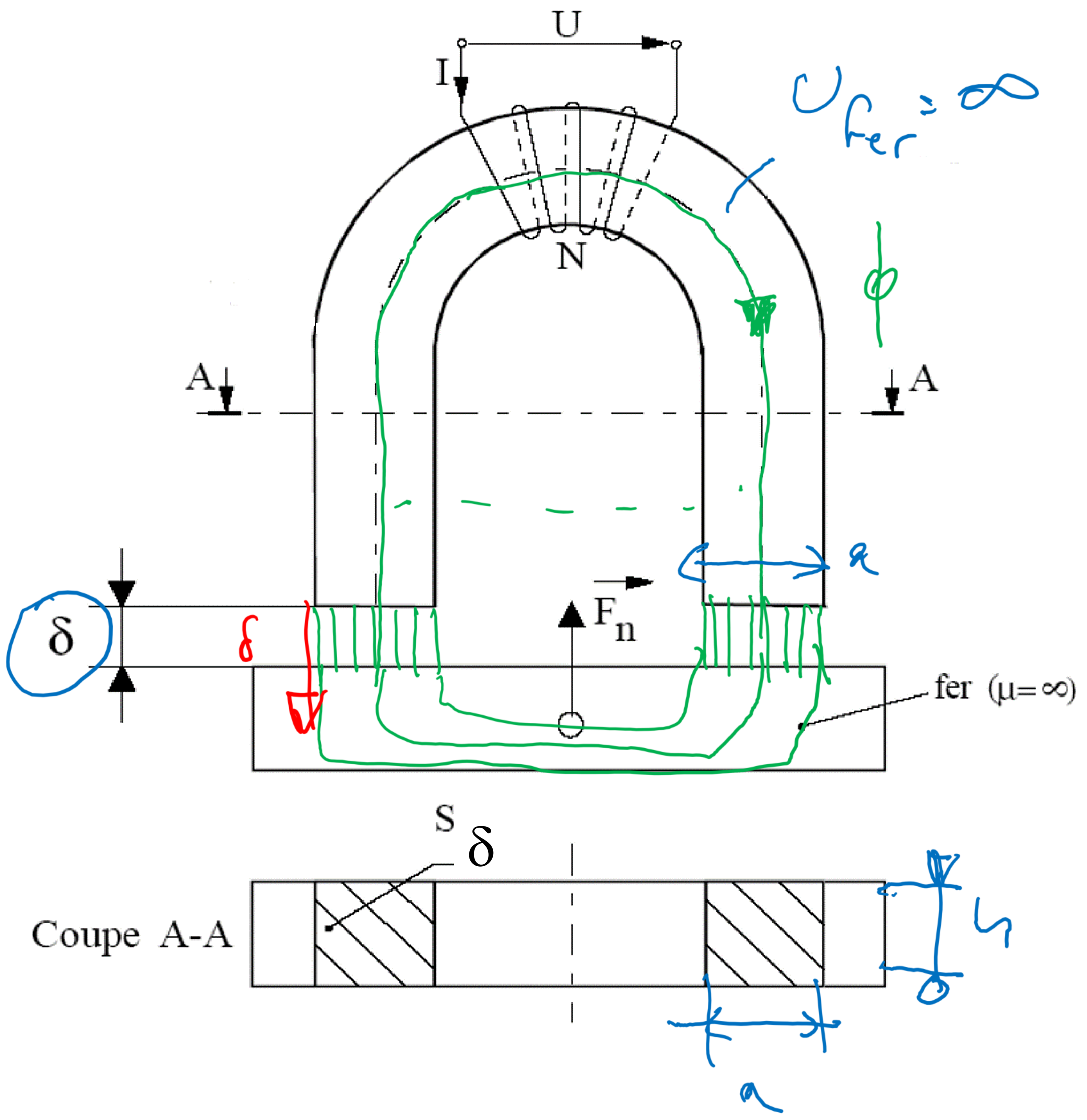
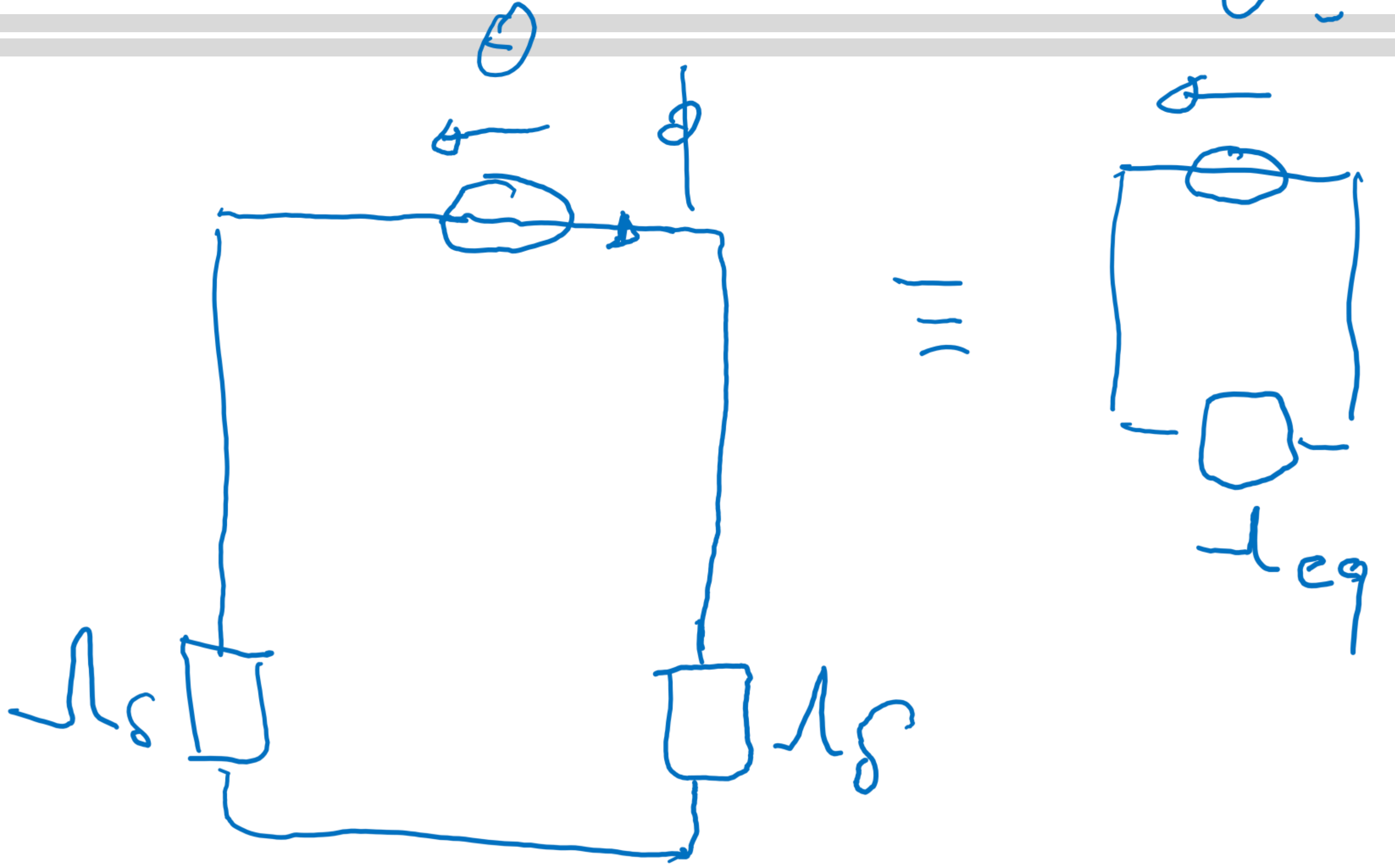
Conversion électromécanique

