

EPFL

Transformateur monophasé

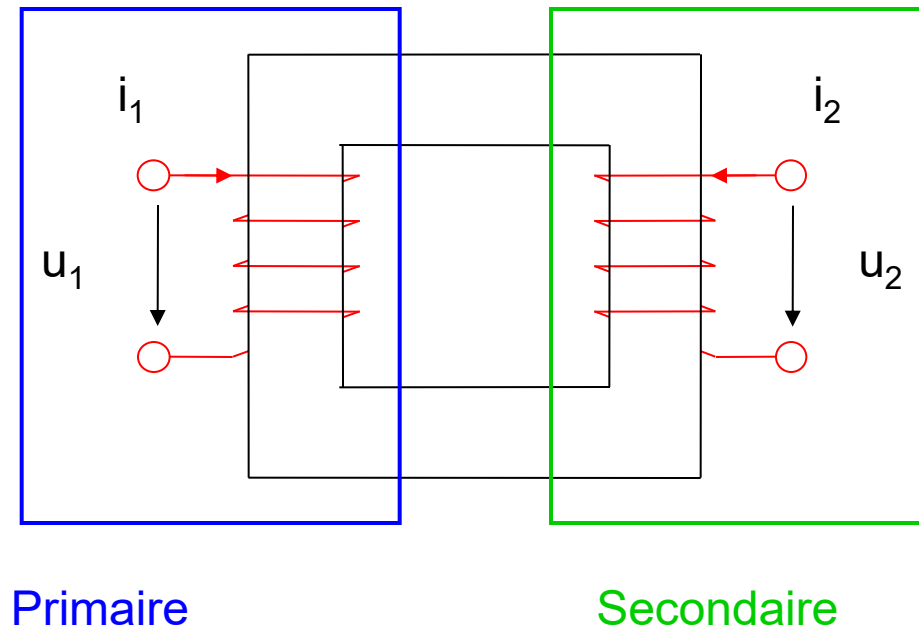
Actionneurs et systèmes électromagnétiques

