

Please select a slot on moodle for the exam !  
(January 13 – 14)

# Selected Topics in Advanced Optics

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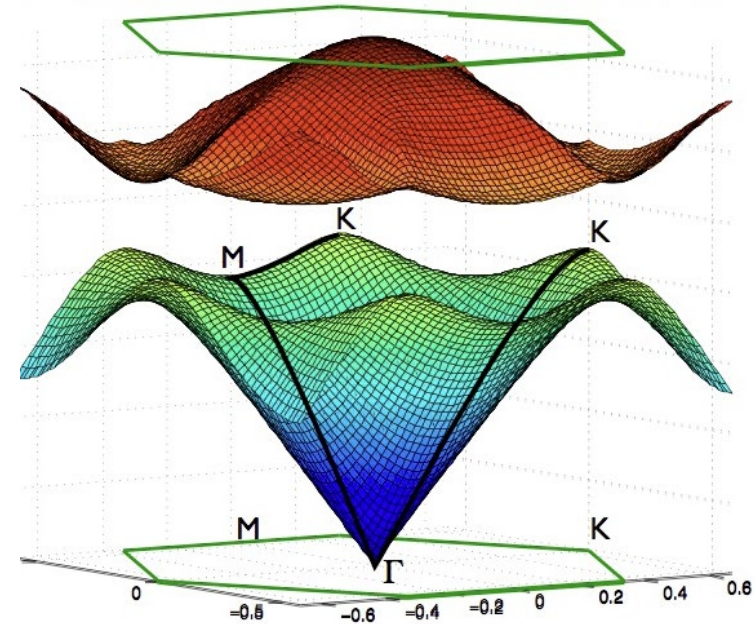
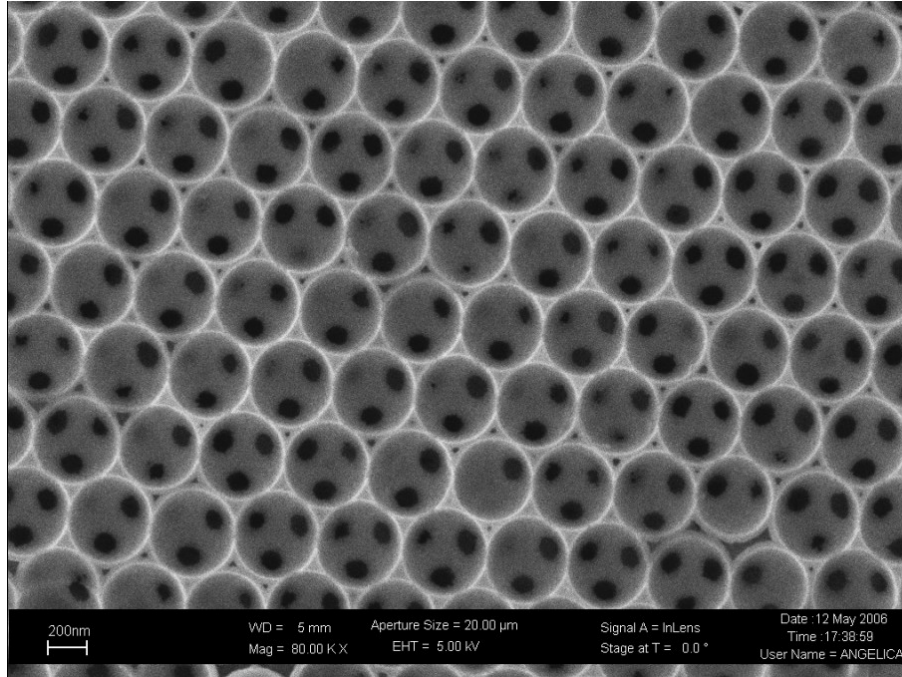
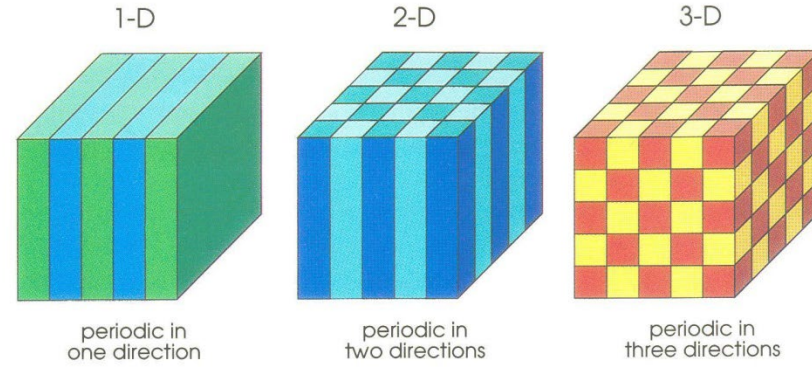
## Week 10 – part 1

Olivier J.F. Martin  
Nanophotonics and Metrology Laboratory

**EPFL**

# Gratings, stratified media and photonic crystals

## Part III – Photonic crystals

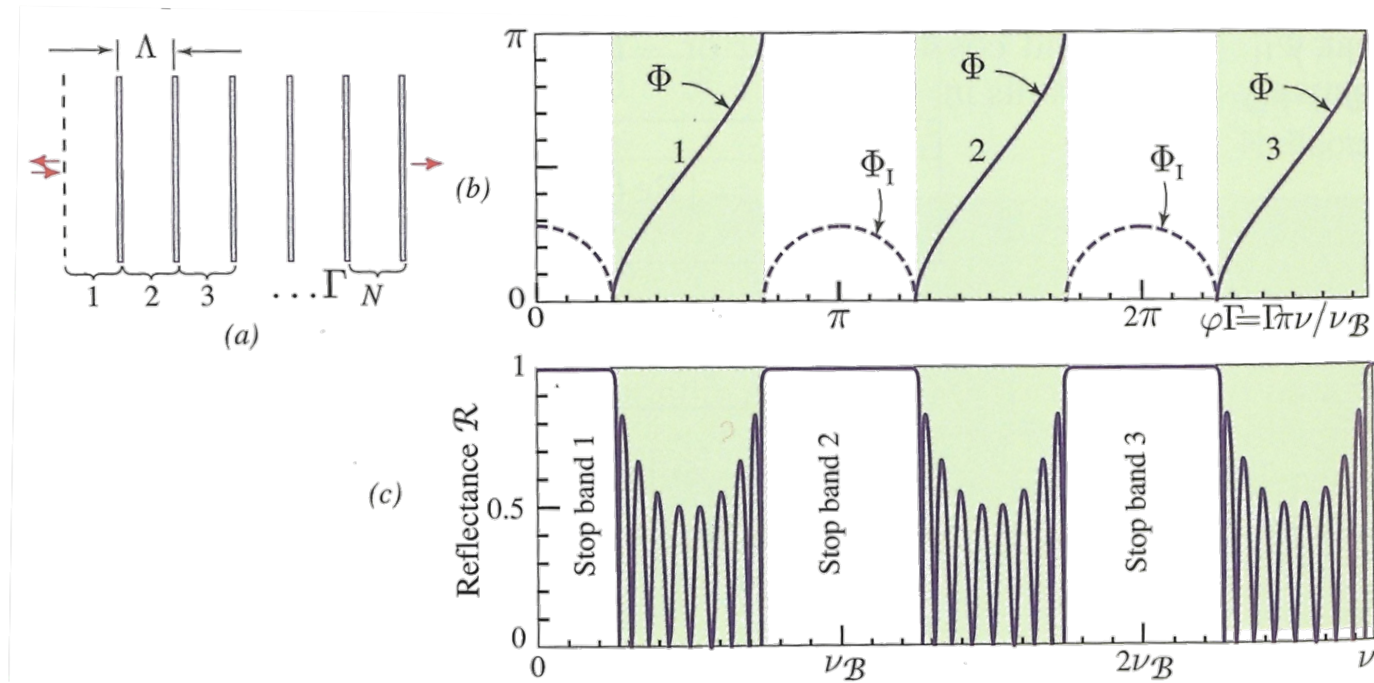


## Some references

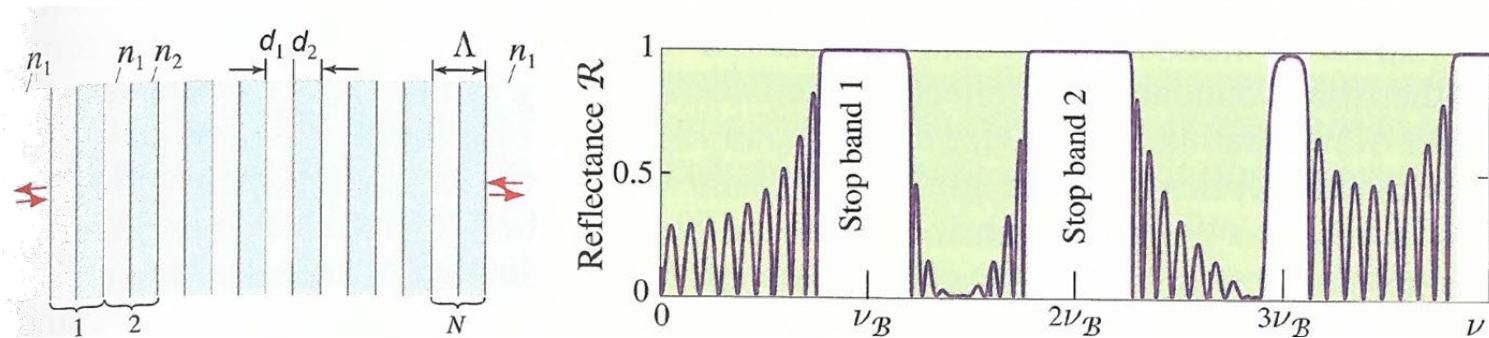
- B.E.A. Saleh & M.C. Teich, Fundamental of photonics 2<sup>nd</sup> Ed. (Wiley, Hoboken, 2007), Chapter 7.
- J.D. Joannopoulos *et al.*, Photonic crystals, Molding the flow of light (Princeton University Press, 1995).
- K. Sakida, Optical properties of photonic crystals (Springer, Berlin, 2001)

# Bragg gratings – total reflection regime

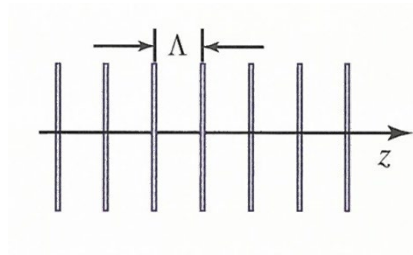
- Stack of partially reflective mirrors:



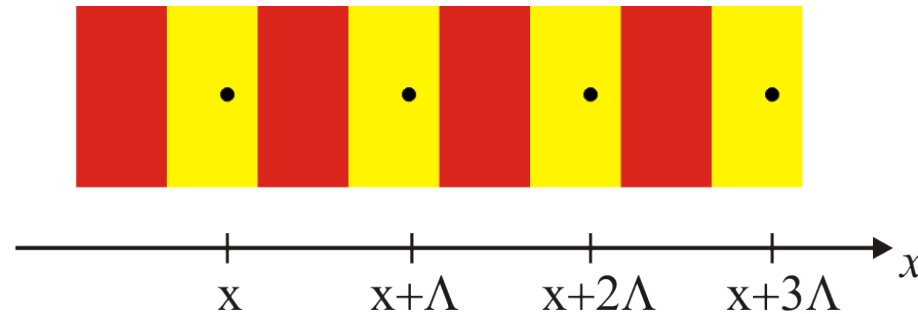
- Dielectric Bragg grating:



# 1D photonic crystal



- "Crystal" with periodicity  $\Lambda$
- Reciprocal lattice vector:  $G = g = 2\pi / \Lambda$
- We are looking at solutions that satisfy the periodicity of the crystal
- In a periodic system, a wave at a point  $x$  differs from its value a period  $\Lambda$  away by a complex constant  $C$  :



$$\frac{u(x + \Lambda)}{u(x)} = \frac{u(x + 2\Lambda)}{u(x + \Lambda)} = \dots = \frac{u(x + m\Lambda)}{u(x + (m-1)\Lambda)} = C \quad u(x + m\Lambda) = C^m u(x)$$

$$C = e^{-j\beta\Lambda}$$

## 1D photonic crystal

- We now consider a function:  $R(x) = e^{j\beta x} u(x)$
- Which is periodic with period  $\Lambda$ :  $R(x + \Lambda) = e^{j\beta(x+\Lambda)} u(x + \Lambda) = R(x)$
- And can be written as a Fourier series:

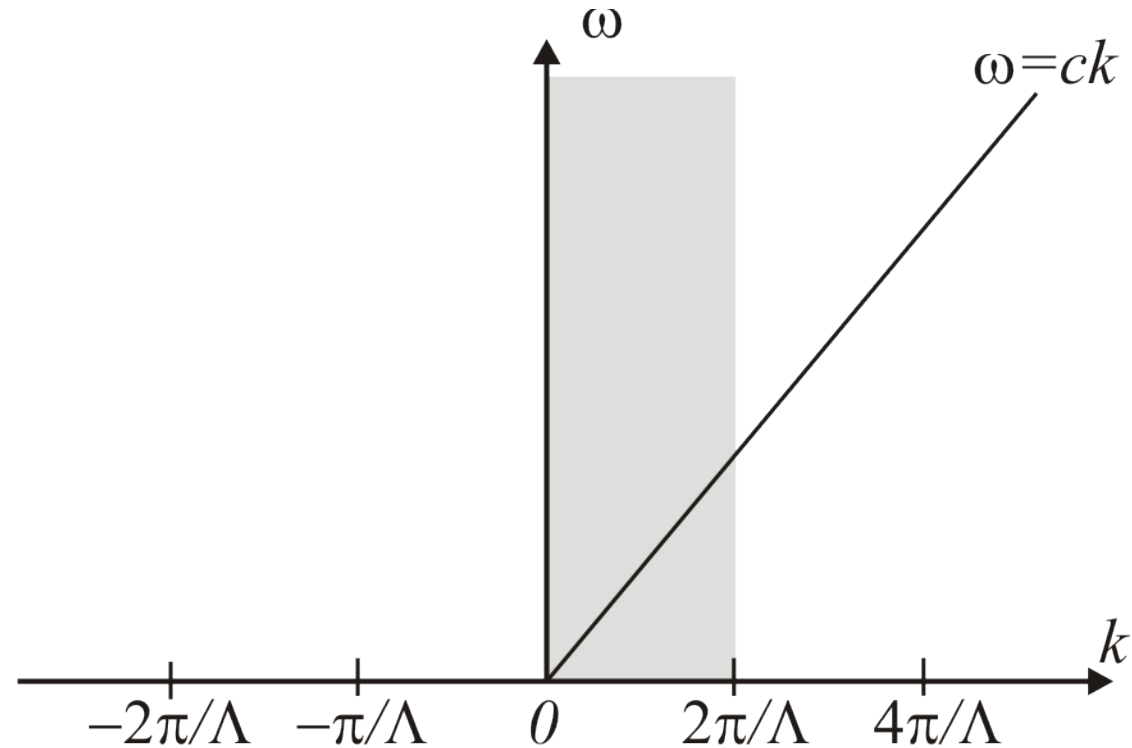
$$R(x) = \sum_{n=-\infty}^{\infty} A_n e^{-j(2n\pi/\Lambda)x}$$

- We obtain an expression for a wave in a periodic structure:

$$u(x) = \sum_{n=-\infty}^{\infty} A_n e^{-j(\beta+2n\pi/\Lambda)x} = \sum_{n=-\infty}^{\infty} A_n e^{-j\beta_n x} \quad \beta_n = \beta + \frac{2n\pi}{\Lambda}$$

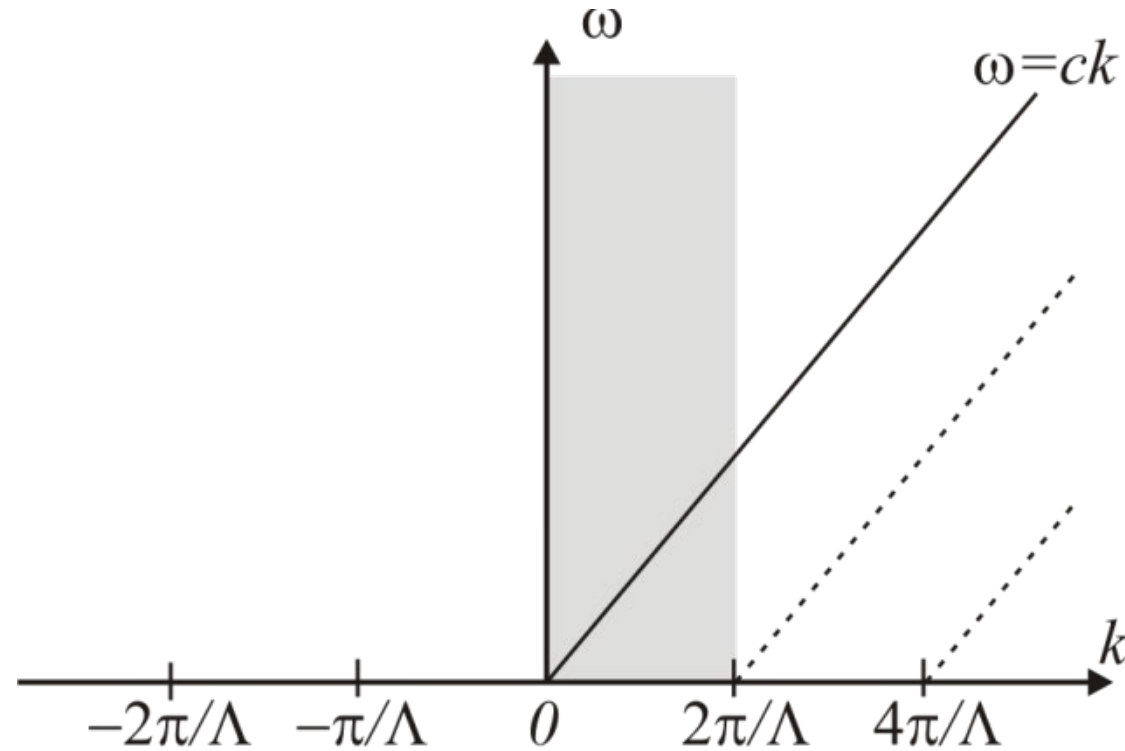
# 1D photonic crystal – Reciprocal lattice

$$u(x) = \sum_{n=-\infty}^{\infty} A_n e^{-j(\beta+2n\pi/\Lambda)x} = \sum_{n=-\infty}^{\infty} A_n e^{-j\beta_n x} \quad \beta_n = \beta + \frac{2n\pi}{\Lambda}$$