Actionneurs et systèmes électromagnétiques

Exemple: électroaimant



$\Theta = NI$



$$\Lambda_{eq} = \frac{\Lambda_\delta}{2}$$

$$\Lambda_{eq} = \frac{1}{\frac{1}{\Lambda_\delta} + \frac{1}{\Lambda_\delta}}$$

$$\Lambda_\delta = \frac{\mu S}{l} = \frac{\mu_0 a \cdot h}{\delta}$$

$$L_{11} = \frac{\Psi_{11}}{i_1} = \frac{N_1 \phi_{11}}{i_1} = \frac{N \phi}{I} = \frac{N \Lambda_{eq} \cdot NI}{I} = N^2 \Lambda_{eq}$$

$$L = N^2 \frac{\mu_0 a h}{2\delta}$$

Expression de la force

$$U = Ri + \frac{d\varphi}{dt} \qquad \varphi = Li$$

$$U = Ri + \frac{d}{dt}(Li) = Ri + \underbrace{L\frac{di}{dt}}_{I = \text{cte} \to 0} + i\frac{dL}{dt}$$

$$U = RI + I\frac{dL}{dt} \quad \Big\| \cdot I \quad \Rightarrow \quad \underbrace{UI}_{P_{el}} = \underbrace{RI^2}_{P_{joule}} + \underbrace{I^2\frac{dL}{dt}}_{P_{mag} + P_{mec}}$$

$$W_{mag} + W_{mec} = \int_t I^2 \frac{dL}{dt} \cdot dt = \int I^2 dL = LI^2$$

$$W_{mag} = \frac{1}{2}LI^2 \qquad W_{mec} = LI^2 - \frac{1}{2}LI^2 = \frac{1}{2}LI^2$$