Fonctionnalité

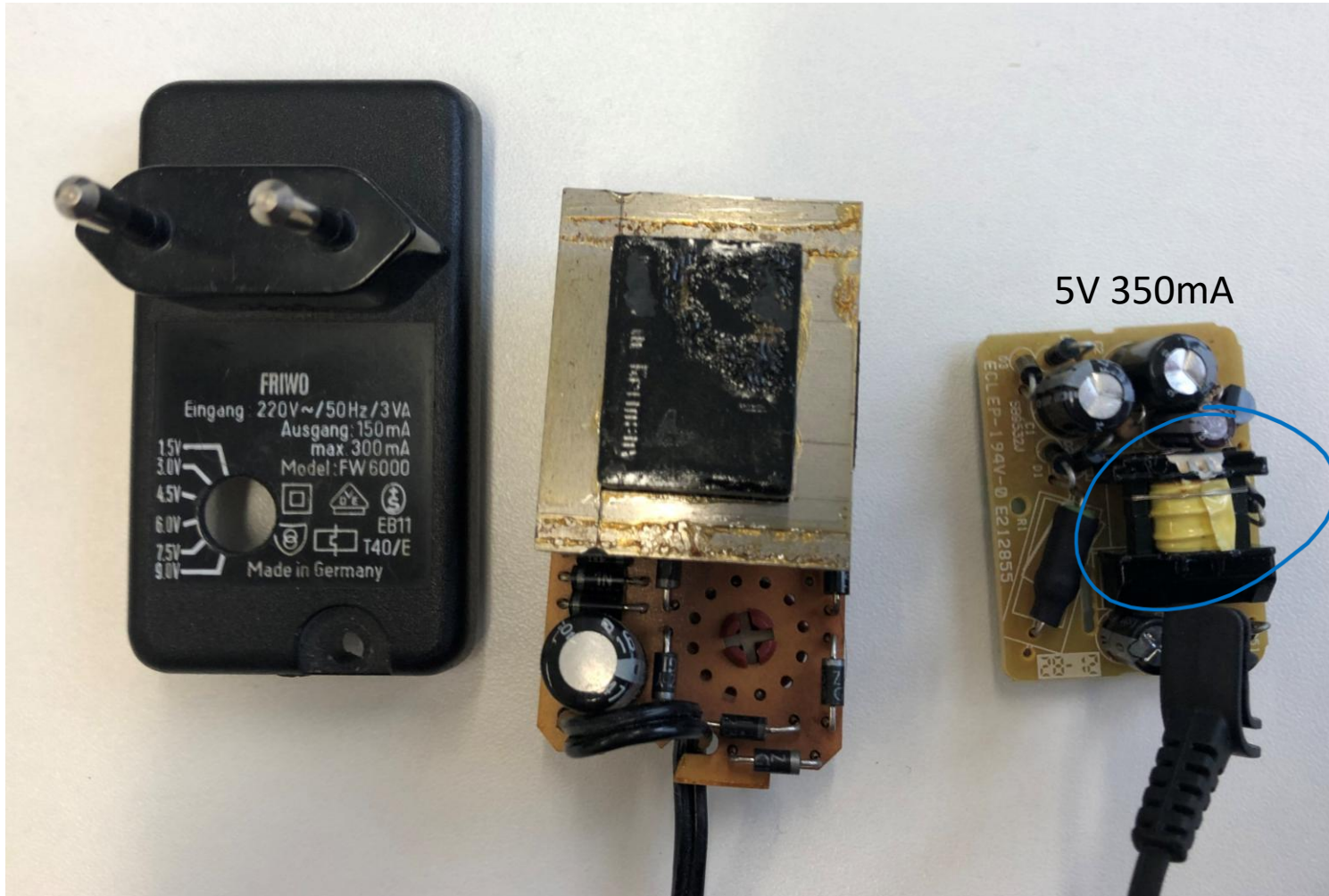


$$L = N^2 \mathcal{L} = N^2 \cdot \frac{\mu \cdot S}{l}$$

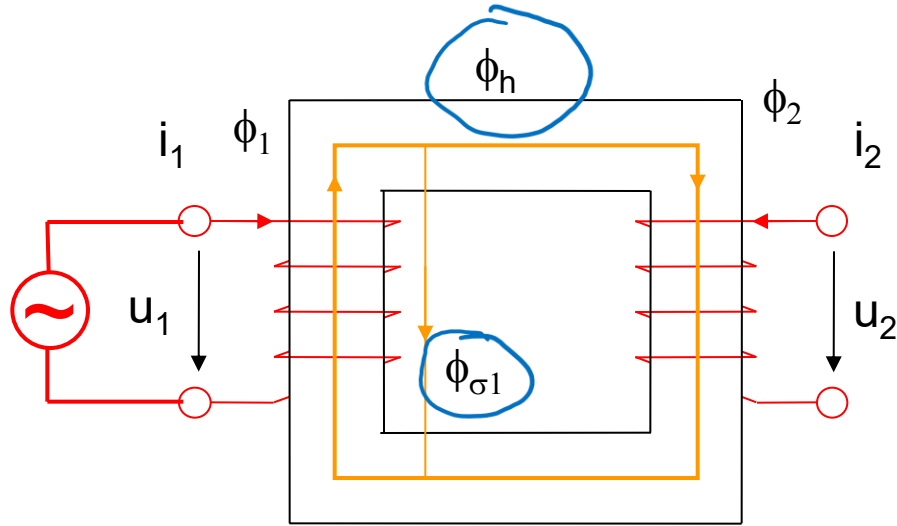
Principe



Evolution



Principe de fonctionnement



$$u_1 = R_1 i_1 + \frac{d\psi_1}{dt} = R_1 i_1 + N_1 \frac{d\phi_1}{dt}$$

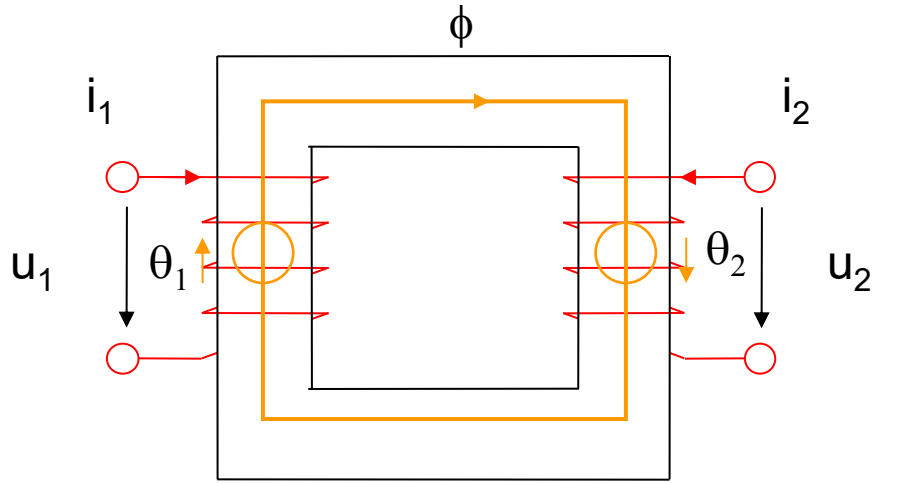
$$u_2 = \cancel{R_2 i_2} + \frac{d\psi_2}{dt}$$

$$\psi_2 = \cancel{L_{22} i_2} + L_{12} i_1$$

$$i_2 = 0$$

$$u_2 = \frac{d\psi_2}{dt} \approx \frac{d(L_{12} i_1)}{dt}$$