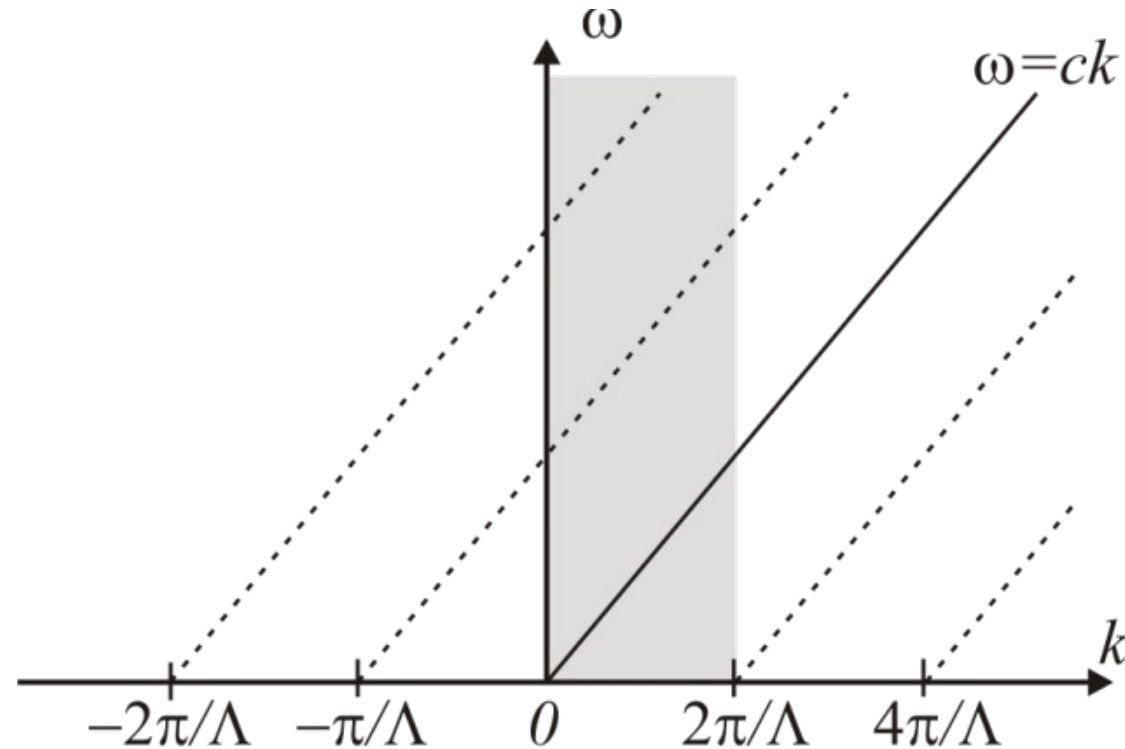
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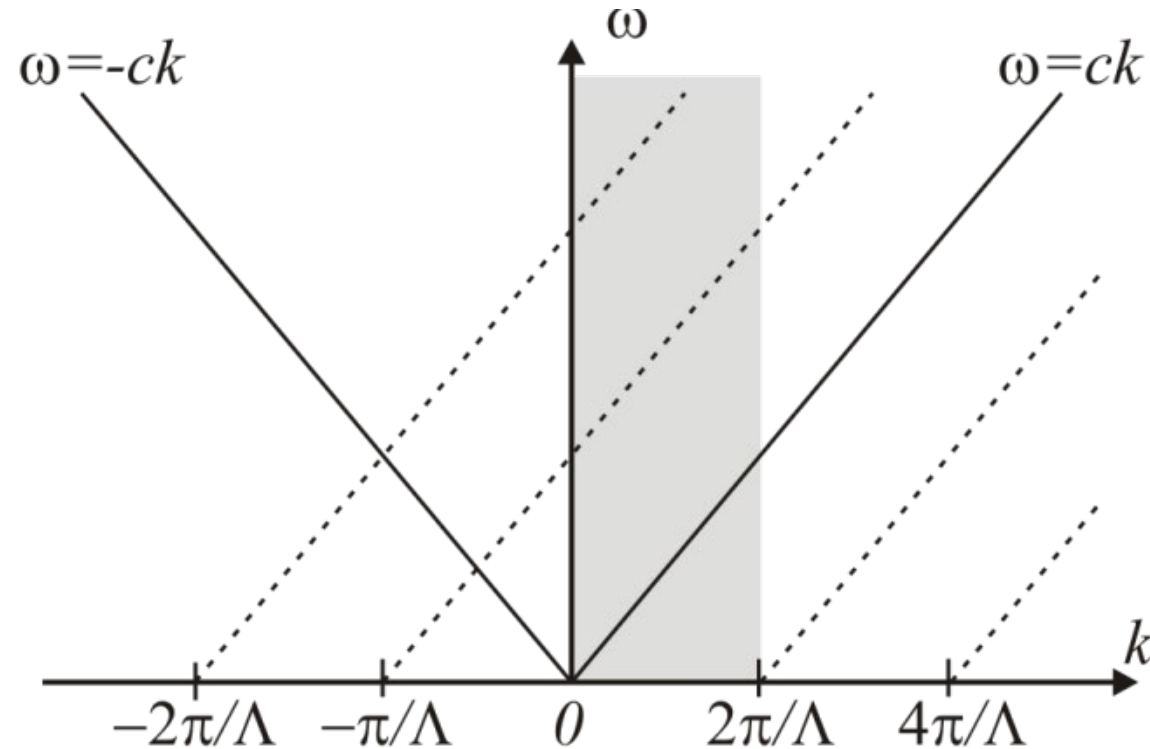
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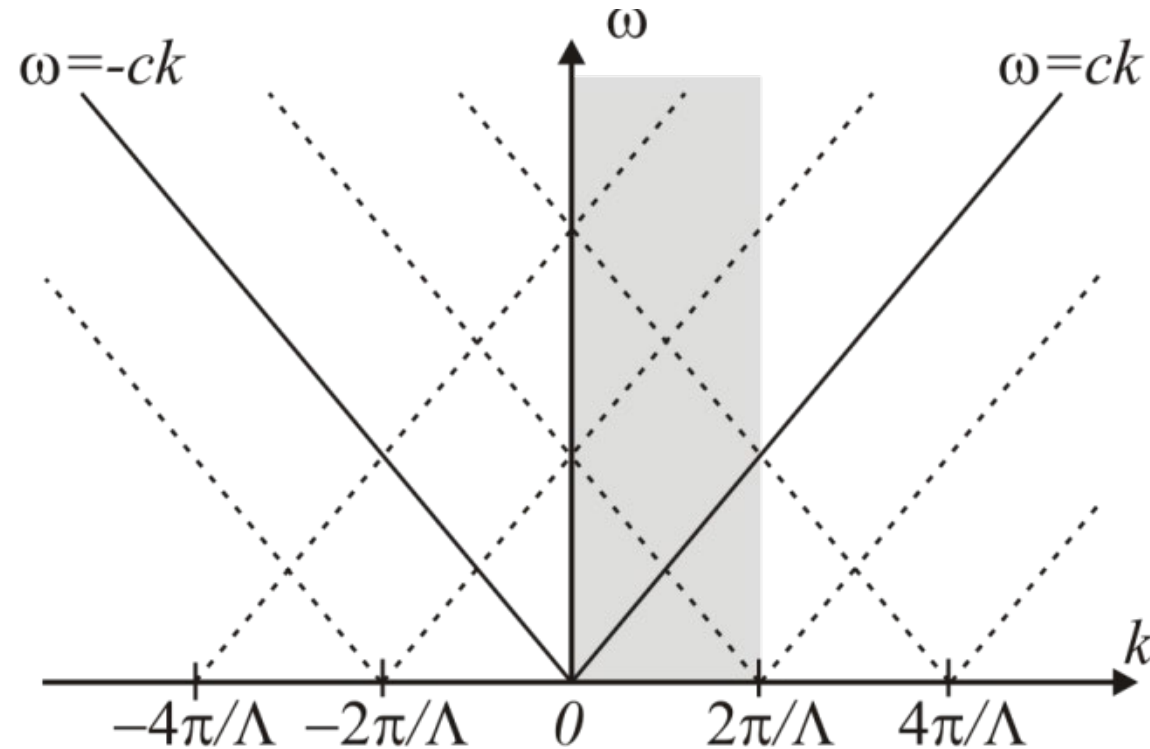
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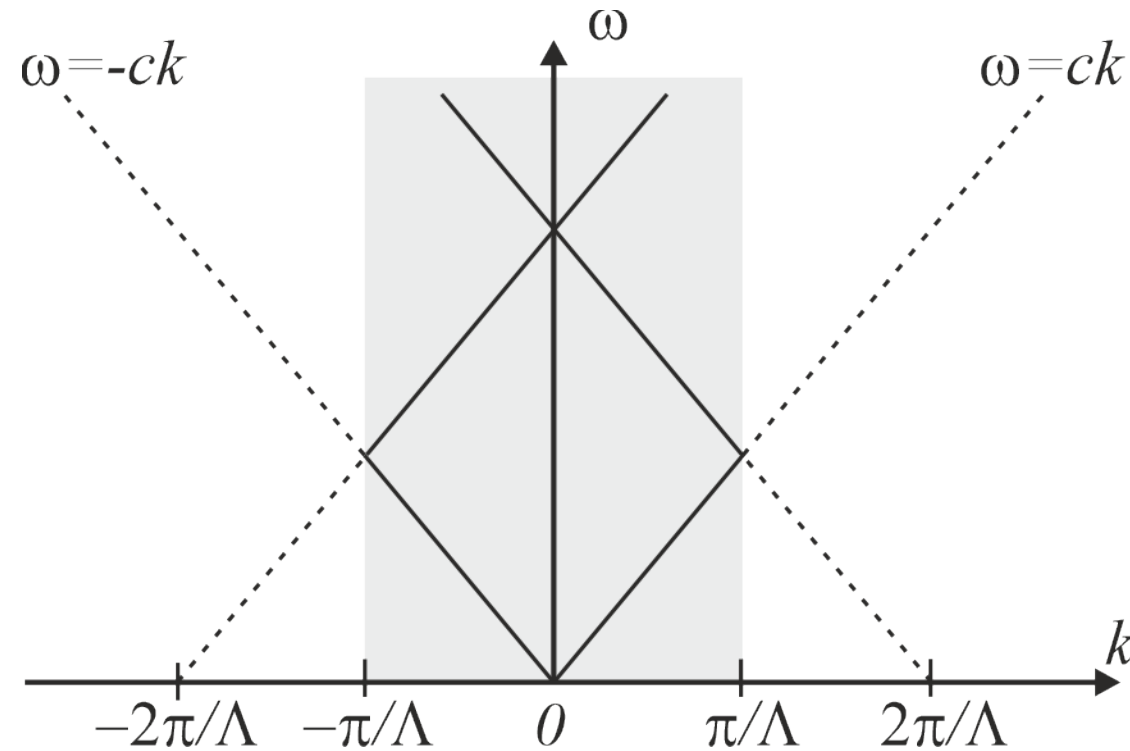
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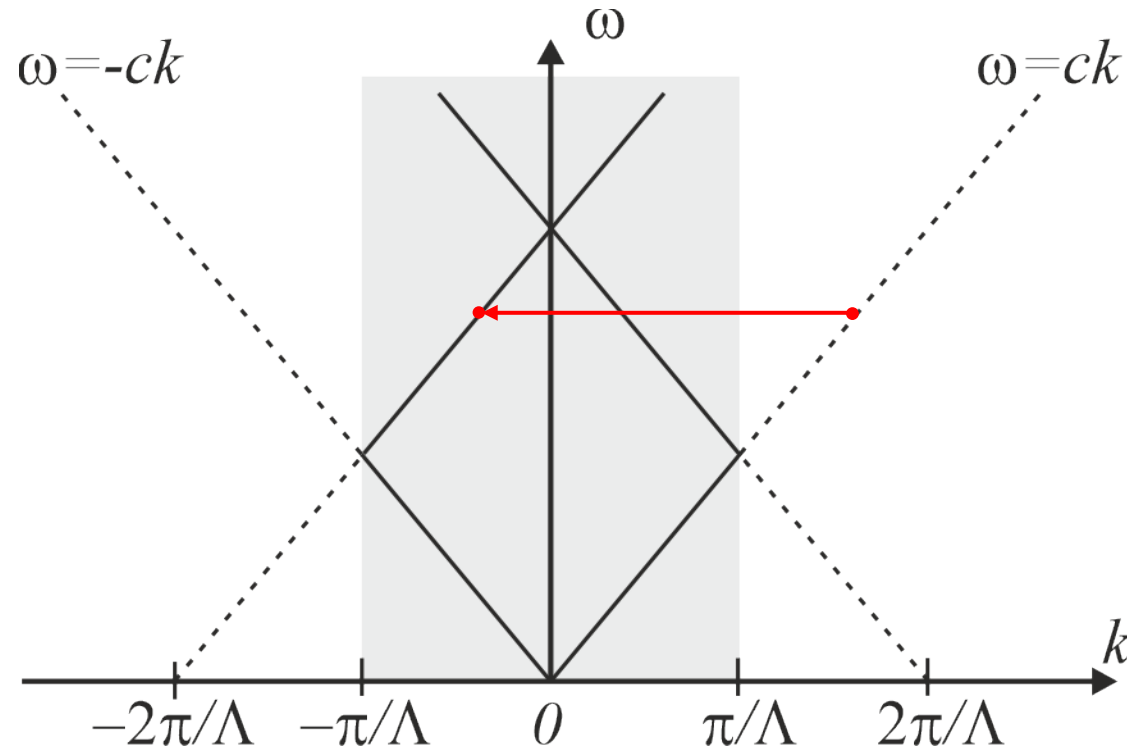
# 1D photonic crystal – Reciprocal lattice

- Traditionally, one represents the first Brillouin zone between  $-\pi/\Lambda$  and  $\pi/\Lambda$  :



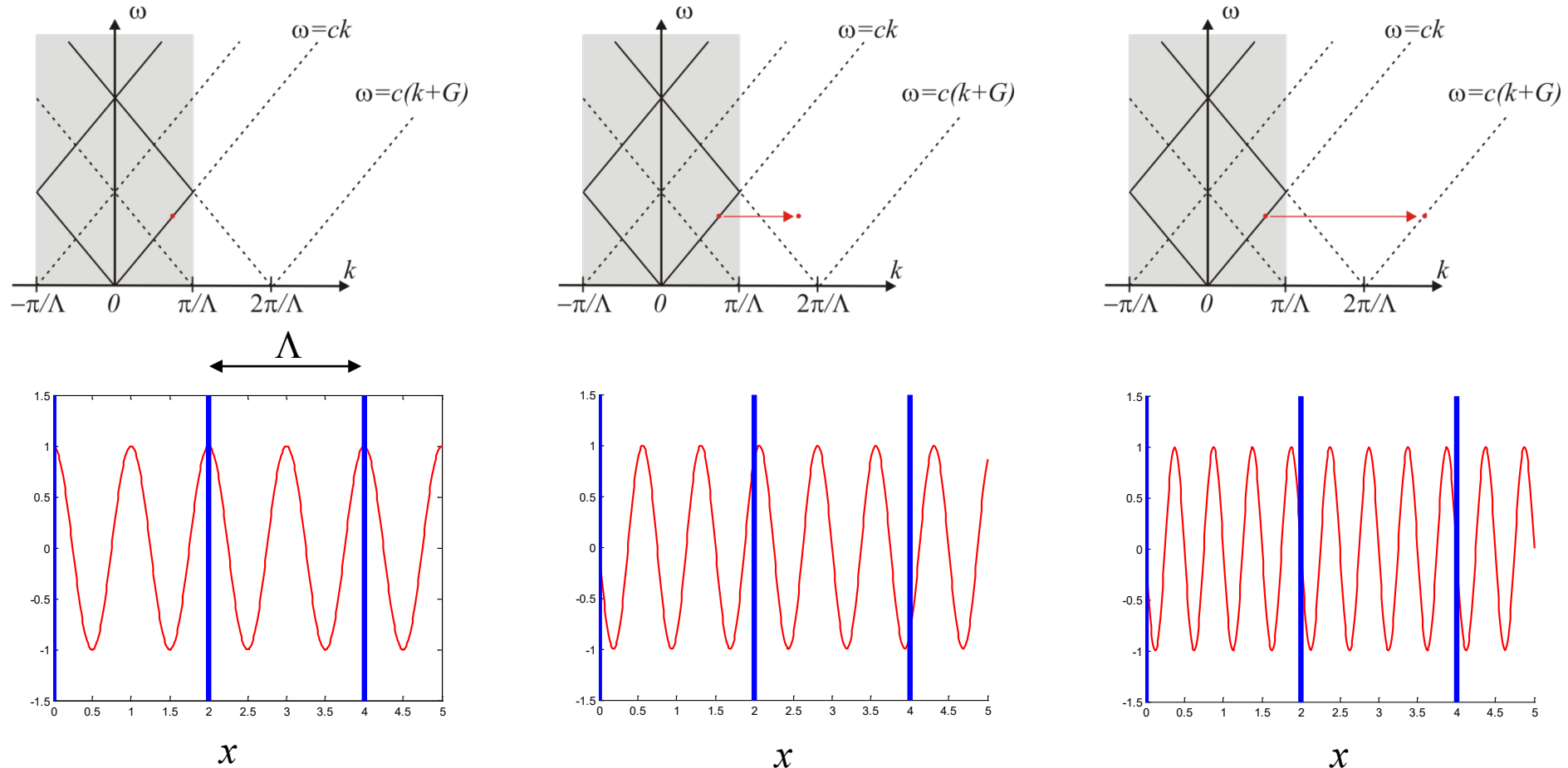
# 1D photonic crystal – Reciprocal lattice

- All the solutions from the wave equation that satisfy periodicity can be represented in the first Brillouin zone:



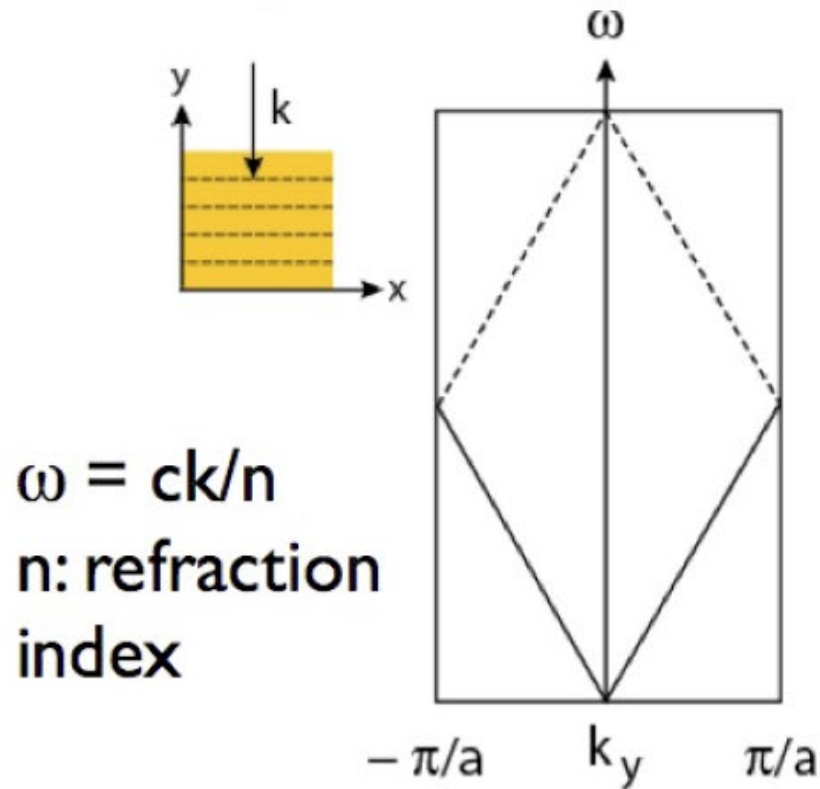
# 1D photonic crystal – Reciprocal lattice

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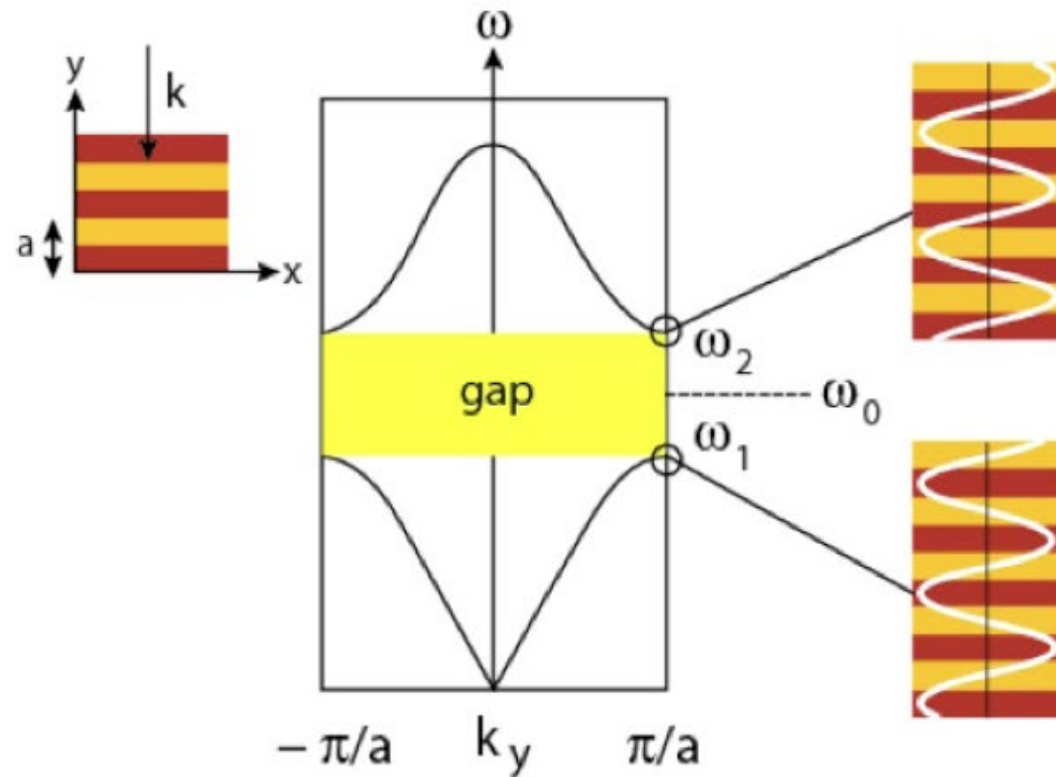


# 1D photonic crystal – Band structure

- Free space



- Photonic crystal (dielectric/air)



- Two types of modes are created:

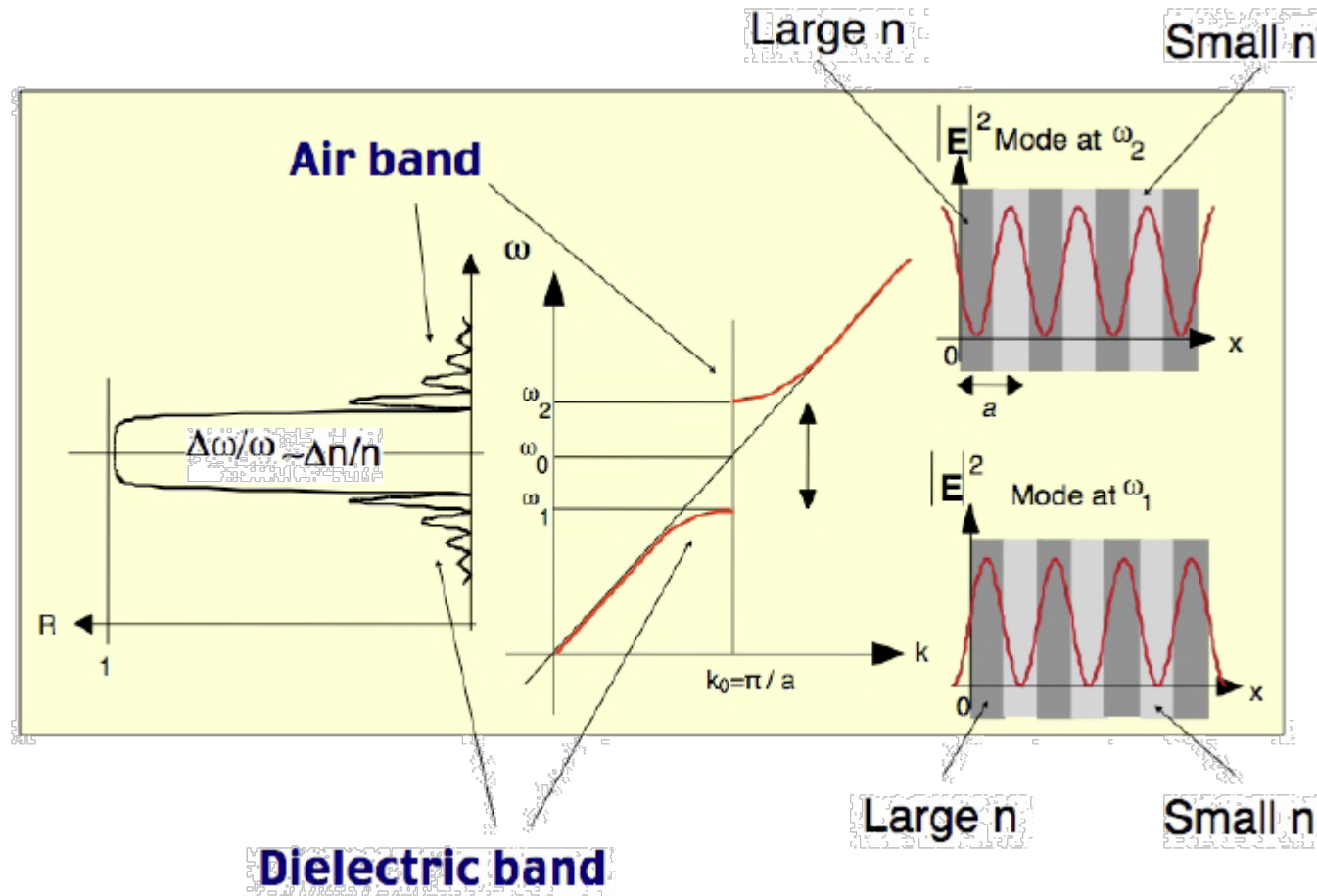
- Air modes (field in the low index – orange – region)
- Dielectric modes (field in the high index – red – region)



gap

# 1D photonic crystal – Band structure and reflectance

- Very high reflectivity is observed in the bandgap, as optical modes cannot exist in the crystal at those frequencies
- The dispersion is bent close to the edge of the bandgap (modified group velocity!)