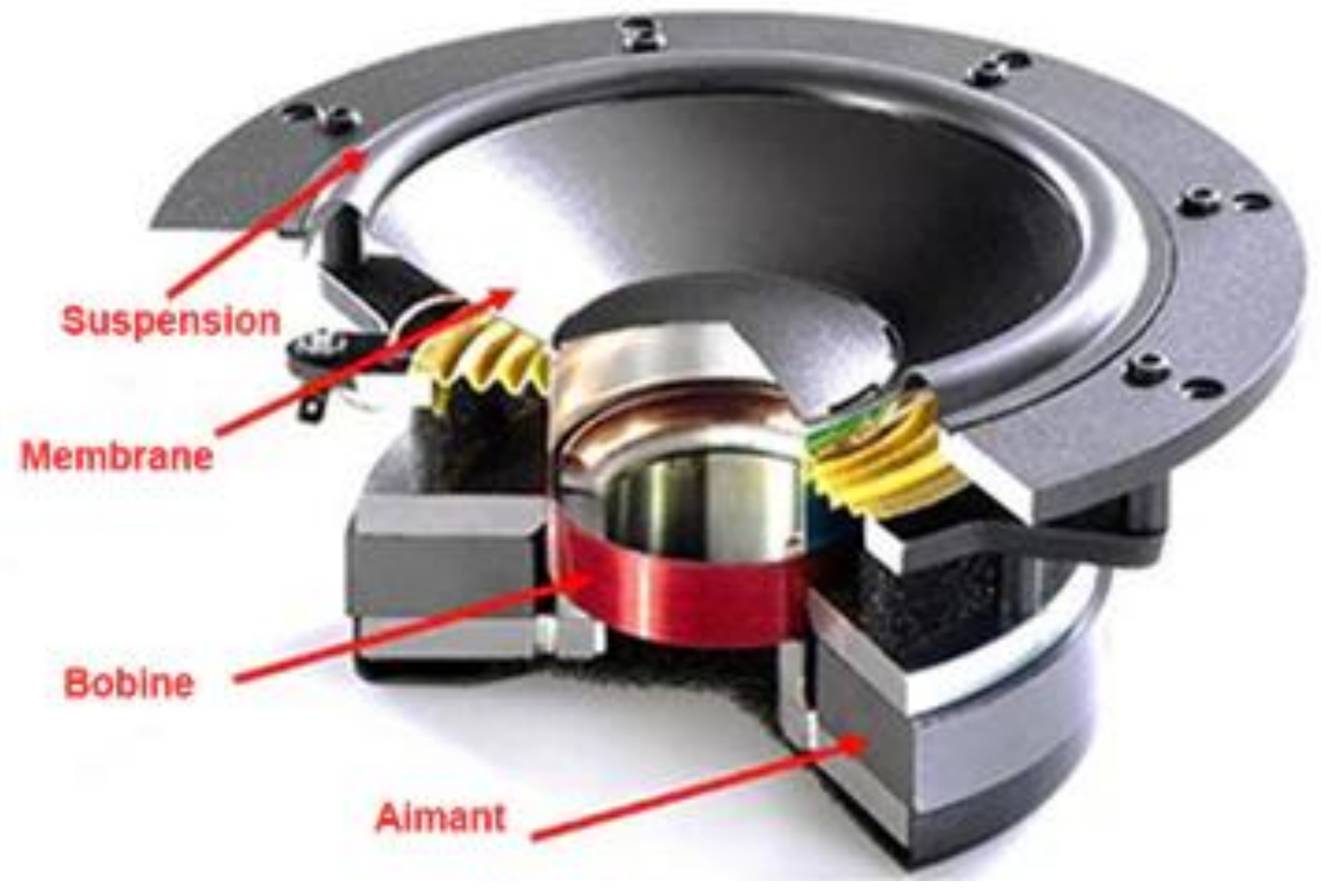
$$W_{mec} = \int F(\delta) \cdot d\delta \Rightarrow F(\delta) = \frac{dW_{mec}}{d\delta} = \frac{1}{2}\frac{dL}{d\delta} \cdot I^2 = -\frac{1}{4}\frac{N^2 \mu_0 a h}{\delta^2} I^2$$

