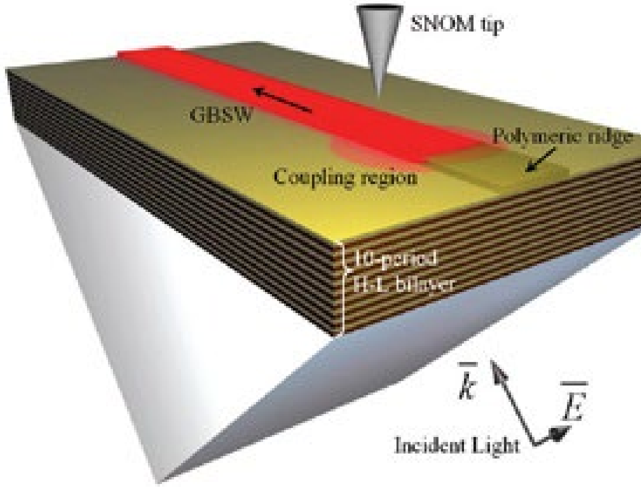
Selected Topics in Advanced Optics

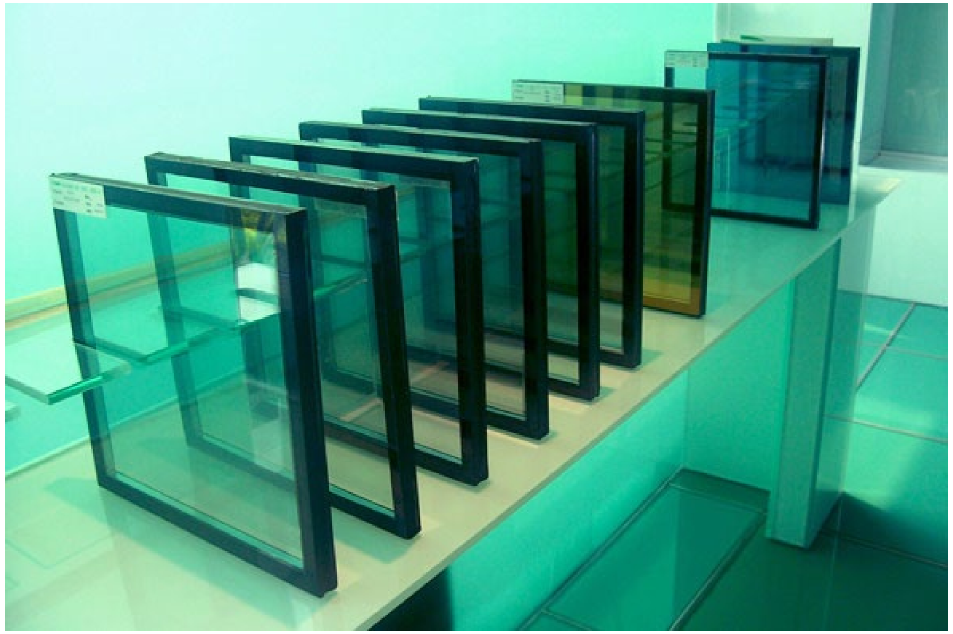
Week 9 – part 1

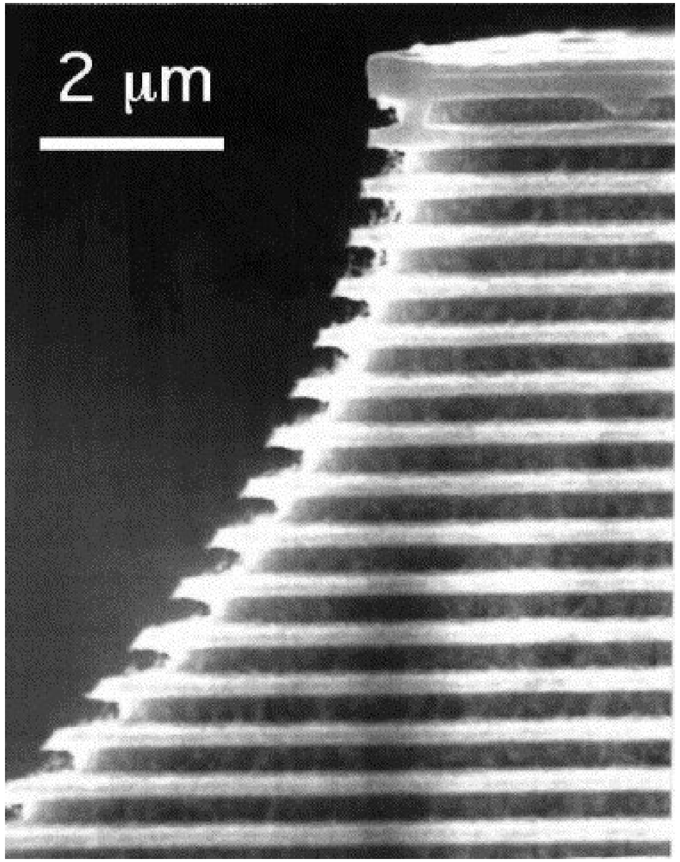
Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

Gratings, stratified media and photonic crystals
Part II – Stratified media







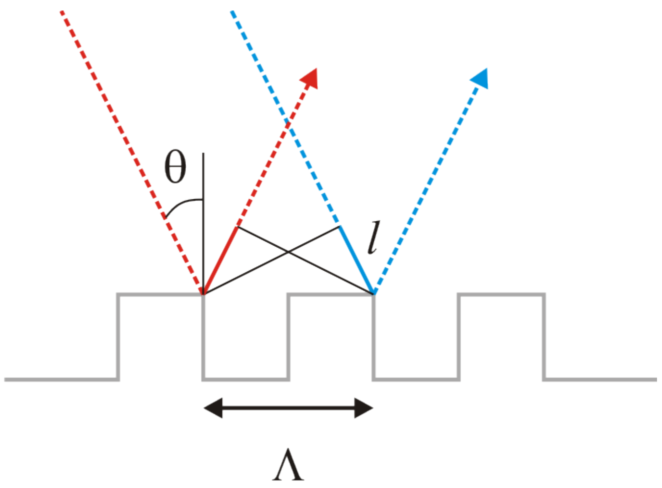
Olivier J.F. Martin

Some references

- B.E.A. Saleh & M.C. Teich, Fundamental of photonics 2nd Ed. (Wiley, Hoboken, 2007), Chapter 6.
- H.A. Macleod, Thin-film optical filters 5th Ed. (CRC Press, Boca Raton, 2018).
- M. Born & E. Wolf, Principles of optics 6th Ed. (Pergamon, Oxford, 1980).

(Flat) Bragg grating – Interference perspective

- One can obtain the grating law by requiring that waves incident a Λ apart on the structure are reflected in phase:



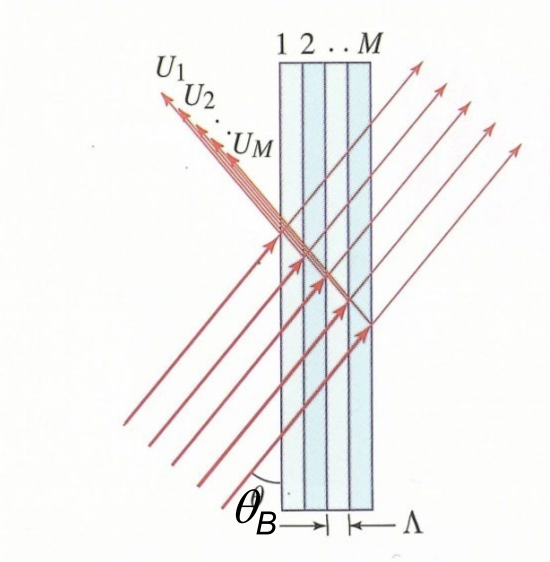
- The optical path difference between the red and blue rays must be a multiple of the wavelength

$$2\Lambda \sin\theta = q\lambda \quad q = 0, 1, 2, \ldots$$

$$\boxed{\sin\theta = q\frac{\lambda}{2\Lambda} \quad q = 0, 1, 2, \ldots}$$

(Stratified) Bragg grating – Interference perspective

- Set of uniformly spaced parallel partially reflective planar mirrors:



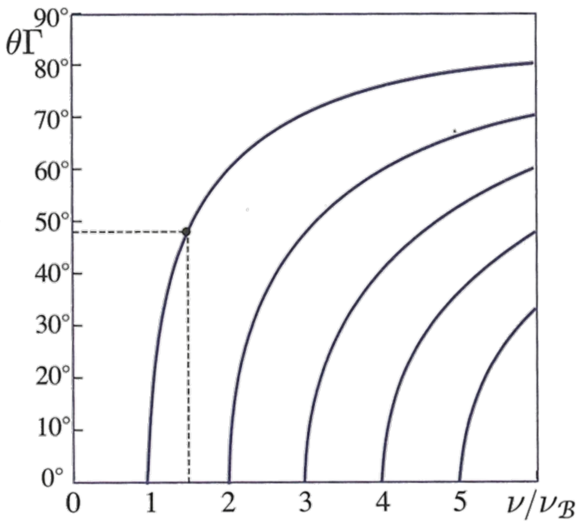
- Maximum reflection when

$$\cos\theta = q\frac{\lambda}{2\Lambda} = q\frac{\omega_B}{\omega} = q\frac{\nu_B}{\nu}$$ Bragg condition

$$\nu_B = \frac{c}{2\Lambda} \qquad \omega_B = \frac{\pi c}{\Lambda}$$ Bragg frequency

- At normal incidence (θ=0) reflectance peaks occur at multiples of the Bragg frequency
- Depending on the frequency ν, there may be 0, 1 or more angles satisfying the Bragg condition
- Bragg angle:

$$\theta_B = \pi/2 - \theta = \sin^{-1}\left(\lambda/2\Lambda\right)$$



Olivier J.F. Martin

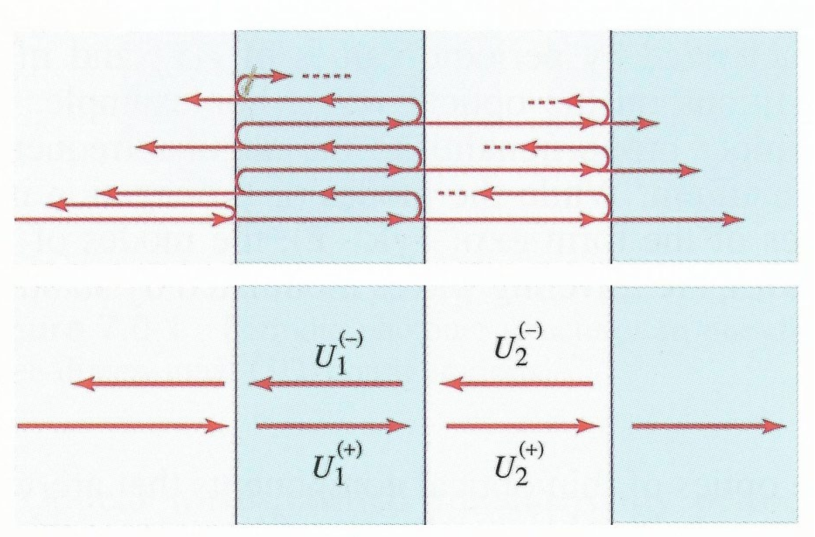
Selected Topics in Advanced Optics

Week 9 – part 2

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory
EPFL

Matrix theory of multilayer optics

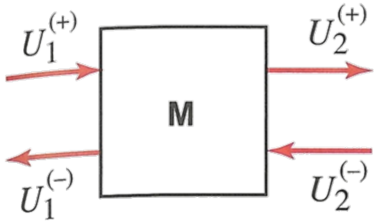
- Stratified or multilayered medium
- Normal incidence from the left
- The multiple transmitted/reflected amplitudes in each layer can be replaced by one amplitude in forward and one in backward directions



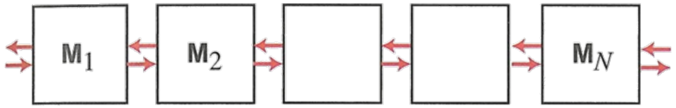
- Fresnel coefficients determine reflection/transmission <u>for a single interface</u>
- Objective: determine the reflection/transmission <u>of the entire structure</u>

Matrix theory of multilayer optics – Two different approaches

- Wave-transfer matrix $\mathbf{M}$ (left/right):



$$\begin{bmatrix} U_2^{(+)} \\ U_2^{(-)} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_1^{(+)} \\ U_1^{(-)} \end{bmatrix}$$

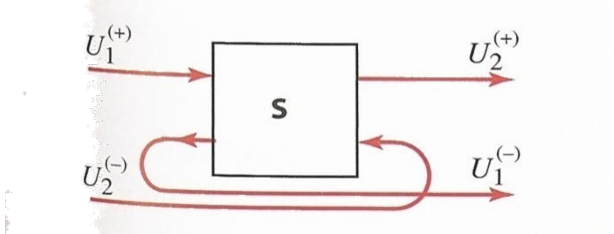


$$\mathbf{M} = \mathbf{M}_N \cdot \ldots \cdot \mathbf{M}_2 \cdot \mathbf{M}_1$$

concatenation of N elements

(multiplication order!)