Transformateur Idéal



$$\mu_{\text{fer}} = \infty$$

Pas de pertes fer

$$R_1 = R_2 = 0$$

$$\phi_1 = \phi_2 = \phi$$

$$u_1 = \frac{d\psi_1}{dt}$$

$$u_2 = \frac{d\psi_2}{dt}$$

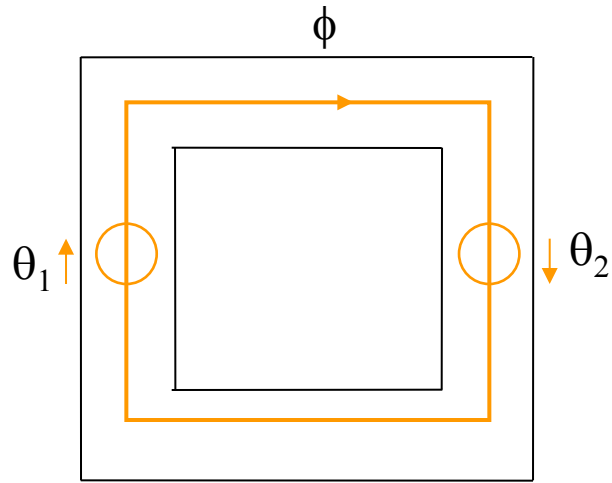
$$\psi_1 = N_1 \phi_1 = N_1 \phi \quad \rightarrow \quad u_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\phi}{dt}$$

$$\psi_2 = N_2 \phi_2 = N_2 \phi \quad \rightarrow \quad u_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{u_1}{N_1} = \frac{u_2}{N_2}$$