Définition

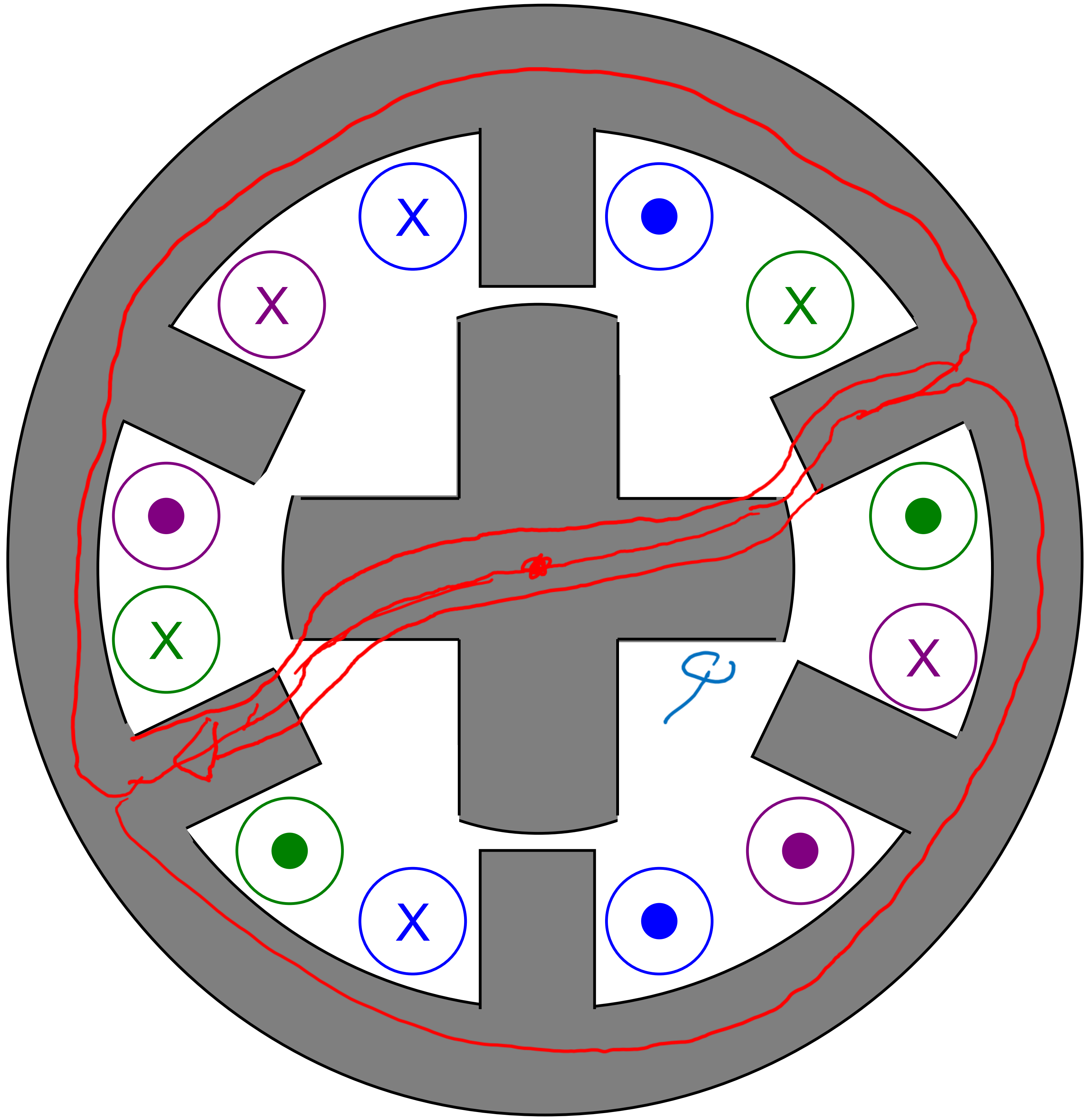
Système électromécanique:

- ensemble de circuits électriques liés mécaniquement ou couplés magnétiquement;
- géométriquement déformable;
- possède un nombre variable *n* de degrés de liberté mécaniques.

Exemples de système électromécanique



Source: conector.fr



Actionneur réluctant

Equations des systèmes électromécaniques

$$u_j = R_j i_j + \frac{d\Psi_j}{dt}$$

(k circuits)

