Problem set 1

Problem 1

Consider a rectangular membrane stretched between fixed supports, subjected to a distributed load p, and experiencing a uniform tension S. The strong form governing membrane's transverse vibrations is: find the scalar quantity $u_3\,(x, y, t) \in C^2(\Omega \times [0, T])$, which represents the transverse displacement of the membrane at point (x, y) and time t, such that the following equation is verified:

$$S\nabla^2 u_3 + p = \rho \ddot{u}_3 \quad \text{in } \Omega\times]0, T[,$$

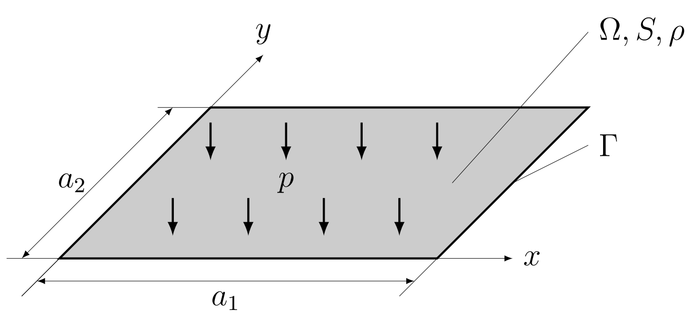
coupled with the following boundary and initial conditions:

$$\begin{cases} u_3(x, y, t) = 0 & (x, y) \in \Gamma,\ t \in]0, T[\\ u_3(x, y, 0) = u_{03}(x, y) & (x, y) \in \Omega \\ \dot{u}_3(x, y, 0) = v_{03}(x, y) & (x, y) \in \Omega. \end{cases}$$

Here the Laplacian operator is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

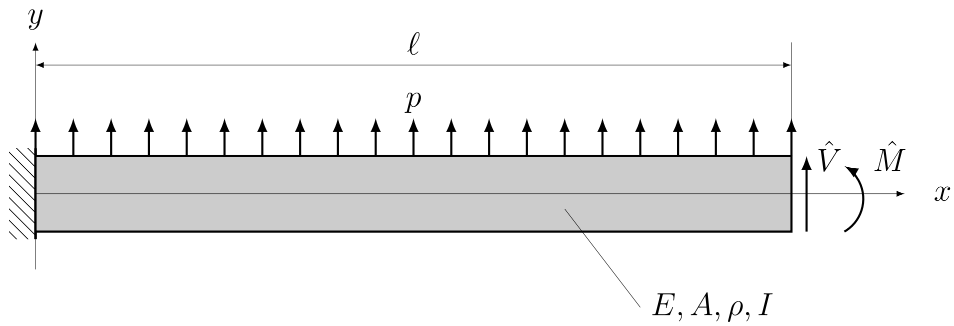
and ρ represents the density, $\Omega =]0, a_1[\times]0, a_2[$ is the domain (rectangle of sides a_1 and a_2) and Γ is the boundary of the membrane. The functions u_{03} and v_{03} specify the initial conditions of the membrane, where u_{03} represents the initial transverse displacement distribution, and v_{03} denotes the initial transverse velocity distribution.



Derive the corresponding weak form of the problem and define the appropriate functional spaces.

Problem 2

Consider a rectilinear beam fixed at one end, subjected to a distributed force p and experiencing a bending moment $\hat{M}$ and a shear force $\hat{V}$ at the free end. The structure is characterised by a length l, a uniform cross-section A of moment of inertia I, a modulus of elasticity E and a mass density ρ.



The strong formulation of the equations of motion for the beam in Timoshenko theory, which accounts for both shear deformation and rotary inertia, is given by:

$$\begin{cases} \nabla_\sigma^T \mathbf{C} \nabla_u \mathbf{u} + \mathbf{f} = \mathbf{M}\ddot{\mathbf{u}} & \forall (x,t) \in]0,\ell[\times]0,T[\\ \mathbf{u}(0,t) = 0 & \forall t \in]0,T[\\ \mathbf{C}\nabla_u \mathbf{u}(\ell,t) = \hat{\mathbf{f}} & \forall t \in]0,T[\\ \mathbf{u}(x,0) = \mathbf{u}_0 & \forall x \in]0,\ell[\\ \dot{\mathbf{u}}(x,0) = \mathbf{v_0} & \forall x \in]0,\ell[\end{cases}$$

where $\mathbf{u}\left(x,t\right) = \begin{bmatrix} u_2(x,t) \\ \theta_3\left(x,t\right) \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} p \\ 0 \end{bmatrix}$, $\hat{\mathbf{f}} = \begin{bmatrix} \hat{V} \\ \hat{M} \end{bmatrix}$, $\mathbf{u}_0\left(x\right) = \begin{bmatrix} u_0(x) \\ \theta_0(x) \end{bmatrix}$, $\mathbf{v}_0 = \begin{bmatrix} v_0(x) \\ \phi_0(x) \end{bmatrix}$, and

$$\nabla_\sigma = \begin{bmatrix} \partial_x & 1 \\ 0 & \partial_x \end{bmatrix}, \quad \nabla_u = \begin{bmatrix} \partial_x & -1 \\ 0 & \partial_x \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} kGA & 0 \\ 0 & EI \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix}.$$

Given the strong form of the governing equations of motion, determine the weak form describing the transversal vibrations of the beam.

Problem 1 is taken from [G] exercise 2.6
Problem 2 is taken from [G] examples 2..1.5 and 2.2.3
[G] Gmür, Dynamique des structures: analyse modale numérique. EPFL Press, 1997