$$\frac{u_1}{u_2} = \frac{N_1}{N_2}$$

Transformateur Idéal



$\mu_{\text{fer}} = \infty$

Pas de pertes fer

$$\rightarrow \theta_1 + \theta_2 = 0$$

$$\frac{u_1}{u_2} = \frac{N_1}{N_2}$$

$\frac{N_2}{N_1} = \overset{\text{rapport de transformation}}{u}$

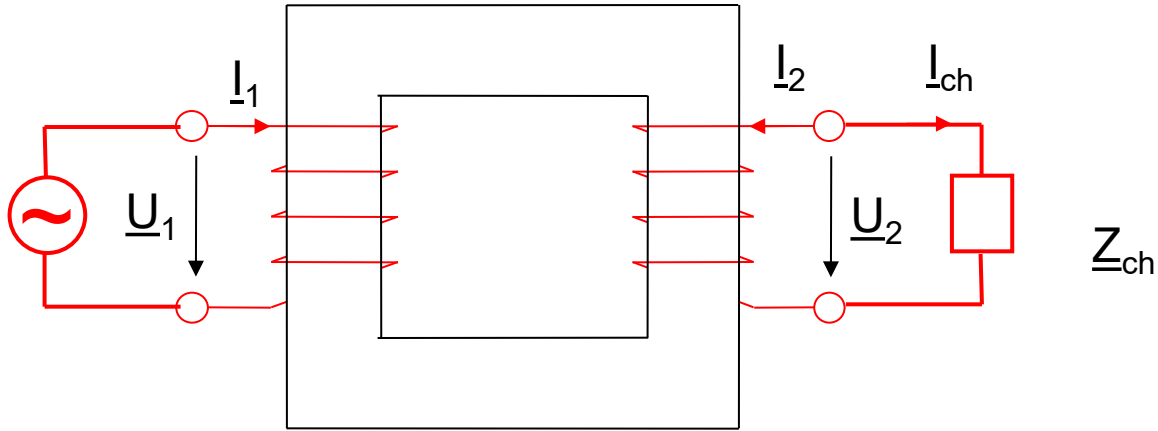
$$N_1 \cdot i_1 + N_2 \cdot i_2 = 0$$

$$N_1 i_1 = -N_2 i_2$$

$$\frac{-i_2}{i_1} = \frac{N_1}{N_2}$$

$$u = \frac{u_1}{u_2} = -\frac{i_2}{i_1}$$

Alimentation sinusoïdale



$$\frac{N_1}{N_2} = \frac{u_1}{u_2} = -\frac{i_2}{i_1} = \ddot{u} \quad \longrightarrow \quad \frac{N_1}{N_2} = \frac{\underline{U}_1}{\underline{U}_2} = -\frac{\underline{I}_2}{\underline{I}_1} = \ddot{u}$$

$$\frac{\underline{U}_1}{\underline{I}_1} = \frac{\frac{N_1}{N_2} \cdot \underline{U}_2}{\frac{N_2}{N_1} \cdot \underline{I}_2} = \left(\frac{N_1}{N_2} \right)^2 \frac{\underline{U}_2}{-\underline{I}_2} = \ddot{u}^2 \frac{\underline{U}_2}{\underline{I}_{ch}} = \ddot{u}^2 \underline{Z}_{ch}$$

Grandeurs rapportées

Tension secondaire rapportée au primaire

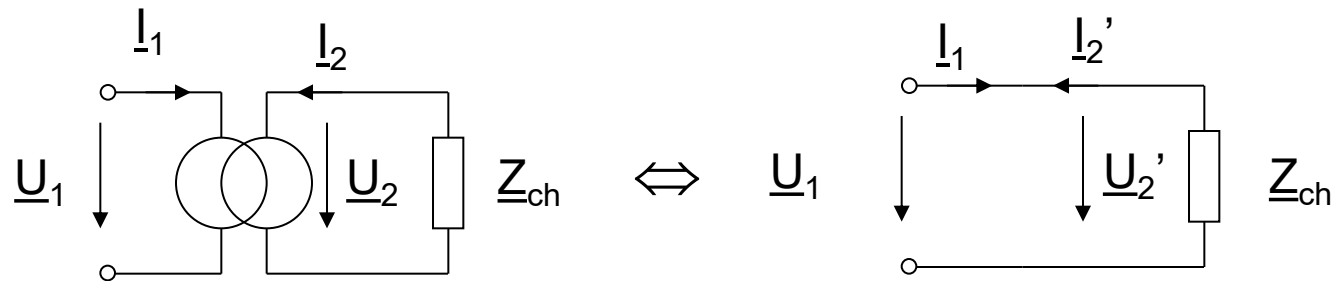
$$\underline{U}_2' = \ddot{u}\underline{U}_2$$