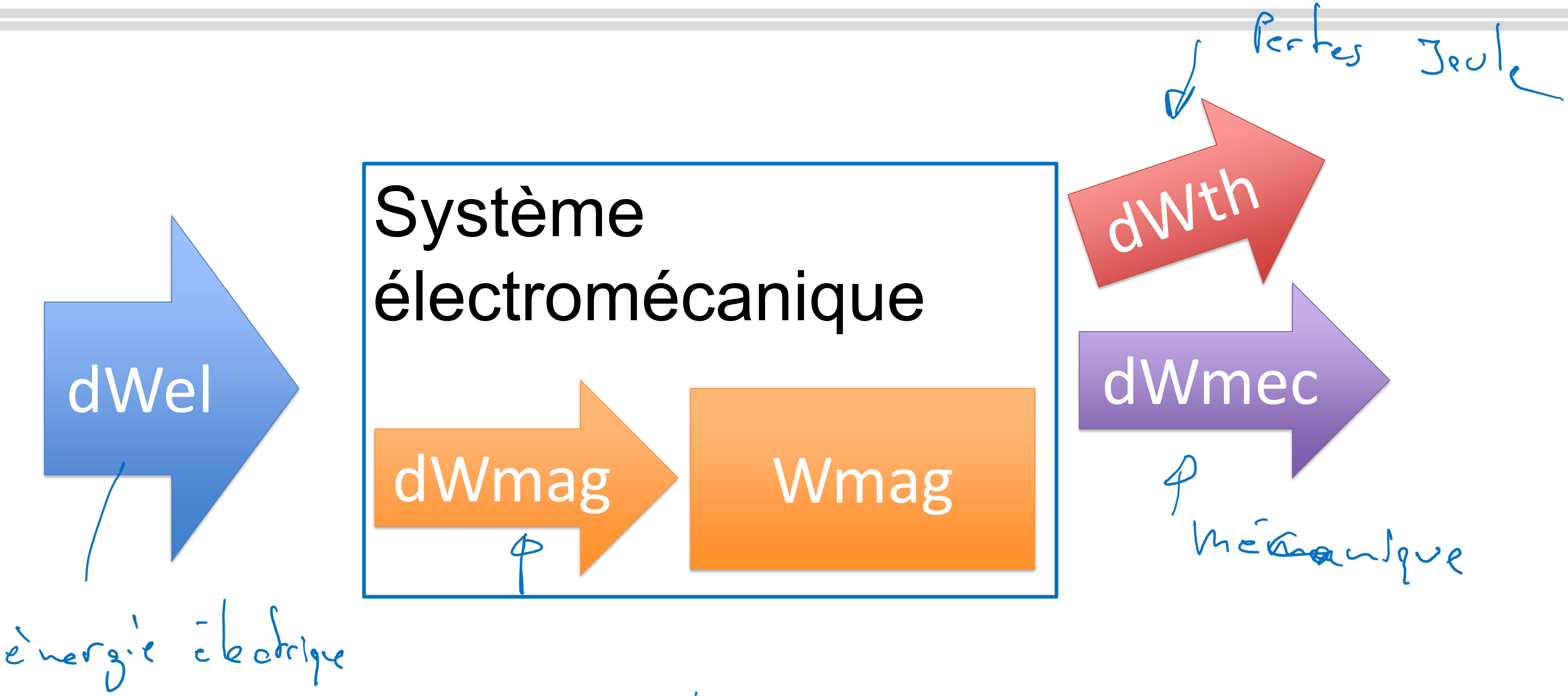
$$\Psi_j = \sum_{p=1}^{k} L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2}$$

(n degrés de liberté)

$$\sum M_m = J_m \frac{d^2 \alpha_m}{dt^2}$$

Bilan d'énergie (moteur)



$$dW_{el} = dW_{mec} + dW_{th} + dW_{mag}$$

Obtention d'une force

$$dW_{el} = dW_{mag} + dW_{th} + dW_{mec}$$

$$dW_{mec} = \sum_{m=1}^{n} F_m dx_m$$

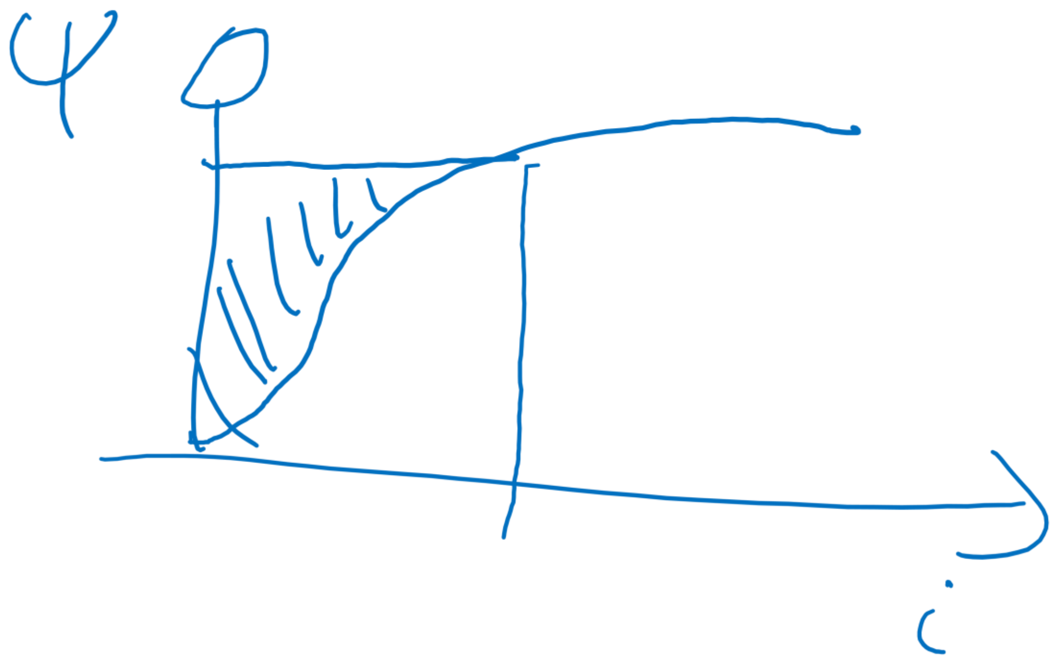
$$dW_{el} = \sum_{j=1}^{k} U_j i_j dt = \sum_{j=1}^{k} R_j i_j^2 dt + \sum_{j=1}^{k} d\Psi_j i_j$$

