Solar Energy Conversion Devices and Plants

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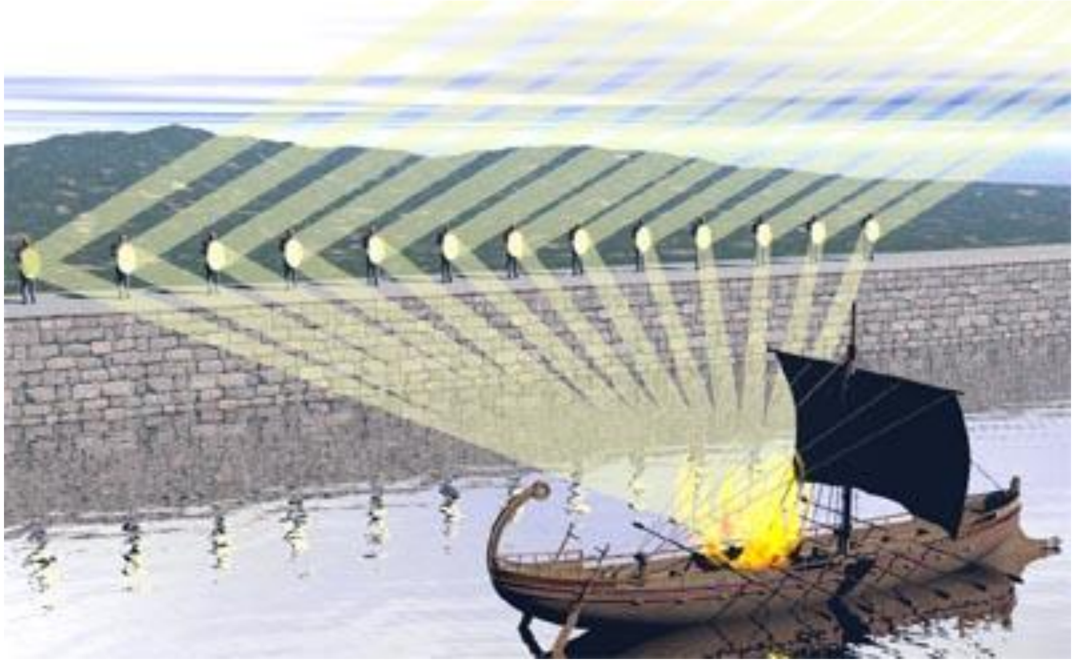
Outline

- Optics for Solar Concentration
  - Basics of geometrical optics
  - Recap: Monte Carlo method
  - Nonimaging optical systems
  - CPC as one basic nonimaging concentrator

Some history

- Archimedes constructed “death ray”, using a giant mirror, or set of mirrors, to set fire to Roman ships attacking his home city of Syracuse in 212 B.C.

http://www.unmuseum.org/burning_mirror.htm



- MIT students tried to reproduce:

http://web.mit.edu/2.009/www/experiments/deathray/10_ArchimedesResult.html

Some history

- Truncated Cone Reflector
  - Developed in 1878, called axicon
  - Boils water to drive steam power cycle
  - Seen as an improvement over point focus concentrators such as parabolic dishes

