$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Week 4
(Light Scattering)

Stavros Athanasiou

Lausanne, 30 Sep 2025

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Course Timeline

Week 1
- Introduction
- Ray Optics
- Wave Optics

Week 2
- EM Optics

Week 3
- Material Response
- Optical Constants

Week 4
- Light Scattering
- Mie Theory

Week 5
- Localized Surface Plasmons

Week 6
- Propagating Surface Plasmons

Week 7
- Plasmonic
- Applications

Week 8
- Gratings

Week 9
- Stratified Media
- Transfer Matrix

Week 10
- Photonic Crystals

Week 11
- Metamaterials

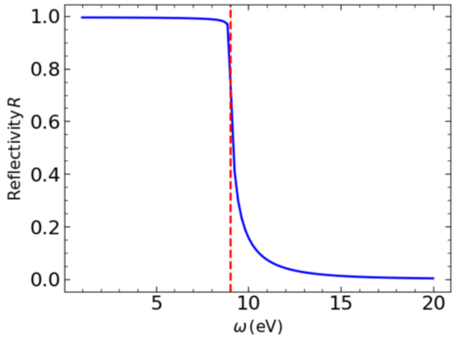
Week 12
- Metasurfaces

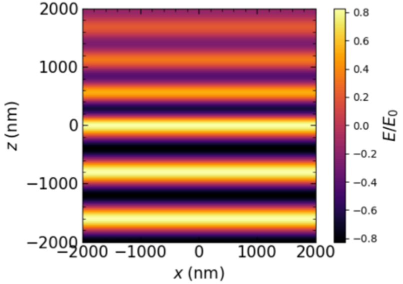
Week 13
- Applications of Metamaterials

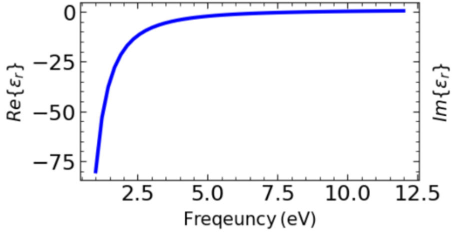
Week 14
- Lab Tour
- **No Exercise Session**

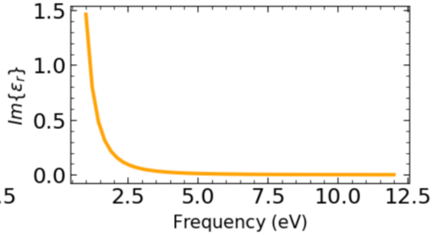
Last Week : Material Properties and Optical Constants

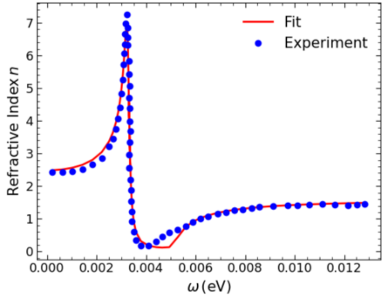
1. Propagation in Lossy Media
2. The Drude Model
3. Reflection from a Metal Planar Surface
4. The Lorentz Model in Action











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Small Error in Solutions

In Suggested Solutions, I used `drude` instead of `get_drude_permittivity` (valid for Exercises 3.2 and 3.3)

```python
def get_drude_permittivity(omega: float, omega_p: float, gamma: float):
    """
    Implementation of the Drude model.

    Arguments:
    ----------
    omega : float
        frequency
    omega_p : float
        the plasma frequency
    gamma : float
        the damping rate

    Returns:
    --------
    eps_r : complex
        the complex relative permittivity
    """

    # --- write your code here --- #

    denom = omega**2.0 + 1.j * gamma * omega
    eps_r = 1.0 - omega_p**2.0 / denom

    # --------------------------- #

    return eps_r
```

```python
# plot the permittivity for silver

fig, ax = plt.subplots( figsize=(10,3), nrows=1, ncols=2 )

# --- write your code here --- #

omega_p = 9.013   # eV
gamma = 0.018    # eV

omega = np.linspace( 1.0, 12.0, 50)
eps_r = drude( omega=omega, omega_p=omega_p, gamma=gamma )
ax[0].plot( omega, np.real(eps_r), c='blue', linewidth=3 )
ax[1].plot( omega, np.imag(eps_r), c='orange', linewidth=3 )
```

```python
omegas = np.linspace( 1.0, 20.0, 100 )
epsilon_r = drude(omega=omegas, omega_p=omega_p, gamma=gamma)
n2 = np.sqrt( epsilon_r, dtype=complex )

refl = np.abs( rTM(theta=0, n1=n1, n2=n2) )**2
lams = 1239.8 / omegas
ax.plot( lams, refl, c='b', linewidth=2)

lambda_p = 1239.8 / omega_p
ax.axvline( x=lambda_p, c='r', linestyle='dashed', linewidth=2 )
```

Lorentz and Drude Models

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These models serve as useful phenomenological, classical descriptions of material response. However, they do not provide the fundamental microscopic mechanisms underlying the physics.