- Scattering matrix $\mathbf{S}$ (input/output):



$$\begin{bmatrix} U_2^{(+)} \\ U_1^{(-)} \end{bmatrix} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} \begin{bmatrix} U_1^{(+)} \\ U_2^{(-)} \end{bmatrix}$$

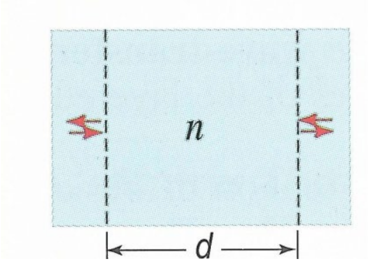
related to the physical parameters

of the system, but $\mathbf{S} \neq \mathbf{S}_N \cdot \ldots \cdot \mathbf{S}_2 \cdot \mathbf{S}_1$

Olivier J.F. Martin

Example: propagation through a homogeneous medium

- Slice of medium with index n and thickness d:

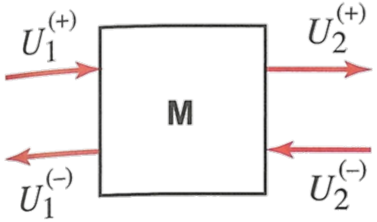


$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(j\varphi) \end{bmatrix} \qquad \varphi = nk_0 d$$

$$S = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(-j\varphi) \end{bmatrix}$$

Conservation relations for lossless media

- In lossless media, incoming and outgoing optical powers must be equal, furthermore the power is related to the amplitude square:



$$\left|U_1^{(+)}\right|^2 + \left|U_2^{(-)}\right|^2 = \left|U_2^{(+)}\right|^2 + \left|U_1^{(-)}\right|^2$$

- This must hold for any combination of incoming amplitudes
- For the scattering matrix:

$$|t_{12}| = |t_{21}| \equiv |t| \quad |r_{12}| = |r_{21}| \equiv |r| \quad |t|^2 + |r|^2 = 1 \quad t_{12} / t_{21}^* = -r_{12} / r_{21}^*$$

- For the transfer matrix:

$$|D| = |A| \quad |C| = |B| \quad |A|^2 - |B|^2 = 1$$

$$\det \mathbf{M} = C / B^* = A / D^* = t_{12} / t_{21} \quad |\det \mathbf{M}| = 1$$

Conservation relations for lossless reciprocal media

- If the medium is also reciprocal (input/output can be exchanged, as is almost always true), the conditions are simpler:

$$t_{12} = t_{21} \equiv t \qquad r_{12} = r_{21} \equiv r$$

- For the scattering matrix:

$$\boxed{|t|^2 + |r|^2 = 1 \quad t/r = -(t/r)^* \qquad \arg\{t\} - \arg\{r\} = \pm\pi/2}$$

$$\mathbf{S} = \begin{bmatrix} t & r \\ r & t \end{bmatrix}$$

- For the transfer matrix:

$$\boxed{A = D^* \qquad B = C^* \qquad |A|^2 - |B|^2 = 1 \qquad \det\mathbf{M} = 1}$$

$$\mathbf{M} = \begin{bmatrix} 1/t^* & r/t \\ r^*/t^* & 1/t \end{bmatrix}$$

Equivalence between scattering and transfer matrices

- Since we cannot cascade scattering matrices, we must use transfer matrices to build up the response of the system and hence need the equivalence between $\mathbf{M}$ and $\mathbf{S}$:

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{t_{21}} \begin{bmatrix} t_{12}t_{21} - r_{12}r_{21} & r_{21} \\ -r_{12} & 1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} AD - BC & B \\ -C & 1 \end{bmatrix}$$

- Procedure for a stack with N layers:
  - Use Fresnel coefficients to compute all the S matrices
  - Convert S matrices into M matrices
  - Concatenate M matrices
  - Convert the resulting M matrix into a S matrix to obtain the amplitude transmittance and reflectance of the stack

Selected Topics in Advanced Optics

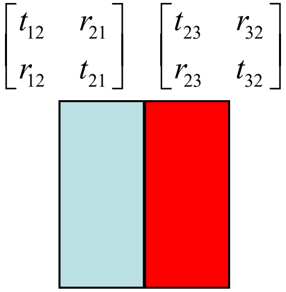
Week 9 – part 3

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

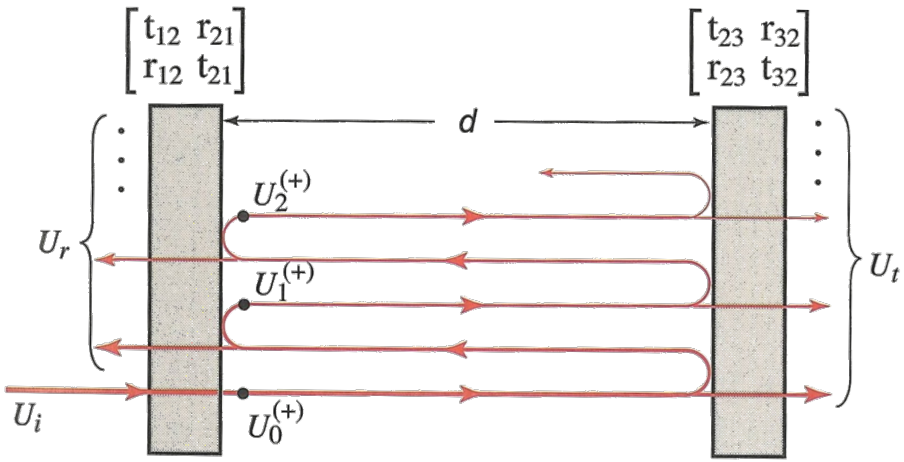
Example: Two-cascaded systems

- Multiplication of $\mathbf{M}$ matrices and conversion into $\mathbf{S}$ matrix leads to



$$t_{13} = \frac{t_{12} t_{23}}{1 - r_{21} r_{23}} \qquad r_{13} = r_{12} + \frac{t_{12} t_{21} r_{23}}{1 - r_{21} r_{23}}$$

- With additional free space propagation (Airy formula)

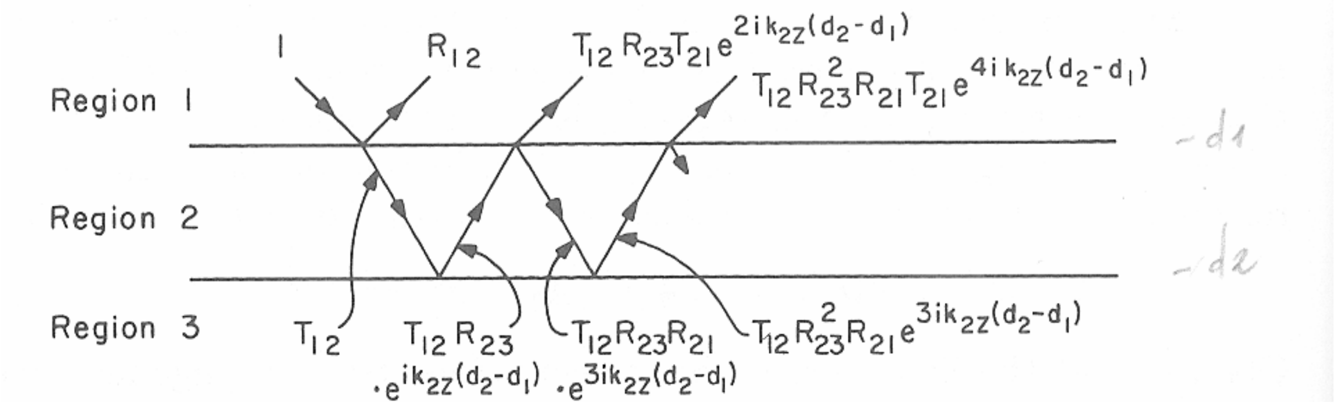


$$t_{13} = \frac{t_{12} t_{23} \exp(-j\varphi)}{1 - r_{21} r_{23} \exp(-j2\varphi)} \qquad r_{13} = r_{12} + \frac{t_{12} t_{21} r_{23} \exp(-j2\varphi)}{1 - r_{21} r_{23} \exp(-j2\varphi)}$$

Two-cascaded systems

- The response can also be obtained as a geometric progression:

$$\tilde{r}_{12} = r_{12} + t_{12} r_{23} t_{21} e^{2ik_{2z}(d_2-d_1)} + t_{12} r_{23}^2 r_{21} t_{21} e^{2ik_{2z}(d_2-d_1)} + \dots$$

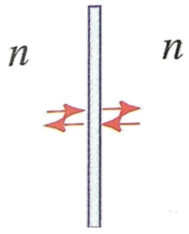


$$s_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$\lim_{n\to\infty} s_n = \frac{a}{1-r}$$

Some propagation examples

- Beamsplitter (lossless)



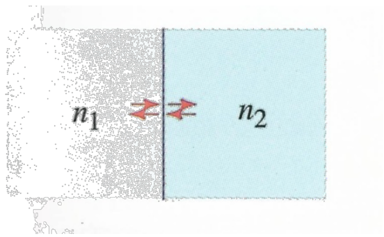
$$\mathbf{S} = \begin{bmatrix} |t| & j|r| \\ j|r| & |t| \end{bmatrix}$$

$$|t|^2 + |r|^2 = 1$$

$$\mathbf{M} = \frac{1}{|t|}\begin{bmatrix} 1 & j|r| \\ -j|r| & 1 \end{bmatrix}$$

Perfect mirror : $|r| = 1$ and $|t| = 0$

- Single dielectric boundary

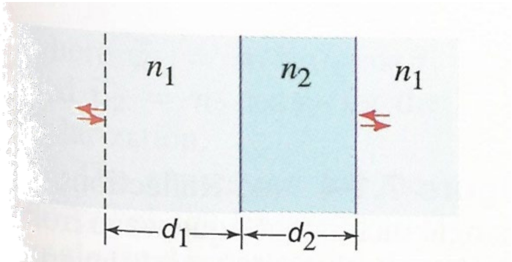


$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{n_1 + n_2}\begin{bmatrix} 2n_1 & n_2 - n_1 \\ n_1 - n_2 & 2n_2 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2n_2}\begin{bmatrix} n_2 + n_1 & n_2 - n_1 \\ n_2 - n_1 & n_2 + n_1 \end{bmatrix}$$

Some propagation examples

- Propagation followed by transmission through a slab



$$\varphi_i = n_i k_0 d_i$$

$$\mathbf{M} = \mathbf{M}_2 \cdot \mathbf{M}_1 =$$

$$\frac{1}{4n_1n_2}\begin{bmatrix} (n_1+n_2)\exp(-j\varphi_2) & (n_1-n_2)\exp(j\varphi_2) \\ (n_1-n_2)\exp(-j\varphi_2) & (n_1+n_2)\exp(j\varphi_2) \end{bmatrix} \cdot \begin{bmatrix} (n_2+n_1)\exp(-j\varphi_1) & (n_2-n_1)\exp(j\varphi_1) \\ (n_2-n_1)\exp(-j\varphi_1) & (n_2+n_1)\exp(j\varphi_1) \end{bmatrix}$$

$$A = D^* = \frac{1}{t^*} = \frac{1}{4n_1n_2}\left[(n_1+n_2)^2\exp(-j\varphi_2) - (n_2-n_1)^2\exp(j\varphi_2)\right]\exp(-j\varphi_1)$$

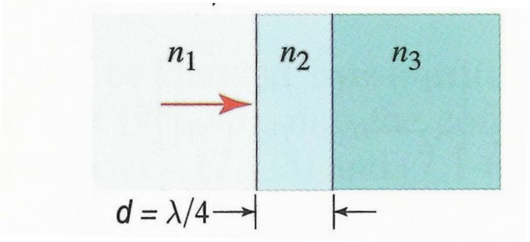
$$B = C^* = \frac{r}{t} = \frac{1}{4n_1n_2}\left(n_2^2 - n_1^2\right)\left[\exp(-j\varphi_2) - \exp(j\varphi_2)\right]\exp(j\varphi_1)$$