Courant secondaire rapportée au primaire

$$\underline{I}_2' = \frac{\underline{I}_2}{\ddot{u}}$$

Impédance secondaire rapportée au primaire

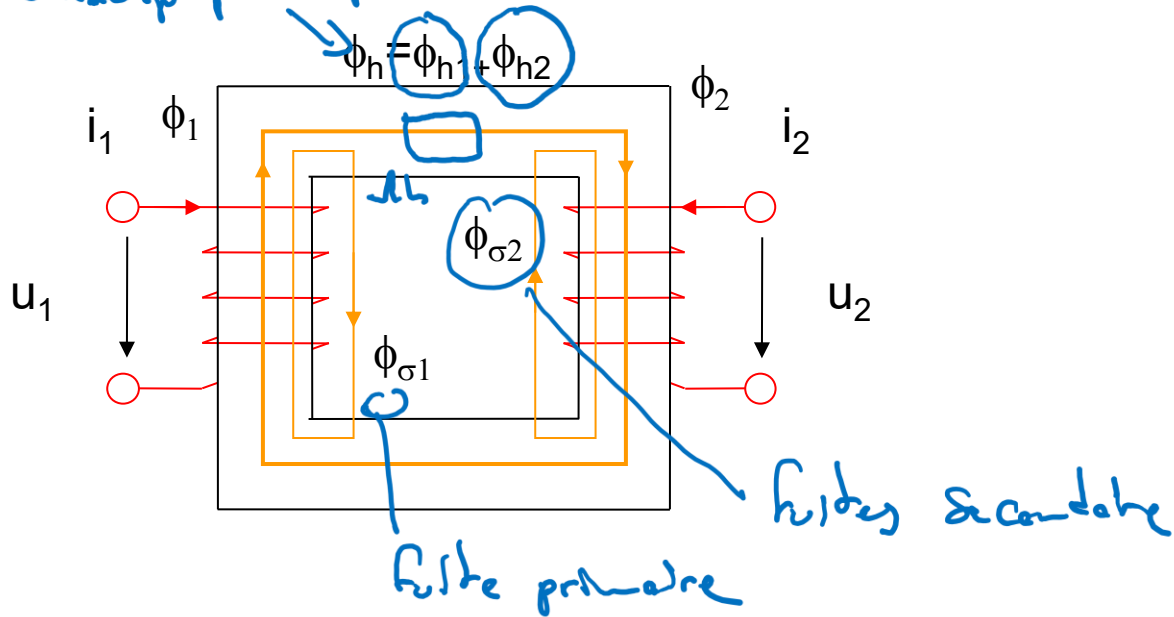
$$\underline{Z}_{ch}' = \ddot{u}^2 \underline{Z}_{ch}$$



Dans un transformateur idéal $\underline{I}_2' = -\underline{I}_1$ et $\underline{U}_2' = \underline{U}_1$

Transformateur réel: Flux

Champs principaux



$$\phi_1 = \phi_{\sigma 1} + \phi_h$$

$$\phi_2 = \phi_{\sigma 2} + \phi_h$$

Handwritten equation for ϕ_2 (labeled "mesure" des 1):

$$\phi_2 = \underbrace{\phi_{h1}}_{\text{champ principal 1}} + \phi_{\sigma 2} + \underbrace{\phi_{h2}}_{\text{champ principal 2}}$$

Additional handwritten notes: "propre 1" and "lignes 1".