# Selected Topics in Advanced Optics

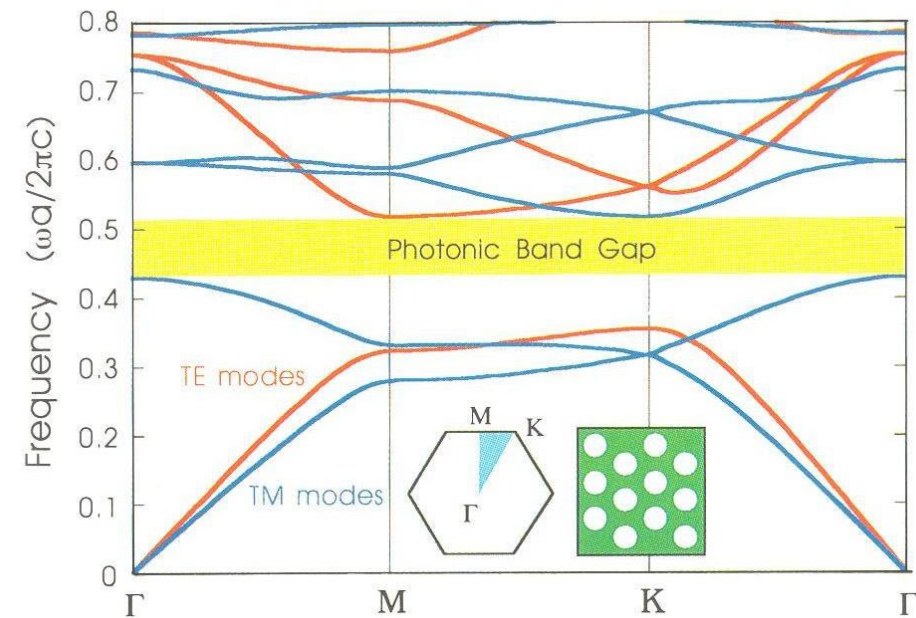
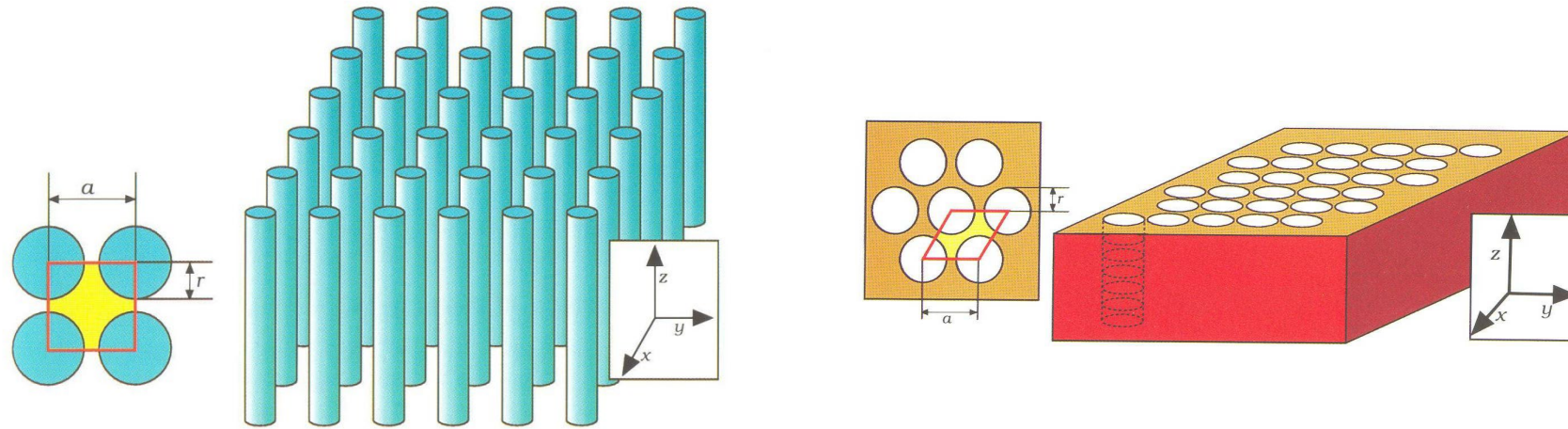
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## Week 10 – part 2

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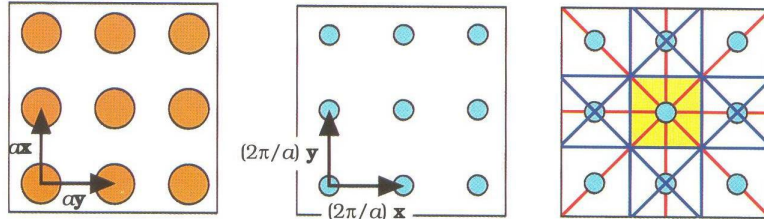


# 2D photonic crystal



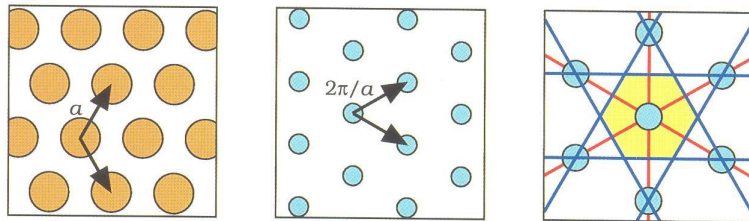
# 2D photonic crystal – Reciprocal lattice

- Square lattice:



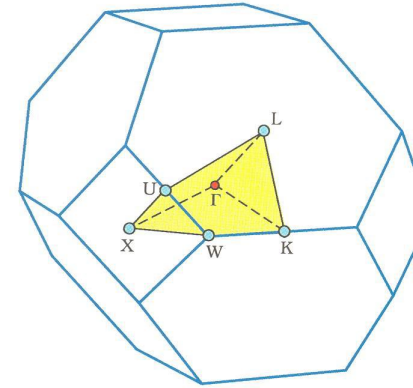
**Figure 2** The square lattice. On the left is the network of lattice points in real space. In the middle is the corresponding reciprocal lattice. On the right is the construction of the Brillouin zone: taking the center point as the origin, we draw the lines connecting the origin to other lattice points (red), their perpendicular bisectors (blue), and highlight the square boundary of the Brillouin zone (yellow).

- Triangular lattice:



**Figure 3** The triangular lattice. On the left is the network of lattice points in real space. In the middle is the corresponding reciprocal lattice, which in this case is a rotated version of the original. On the right is the Brillouin zone construction. In this case, the Brillouin zone is a hexagon centered around the origin.

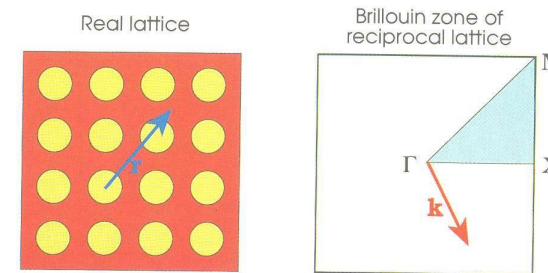
- Things are more complicated in 3D !



**Figure 4** The Brillouin zone for the face-centered cubic lattice. The reciprocal lattice is a body-centered cubic lattice, and the Brillouin zone is a truncated octahedron with center at  $\Gamma$ . Also shown are some of the names which are traditionally given to the special directions in the zone. The irreducible Brillouin zone is the yellow polyhedron with corners at  $\Gamma$ , X, U, L, W, and K.

- Irreducible Brillouin zone:

The Brillouin zone possesses the symmetries of the lattice, hence not the entire zone needs to be considered:

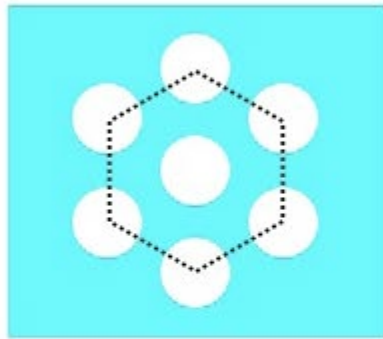


**Figure 5** Left: A photonic crystal made using a square lattice. An arbitrary vector  $\mathbf{r}$  is shown. Right: The Brillouin zone of the square lattice, centered at the origin ( $\Gamma$ ). An arbitrary wave vector  $\mathbf{k}$  is shown. The irreducible zone is the light blue triangular wedge. The special points at the center, corner, and face are conventionally known as  $\Gamma$ , M, and X.

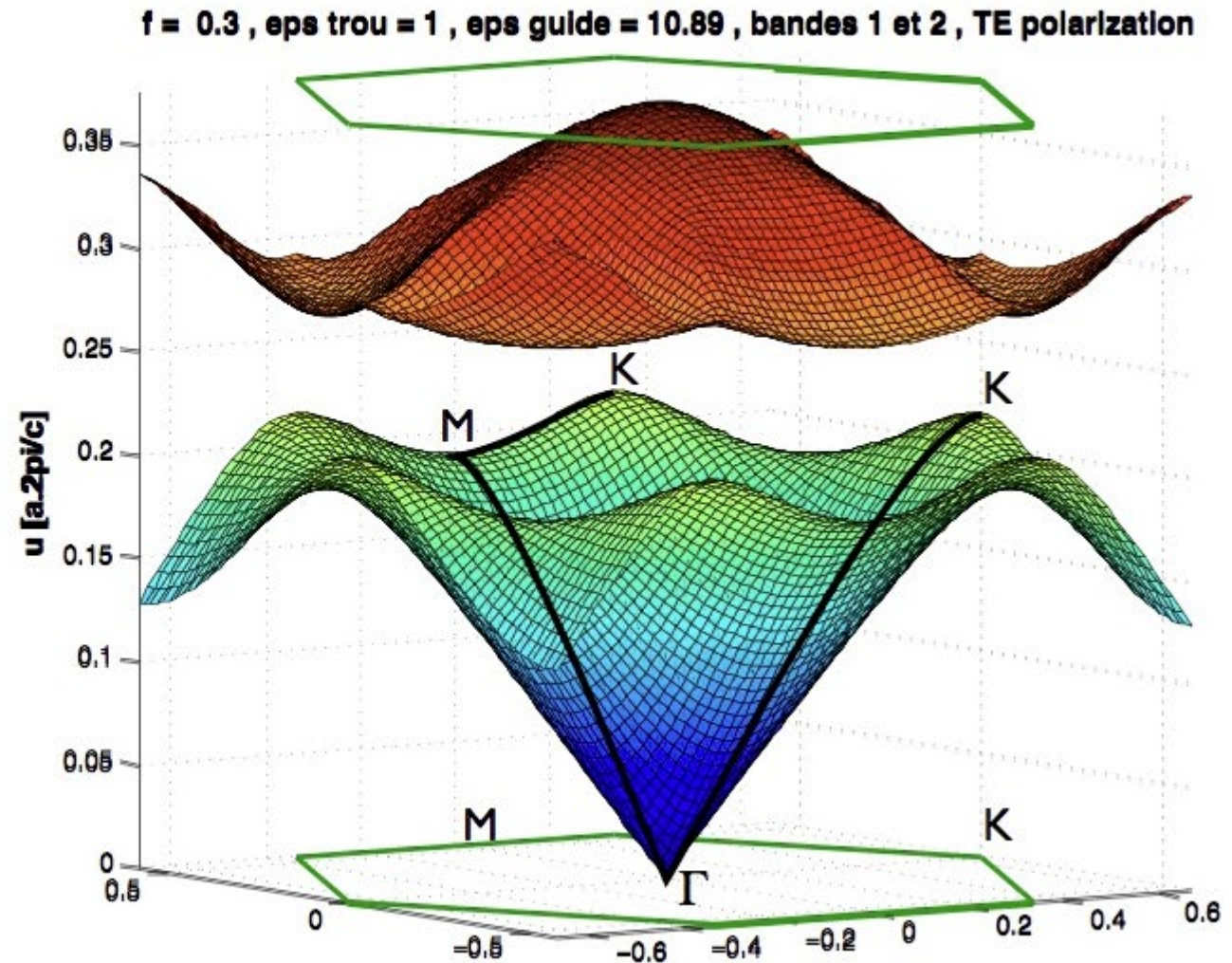
- Specific points in the Brillouin zone are given a name  $\Gamma$  (= center), X, M, etc.... They appear in the band diagram and represent waves propagating in specific directions

## 2D Photonic crystal – Band diagram

- The complete dispersion diagram represents a surface...

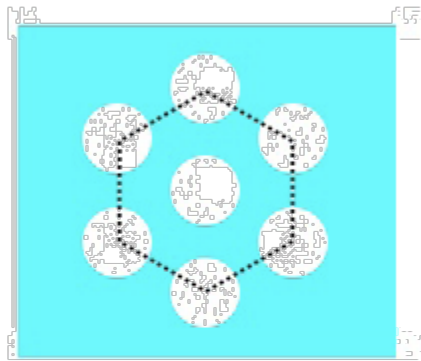


Triangular lattice of holes

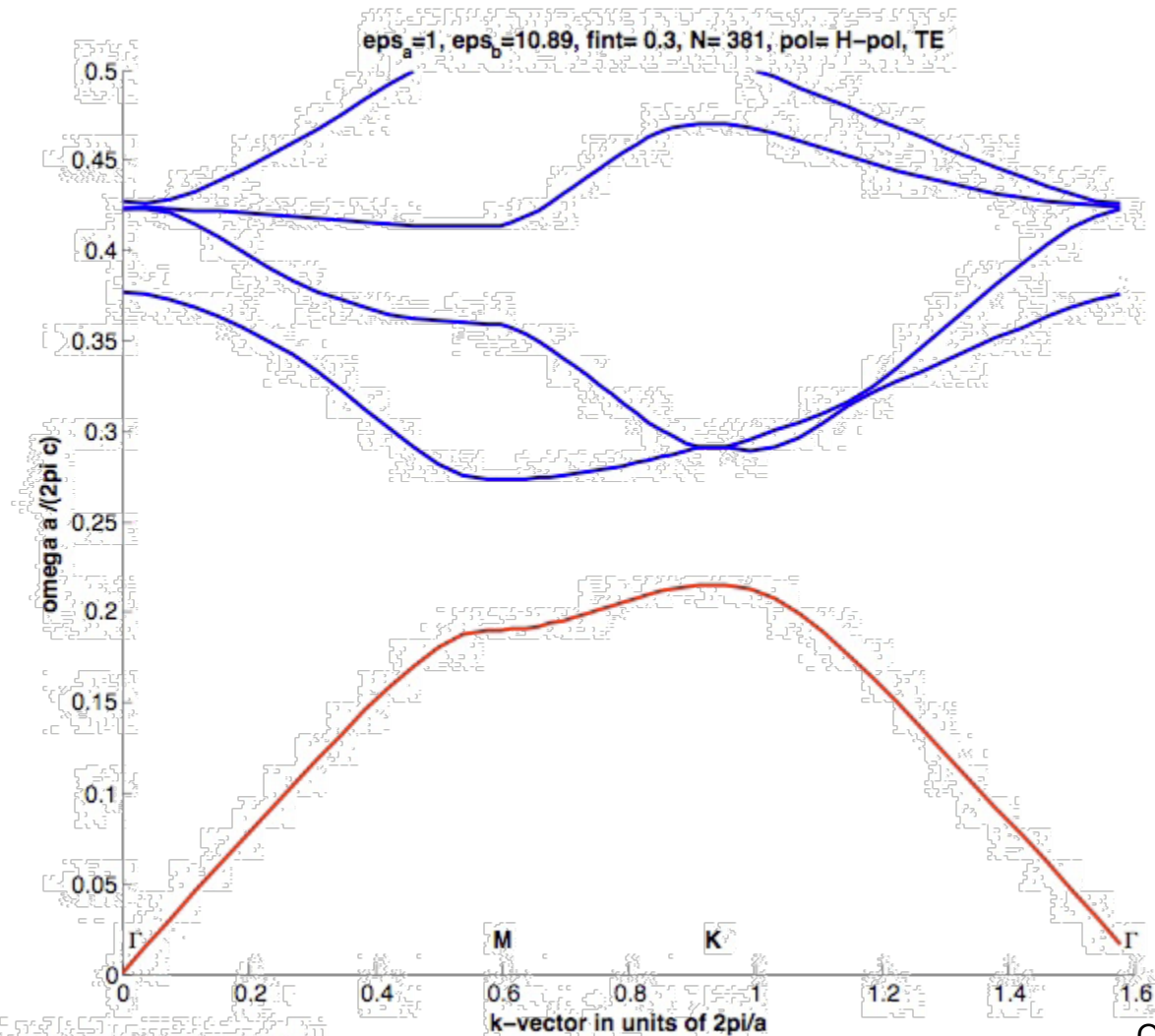


## 2D Photonic crystal – Band diagram

- ... but only specific directions are usually represented

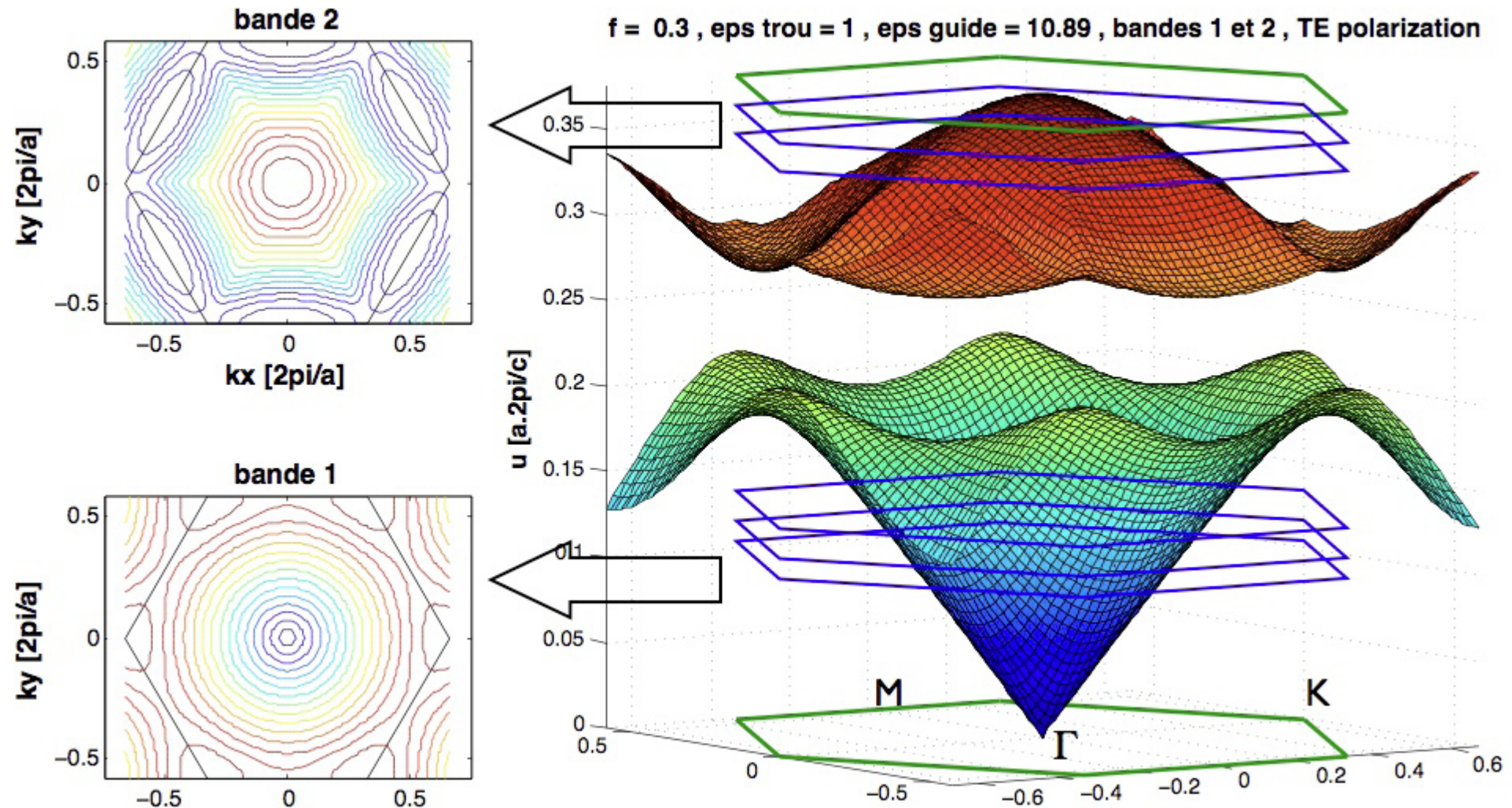


Triangular lattice of holes



## 2D Photonic crystal – Band diagram

- Another representation is equifrequency curves/surfaces:



# Selected Topics in Advanced Optics

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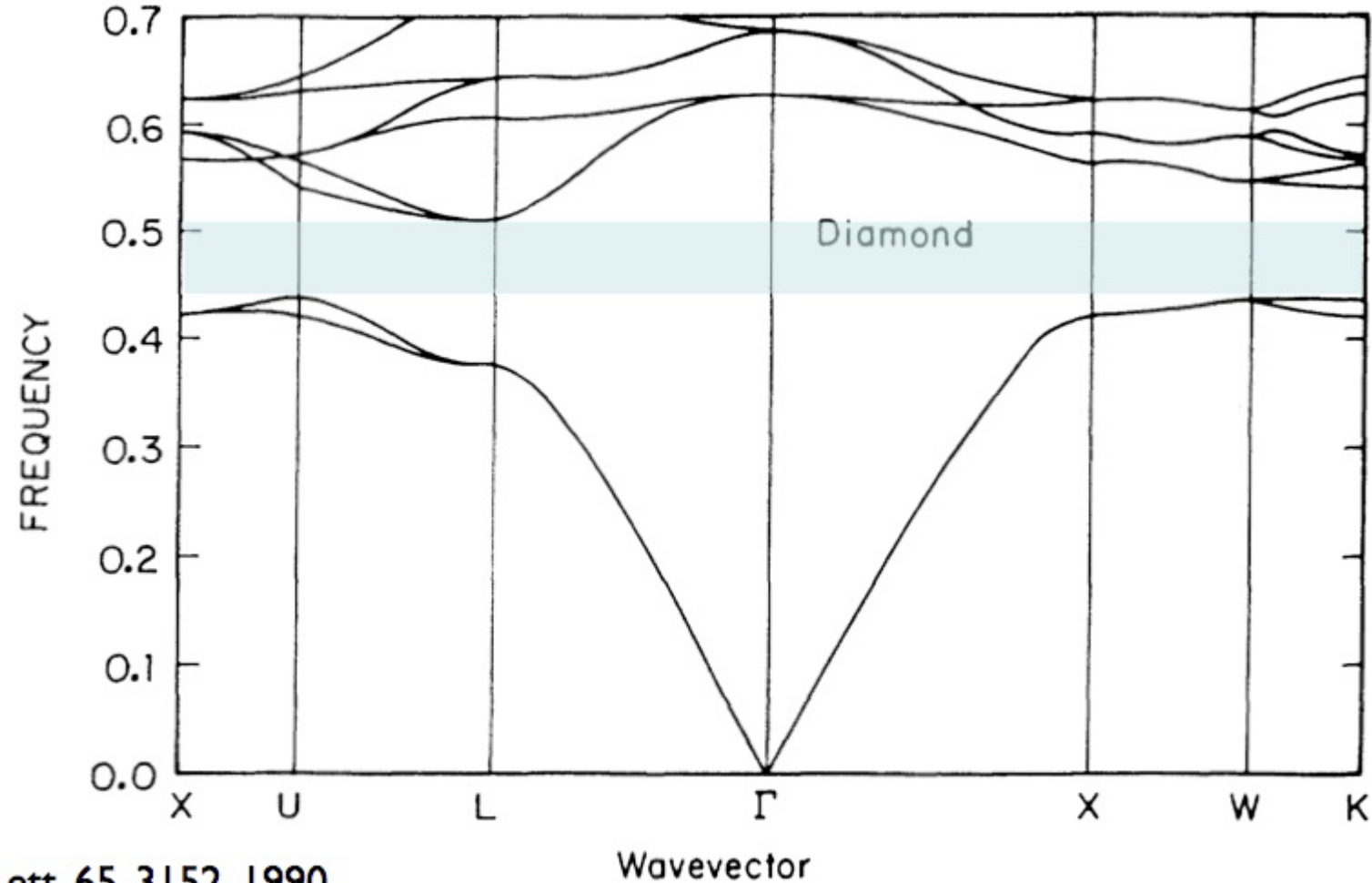
## Week 10 – part 3

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## Example of full bandgap

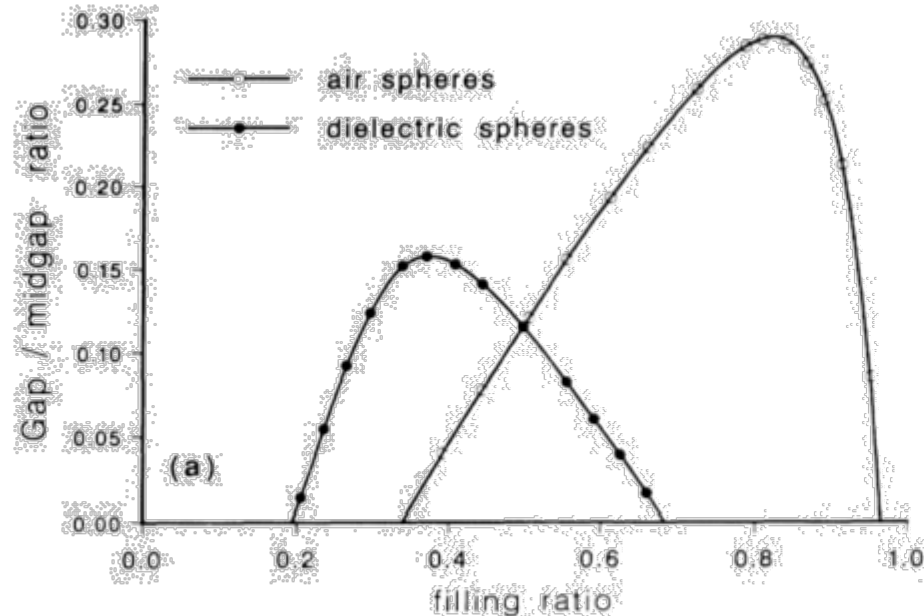
- Dielectric spheres on a diamond lattice



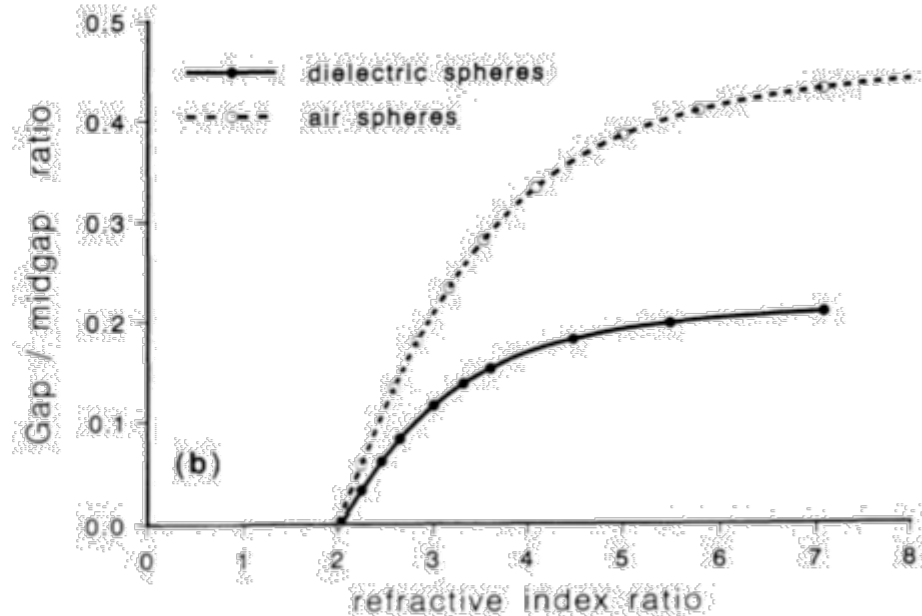
Phys. Rev. Lett. 65, 3152, 1990

## Example of full bandgap

- Dielectric spheres on a diamond lattice
- The existence of a gap depends on the filling factor and on the dielectric contrast between the scatterers and the background



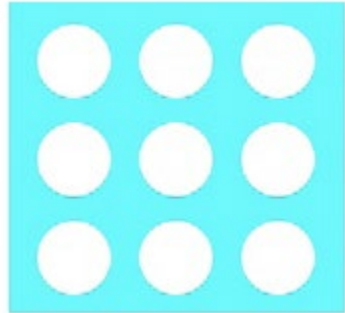
Threshold on filling factor



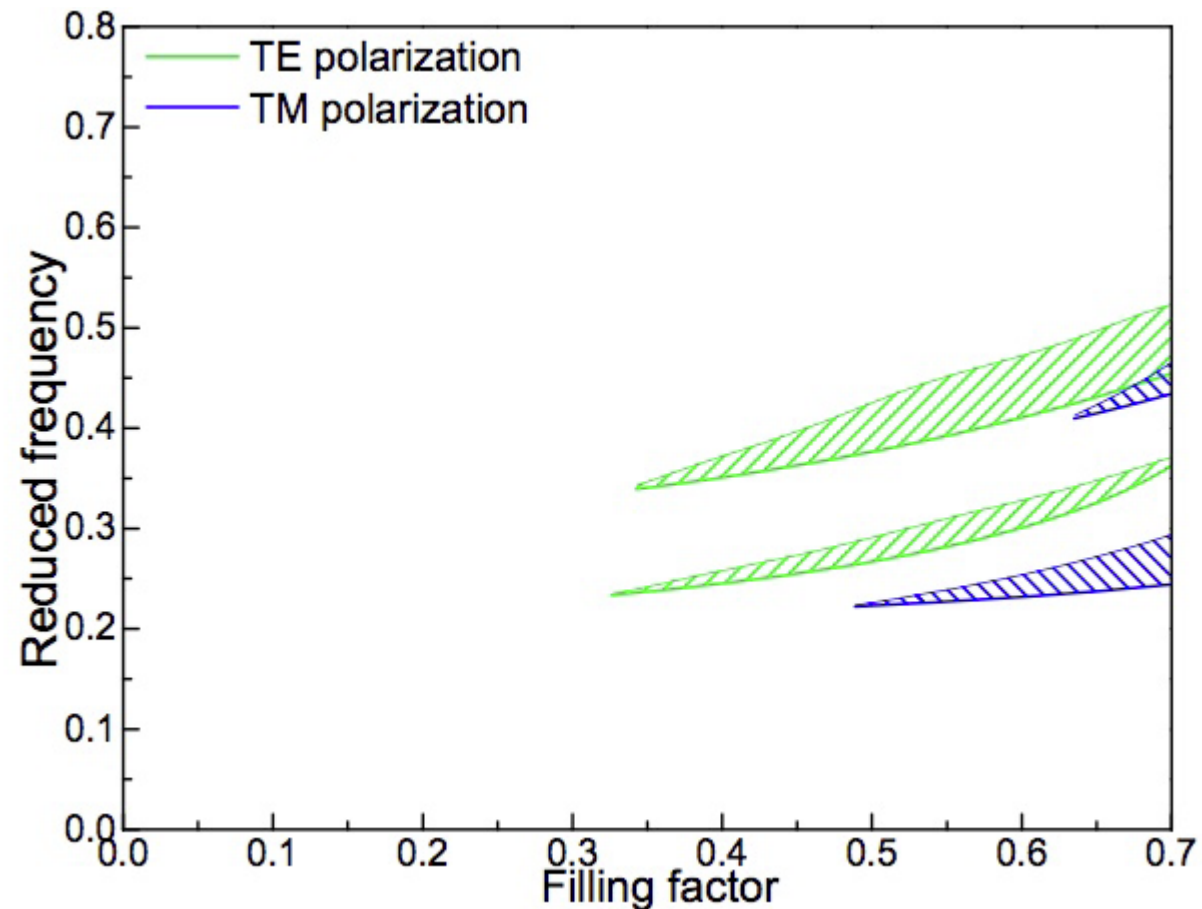
Threshold on index contrast

## Some common 2D lattices

- Square lattice of holes



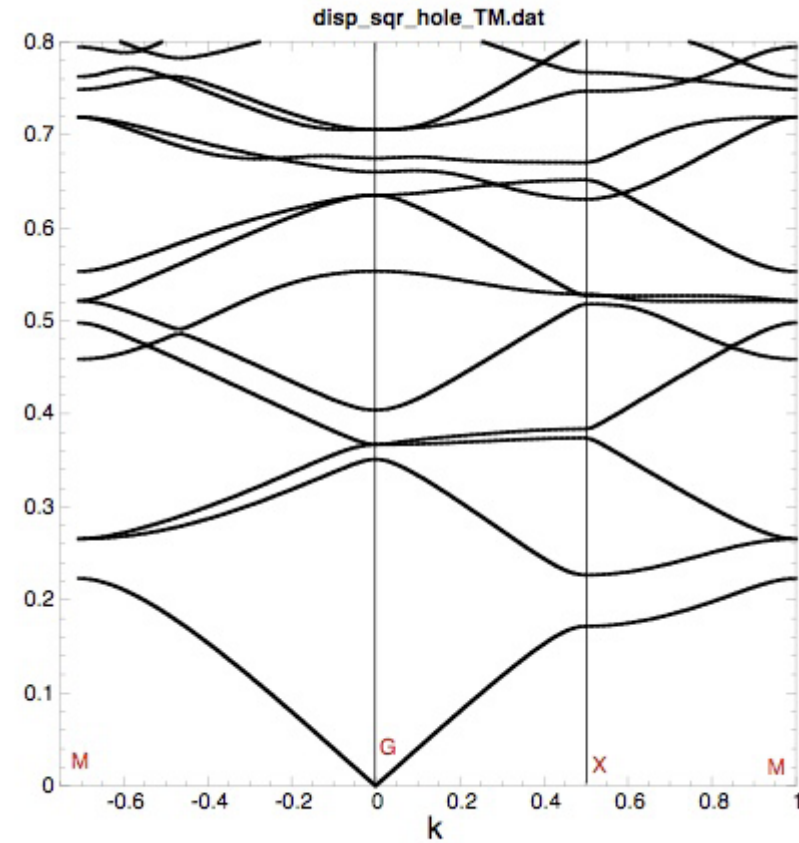
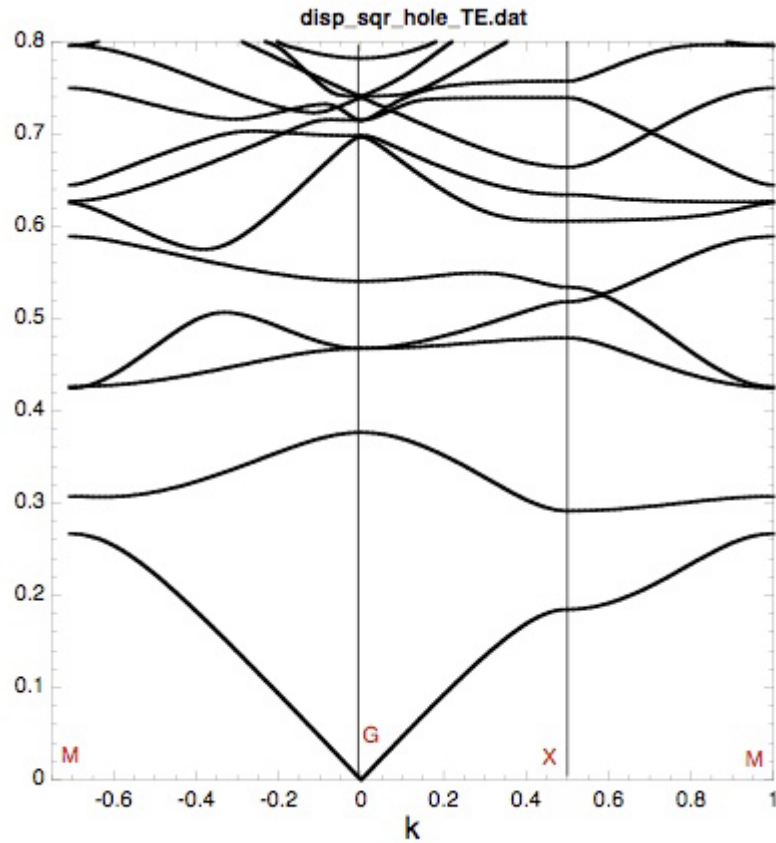
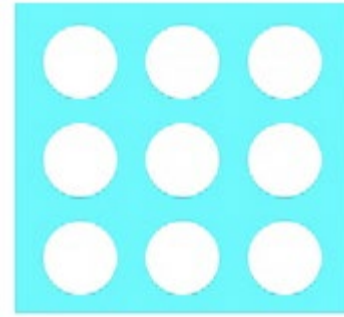
filling factor =  $f_{\text{air}}$   
 $n=3.36$