- Satisfies the properties of a lossless reciprocal medium
- Overall transmission:

$$t = \exp(-j\varphi_1)\frac{4n_1n_2\exp(-j\varphi_2)}{(n_1+n_2)^2 - (n_1-n_2)^2\exp(-j2\varphi_2)}$$

Some propagation examples

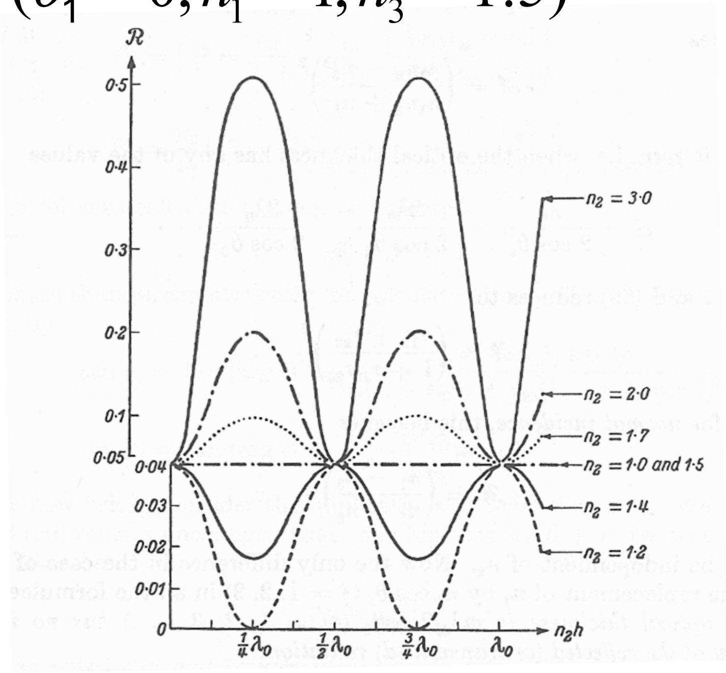
- Quarter-wave film (anti-reflection coating)



$$n_2 = \sqrt{n_1 n_3}$$

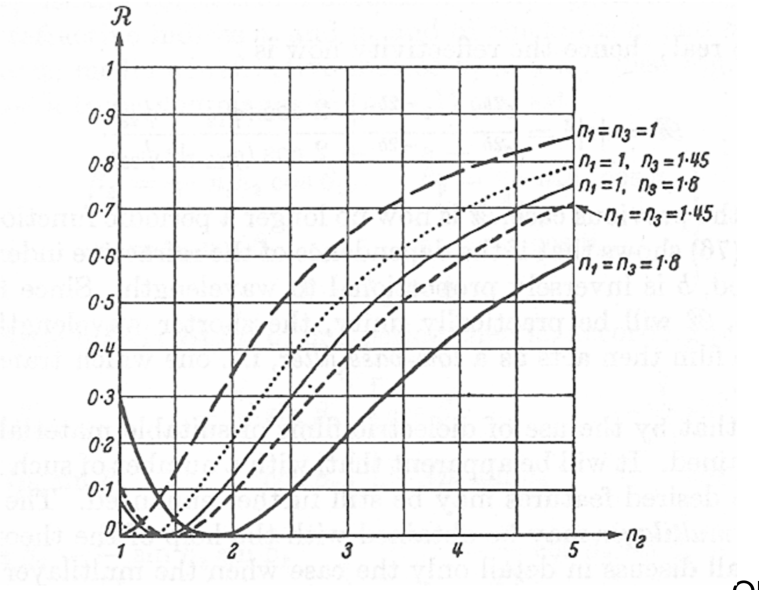
Reflectivity at normal incidence vs. optical thickness:

$$(\theta_1 = 0, n_1 = 1, n_3 = 1.5)$$



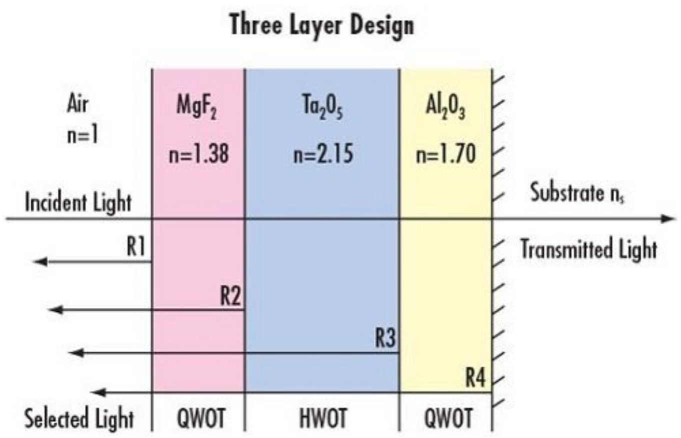
Reflectivity at normal incidence of a quarter-wave film:

$$(n_2 h = \lambda_0 / 4)$$



Olivier J.F. Martin

Some technology considerations



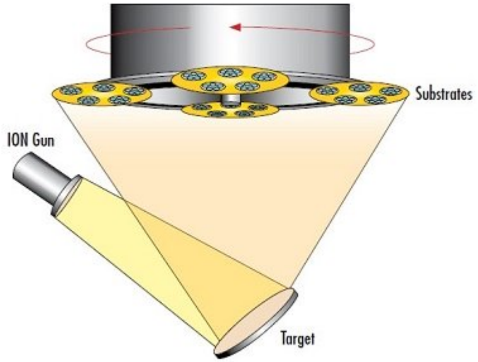


Figure 5: During ion beam sputtering (IBS), a strong electric field accelerates ions from an ion gun onto the target, which releases more ions that deposit a dense thin film coating on the rotating substrates

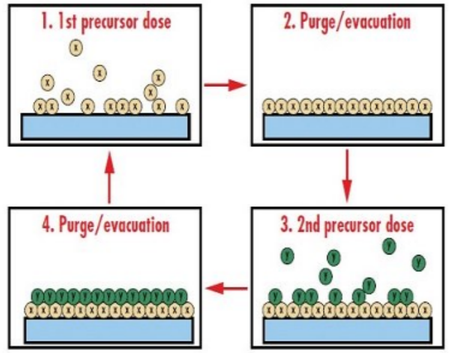


Figure 6: During atomic layer deposition (ALD), individual thin film layers are deposited by exposing the optics to different gaseous precursors, which results in a high level of control of layer thickness independent of the surface geometry of the optics

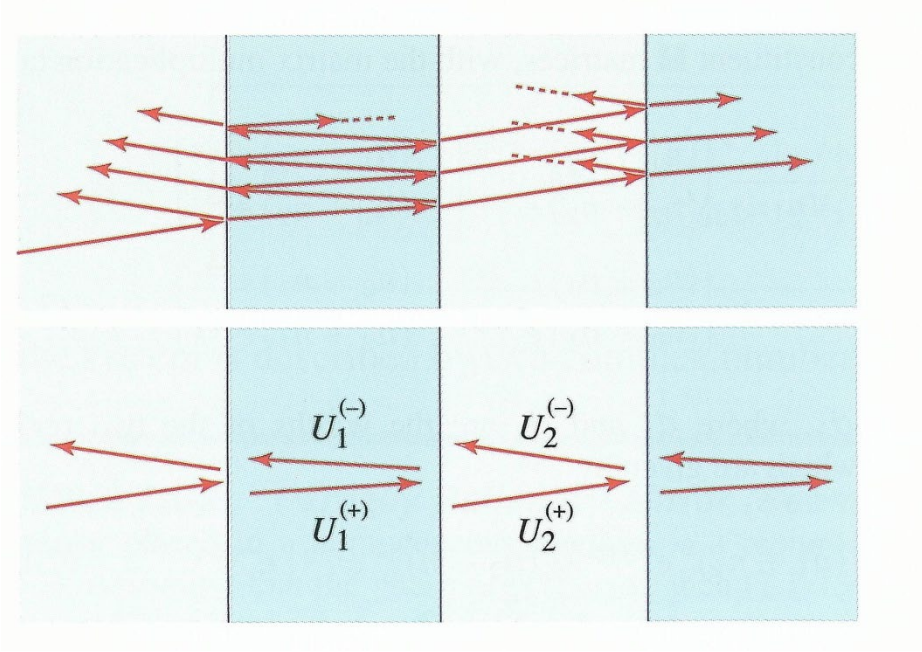
	Evaporative	Evaporative with IAD	Plasma Sputtering	IBS	ALD
Spectral Performance	Low	Medium	High	High-Very High	Very High
Coating Stress	Low	Medium	High	Very High	High
Repeatability	Medium	Medium	High	Very High	Very High
Process Time	Slow	Slow	Intermediate	Very Slow	Very Slow
Non-Flat Geometry Capabilities	Better	Better	Good	Bad	Best
Relative Price	$	$	$$	$$$	$$$

***Table 1:** Comparison of different coating technologies (IAD: ion assisted deposition, IBS: ion beam sputtering, ALD: atomic layer deposition[4]*

Edmund Optics

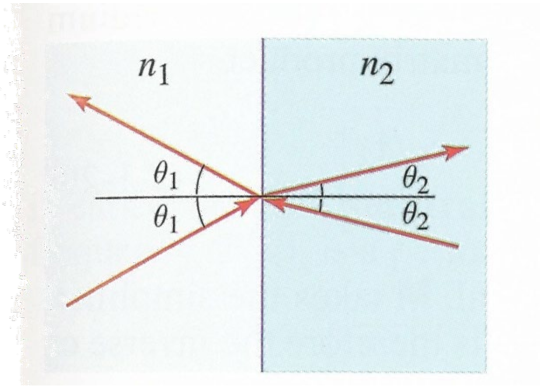
Off-axis propagation in layered media

- Within a specific layer, all forward waves are parallel and all backward waves are parallel
- Now the Fresnel coefficients for a single interface $t_{12}, t_{21}, r_{12}, r_{21}$ depend on the angle of incidence <u>and the polarization</u>; the rest of the calculation remains the same



Off-axis propagation in layered media – example

- Single boundary



$$\mathbf{S} = \begin{bmatrix} t_{12} & r_{21} \\ r_{12} & t_{21} \end{bmatrix} = \frac{1}{\tilde{n}_1 + \tilde{n}_2} \begin{bmatrix} 2a_{12}\tilde{n}_1 & \tilde{n}_2 - \tilde{n}_1 \\ \tilde{n}_1 - \tilde{n}_2 & 2a_{21}\tilde{n}_2 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2a_{21}\tilde{n}_2} \begin{bmatrix} \tilde{n}_1 + \tilde{n}_2 & \tilde{n}_2 - \tilde{n}_1 \\ \tilde{n}_2 - \tilde{n}_1 & \tilde{n}_1 + \tilde{n}_2 \end{bmatrix}$$

$$\text{TE}: \ \tilde{n}_1 = n_1 \cos\theta_1 \quad \tilde{n}_2 = n_2 \cos\theta_2 \quad a_{12} = a_{21} = 1$$

$$\text{TM}: \ \tilde{n}_1 = n_1 \sec\theta_1 \quad \tilde{n}_2 = n_2 \sec\theta_2 \quad a_{12} = \cos\theta_1 / \cos\theta_2 = 1/a_{21}$$