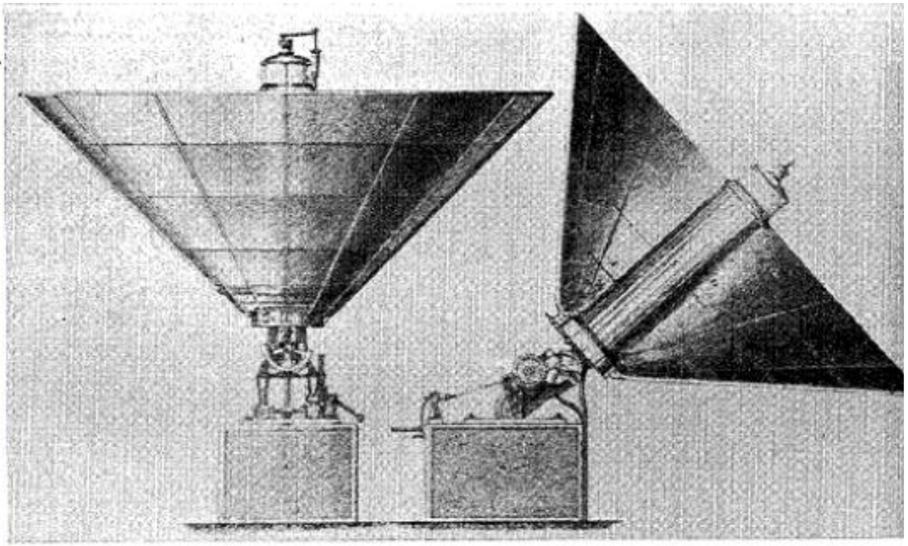


Fig. 1.1 August Mouchot's multiple-tube sun-heat absorber of 1878.

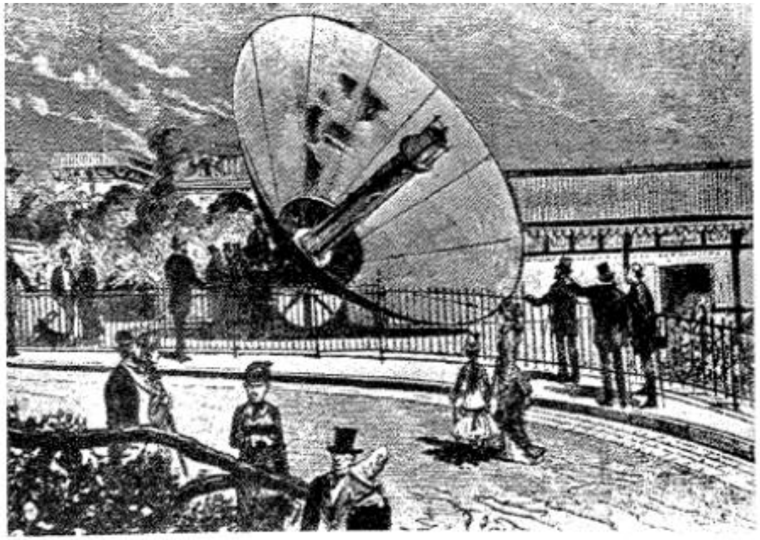


Fig. 1.2 The first large solar collector of the axicon type was exhibited by August Mouchot in 1878.

Source: "Applied Solar Energy: An Introduction", Meineland Meinel, Addition Wesley 1977

Some history

- Ericsson Solar Hot Air Engine
  - Developed in 1883 by John Ericsson
  - Called Ericsson Cycle
  - Parabolic trough design, also line focus

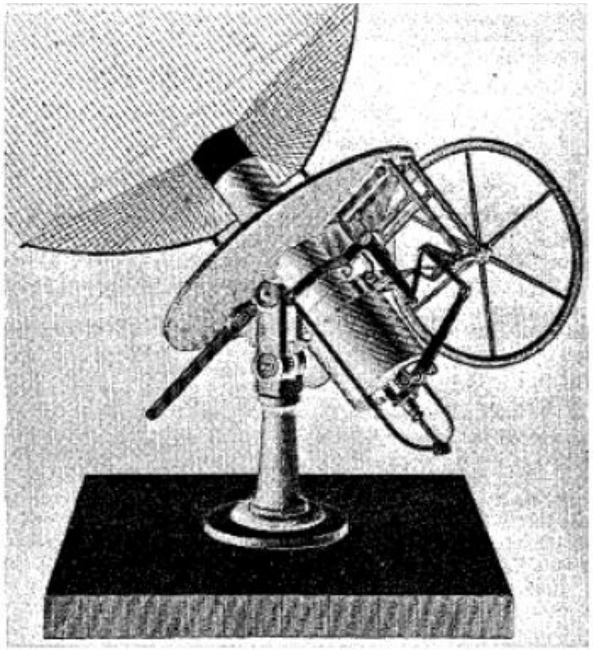


Fig. 1.4 Solar-powered hot-air engine tested by Ericsson nearly 90 years ago.