The Lorentz provides a classical picture that gives a microscopic description of bound electrons, and explains features such as:

- Origin of dispersion
- Role of losses
- Dependence of the permittivity on density and frequency

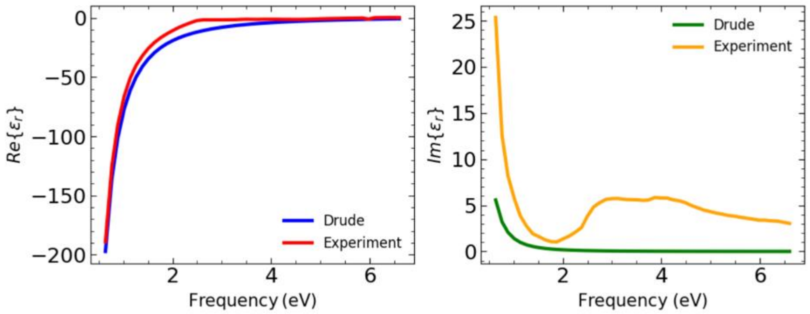
The damping rate is treated as a fixed parameter that accounts for:

- Radiative relaxation (eg spontaneous emission)
- Energy transfer due to collisions between atoms

Benisty et al., Introduction to Nanophotonics, Oxford University Press (2022)

Fitting the Permittivity

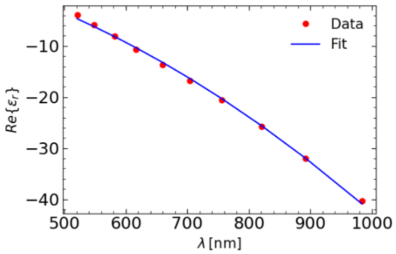
From last week:

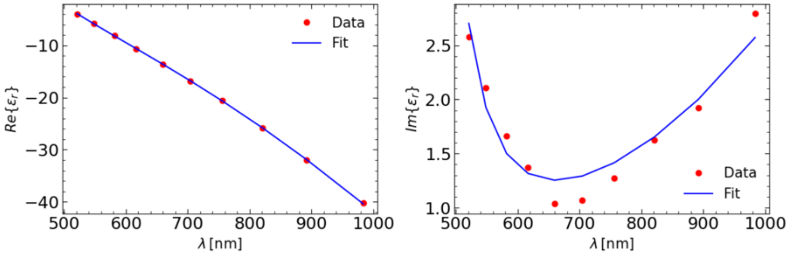


Can we do better?

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}.$$

$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} - \frac{\Delta\epsilon\Omega_L^2}{\omega^2 - \Omega_L^2 + i\Gamma_L\omega}$$





Ref: Vial et al, Phys Rev B 71, 085416 (2005)

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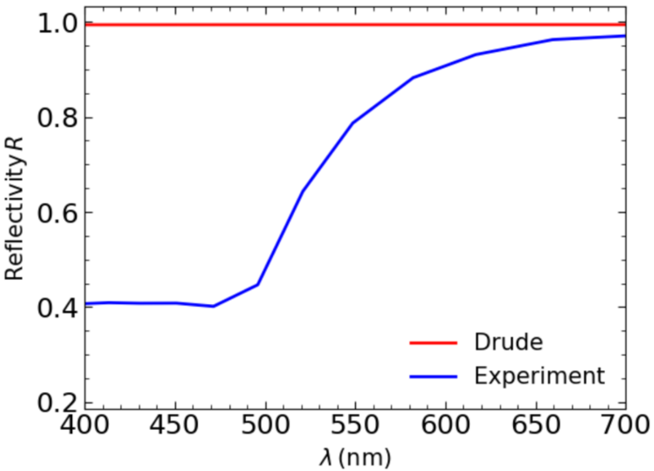
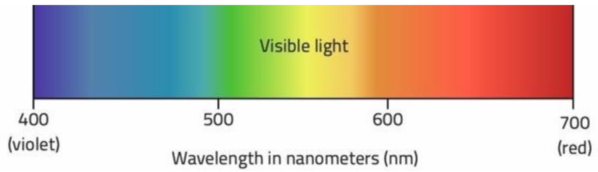
Reflection from a gold surface

Use permittivity from Drude and experiment. Compare!