Selected Topics in Advanced Optics

Week 9 – part 4

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

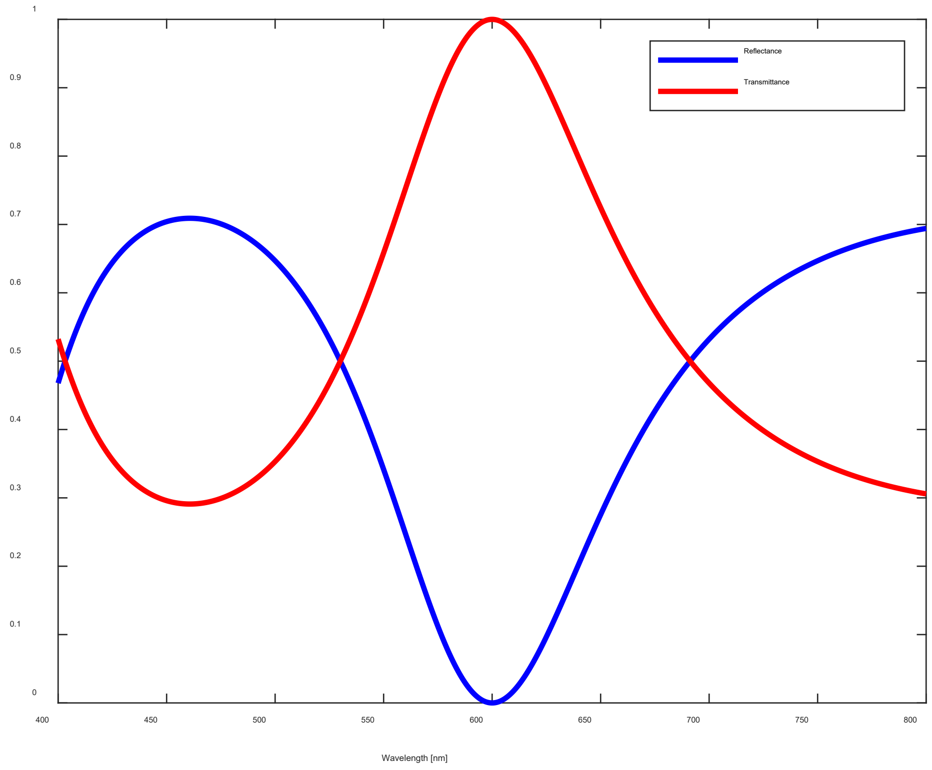
Optical filters

- Filters are often realized by a combination of $\lambda/2$ and $\lambda/4$ layers of high and low refractive indices
- They operate for a specific wavelength (where the $\lambda/2$, $\lambda/4$ conditions are satisfied)
- We define H and L as $\lambda/4$ layers of high and low index material
- The thickness is always given with respect to the effective wavelength in that layer
- In the following, the filters are designed for λ_0=600 nm