~~$\mu_{\text{fer}} = \infty$~~

Pas de pertes fer

~~$R_1 = R_2 = 0$~~

→ Pas de saturation =>

$$\phi_h = \phi_{h1} + \phi_{h2}$$

Avec:

$$\phi_{h1} = \Lambda_h N_1 i_1$$

$$\phi_{h2} = \Lambda_h N_2 i_2$$

$$\phi_h = \Lambda_h (N_1 i_1 + N_2 i_2)$$

Θ_2

Inductances

$$\phi_h = \Lambda_h (N_1 i_1 + N_2 i_2)$$

$$\psi_1 = N_1 \phi_1 = N_1 \phi_{\sigma 1} + N_1 \phi_h$$

$$\psi_2 = N_2 \phi_2 = N_2 \phi_{\sigma 2} + N_2 \phi_h$$

Inductances de fuite

$$L_{\sigma 1} = \frac{N_1 \phi_{\sigma 1}}{i_1} = N_1^2 \Lambda_{\sigma 1}$$

$$L_{\sigma 2} = \frac{N_2 \phi_{\sigma 2}}{i_2} = N_2^2 \Lambda_{\sigma 2}$$

Inductances principales

$$L_{h1} = \frac{N_1 \phi_{h1}}{i_1} = N_1^2 \Lambda_h$$

$$L_{h2} = \frac{N_2 \phi_{h2}}{i_2} = N_2^2 \Lambda_h$$

$$L_{12} =$$

$$L = \frac{\psi_{\text{mesuré}}}{i_{\text{créé}}}$$

$$L_{12} = \frac{\psi_{12 \text{ mesuré}}}{i_{2 \text{ créé}}}$$

Equations de tension induite

$$U_2 = R_2 i_2 + \frac{d\psi_2}{dt}$$

$$U_2 = R_2 i_2 + N_2 \left(\frac{d\phi_{\sigma 2}}{dt} + \frac{d\phi_h}{dt} \right)$$

$$U_2 = R_2 i_2 + \frac{N_2}{N_2} L_{\sigma 2} \frac{di_2}{dt} + N_2 \frac{d\phi_h}{dt}$$

$$U_2 = R_2 i_2 + L_{\sigma 2} \frac{di_2}{dt} + N_2^2 \Lambda_h \frac{di_2}{dt} + N_2 N_2 \Lambda_h \frac{di_2}{dt}$$

$$U_2 = R_2 i_2 + L_{\sigma 2} \frac{di_2}{dt} + L_{h1} \left(\frac{di_2}{dt} + \frac{N_2}{N_2} \frac{di_2}{dt} \right)$$