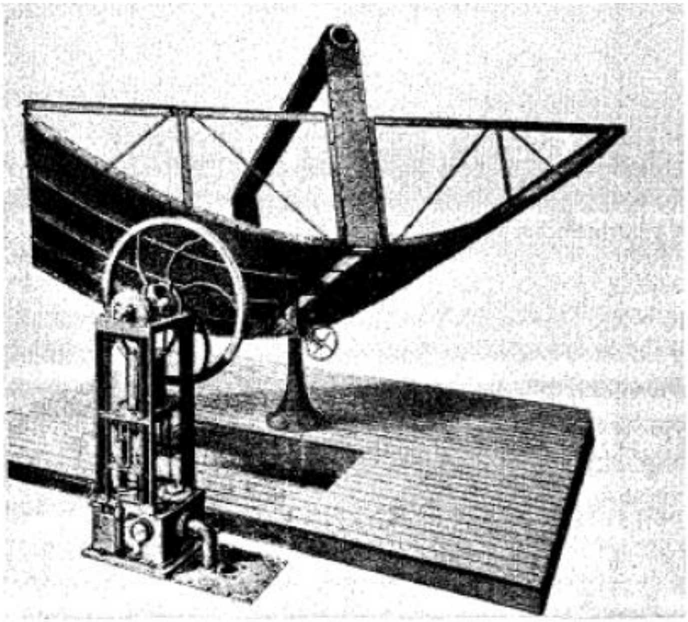


Fig. 1.5 Ericsson's solar collector of 1883 used a parabolic cylinder to focus on the absorbi tube mounted above the mirror.

Source: "Applied Solar Energy: An Introduction", Meineland Meinel, Addition Wesley 1977

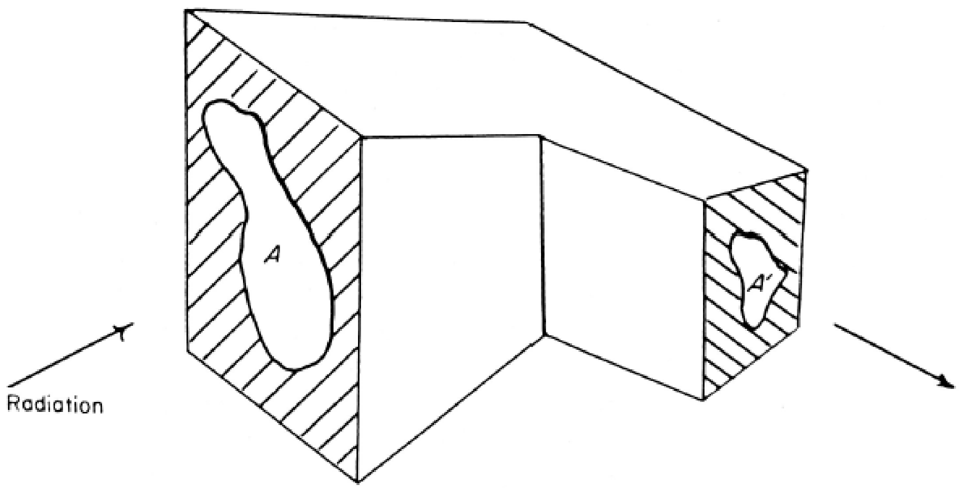
Intro

- We want to increase the power density of the solar radiation: focus the image of the sun with an image-forming system (like a lens or mirror) and the result is an increased power density
- Design an image-forming optical system of very large numerical aperture, i.e. small aperture ratio or f-number
- Has led to a class of concentrator with large aberration, if used as image-forming system → field of nonimaging optics
- What is the theoretical maximum concentration?
- Is it achievable in practice?