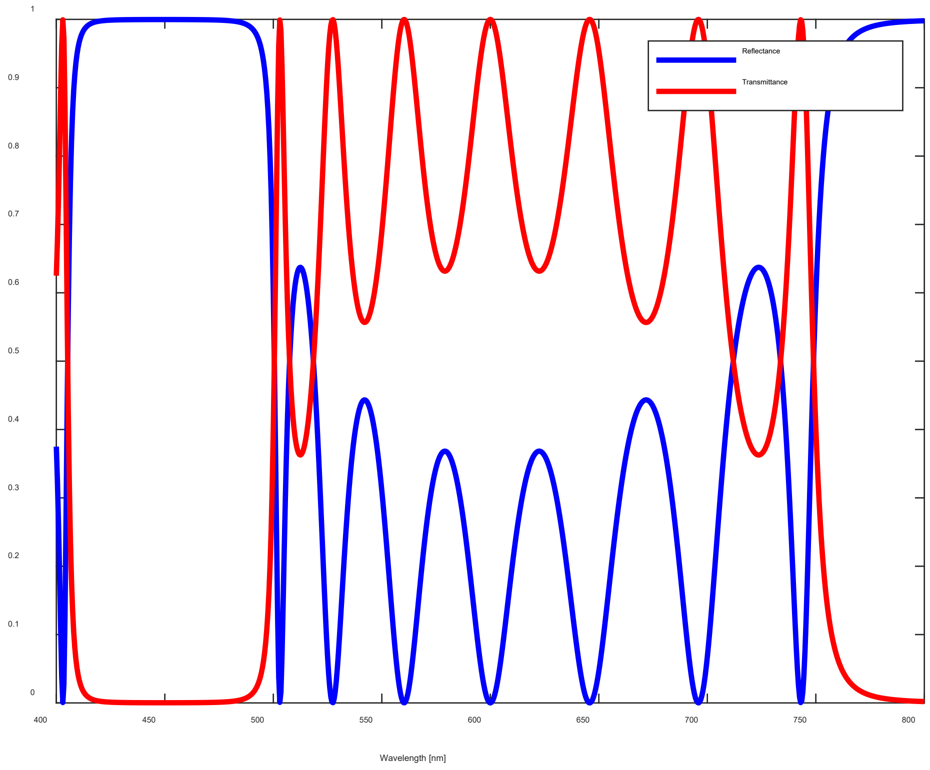
Transmittance filter

- Sequence $(HHL)^k HH$, light within the pass-band is transmitted

K = 2



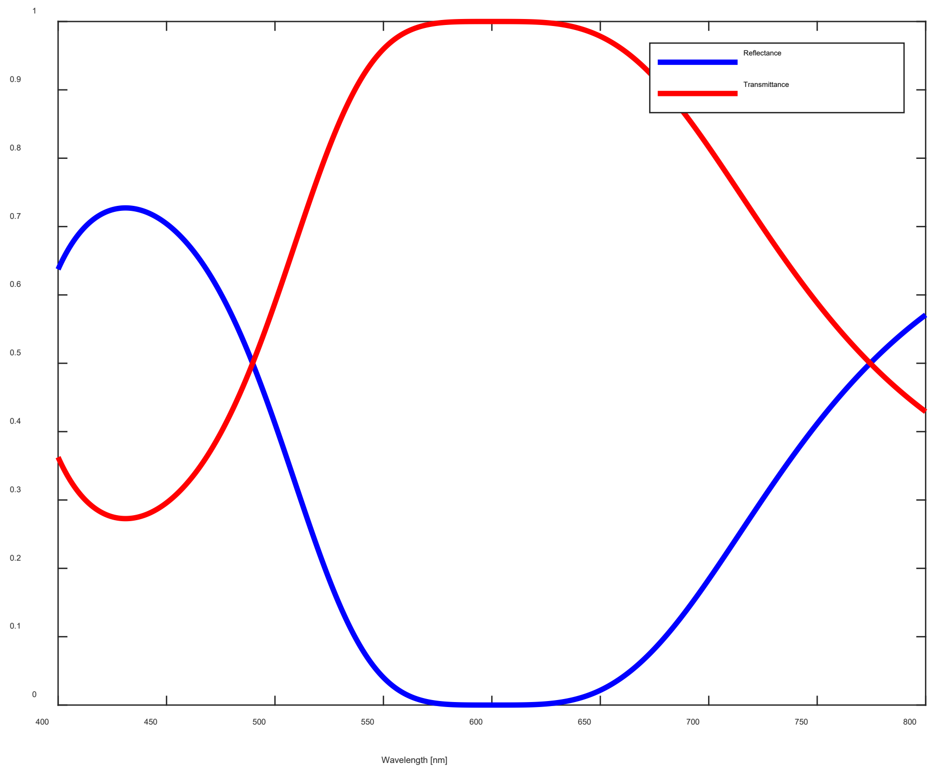
K = 8



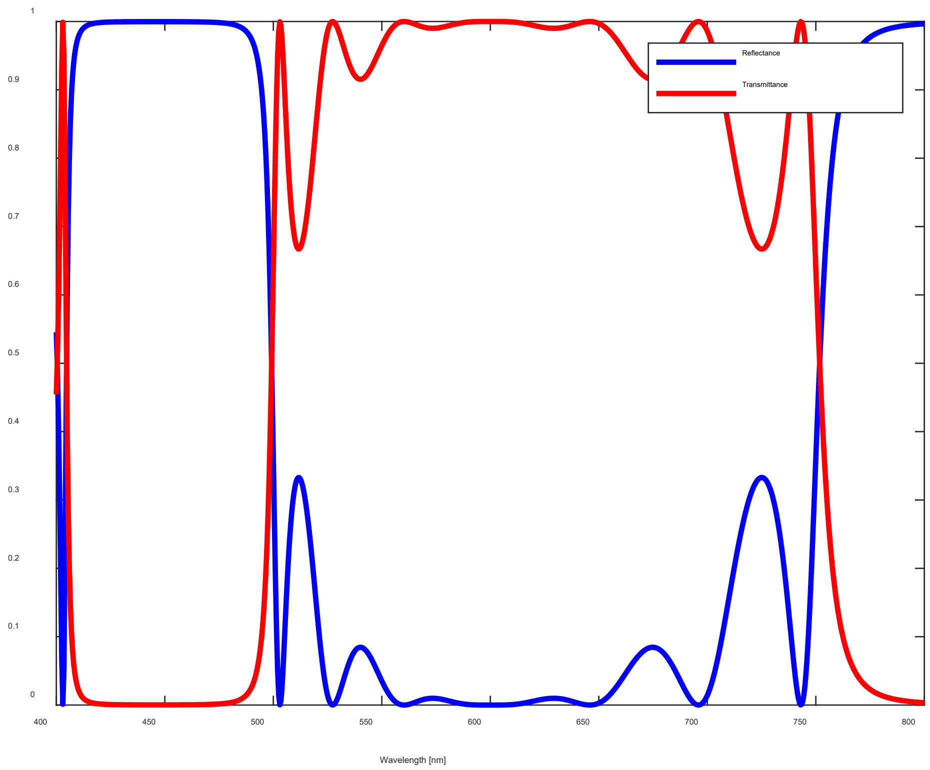
Inverted transmittance filter

- Sequence $(HLL)^kH$, light within the pass-band is transmitted

K = 2



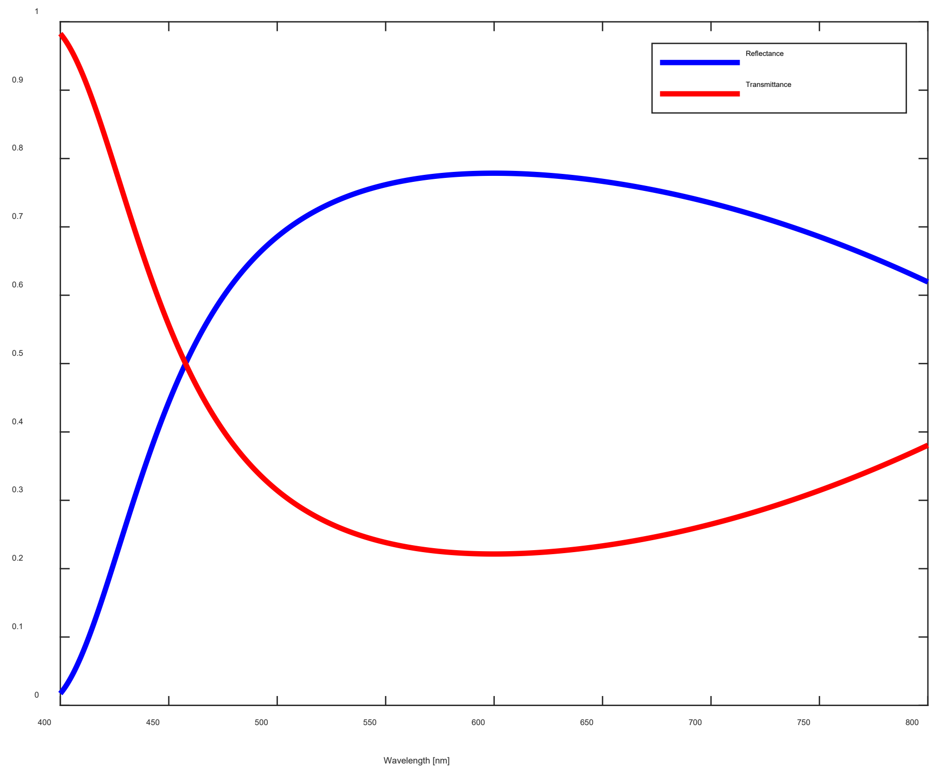
K = 8



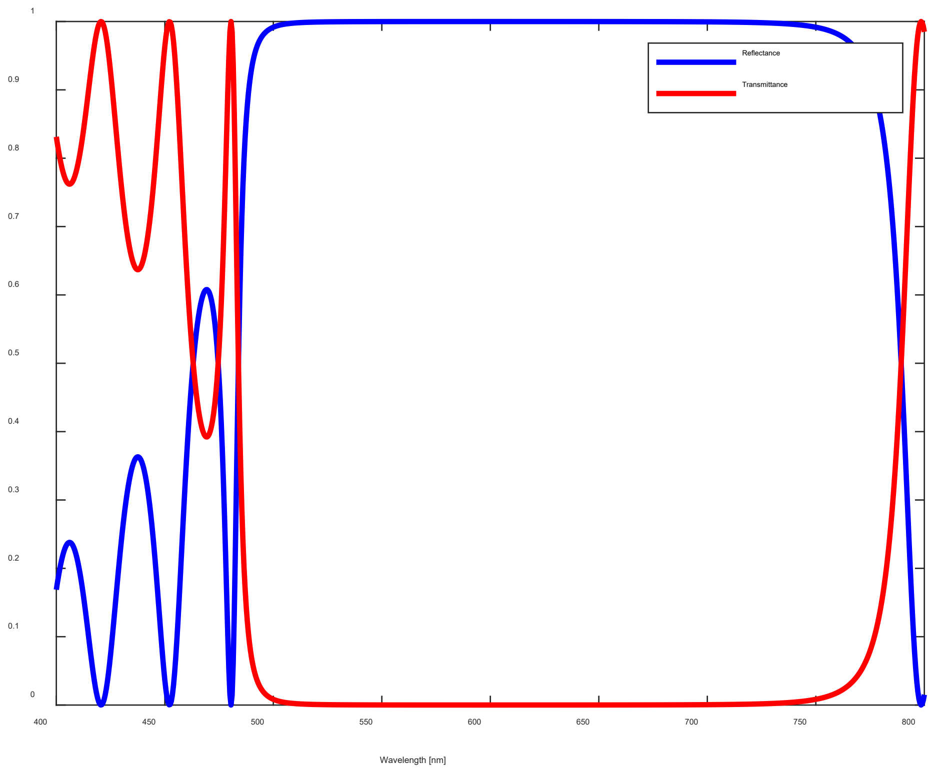
Reflectance filter

- Sequence $(HL)^kH$, light within the pass-band is reflected

K = 2



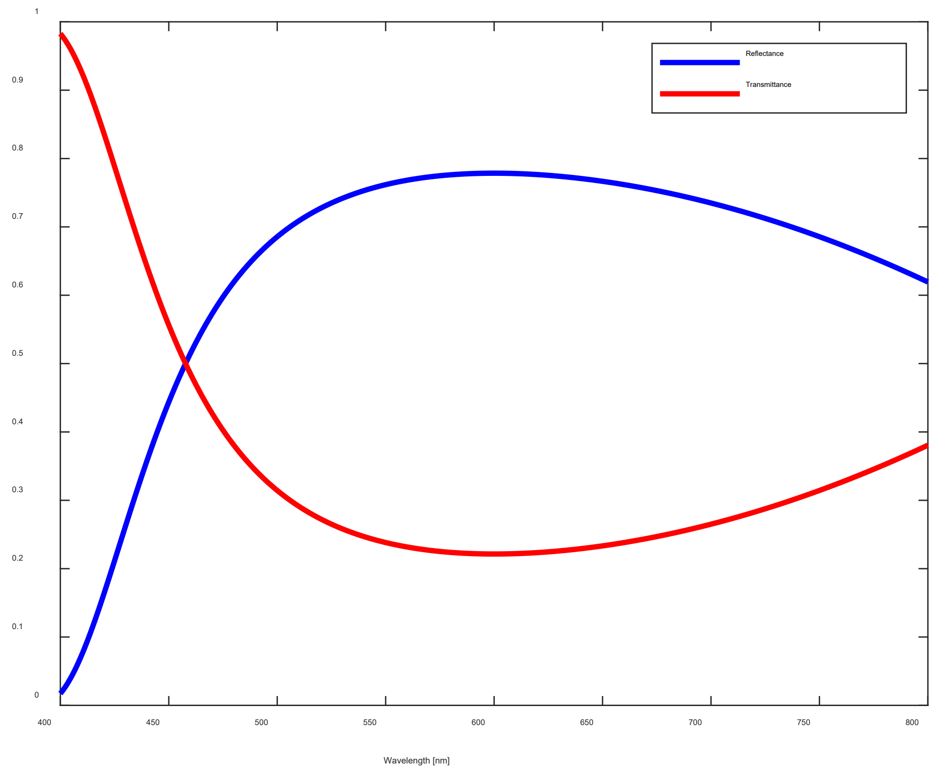
K = 8



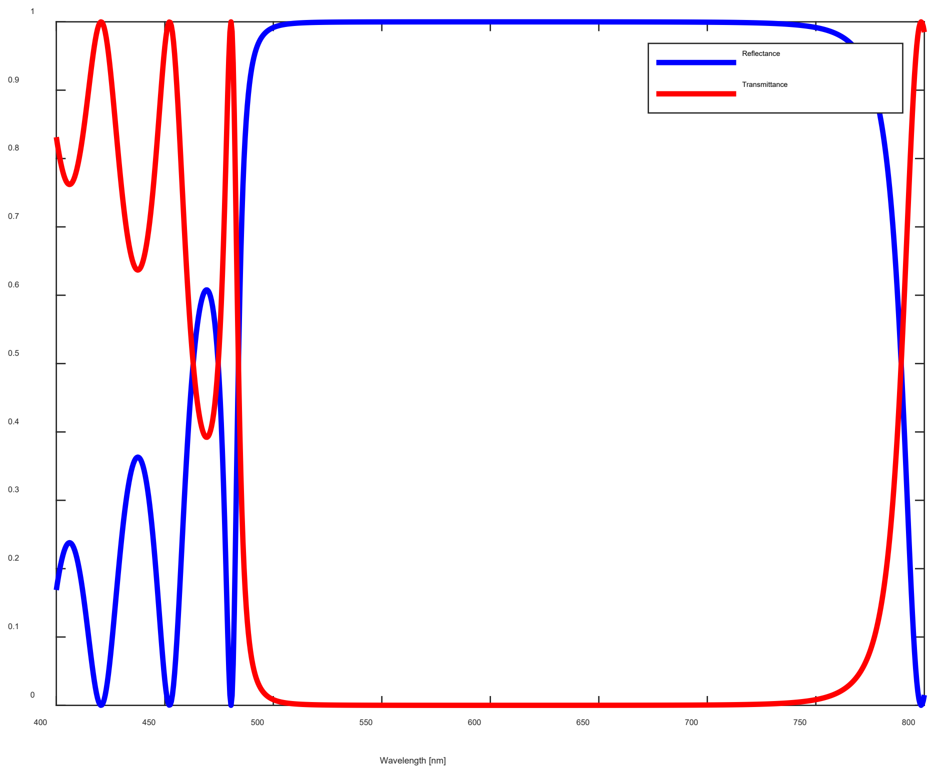
Inverse reflectance filter

- Sequence $(LH)^kL$, light within the pass-band is reflected

K = 2



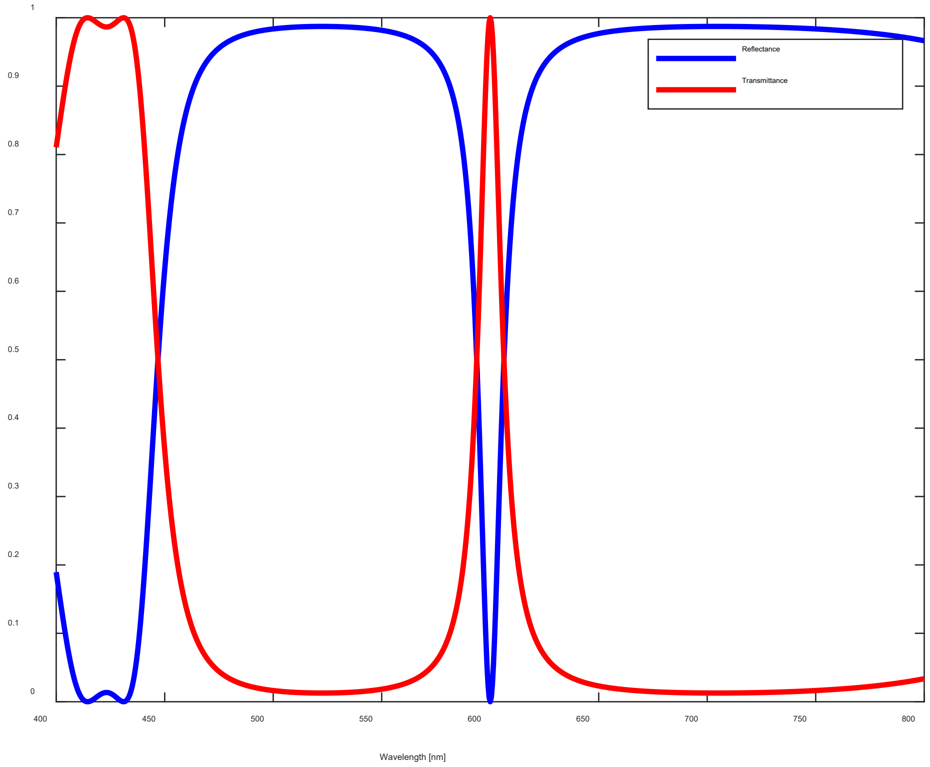
K = 8



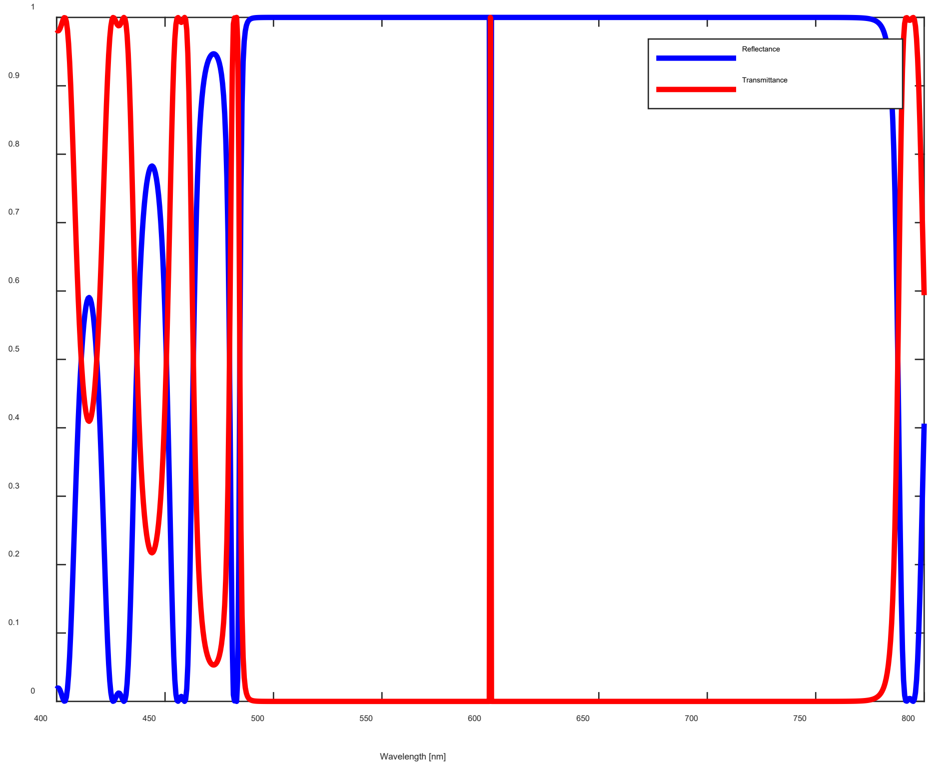
Fabry-Perot etalon

- Two highly efficient reflectors separated by a dielectric resonator LL
- Sequence $(HL)^kHLLH(LH)^k$, extremely selective transmittance

K = 2



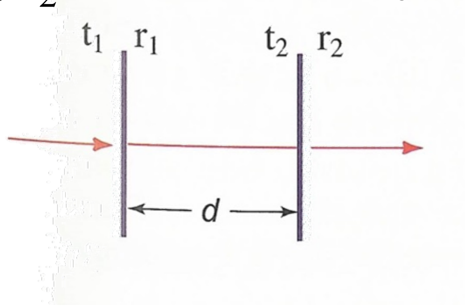
K = 8



Olivier J.F. Martin

Fabry-Perot etalon – mirror

- Interferometer made of two parallel, highly reflective, mirrors with amplitude transmittances and reflectances t_1, t_2, r_1, r_2 separated by a space d



- Cascading the elements:

$$\mathbf{M} = \begin{bmatrix} 1/t_2^* & r_2/t_2 \\ r_2^*/t_2^* & 1/t_2 \end{bmatrix} \cdot \begin{bmatrix} \exp(-j\varphi) & 0 \\ 0 & \exp(j\varphi) \end{bmatrix} \cdot \begin{bmatrix} 1/t_1^* & r_1/t_1 \\ r_1^*/t_1^* & 1/t_1 \end{bmatrix} \quad \varphi = nk_0 d$$