## Some common 2D lattices

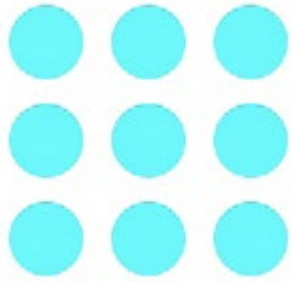
- Square lattice of holes

filling factor =  $f_{\text{air}} = 50\%$   
 $n=3.36$

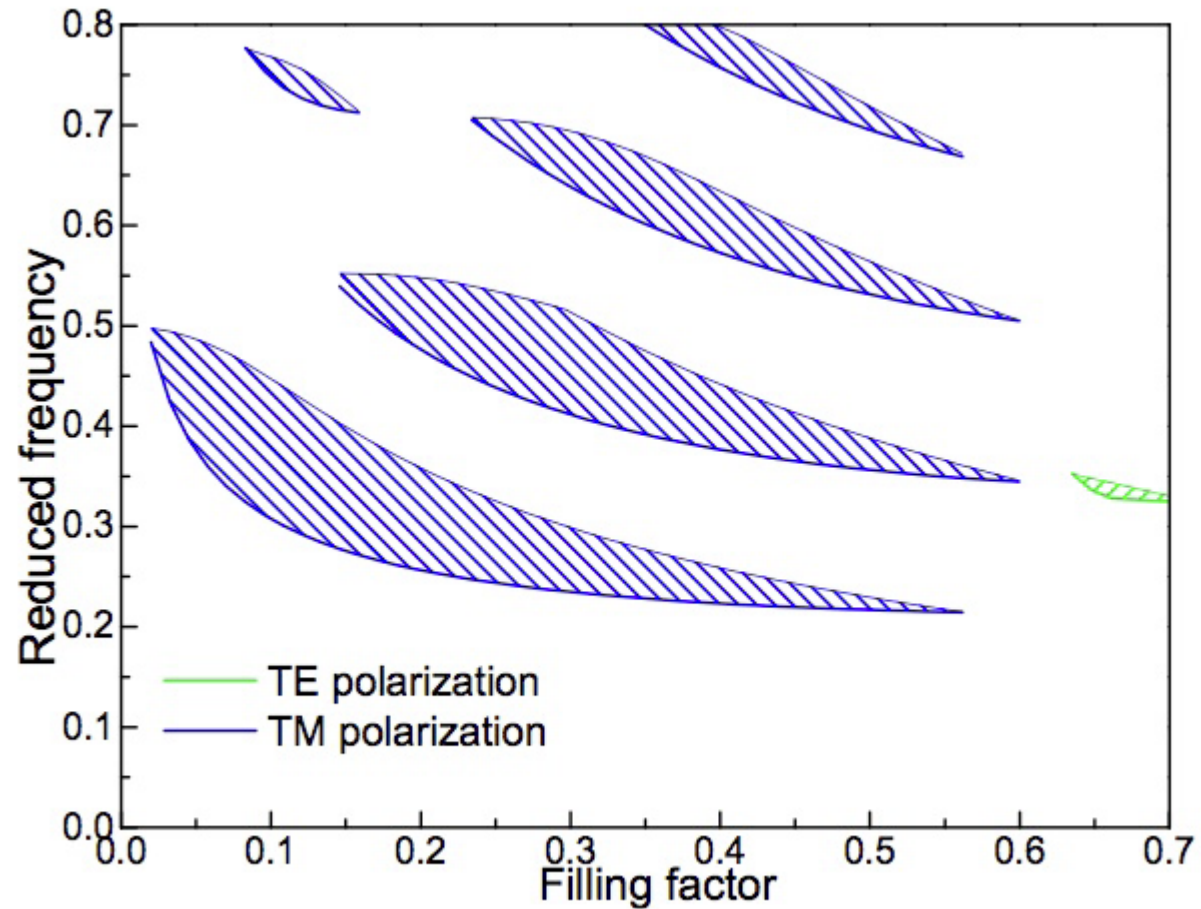


## Some common 2D lattices

- Square lattice of pillars



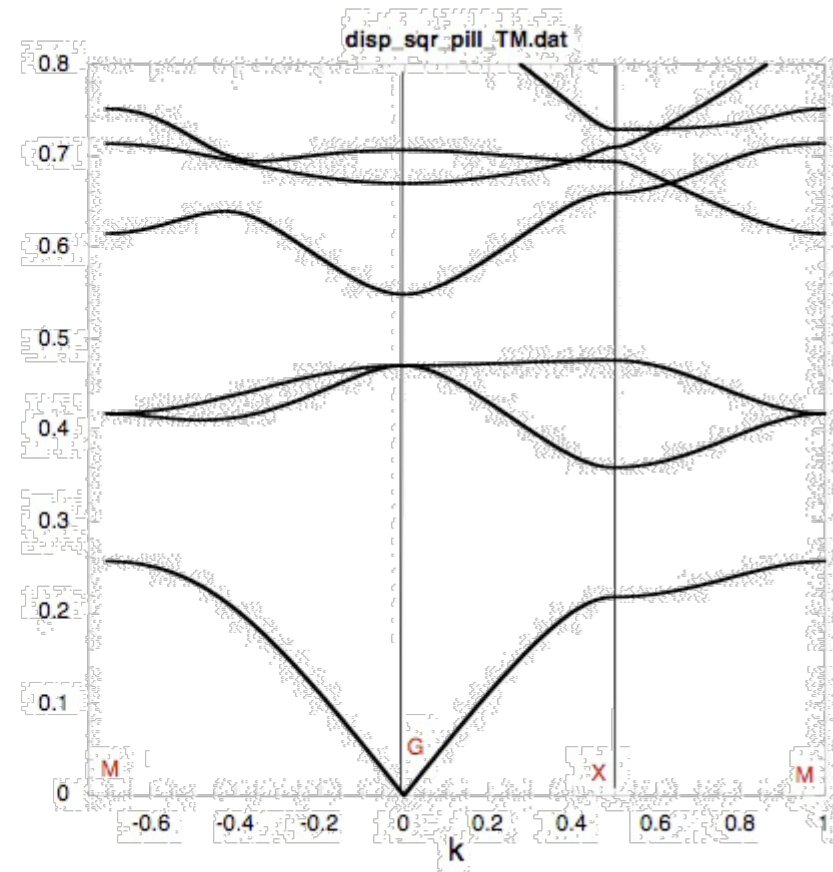
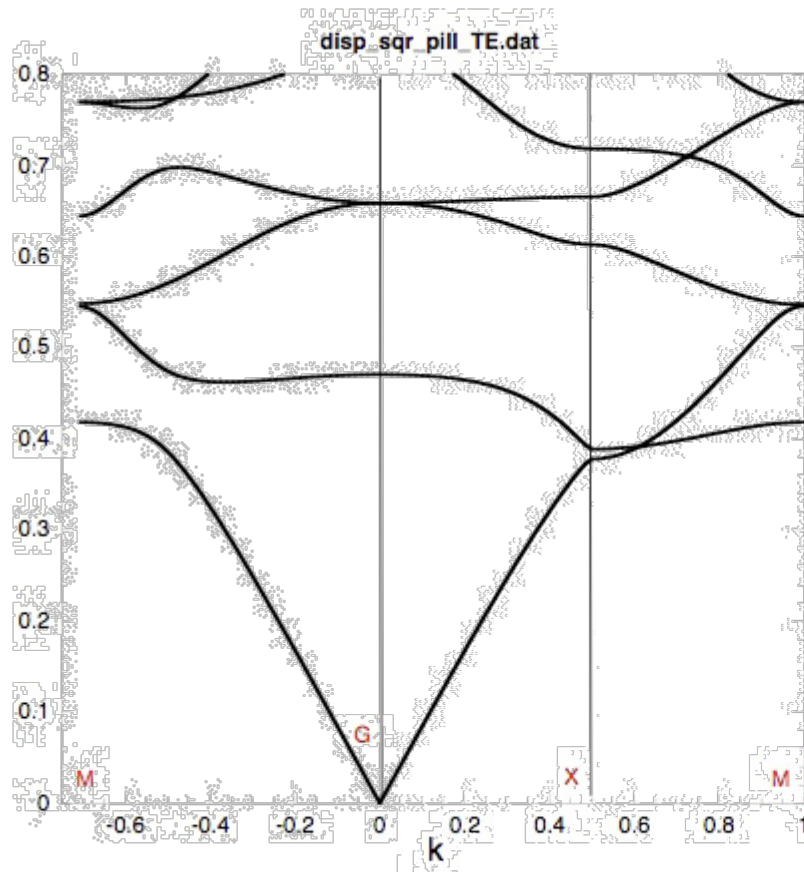
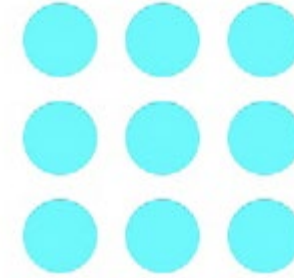
filling factor =  $f_{\text{diel}}$   
 $n=3.36$



# Some common 2D lattices

- Square lattice of pillars

filling factor =  $f_{\text{diel}} = 20\%$   
 $n=3.36$



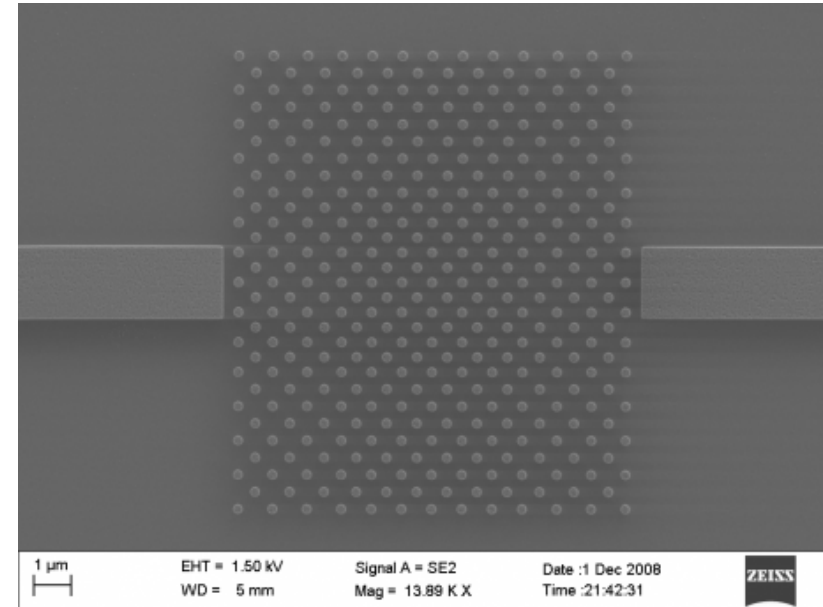
## Scaling laws

- Since the phenomenon behind photonic bands is interference, for a specific dielectric contrast, a given band structure can be produced for different frequencies by scaling the dimensions:

$$\text{Energy : } u = \frac{a}{\lambda} = \frac{\omega a}{2\pi c} \quad \text{Wave vector : } \tilde{k} = \frac{ka}{2\pi} \quad a : \text{period}$$



Microwave frequencies



Optical frequencies

## Band structure

- Always shown in the reciprocal (Fourier) space and limited to the first Brillouin zone
- The forbidden bands in different directions usually do not overlap (i.e. the crystal properties are different for waves propagating in different directions)
- The forbidden bands for both polarizations do not usually overlap (i.e. the crystal properties are different for waves propagating in the same direction, but with different polarizations)
- A full bandgap (for any polarization and propagation direction) rarely exists; it only appears for strong dielectric contrasts (i.e. strong scattering)

# Selected Topics in Advanced Optics

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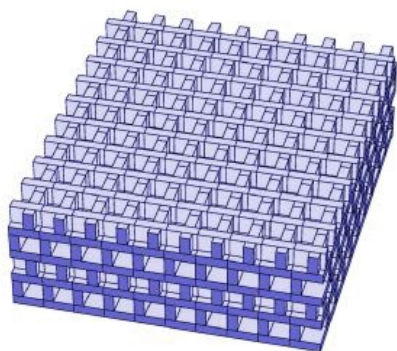
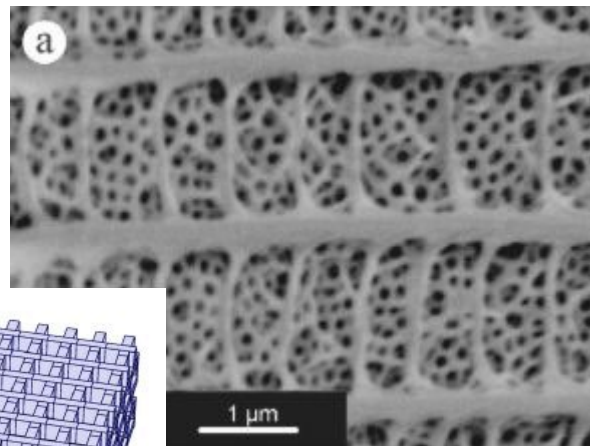
## Week 10 – part 4

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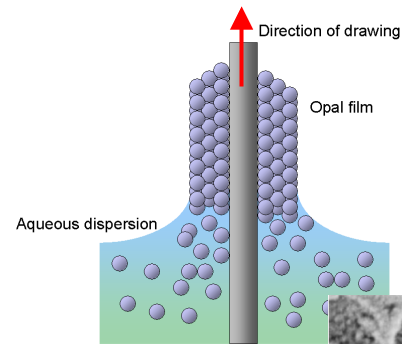
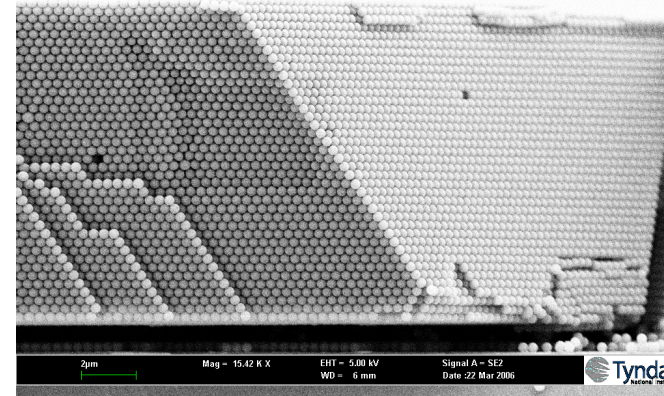
# 3D photonic crystals

- The most striking 3D photonic crystals exist in nature:



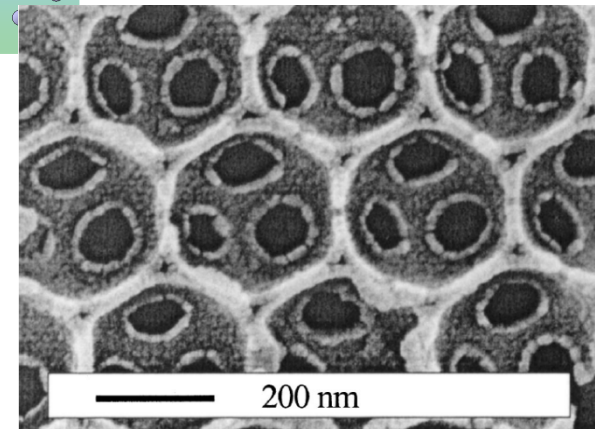
Butterflies

- Or are being replicated in the laboratory:



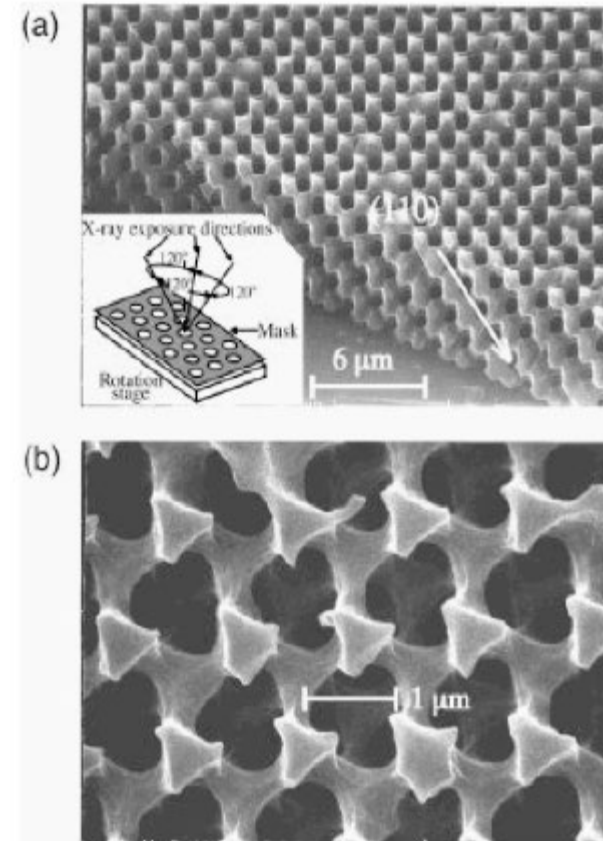
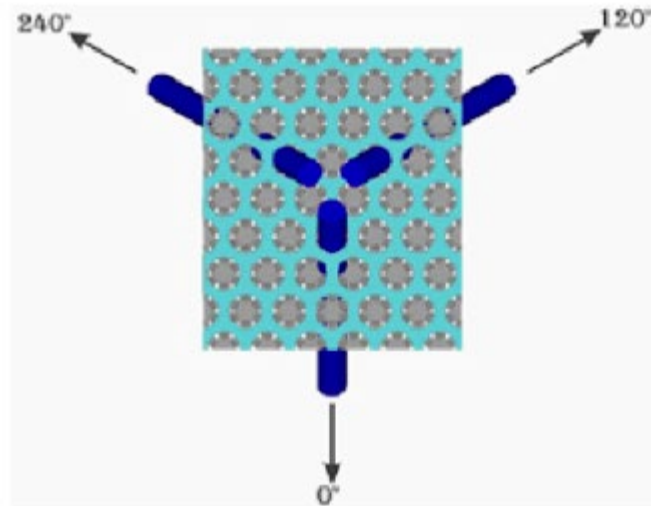
Opal structures  
(Tyndall Lab):

Inverted opal (TiO<sub>2</sub>):



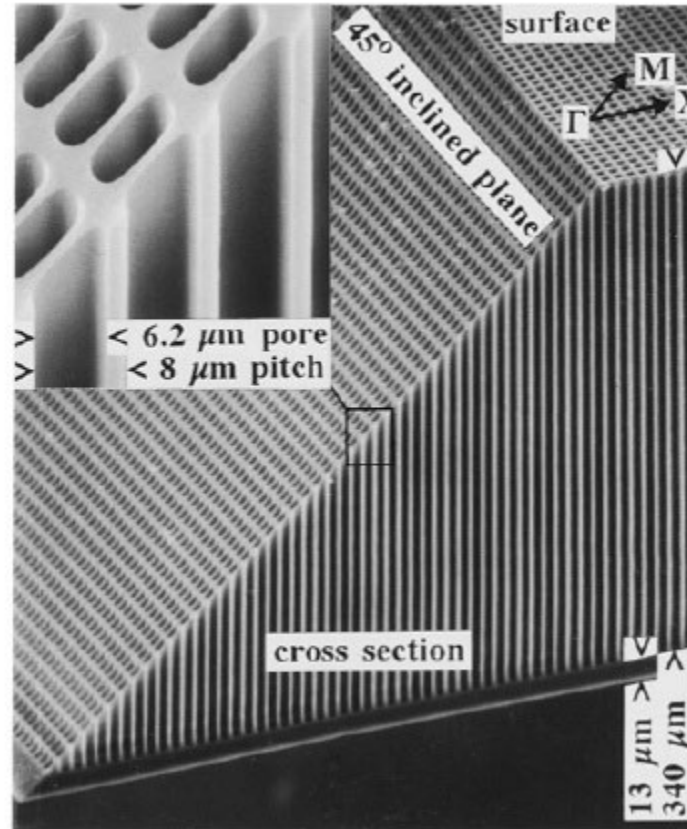
# Examples of 3D photonic crystals and their fabrication

- Yablonovite (etching)



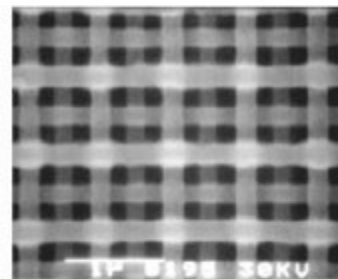
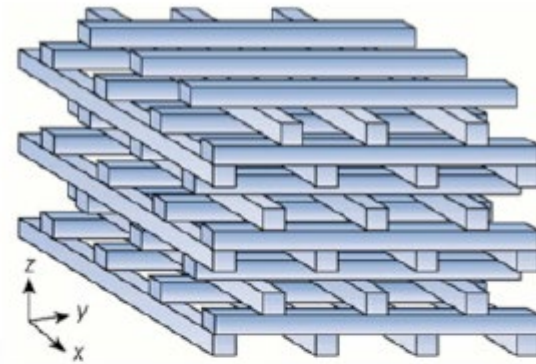
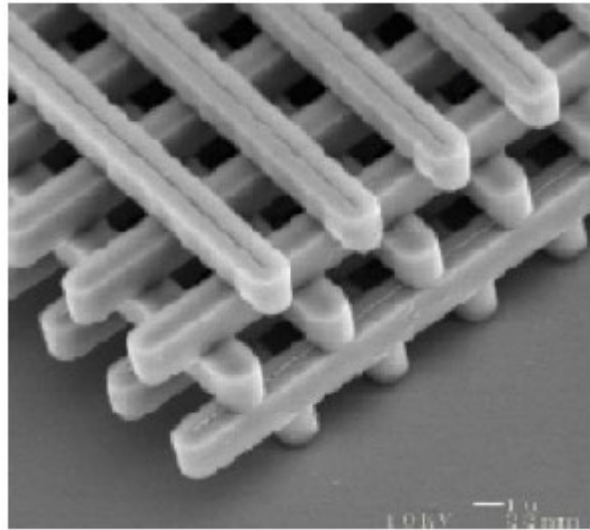
## Examples of 3D photonic crystals and their fabrication

- Very sophisticated deep etching techniques have been developed to realize photonic crystals

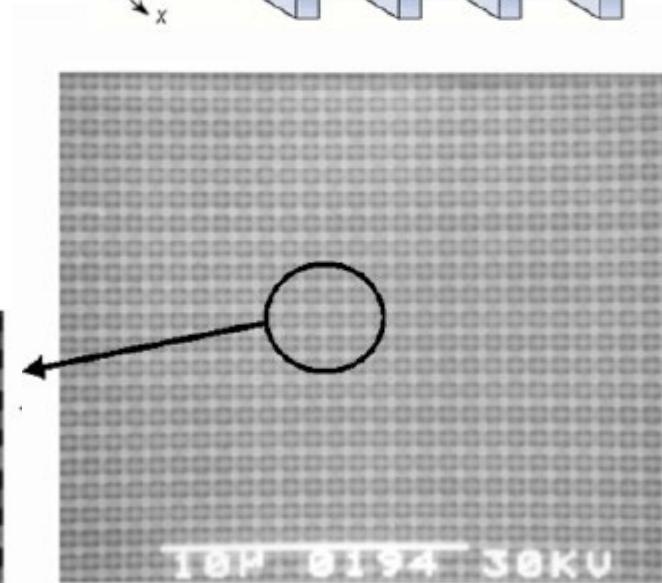


# Examples of 3D photonic crystals and their fabrication

- Wood pile (microfabrication)



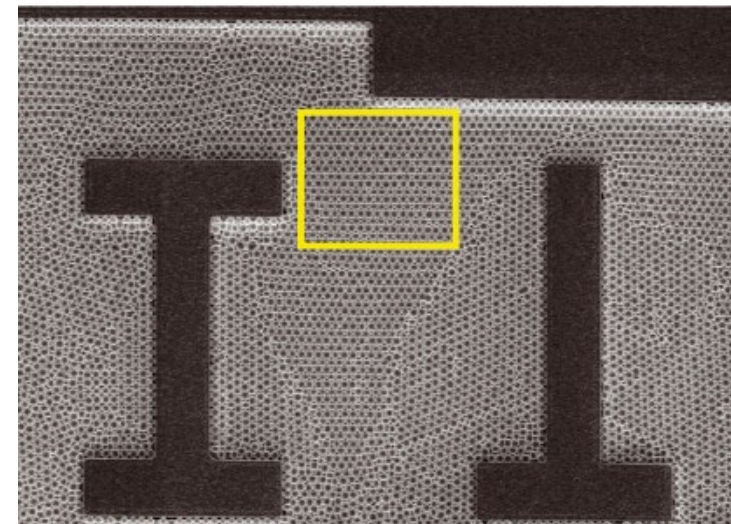
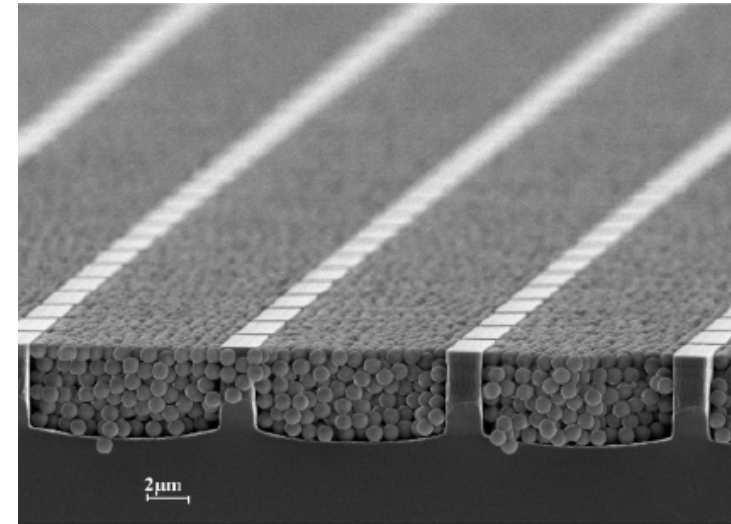
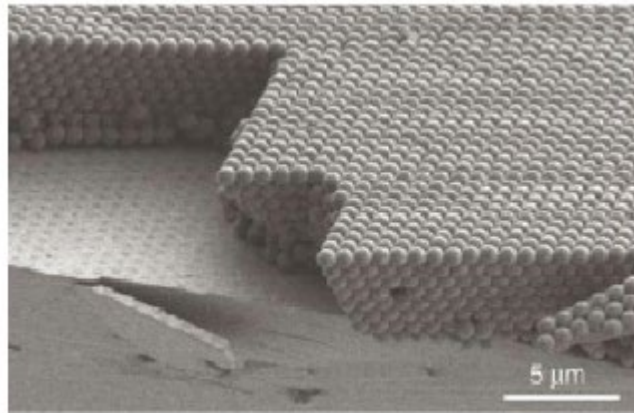
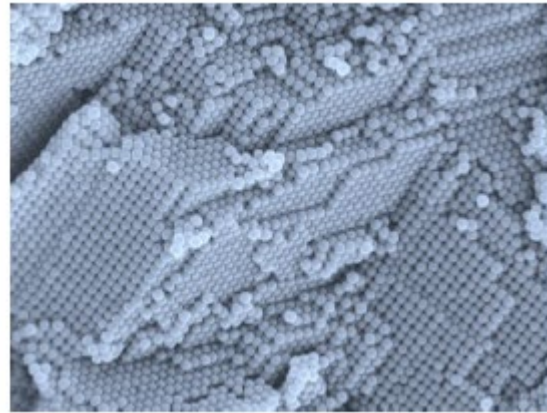
0.7 $\mu\text{m}$