$$dW_{th} = \sum_{j=1}^{k} R_j i_j^2 dt$$

$$W_{mag} = \sum_{j=1}^{k} \int_0^{\Psi_j} i_j d\Psi_j$$

$$dW_{mag}(\Psi_j, x_m) = \sum_{j=1}^{k} \frac{\partial W_{mag}}{\partial \Psi_j} d\Psi_j + \sum_{m=1}^{n} \frac{\partial W_{mag}}{\partial x_m} dx_m = \sum_{j=1}^{k} i_j d\Psi_j + \sum_{m=1}^{n} \frac{\partial W_{mag}}{\partial x_m} dx_m$$

$dW'_{mag}(i_j, x_m)$



Obtention d'une force

degrés de liberté

$$\sum_{m=1}^{n} \frac{\partial W_{mag}}{\partial x_m} dx_m + \sum_{m=1}^{n} F_m dx_m = 0$$

$\to 0$

$$\sum_{m=1}^{n} \left(\frac{\partial W_{mag}}{\partial x_m} + F_m \right) dx_m = 0$$

$$F_m = -\frac{\partial W_{mag}}{\partial x_m} = -\frac{dW_{mag}}{dx_m}\bigg|_{\Psi_j = \text{cte}}$$

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \frac{dW'_{mag}}{dx_m}\bigg|_{i_j = \text{cte}}$$

Méthodologie

- Bilan d'énergie:

$$dW_{el} = dW_{mag} + dW_{th} + dW_{mec}$$

- Méthode de la dérivée de la coénergie

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \left.\frac{dW'_{mag}}{dx_m}\right|_{\mathrm{i_j=cte}}$$

Force électromagnétique

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \left.\frac{dW'_{mag}}{dx_m}\right|_{\mathrm{i_j=cte}}$$

Cas général

$$W'_{mag} = \sum_{j=1}^{k} \int_0^{i_j} \Psi_j di_j$$

$$F_m = \sum_{j=1}^{k} \int_0^{i_j} \frac{\partial \Psi_j}{\partial x_m} di_j$$

$$F_m = \frac{\partial W'_{mag}}{\partial x_m}$$

Milieu linéaire

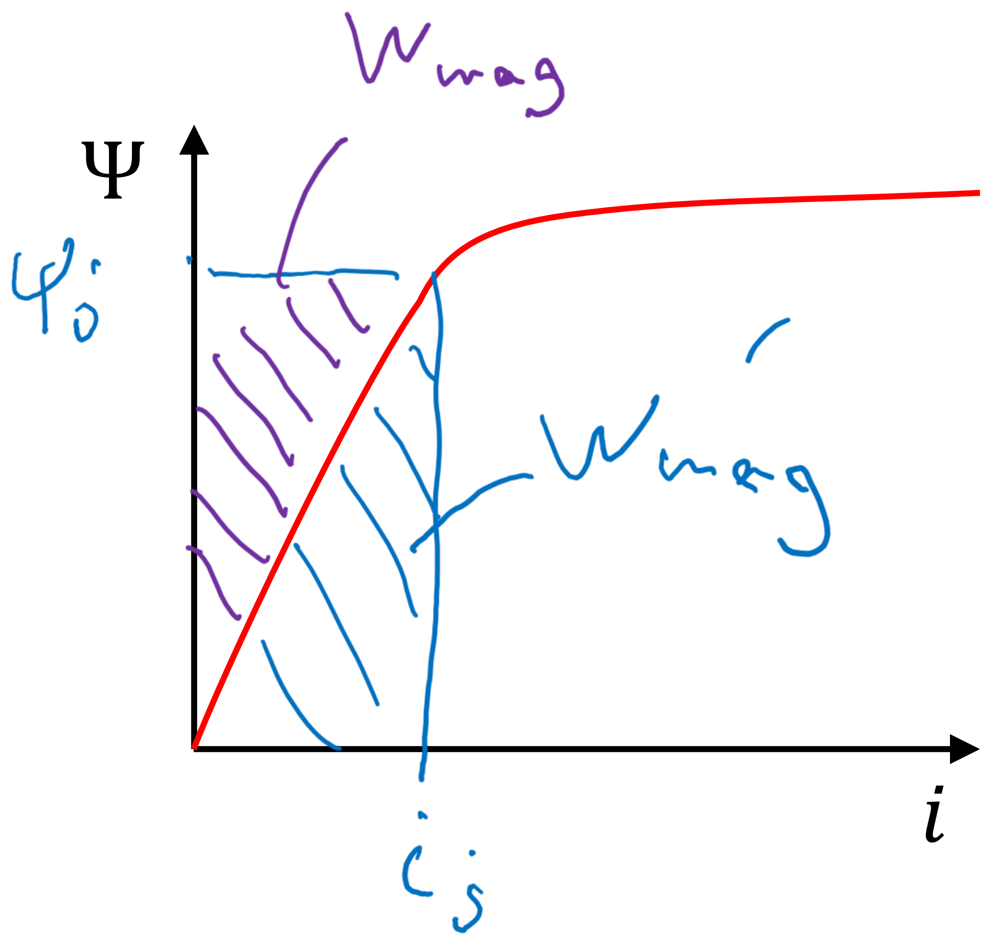
$$W'_{mag} = \frac{1}{2}\sum_{j=1}^{k} \Psi_j \, i_j \qquad \Psi_j = \sum_{p=1}^{k} L_{jp} i_p \qquad F_m = \frac{\partial W'_{mag}}{\partial x_m} = \frac{1}{2}\sum_{j=1}^{k}\sum_{p=1}^{k} \frac{d\mathrm{L}_{jp}}{dx_m} i_j i_p$$