- Overall response (lossless and reciprocal system):

$$\mathbf{M} = \begin{bmatrix} 1/t^* & r/t \\ r^*/t^* & 1/t \end{bmatrix} \quad \text{with } t = \frac{t_1 t_2 \exp(-j\varphi)}{1 - r_1 r_2 \exp(-j2\varphi)}$$

$$|t|^2 + |r|^2 = 1 \quad t/r = -(t/r)^* \quad \arg\{t\} - \arg\{r\} = \pm\pi/2$$

Fabry-Perot etalon – mirror

- Intensity transmittance of the etalon:

$$T = |t|^2 = \frac{|t_1 t_2|^2}{\left|1 - r_1 r_2 \exp(-j2\varphi)\right|^2} = \boxed{\frac{T_{\max}}{1 + (2F/\pi)^2 \sin^2 \varphi}}$$

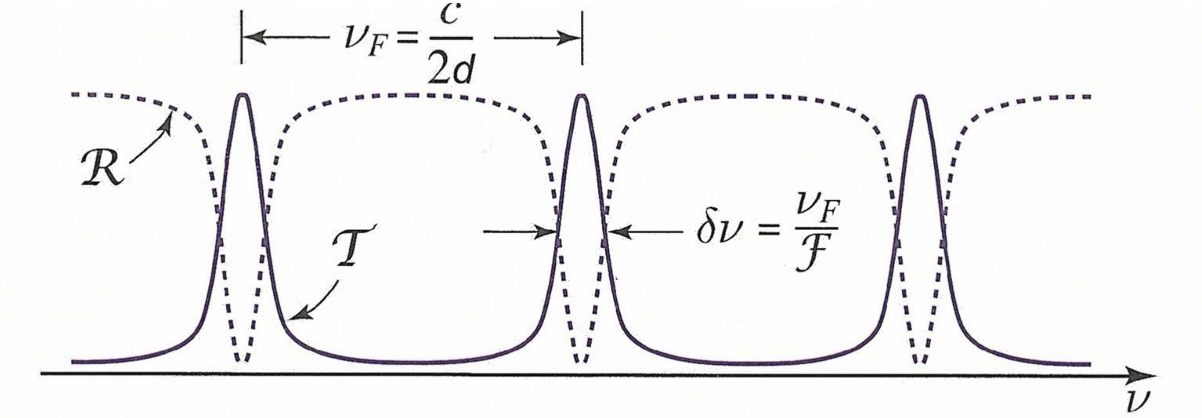
with

$$T_{\max} = \frac{|t_1 t_2|^2}{\left(1 - |r_1 r_2|\right)^2} = \frac{\left(1 - |r_1|^2\right)\left(1 - |r_2|^2\right)}{\left(1 - |r_1 r_2|\right)^2} \qquad F = \frac{\pi\sqrt{|r_1 r_2|}}{1 - |r_1 r_2|} \text{ finesse}$$

- The finesse increases with the reflectance of the mirror
- The transmittance is a periodic function of φ with period π
- The transmittance reaches a maximum when $\varphi = n\pi$

Fabry-Perot etalon – mirror

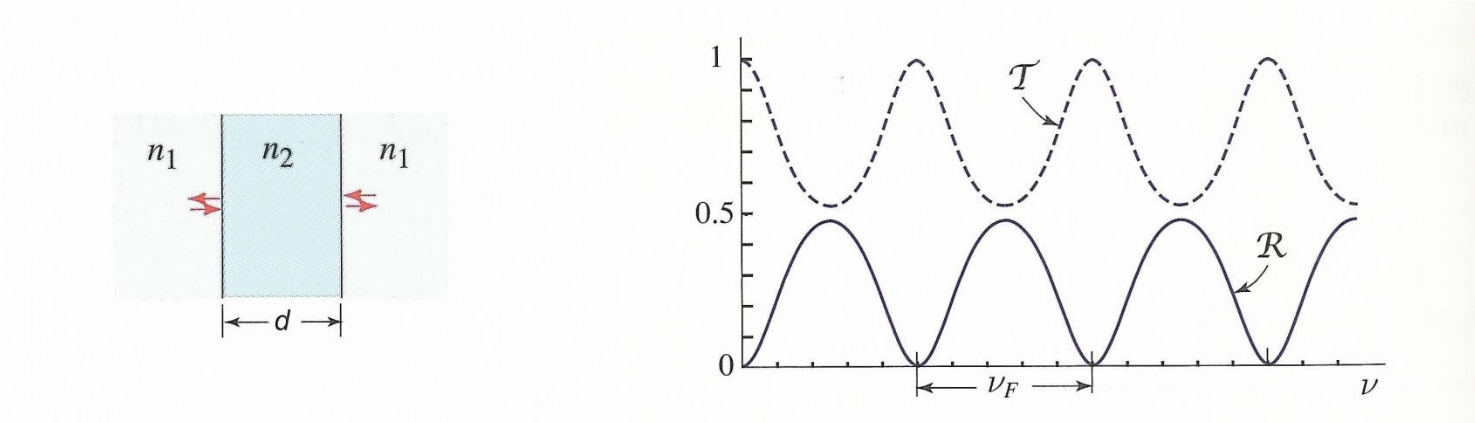
- When the finesse is large, T has sharp peaks



- Free spectral range: $\nu_F = \dfrac{c}{2d} \quad \omega_F = \dfrac{\pi c}{d}$

- Transmittance: $$T(\nu) = \frac{T_{\max}}{1 + (2F/\pi)^2 \sin^2(\pi\nu/\nu_F)}$$

Fabry-Perot etalon – dielectric slab



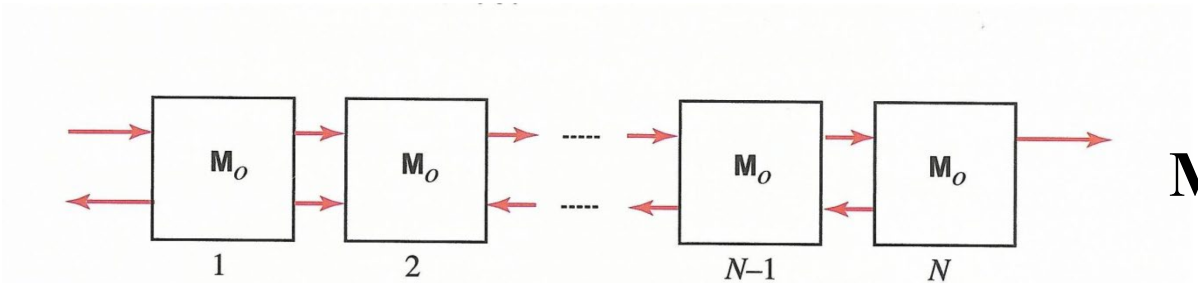
- Amplitude transmittance:

$$t = \frac{4 n_1 n_2 \exp(-j\varphi)}{(n_1 + n_2)^2 - (n_1 - n_2)^2 \exp(-j2\varphi)}$$

$$F = \frac{\pi}{4} \frac{\left| n_2^2 - n_1^2 \right|}{n_1 n_2}$$

Bragg gratings – matrix theory

- Include multiple reflections/transmissions
- Include depletion of the incident wave
- Stack of N identical generic segments, each described with the wave-transfer matrix



$$\mathbf{M}_0 = \begin{bmatrix} 1/t^* & r/t \\ r^*/t^* & 1/t \end{bmatrix}$$

- Assuming a lossless reciprocal medium:

$$|t|^2 + |r|^2 = 1 \qquad t/r = -(t/r)^* \qquad \arg\{t\} - \arg\{r\} = \pm\pi/2$$

- Overall response of the system: $\mathbf{M} = \mathbf{M}_0^N$
- Since $\det \mathbf{M}_0 = 1$ we have:

$$\boxed{\mathbf{M}_0^N = \psi_N \mathbf{M}_0 - \psi_{N-1}\mathbf{I}} \qquad \psi_N = \frac{\sin N\Phi}{\sin\Phi}$$

$$\cos\Phi = \mathrm{Re}\{1/t\}$$

Bragg gratings – matrix theory

- The overall system is also lossless and reciprocal:

$$\mathbf{M}_0^N = \begin{bmatrix} 1/t_N^* & r_N/t_N \\ r_N^*/t_N^* & 1/t_N \end{bmatrix}$$

- Overall transmittance and reflectance:

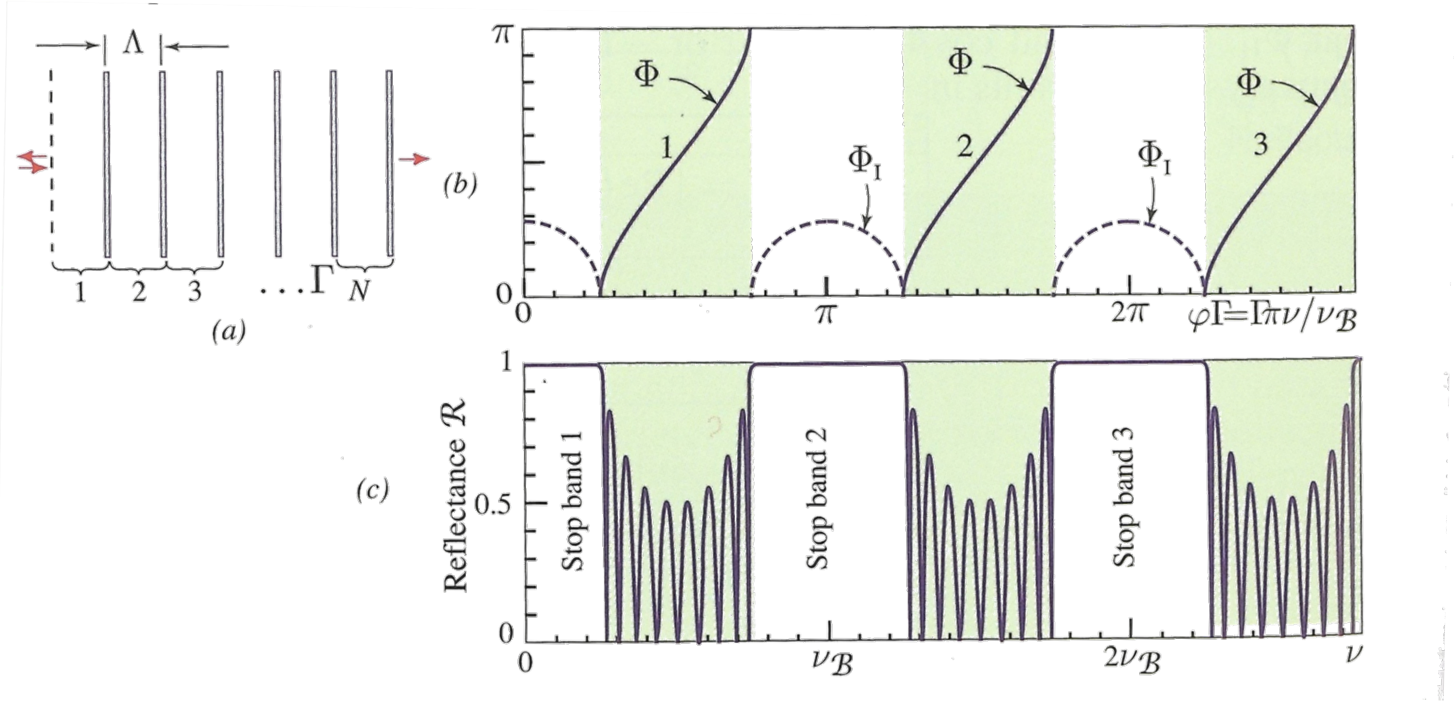
$$T_N = \left|t_N\right|^2 = \frac{T}{T + \psi_N^2 \left(1 - T\right)}$$

$$R_N = 1 - T_N = \frac{\psi_N^2 R}{1 - R + \psi_N^2 R}$$

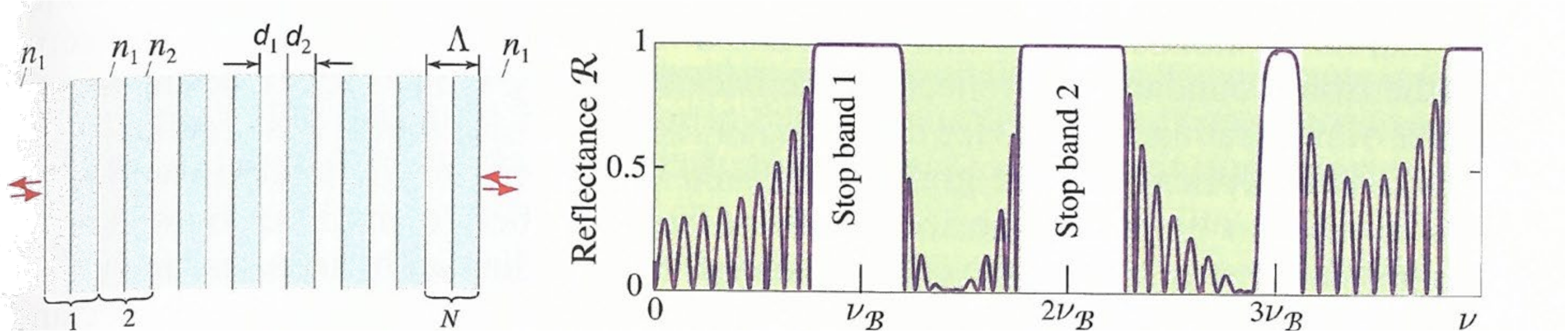
- The reflectance of the stack is related to the reflectance of the single elements via a non-linear relation