10 $\mu\text{m}$

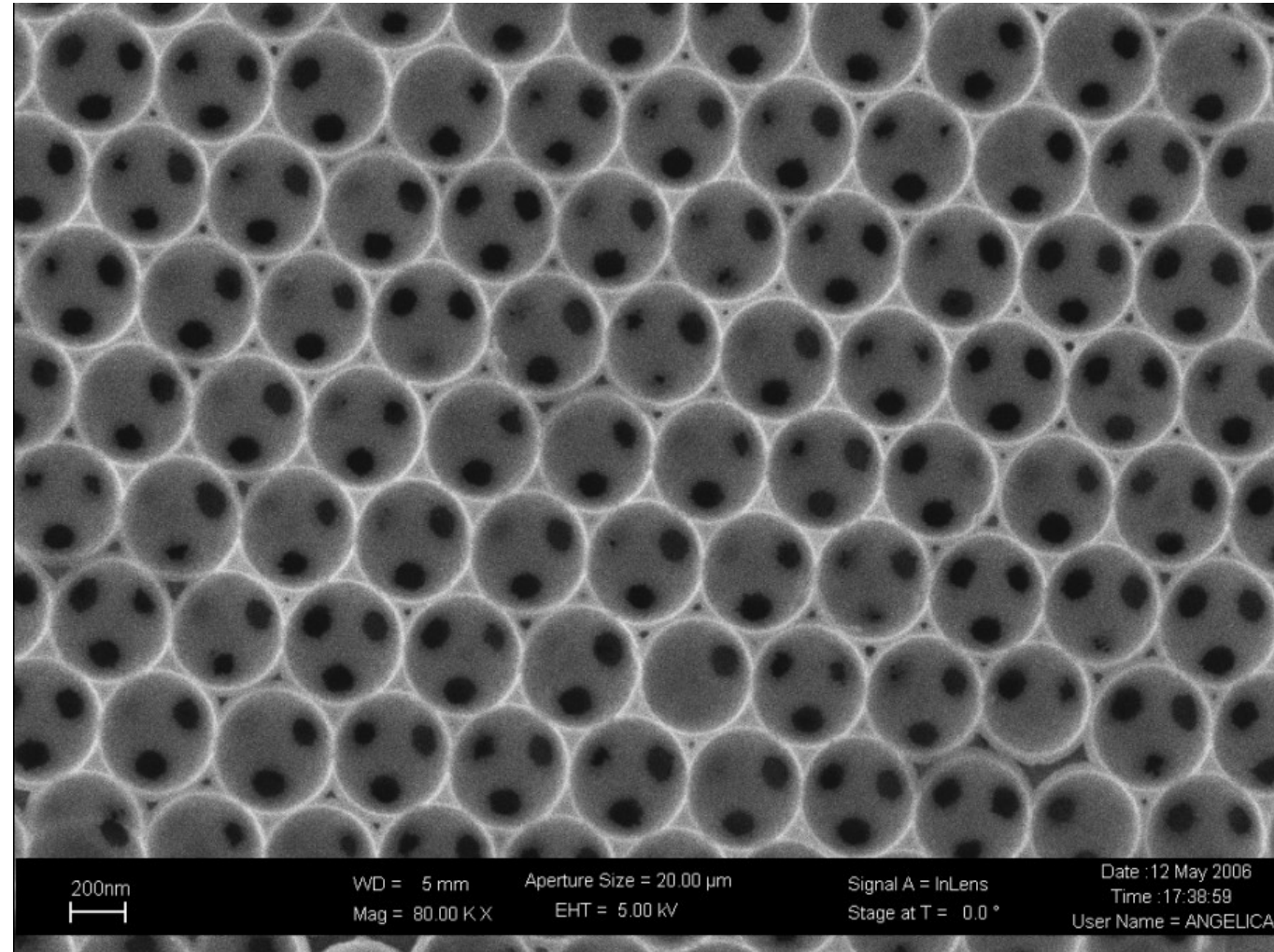
# Examples of 3D photonic crystals and their fabrication

- Opals (Silica or Latex self-assembly)



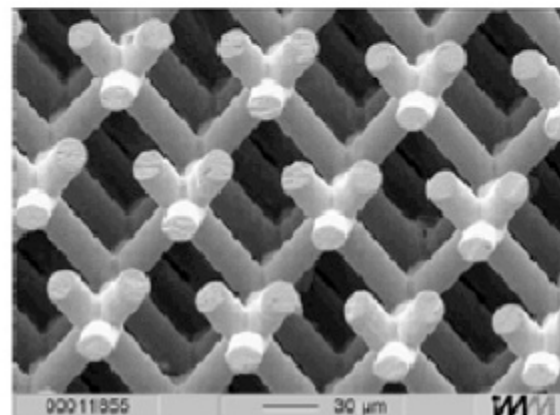
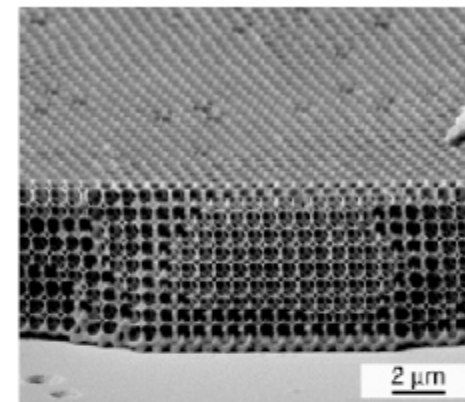
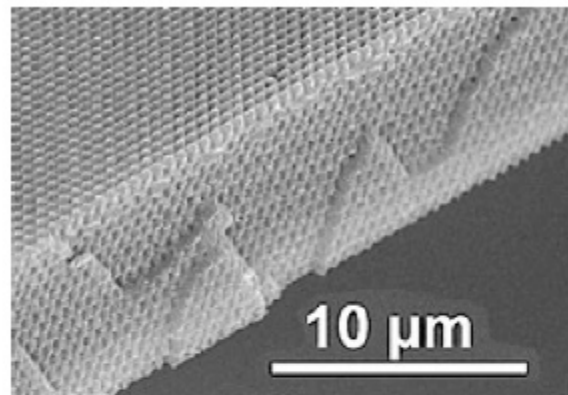
## Different examples of photonic crystals and their fabrication

- Inverted opals (infiltration + selective etching)



# Examples of 3D photonic crystals and their fabrication

- Holography



# Selected Topics in Advanced Optics

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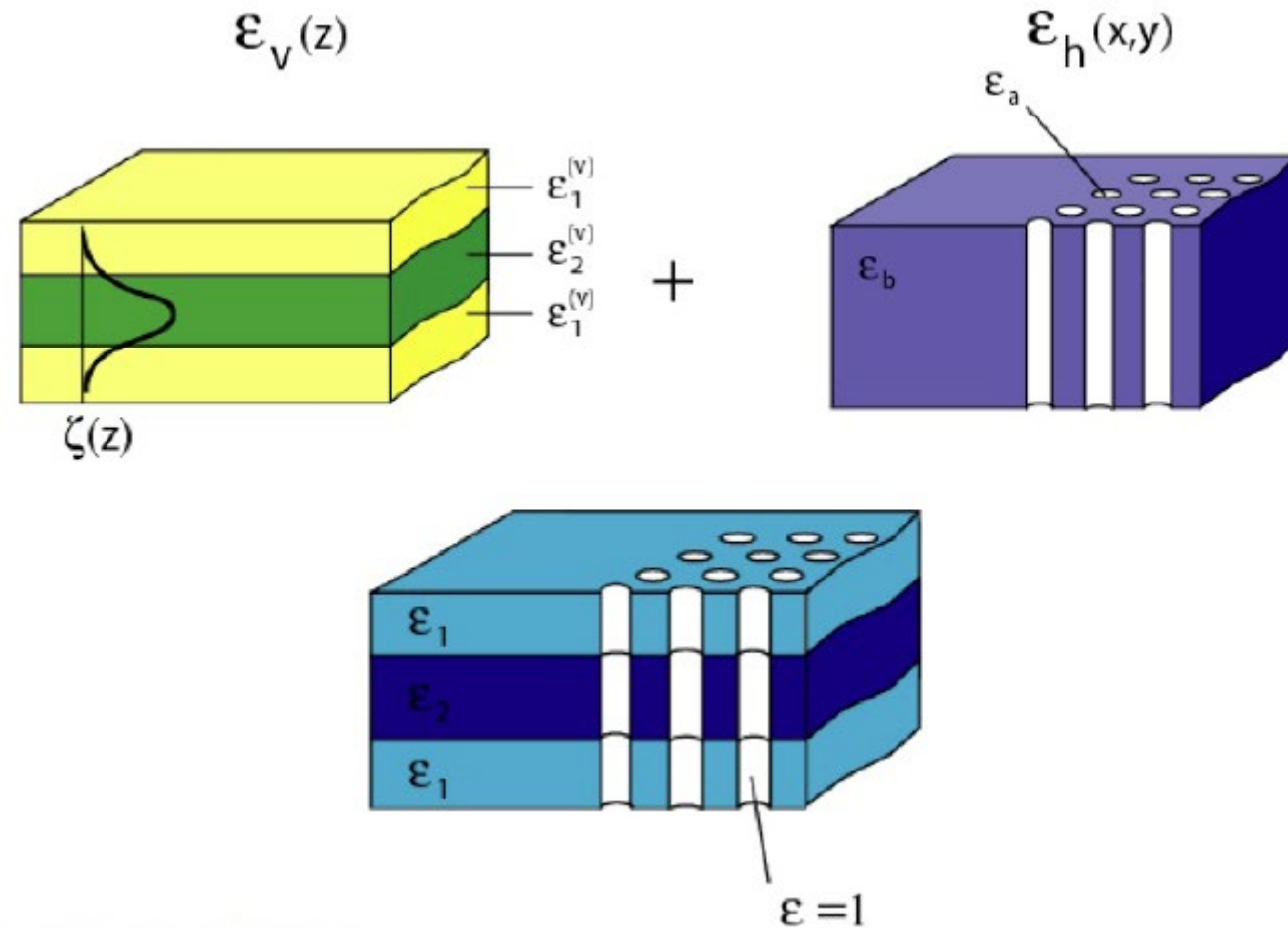
## Week 10 – part 5

Olivier J.F. Martin  
Nanophotonics and Metrology Laboratory

**EPFL**

## Basic photonic crystal components

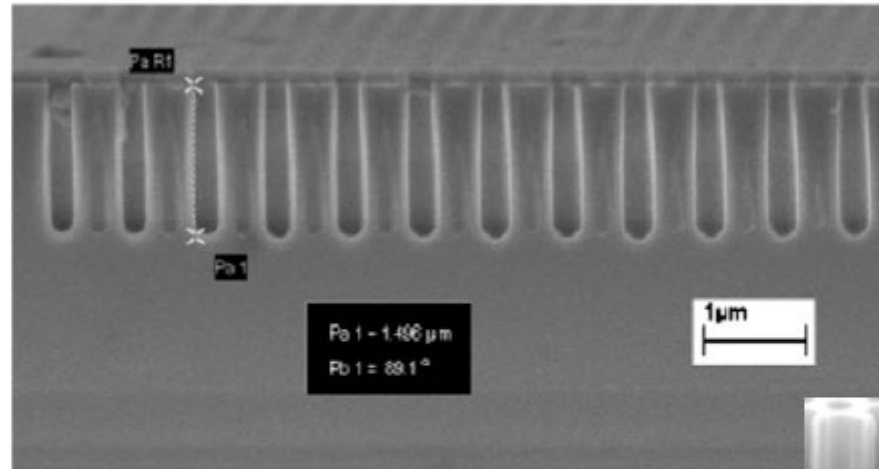
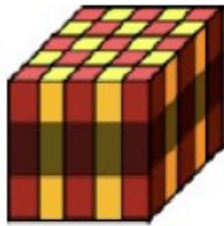
- 2D patterning in planar waveguide (the waveguide provides vertical confinement, the photonic crystal lateral functions)



# Basic photonic crystal components

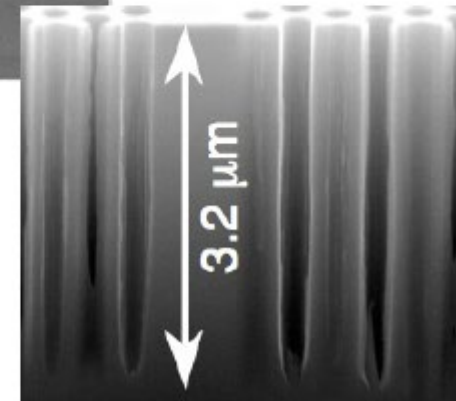
- 2D patterning in planar waveguide

"2+1" D



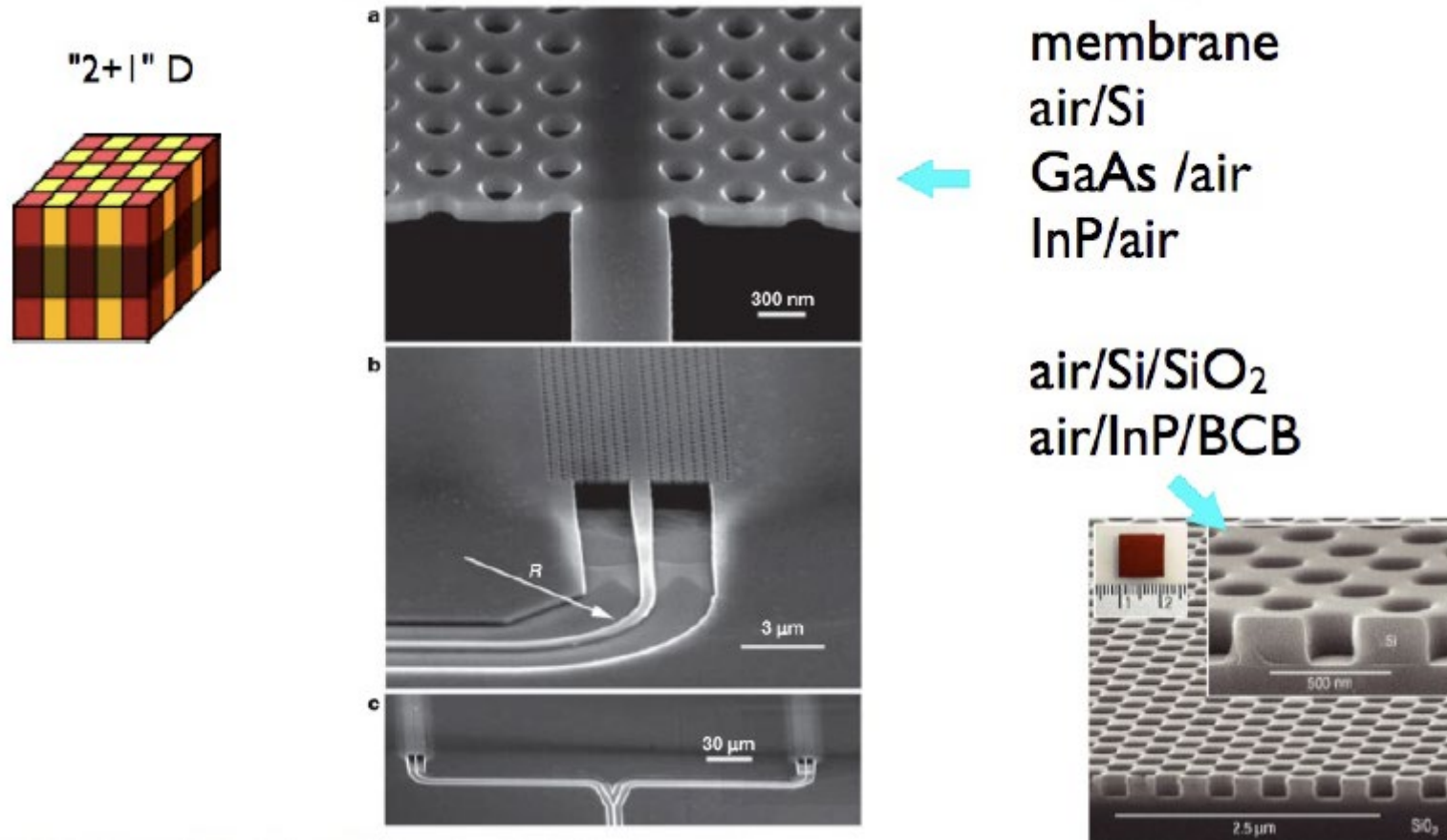
InP /  $\text{Ga}_{1-x}\text{As}_x\text{In}_x\text{P}_{1-x-y}$

GaAs /  $\text{Al}_x\text{Ga}_{1-x}\text{As}$



# Basic photonic crystal components

- Suspended membranes can prevent such leakage by providing high dielectric contrast in the vertical direction

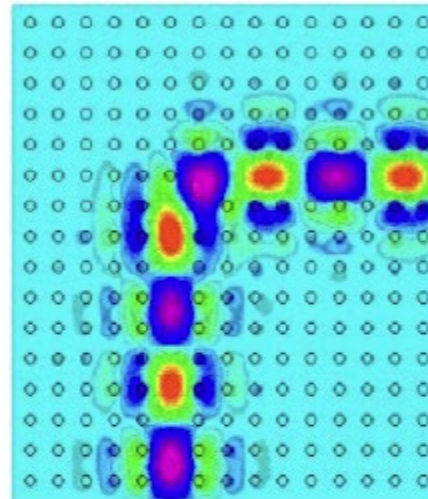


## Defects in infinite periodic lattices

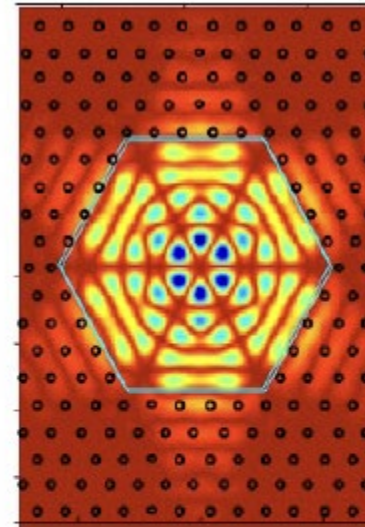
- Like in semiconductors, most interesting effects arise from defects in the periodic lattice
- These defects build “confined states” within the band structure



Bi-dimensional defect  
Planar cavity



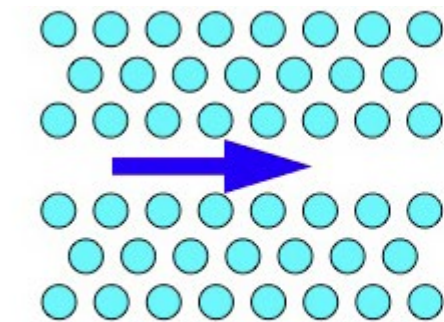
One-dimensional defect  
Wave guide



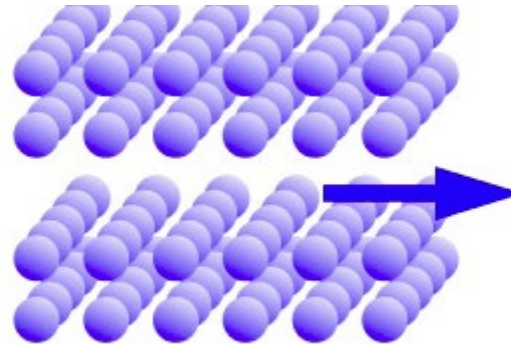
Point defect  
Optical cavity

# Waveguides

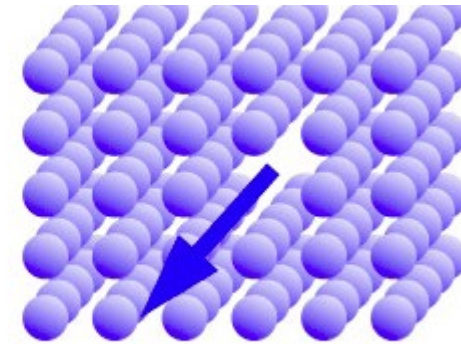
- Extended defect in one specific direction
- The propagation vector can be defined in that direction
- Waveguides with different dimensionalities exist:



1D in a 2D world



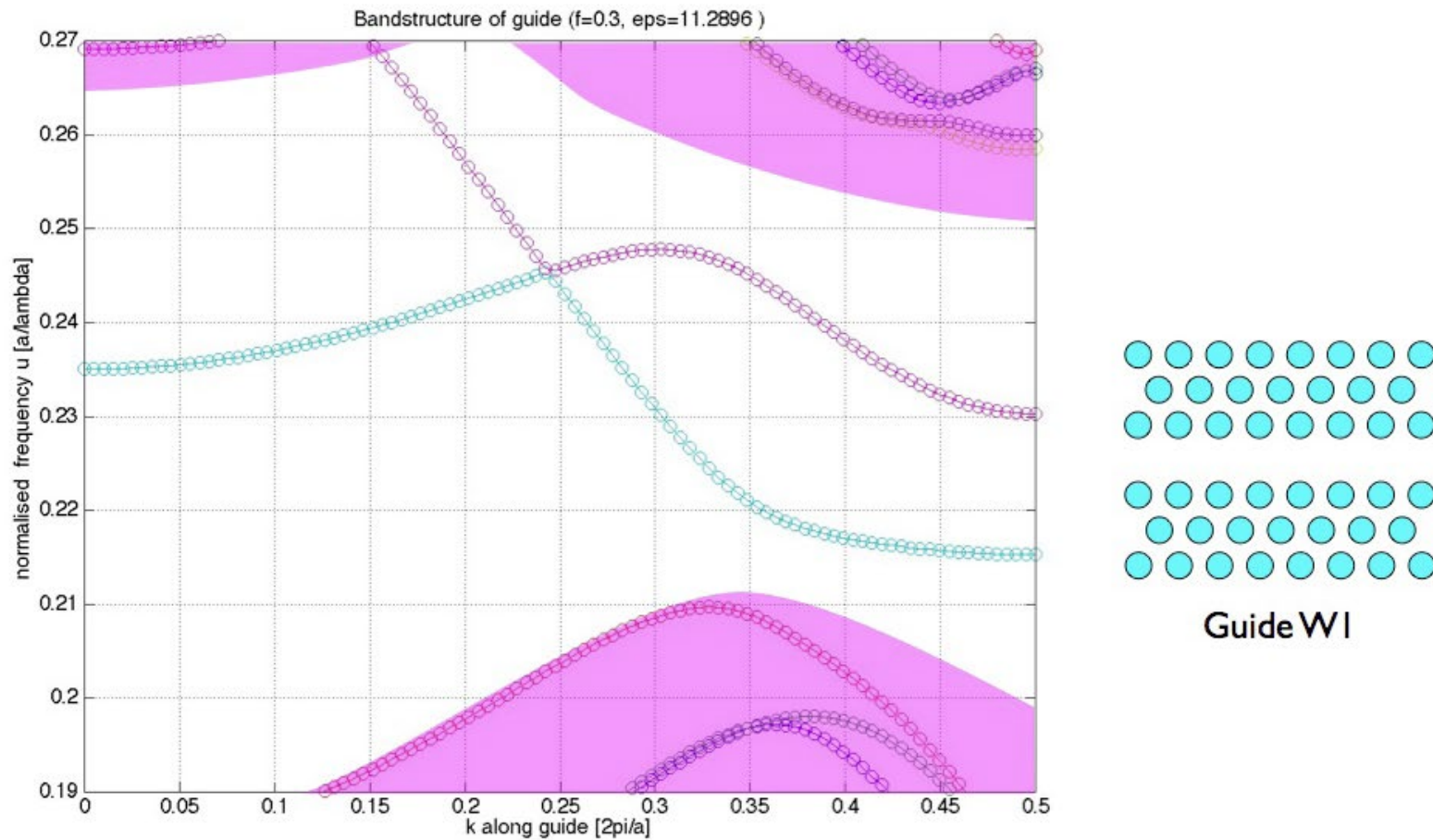
2D in a 3D world



1D in a 3D world

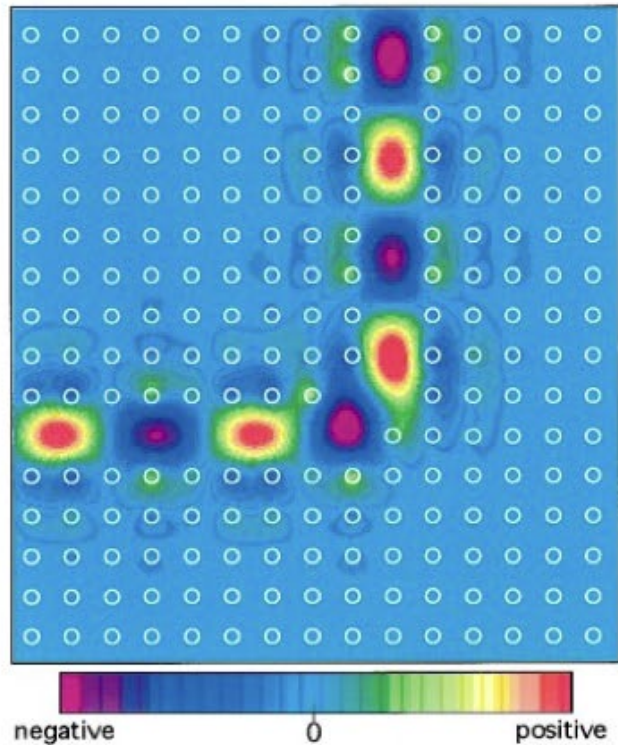
# Waveguides

- The defect (e.g. Missing row(s)) creates a mini-band inside the bandgap, where optical modes can exist

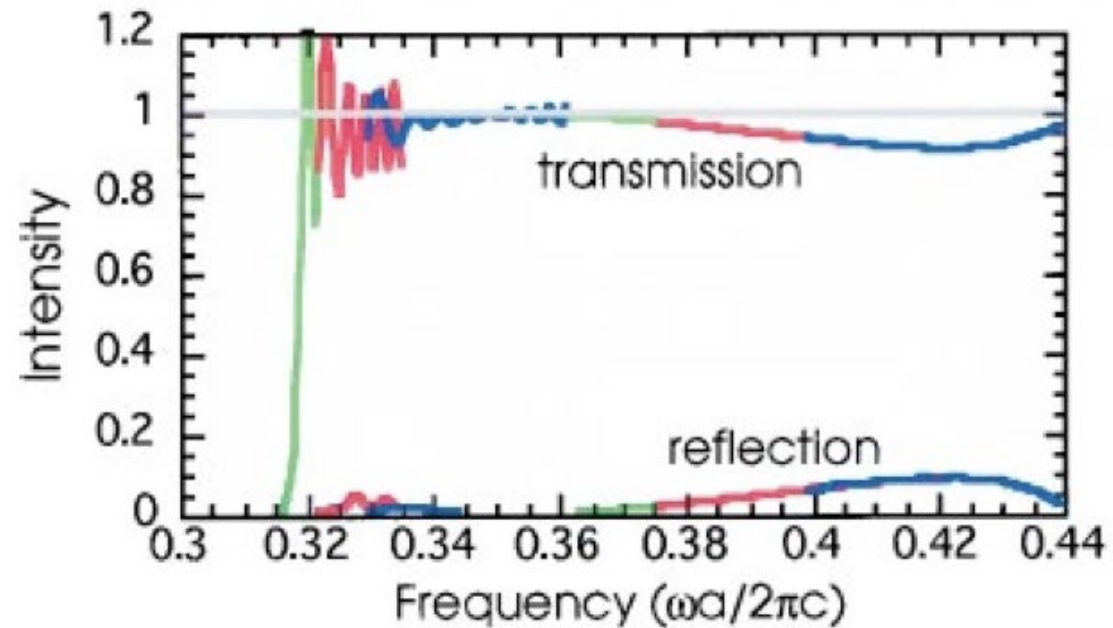


## Waveguides and bends

- Since we operate inside the bandgap, losses are very small, even for very small bending radii
- This cannot be achieved with conventional waveguides



Square lattice of holes



$T \approx 1$  within a reduced frequency range

# Waveguides and bends

- Optimization of the structures using sophisticated algorithms (e.g. genetic optimization) can produce complex optical circuits

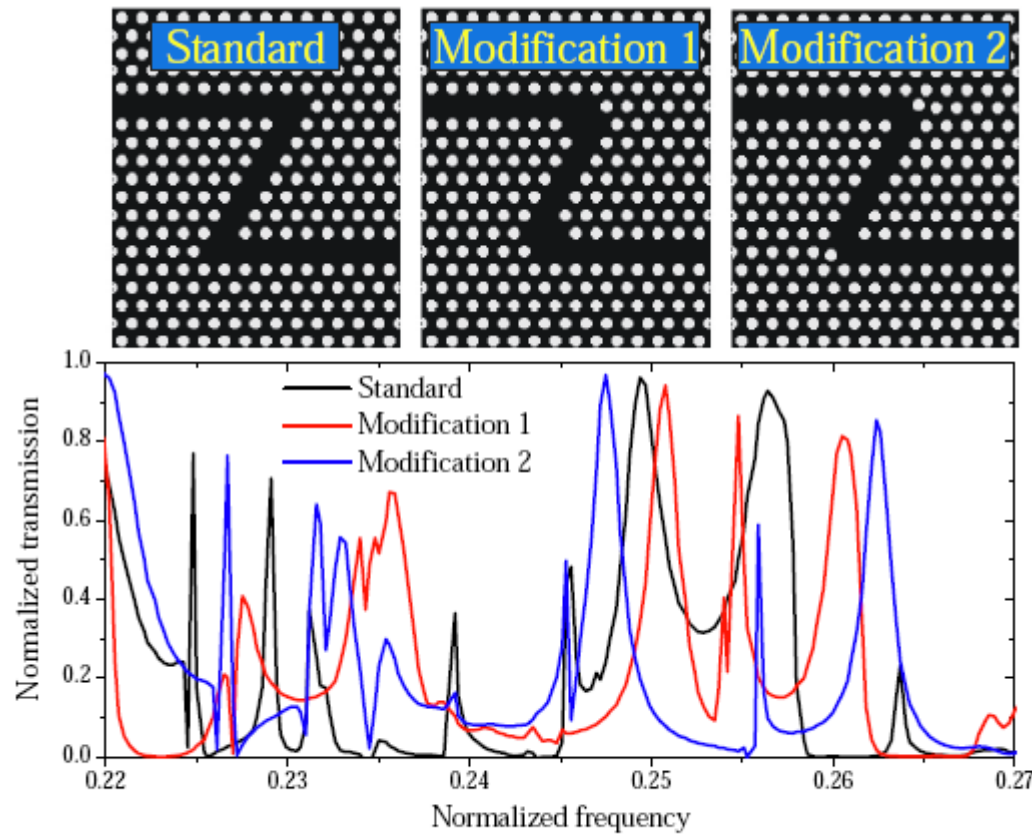


Fig. 1. Top: Standard and two modified Z-bend waveguides. Bottom: Transmission through the bends calculated using a 2D frequency domain finite element model.

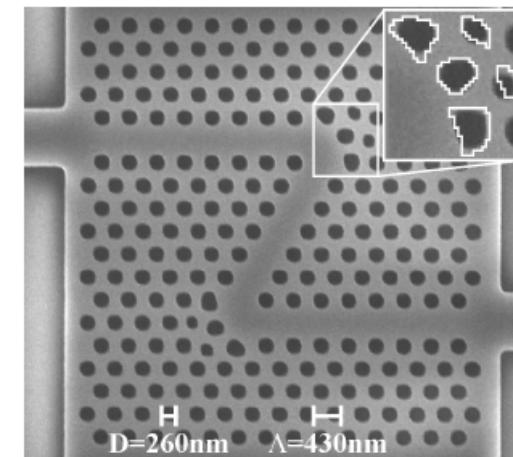
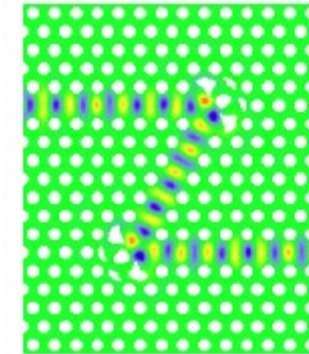
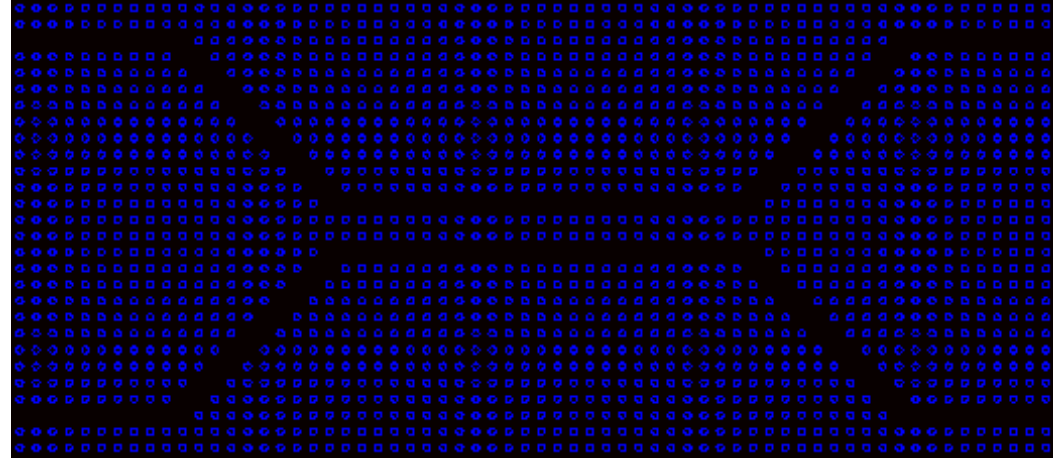


Fig. 4. Scanning electron micrograph of the fabricated Z-bend. The number, shape and size of the holes at each bend are designed using topology optimization. The inset shows a magnified view of the optimized holes as designed (white contour) and actually fabricated.

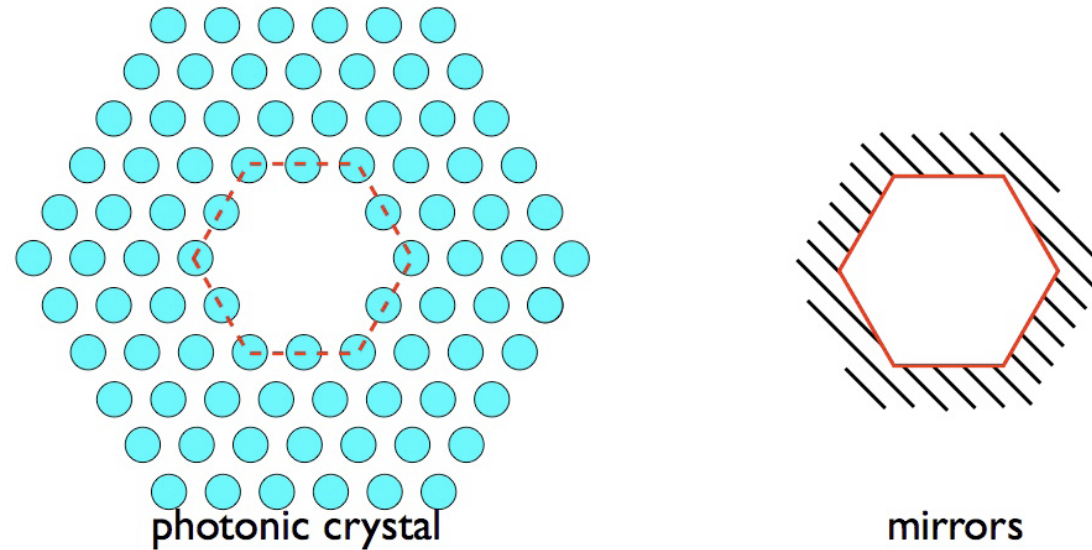
## Waveguides and bends

- Many conventional integrated optics components can be revisited within the framework of photonic crystals

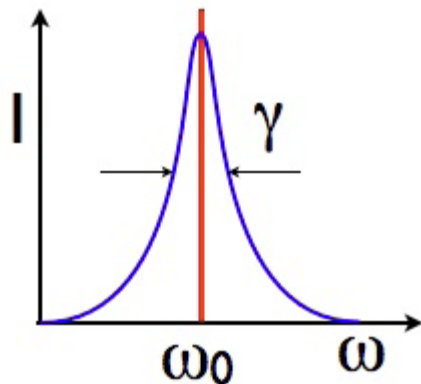


# Optical resonator

- Missing motif and its simple equivalent (in the bandgap)



- Lifetime, linewidth and quality factor:



$$Q = 2\pi \frac{\text{energy stored in cavity}}{\text{energy lost per cycle}} = \frac{\omega_0}{\gamma} = \frac{\omega_0 \tau}{2}$$

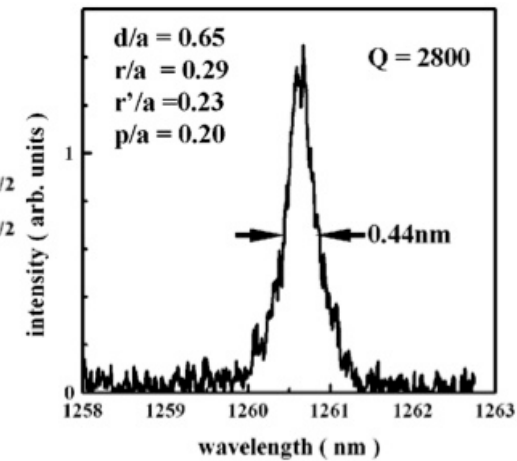
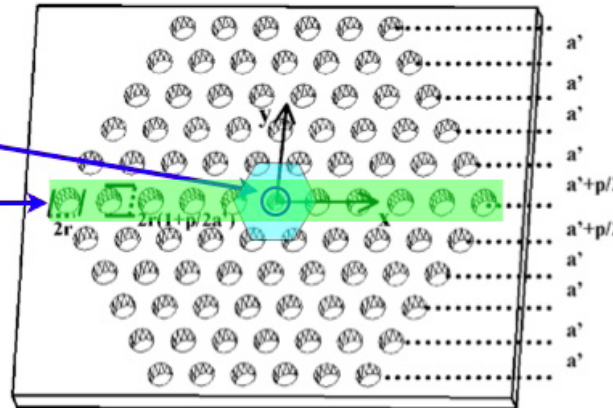
# High Q cavities