$$L_{12} = \frac{N_2 \phi_{h2}}{i_2} = N_2 N_2 \Lambda_{h2}$$

Equations de tension en grandeurs rapportées

$$\ddot{u} = \frac{N_1}{N_2}$$

$$u_2' = \ddot{u} u_2$$

$$i_2' = \frac{i_2}{\ddot{u}}$$

$$R_2' = \ddot{u}^2 R_2$$

$$L_{\sigma 2}' = \ddot{u}^2 L_{\sigma 2}$$

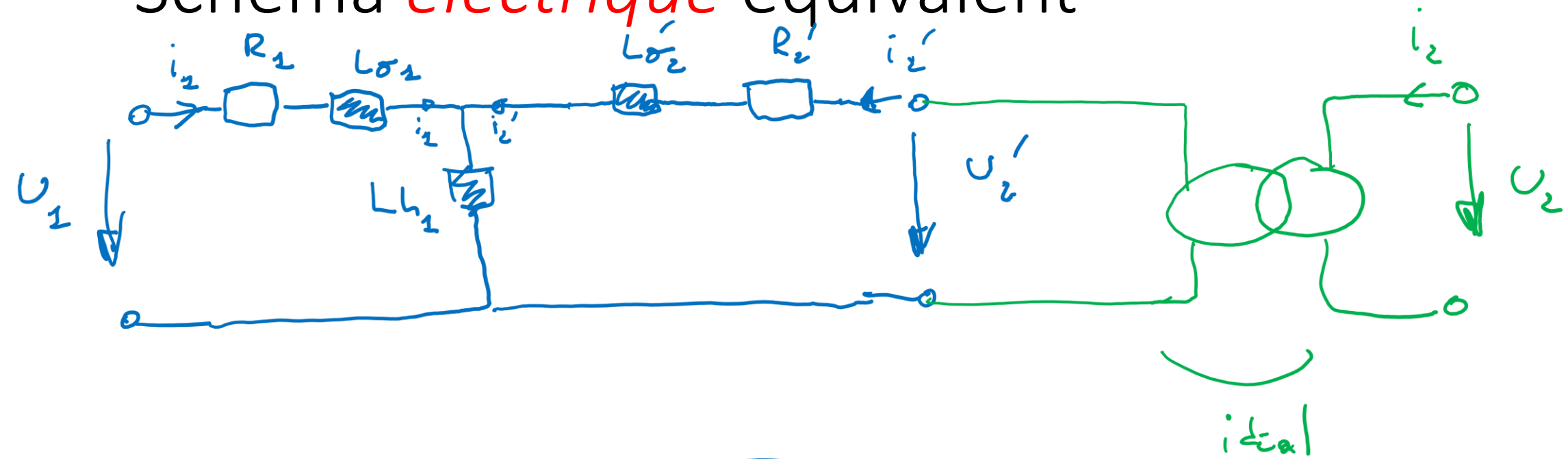
$$L_{h2}' = \ddot{u}^2 L_{h2} = L_{h1}$$

$$u_1 = R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} + L_{h1} \left(\frac{di_1}{dt} + \frac{N_2}{N_1} \frac{di_2}{dt} \right)$$

$$u_2 = R_2 i_2 + L_{\sigma 2} \frac{di_2}{dt} + L_{h2} \left(\frac{di_2}{dt} + \frac{di_2'}{dt} \right)$$

$$u_2' = R_2' i_2' + L_{\sigma 2}' \frac{di_2'}{dt} + L_{h2}' \left(\frac{di_2'}{dt} + \frac{di_2'}{dt} \right)$$

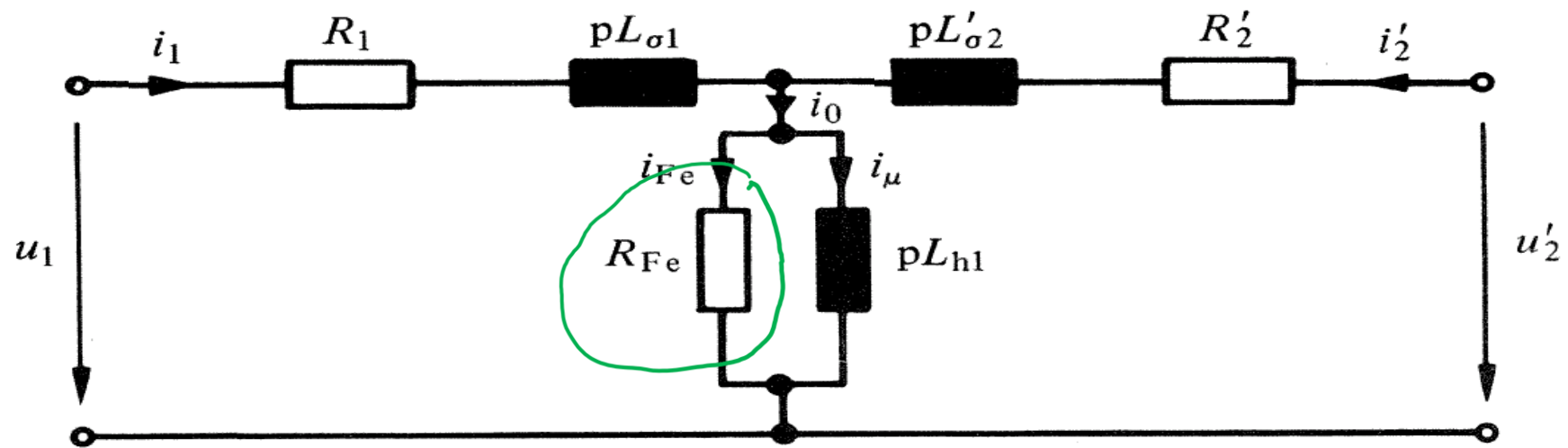
Schéma *électrique* équivalent



$$u_1 = R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} + L_{h1} \left(\frac{di_1}{dt} + \frac{di'_2}{dt} \right)$$