Bragg gratings – total reflection regime

- Stack of partially reflective mirrors:

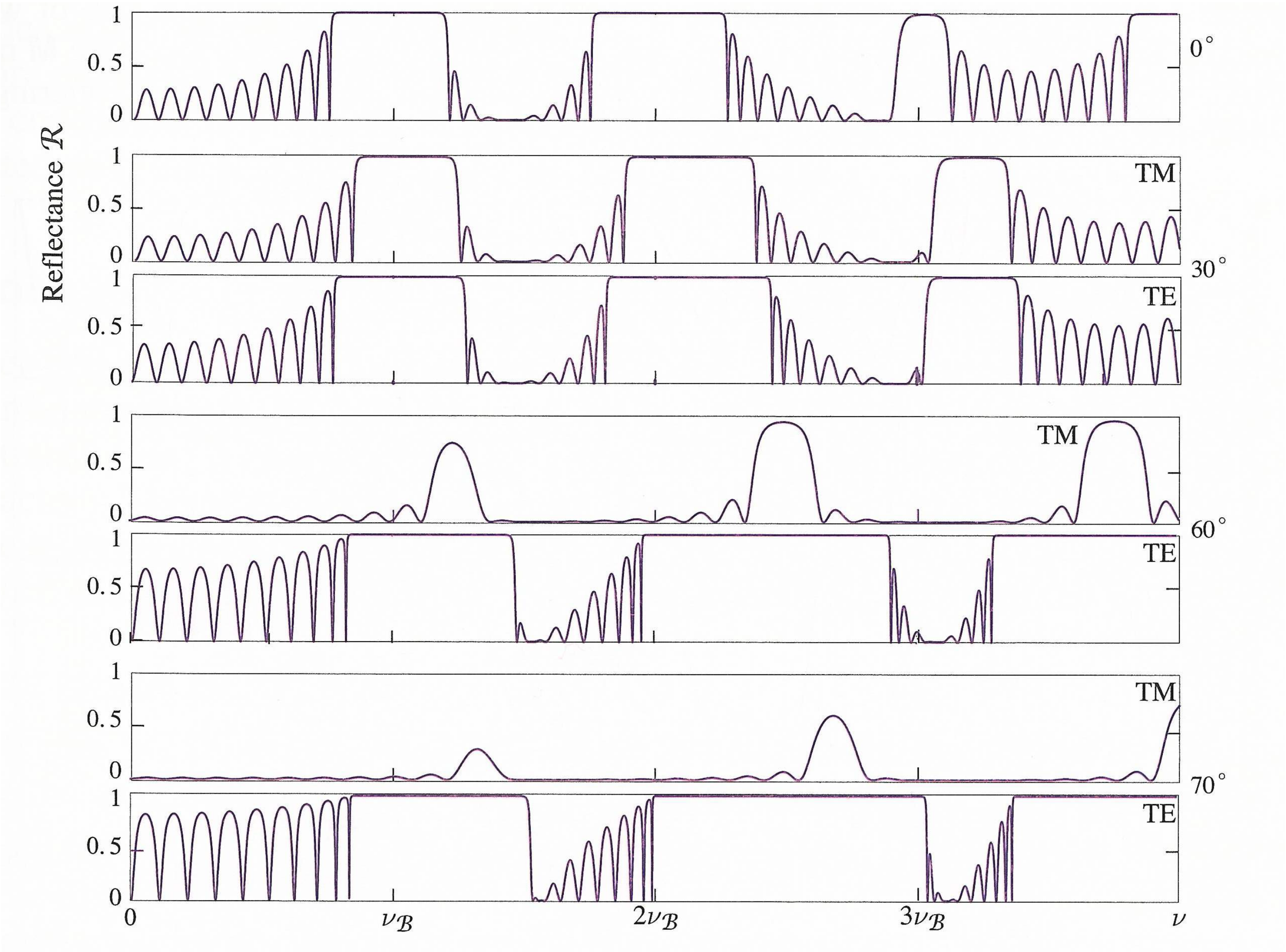


- Dielectric Bragg grating:



Bragg gratings – oblique incidence

- Polarization splitting at non-normal incidence



Selected Topics in Advanced Optics

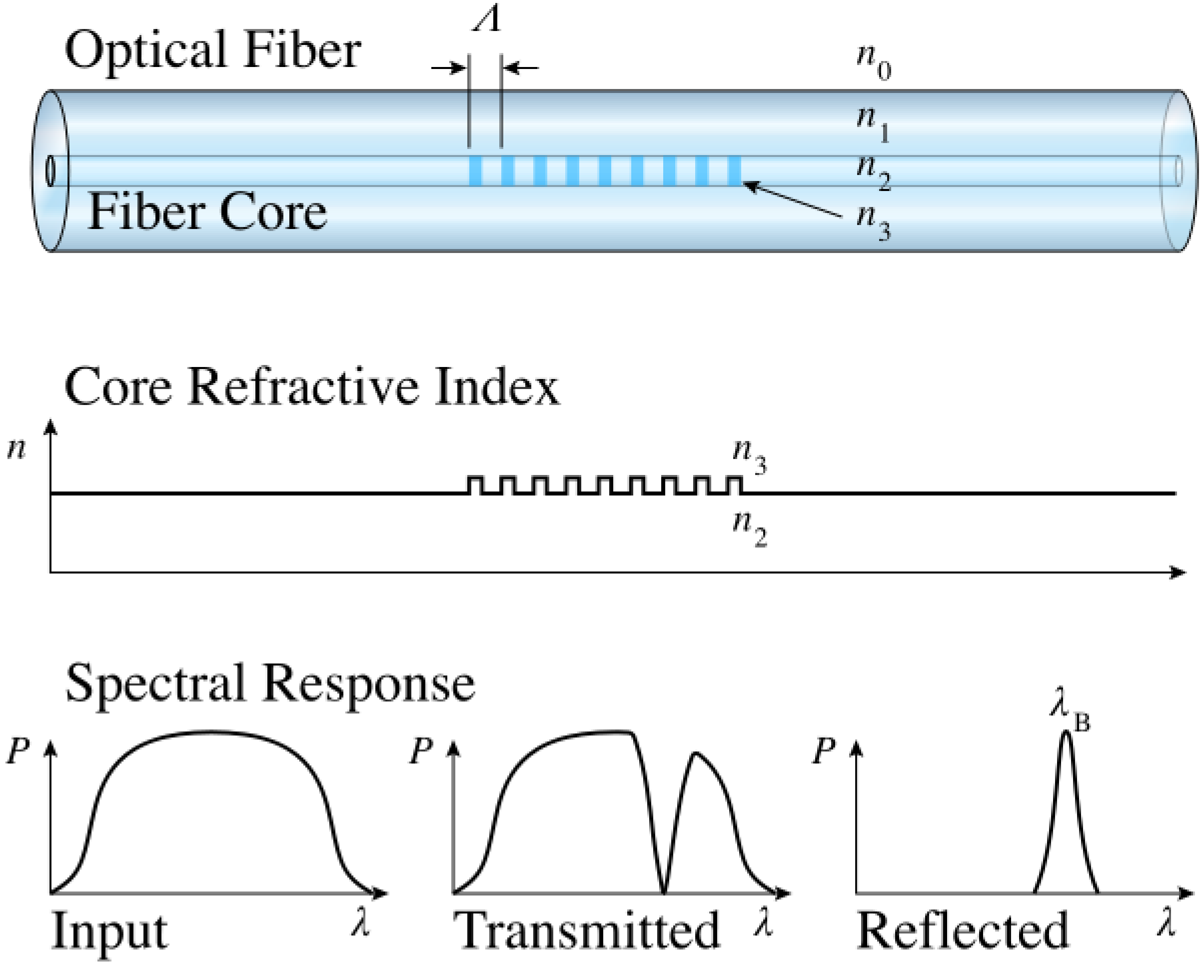
Week 9 – part 5

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

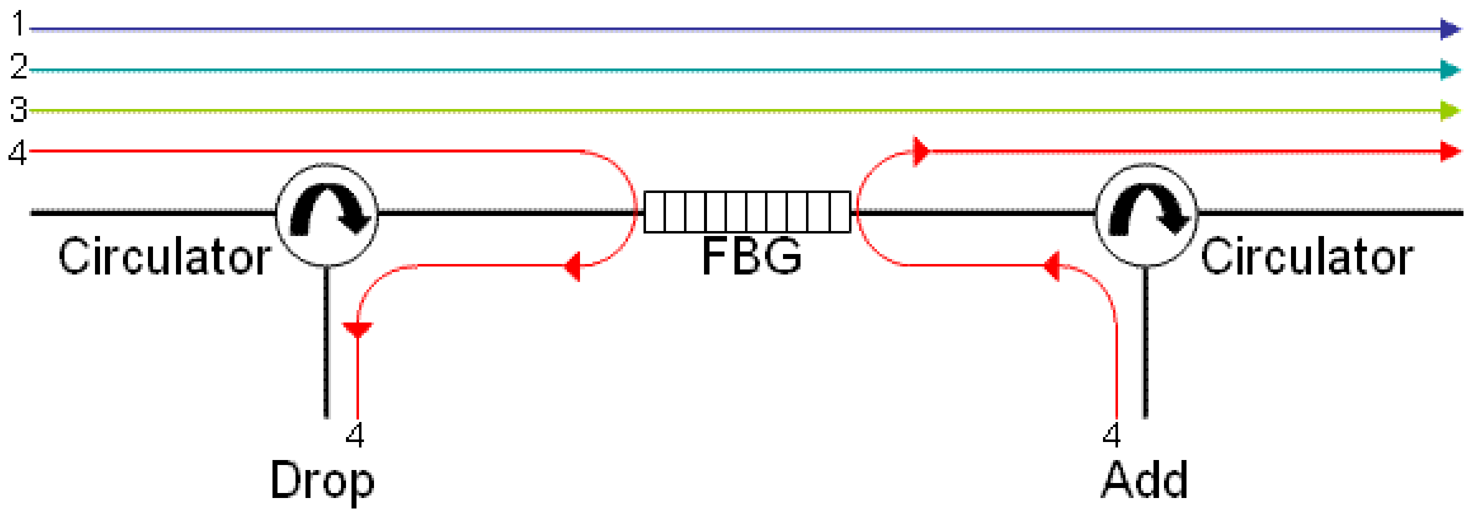
Bragg grating in optical fibre

- A Bragg grating can be inscribed into an optical fibre and used as filter



Bragg grating in optical fibre

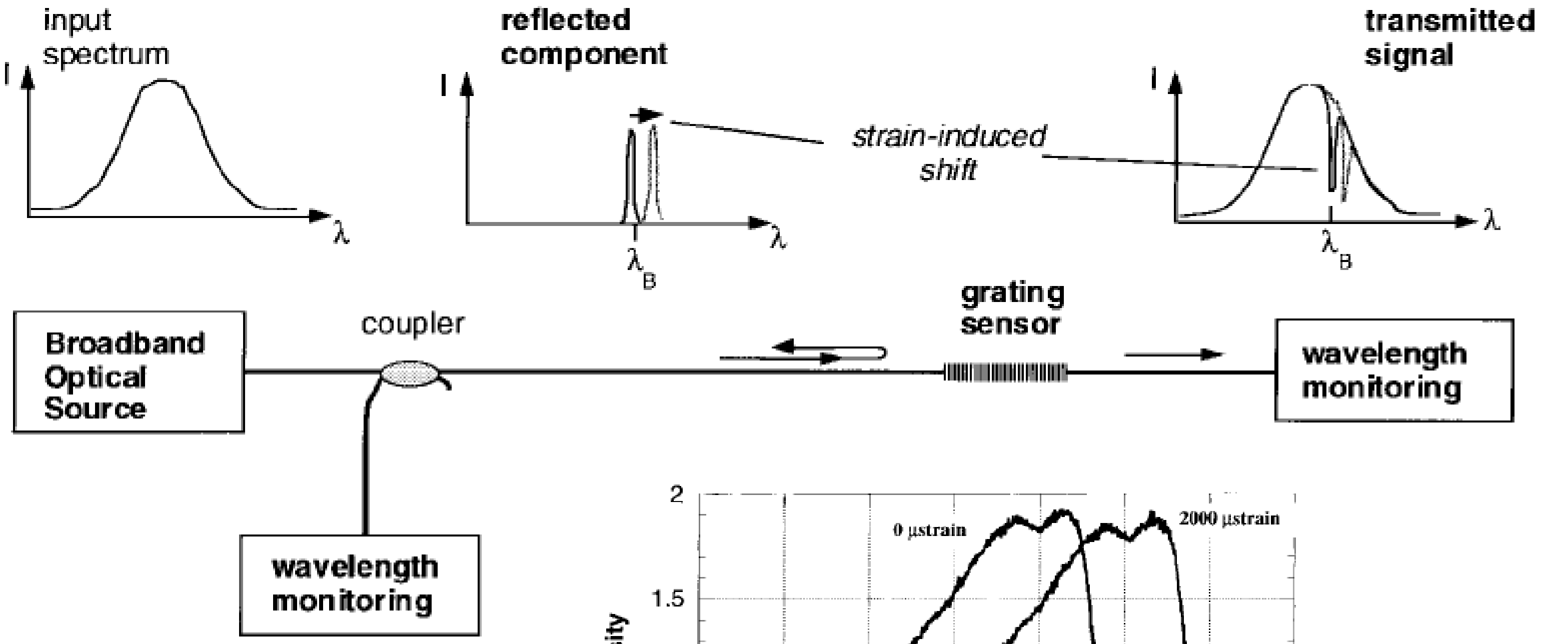
- The filtering function can be used to drop/add a specific wavelength in a WDM network

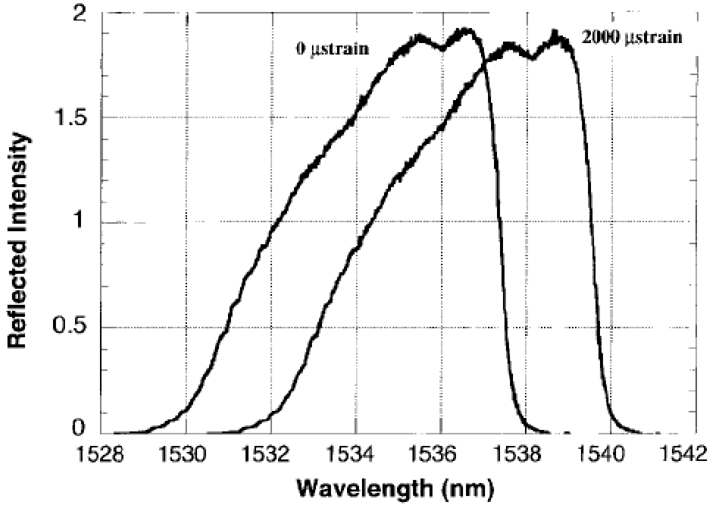


wikimedia

Bragg grating sensor

- Strain or temperature measuremnts





A.D. Kersey et al. J. Lightwave Technol. ***15*** *, 1442-1463 (1997)*

Bragg grating sensor

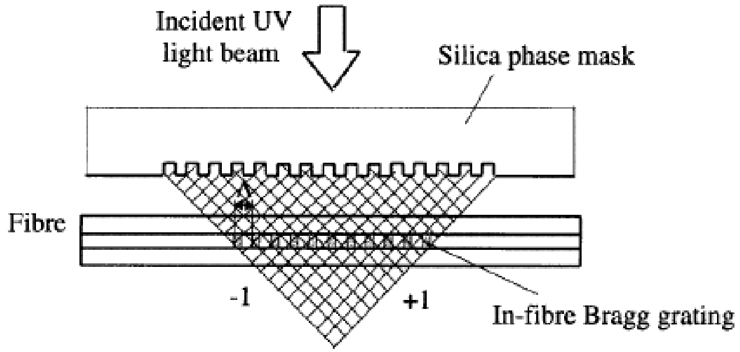
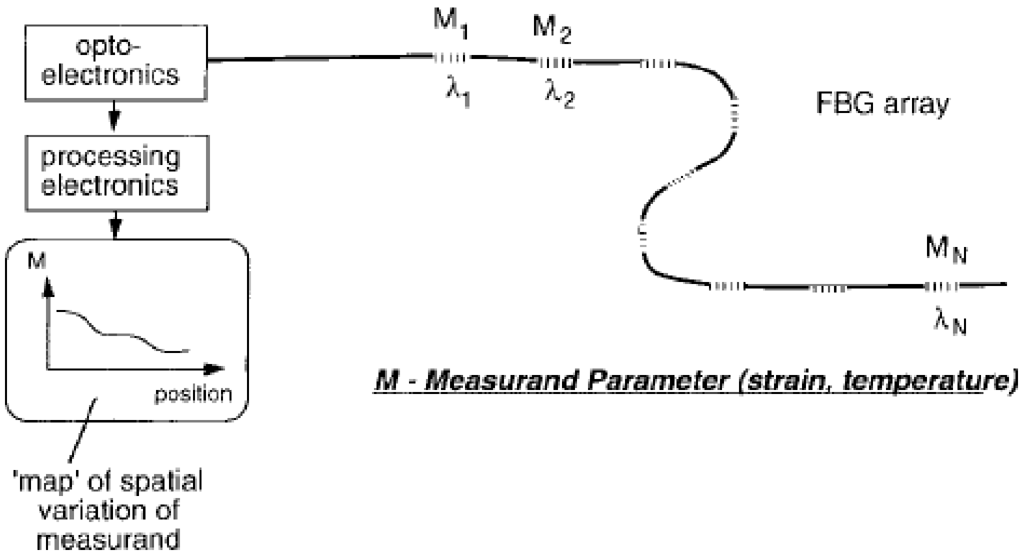


Figure 6. Schematic diagram of the phase-mask writing method.

- Measurements can be distributed along the fibre



A.D. Kersey et al. J. Lightwave Technol. ***15*** *, 1442-1463 (1997)*

Bloch surface wave

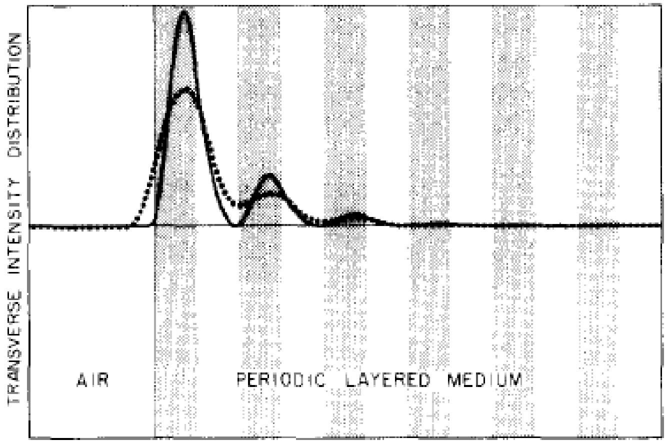
Optical surface waves in periodic layered media[a)]

Pochi Yeh and Amnon Yariv

California Institute of Technology, Pasadena, California 91125

A. Y. Cho

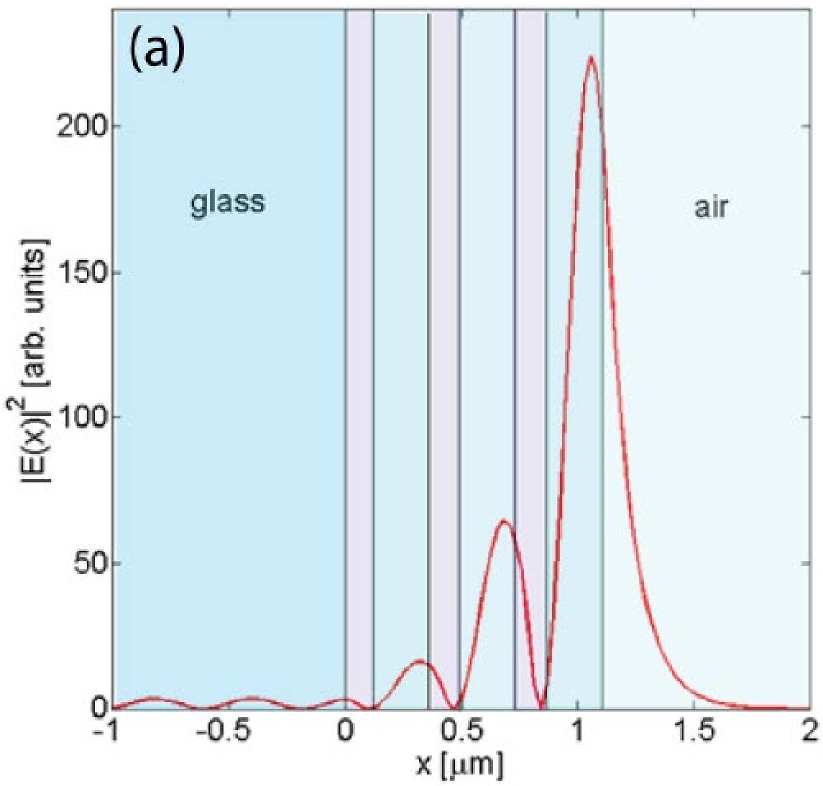
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 24 June 1977; accepted for publication 12 October 1977)



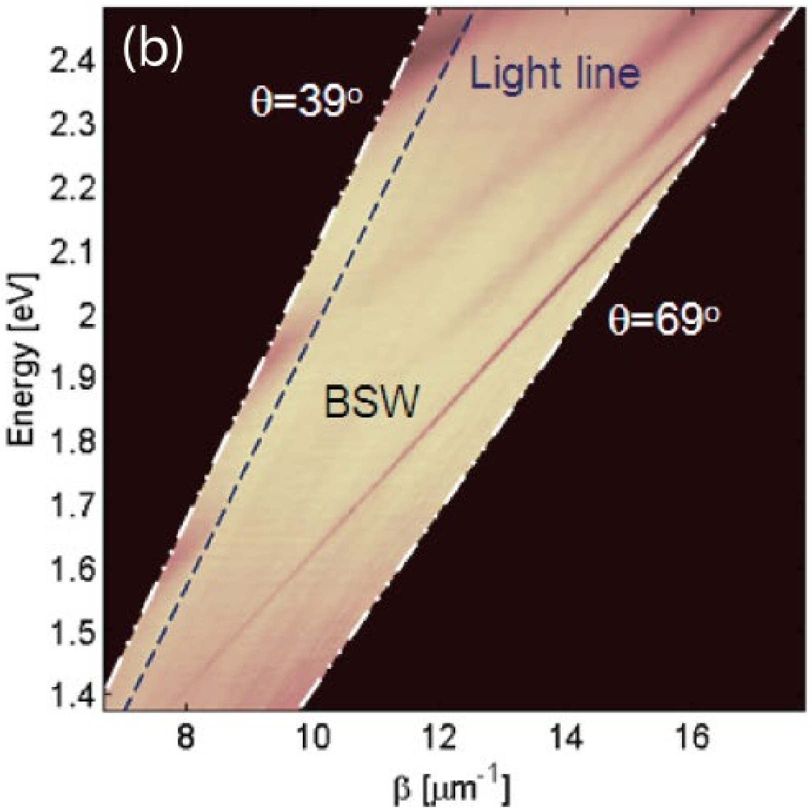
- Confined at the interface between
  a stack of periodic layers and a homogeneous space
- Analogous to surface plasmons…
- … but can exist for both polarizations (surface plasmons exist only for TM polarization) !

Bloch surface wave

- 6 layers of SiO_2 (n=1.48, 275 nm) / Si_xN_{1-x} (n=2.33, 105 nm) at λ=780 nm, fabricated by PECVD
- TE-polarized Bloch surface waves
- ~200 intensity enhancement at the top interface



Measured reflectance



Olivier J.F. Martin