Fine and subtle parameters adjustment

$Q = 2800$

Cavity modified HI

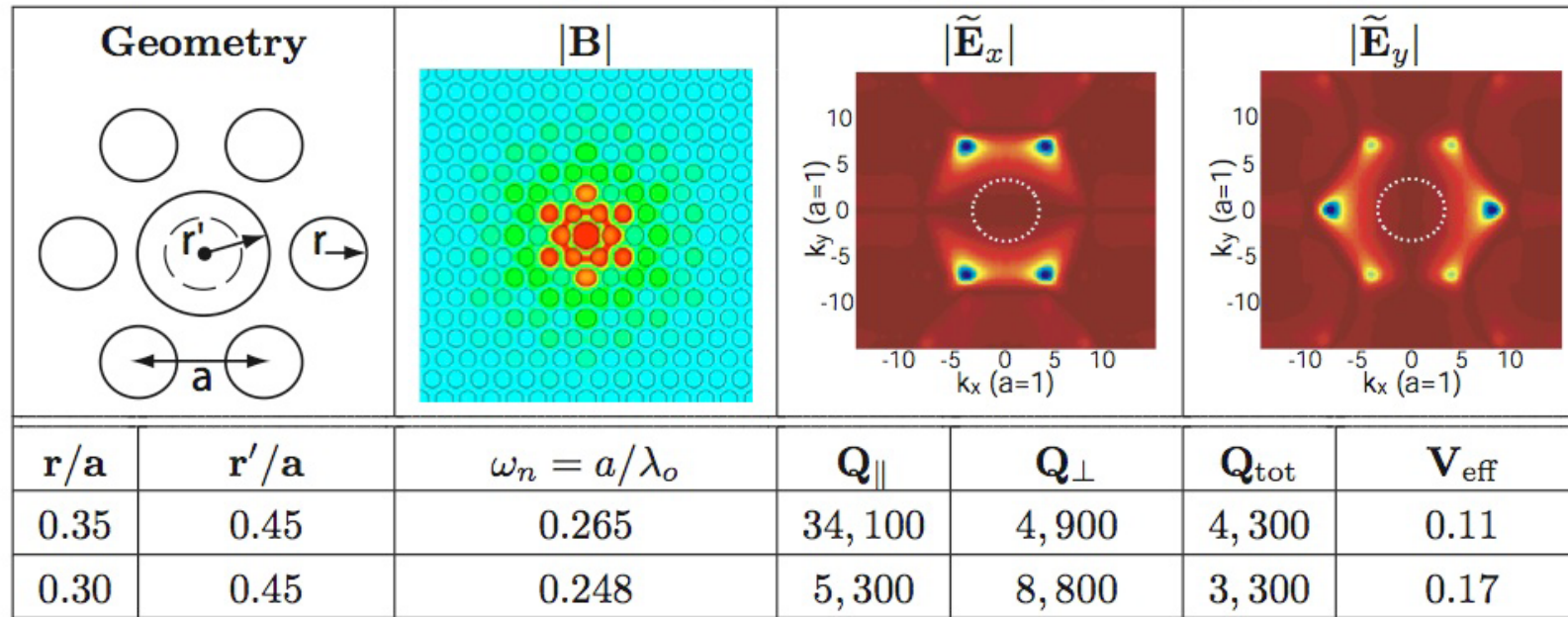
- Small hole
- Dislocation



T. Yoshie et al., Appl. Phys. Lett., 79, 4289 (2001)

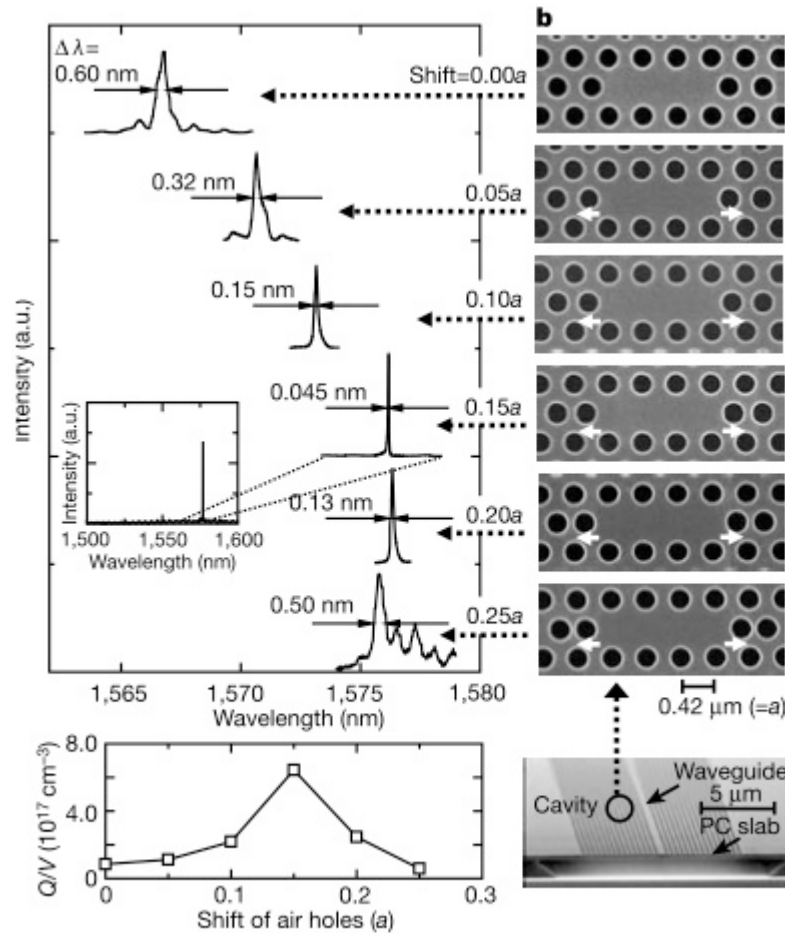
# High Q cavities

- Optimization processes can lead to very high Q and very small modal volume

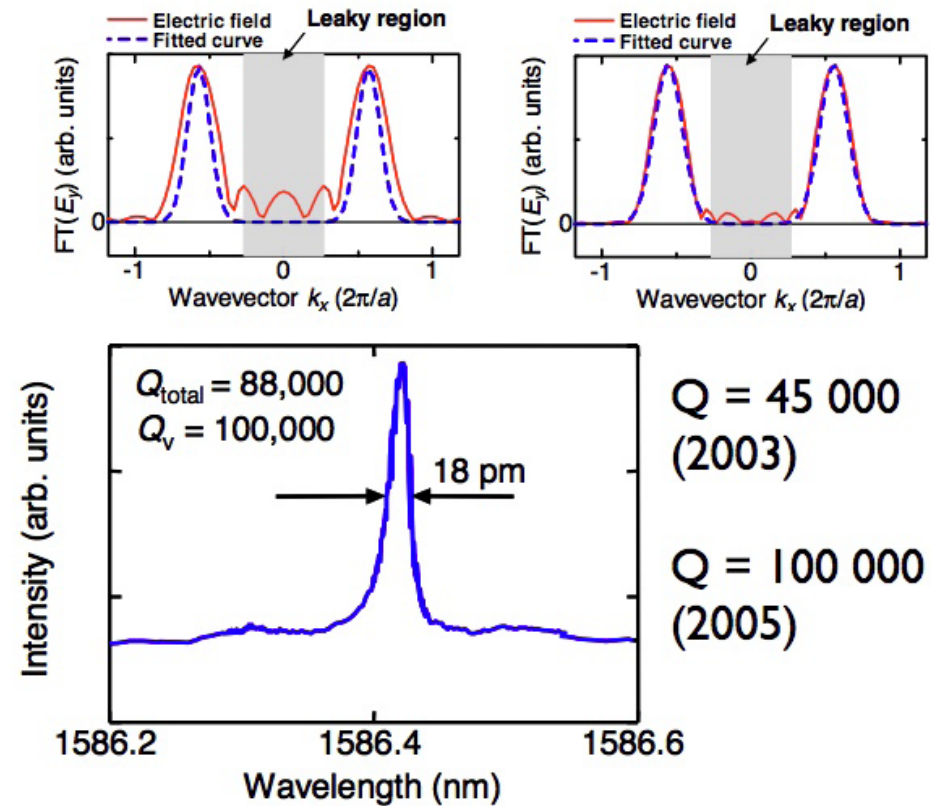


# High Q cavities

- Optimization processes can lead to very high Q and very small modal volume



Y. Akahane et al., Nature, 425, 944 (2003)

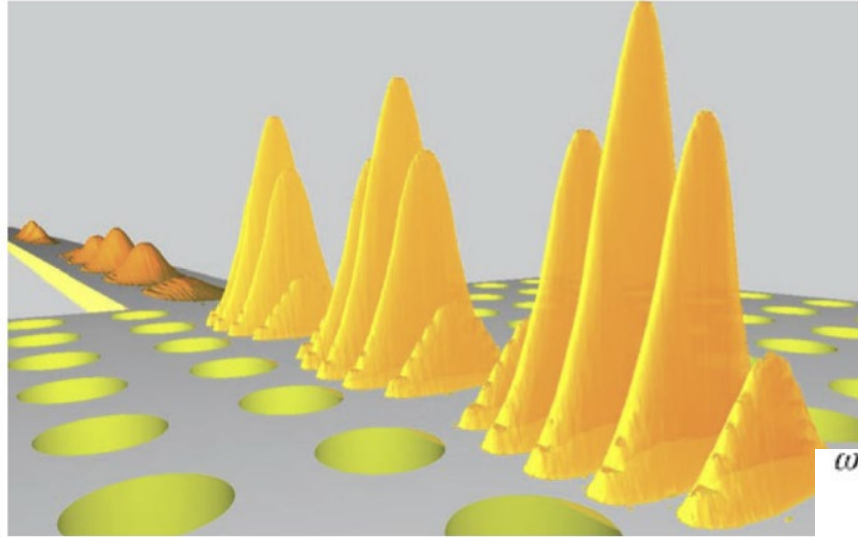


# Slow light

COMMENTARY | FOCUS

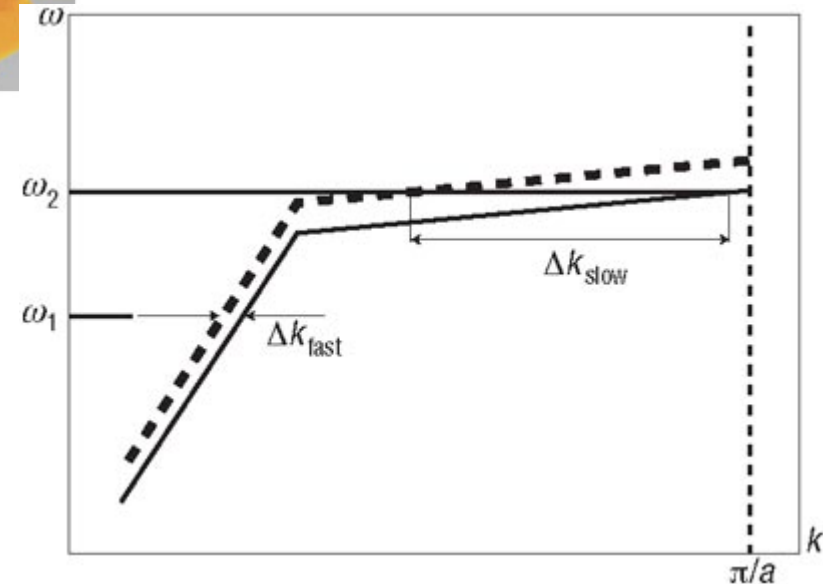
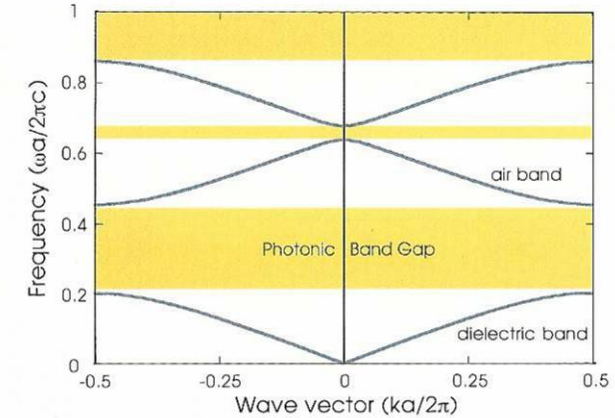
## Why do we need slow light?

THOMAS F. KRAUSS



nature photonics | VOL 2 | AUGUST 2008 | www.nature.com/naturephotonics pp. 448-450

At the edge of the bandgap the group velocity  $d\omega/dk$  can be significantly reduced, reaching almost zero!



# Slow light

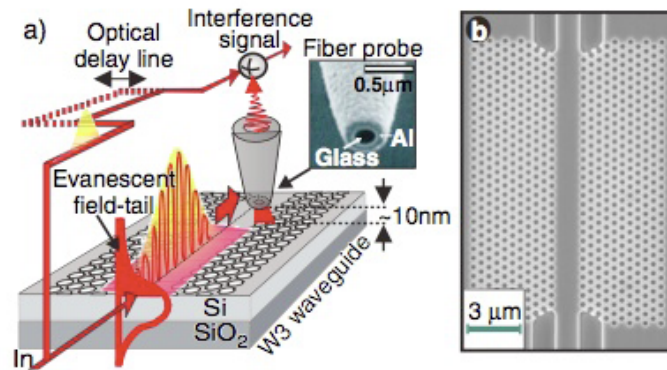
PRL **94**, 073903 (2005)

PHYSICAL REVIEW LETTERS

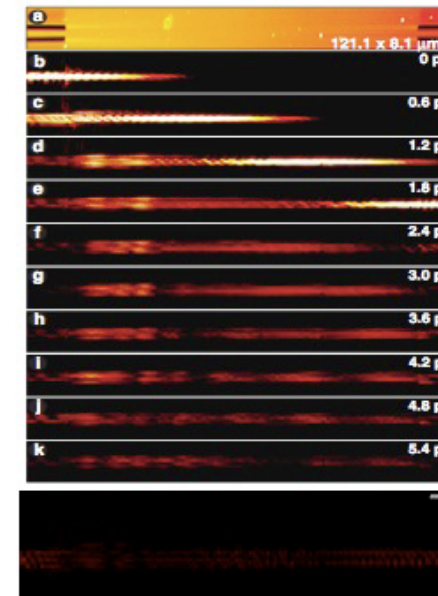
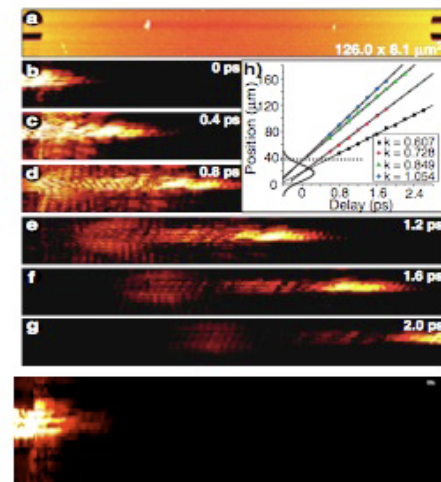
week ending  
25 FEBRUARY 2005

## Real-Space Observation of Ultraslow Light in Photonic Crystal Waveguides

H. Gersen,<sup>1,\*</sup> T. J. Karle,<sup>2</sup> R. J. P. Engelen,<sup>1</sup> W. Bogaerts,<sup>3</sup> J. P. Korterik,<sup>1</sup> N. F. van Hulst,<sup>1</sup> T. F. Krauss,<sup>2</sup> and L. Kuipers<sup>1,4,†</sup>



Time resolved

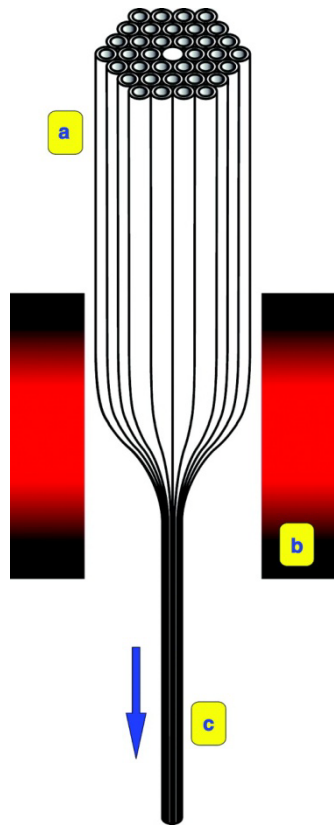


Group velocity  $c_0/1'000$

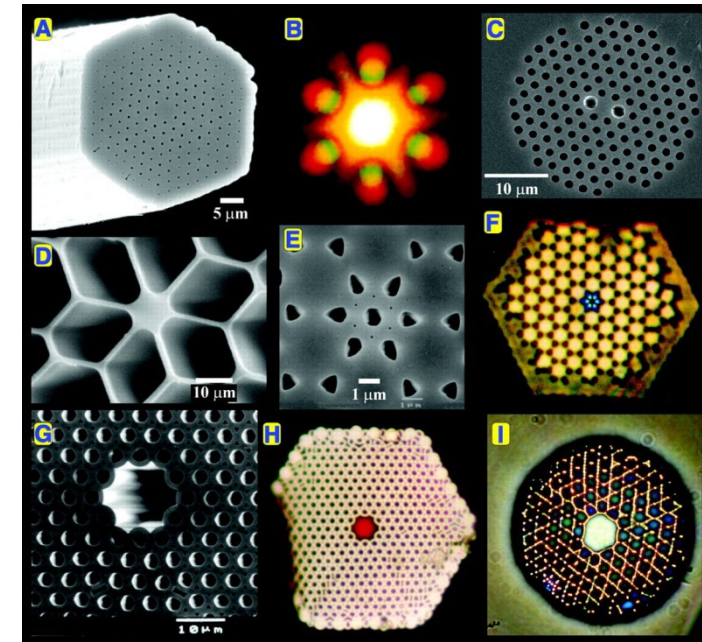
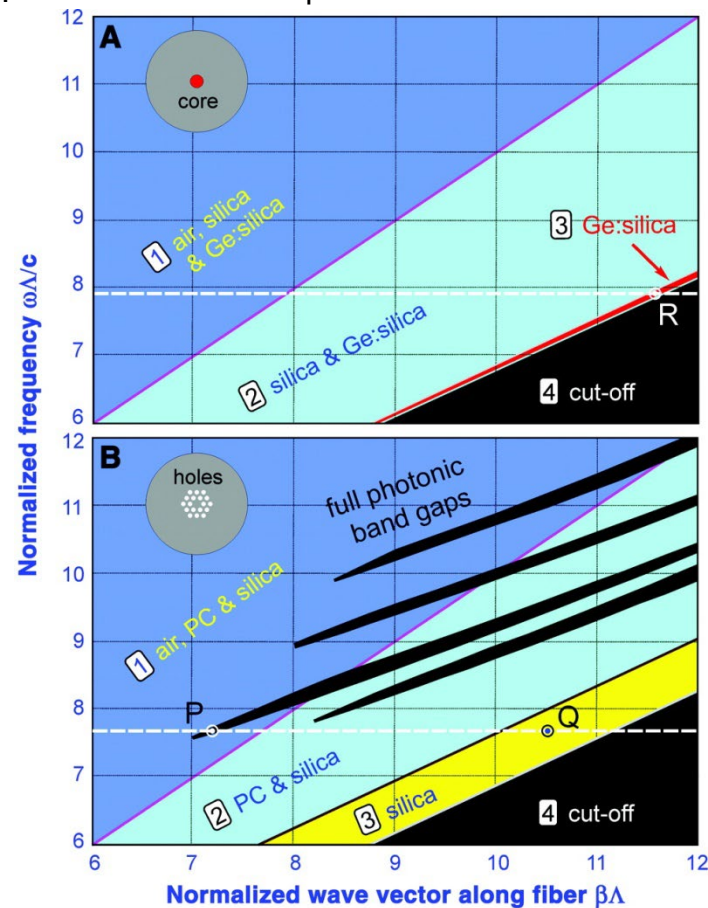
# Photonic fibers

- The photonic bandgap associated with such fibers produces frequency selectivity, high power concentration and can trigger nonlinear effects and anomalous dispersion

Fabrication from a bundle:



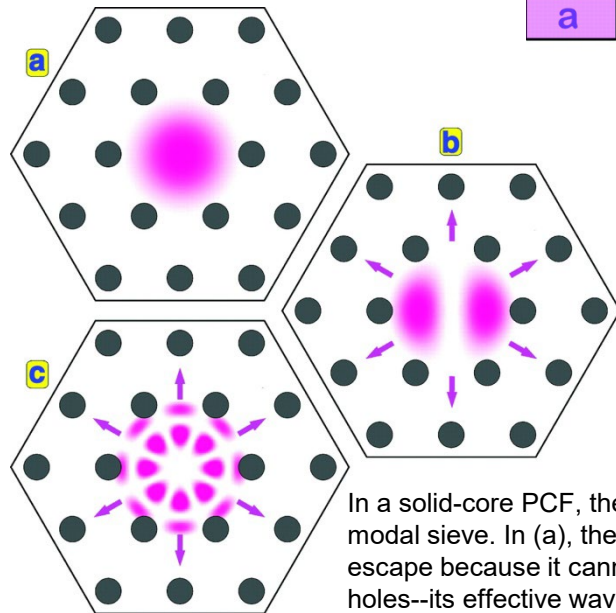
Dispersion relation:



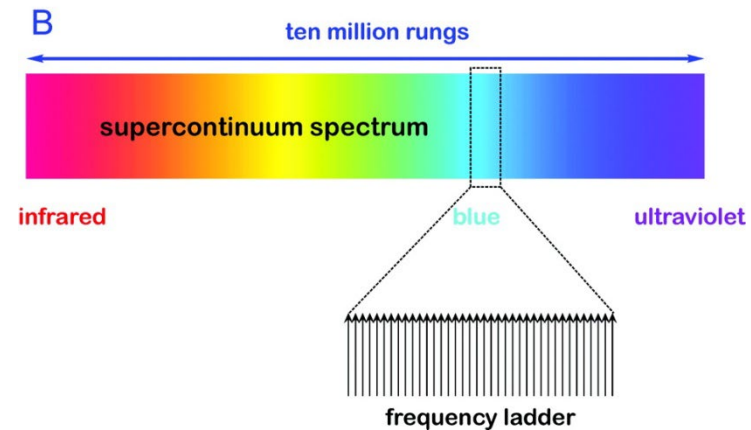
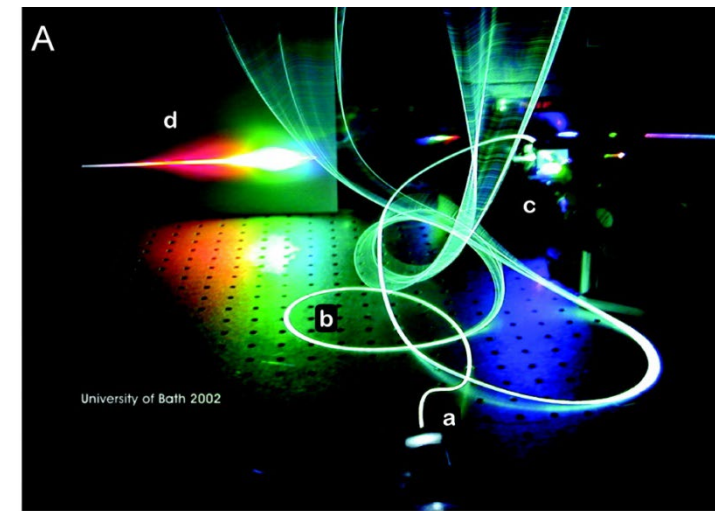
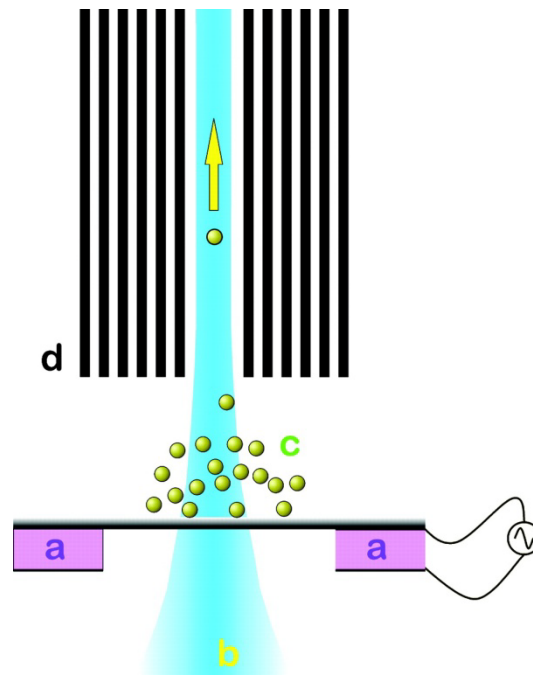
An assortment of optical (OM) and scanning electron (SEM) micrographs of PCF structures. (A) SEM of an endlessly single-mode solid core PCF. (B) Far-field optical pattern produced by (A) when excited by red and green laser light. (C) SEM of a recent birefringent PCF. (D) SEM of a small (800 nm) core PCF with ultrahigh nonlinearity and a zero chromatic dispersion at 560-nm wavelength. (E) SEM of the first photonic band gap PCF, its core formed by an additional air hole in a graphite lattice of air holes. (F) Near-field OM of the six-leaved blue mode that appears when (E) is excited by white light. (G) SEM of a hollow-core photonic band gap fiber. (H) Near-field OM of a red mode in hollow-core PCF (white light is launched into the core). (I) OM of a hollow-core PCF with a Kagomé cladding lattice, guiding white light.

# Photonic fibers

Particle trapping and guidance in a hollow-core PCF (38). The van der Waals forces between the  $\mu\text{m}$ -sized polystyrene particles (c) are broken by making them dance on a vibrating plate (a). The laser beam (b) captures them and entrains them into the hollow-core PCF (d).



In a solid-core PCF, the pattern of air holes acts like a modal sieve. In (a), the fundamental mode is unable to escape because it cannot fit in the gaps between the air holes--its effective wavelength in the transverse plane is too large. In (b) and (c), the higher order modes are able to leak away because their transverse effective wavelength is smaller. If the diameter of the air holes is increased, the gaps between them shrink and more and more higher order modes become trapped in the "sieve."



(A) The supercontinuum spectrum produced from an infrared laser operating at 800 nm and producing 200-fs pulses. The infrared light is launched (a) into highly nonlinear PCF (b) and the supercontinuum is dispersed into its constituent colors at a diffraction grating (d). The resulting spectrum is cast on a screen (c). (B) The supercontinuum spectrum consists of millions of individual frequencies, spaced by the  $\sim 100\text{-MHz}$  repetition rate of the infrared laser. The resulting ladder can be used as a highly accurate "ruler" for measuring frequency