Basics

- Concentration ratio:

$$C = \frac{A}{A'}$$



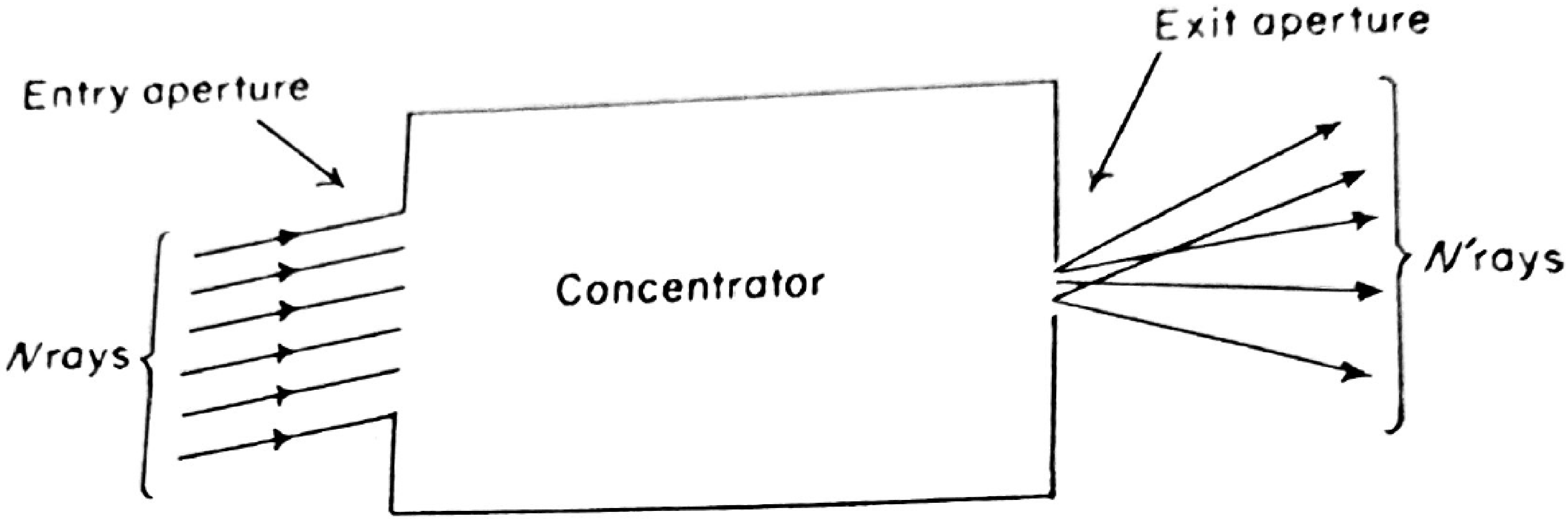
- Class of concentrators in which the beam is compressed only in one dimension: 2D concentrators, linear concentrator
- 2D concentrators do not need to be guided to follow the sun

Basics of geometrical optics

- Concept of geometrical optics:
  - Basic tool to design almost any optical system
  - Use intuitive idea of ray of light, i.e. path along which light energy travels, with interactions at surfaces
  - Reflection on smoot surface:
    - Incident and reflected ray make equal angle with surface normal
    - Ray and surface normal lie in one plane
  - Refraction at surface governed by generalized Snell's law

