is not possible, thus we introduce a branch cut. Whenever we cross it, the sign flips.

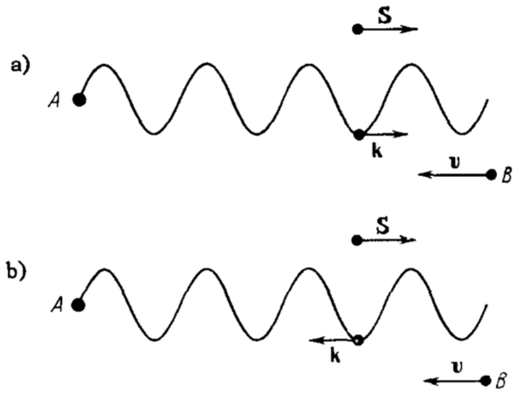


2. **Riemann Surfaces:** Surface-like configurations that cover the entire complex plane.



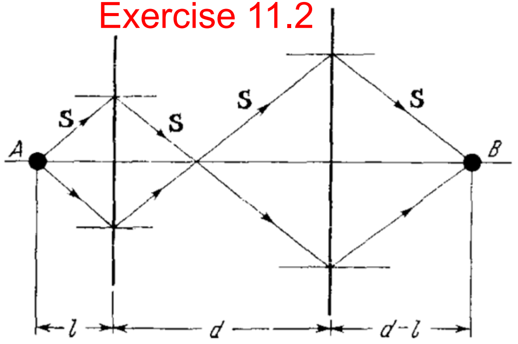
For our case, using Riemann surfaces allows us to cast the square root as a holomorphic function, where both branches (+1 and -1) are represented.

Implications of Negative Refraction

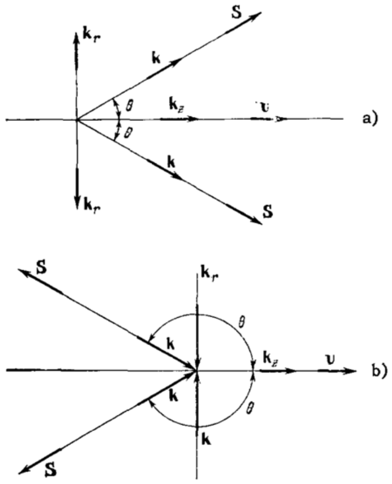


Reversed Doppler Effect

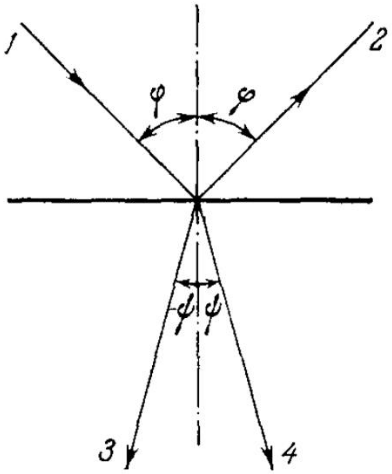
Exercise 11.2



Perfect Lens



Reversed Cherenkov Radiation



Negative Refraction

Viktor G Veselago, *Sov. Phys. Usp.* **10** 509 (1968

Negative Refraction Makes a Perfect Lens

VOLUME 85, NUMBER 18 PHYSICAL REVIEW LETTERS 30 OCTOBER 2000

Negative Refraction Makes a Perfect Lens

J. B. Pendry
Condensed Matter Theory Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
(Received 25 April 2000)

With a conventional lens sharpness of the image is always limited by the wavelength of light. An unconventional alternative to a lens, a slab of negative refractive index material, has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner. Such "superlenses" can be realized in the microwave band with current technology. Our simulations show that a version of the lens operating at the frequency of visible light can be realized in the form of a thin slab of silver. This optical version resolves objects only a few nanometers across.

PACS numbers: 78.20.Ci, 42.30.Wb, 73.20.Mf, 78.66.Bz

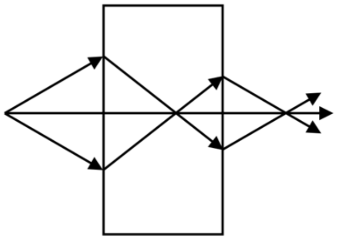
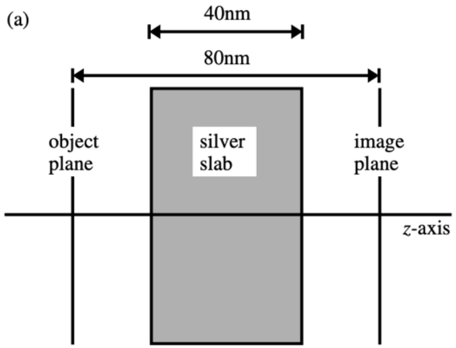
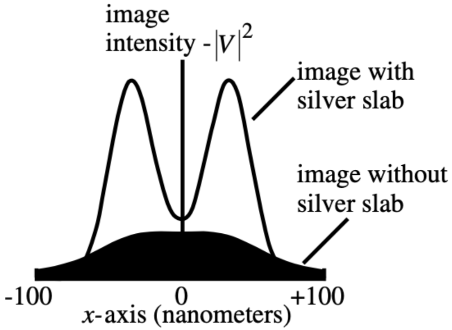


FIG. 1. A negative refractive index medium bends light to a negative angle with the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

(a)



(c)



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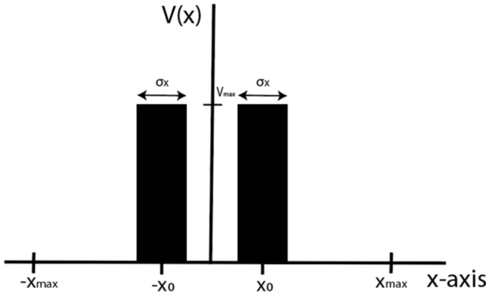
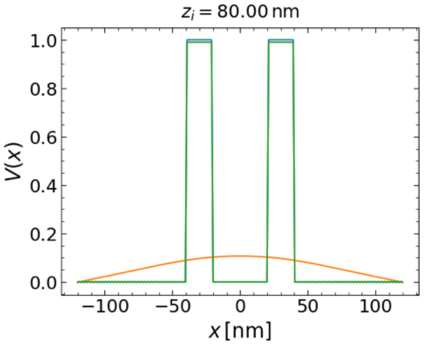
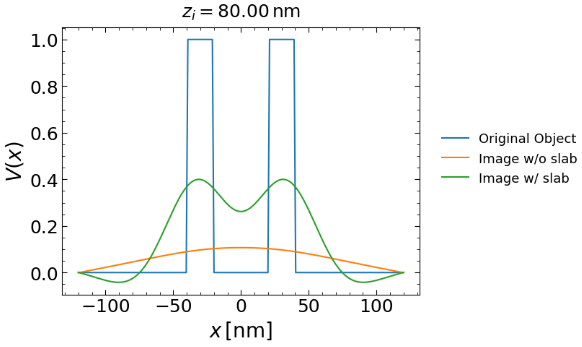




No absorption



With absorption