$$\Psi_1 = L_{11} \cdot i_1 + L_{12} i_2 + L_{13} i_3 + \dots$$

$$L_{jp} = N_j N_p \Lambda_{jp} \qquad \Theta_j = N_j i_j$$

$$N_p = N_p \cdot i_p$$

$$F_m = \frac{1}{2} \sum_{j=1}^{K} \sum_{p=1}^{K} \frac{d\Lambda_{jp}}{dx_m} \cdot \Theta_j \Theta_p$$



Expressions dans un milieu linéaire

- Force:

$$F_m = \frac{1}{2}\sum_{j=1}^{k}\sum_{p=1}^{k}\frac{d\mathrm{L}_{jp}}{dx_m} i_j i_p$$

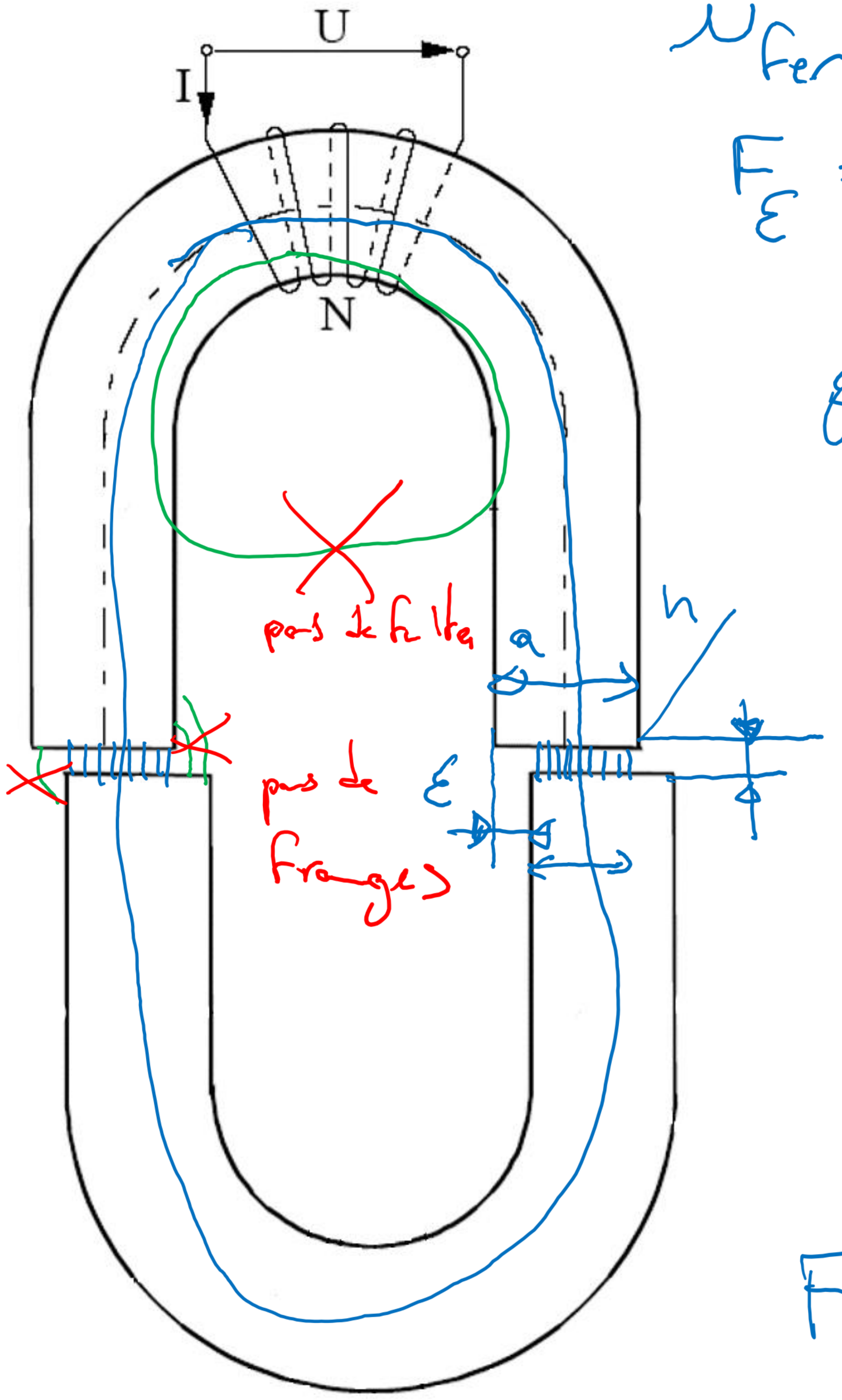
$$F_m = \frac{1}{2}\sum_{j=1}^{k}\sum_{p=1}^{k}\frac{d\Lambda_{jp}}{dx_m} \Theta_j \Theta_p$$