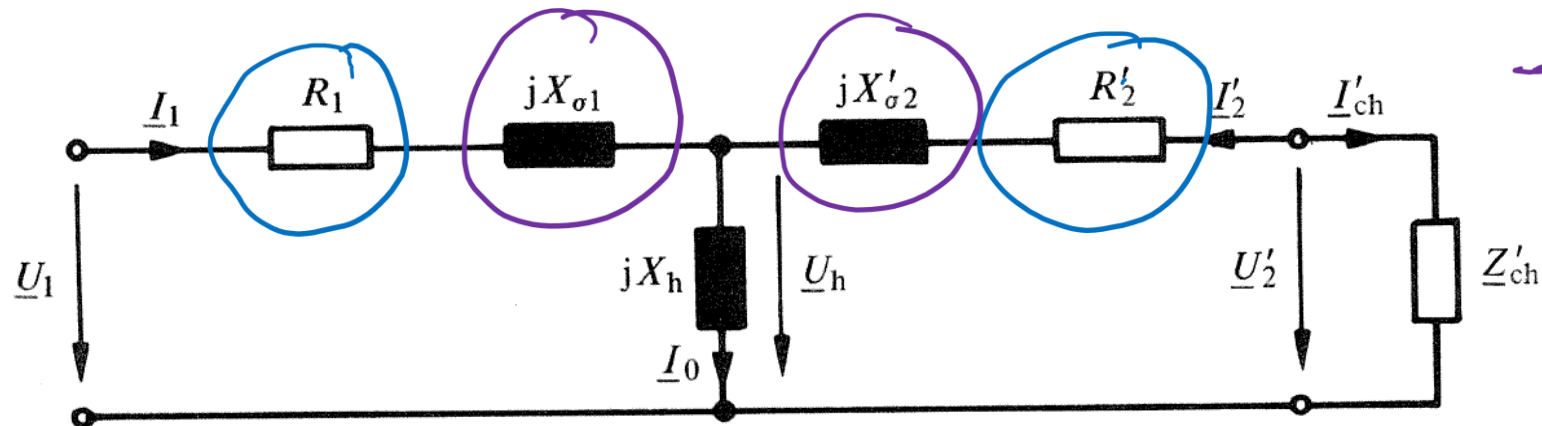
$$u'_2 = R'_2 i'_2 + L'_{\sigma 2} \frac{di'_2}{dt} + L_{h1} \left(\frac{di_1}{dt} + \frac{di'_2}{dt} \right)$$

Pertes fer



Régime permanent (sinus)

$$\begin{aligned} \underline{U}_1 &= R_1 \underline{I}_1 + j\omega L_{\sigma 1} \underline{I}_1 + j\omega L_{h1} (\underline{I}_1 + \underline{I}_2') & \underline{U}_2' &= R_2' \underline{I}_2' + j\omega L_{\sigma 2} \underline{I}_2' + j\omega L_{h1} (\underline{I}_1 + \underline{I}_2') \\ &= R_1 \underline{I}_1 + jX_{\sigma 1} \underline{I}_1 + jX_h (\underline{I}_1 + \underline{I}_2') & & R_2' \underline{I}_2' + jX_{\sigma 2} \underline{I}_2' + jX_h (\underline{I}_1 + \underline{I}_2') \end{aligned}$$



$$X_h = \omega L_h$$

— perdas
— circuit