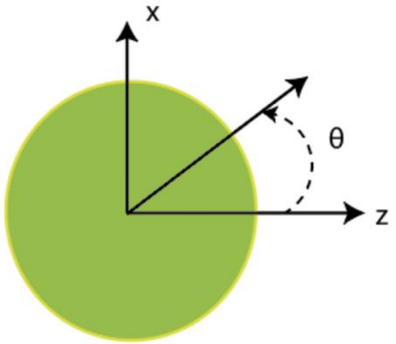


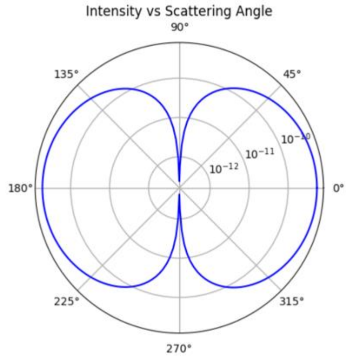


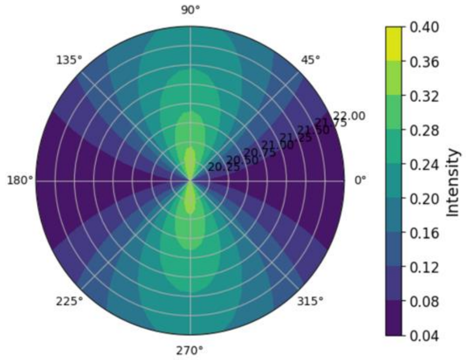
Bonus: Golden Mystery Solved (https://physics.aps.org/articles/v10/s3)

This Week : Light Scattering / 1

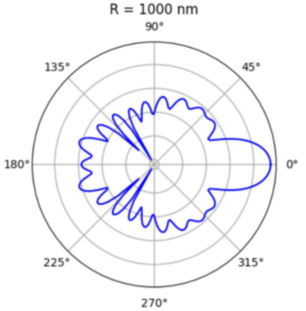
- The near- and far- fields

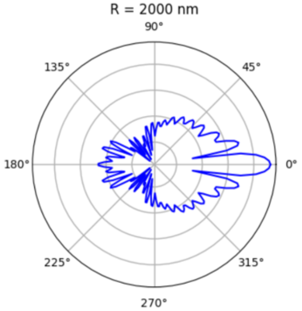






- Modes of the Radiation Pattern





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This Week : Light Scattering / 2

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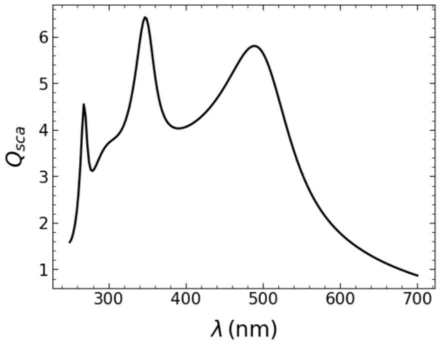
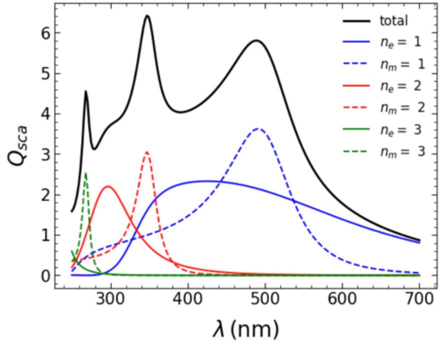
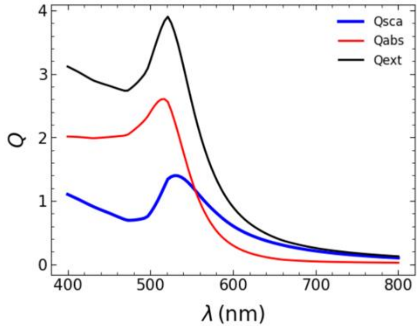
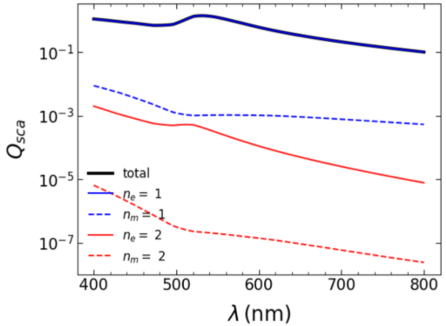
- The Response of Dielectric Particles and Multipole Analysis





- The Response of Metal Particles





Mie theory

Ref: Bohren and Huffman, Absorption and Scattering of Light by Small Particles (1983)

For this week’s exercises:

- week4 notebook
- Mie theory Python implementation notebook
- mie_theory module