- Couple

$$M_m = \frac{1}{2}\sum_{j=1}^{k}\sum_{p=1}^{k}\frac{d\mathrm{L}_{jp}}{d\alpha_m} i_j i_p$$

$$M_m = \frac{1}{2}\sum_{j=1}^{k}\sum_{p=1}^{k}\frac{d\Lambda_{jp}}{d\alpha_m} \Theta_j \Theta_p$$

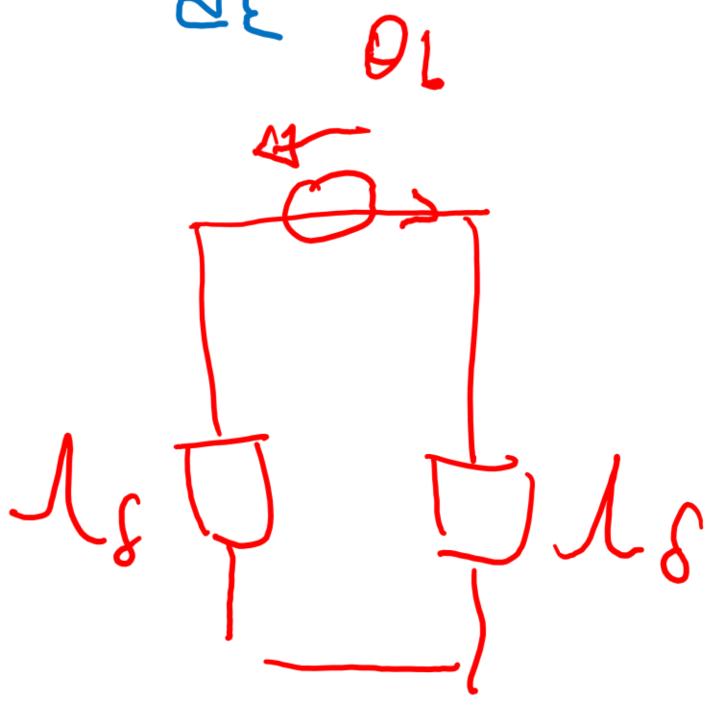
Exemple: force de centrage



$\mu_{fer} = \infty$ pas de fuites, pas de franges

$$F_\varepsilon = \frac{1}{2} \sum_{j=1}^{K} \sum_{p=1}^{K} \frac{d\Lambda_{jp}}{d\varepsilon} \theta_j \theta_p = \frac{1}{2} \frac{d\Lambda_{11}}{d\varepsilon} \cdot \theta_1^2$$

$$\theta_1 = \theta_b = NI$$



$$\Lambda_{11} = \frac{\Phi_{11}}{\theta_1} = \frac{\Phi_{bb}}{\theta_b} = \frac{\Lambda_\delta}{2}$$

$$\Lambda_\delta = \frac{\mu S}{l} = \frac{\mu_0 (a - \varepsilon) h}{\delta}$$

$$\Phi_b = \theta_b \cdot \Lambda_{eq} = \theta_b \cdot \frac{\Lambda_\delta}{2}$$

$$F_\varepsilon = \frac{1}{2} \frac{d\Lambda_\delta}{2 d\varepsilon} \cdot N^2 I^2 = \frac{1}{4} \frac{\mu_0 h (-1)}{\delta} N^2 I^2 = -\frac{\mu_0 h N^2 I^2}{4\delta}$$