Basics of geometrical optics

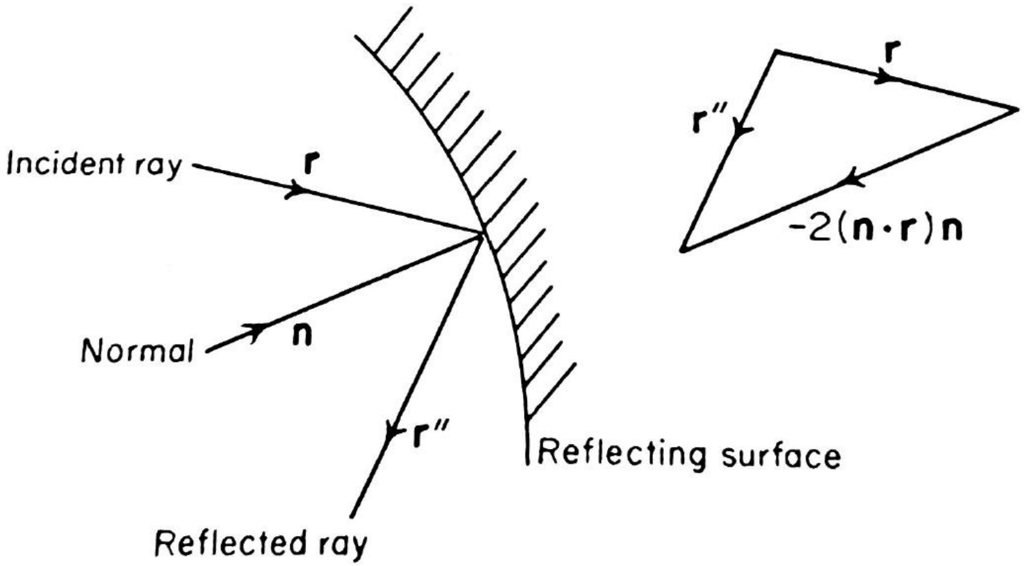
- Concept of geometrical optics:
  - Ray tracing can be used to design concentrators
  - In geometrical optics:
    - Power density of surface is density of ray intersections
    - Total power of surface is sum of rays on surface



Basics of geometrical optics

- Reflection at surface:

$$\hat{\mathbf{r}}'' = \hat{\mathbf{r}} - 2\left(\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}\right)\hat{\mathbf{n}}$$



- Ray tracing procedure for reflection:
  - Find point of incidence
  - Find normal of surface
  - Find direction of reflection

Basics of geometrical optics

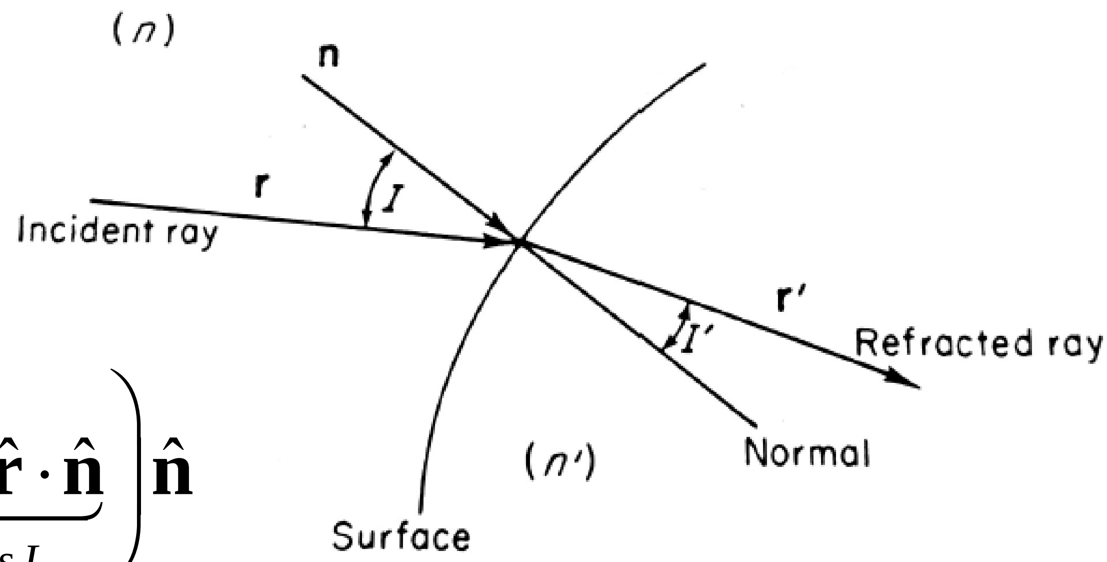
- Refraction at surface (for simple case of non-absorbing media):

$$n'\sin I' = n\sin I$$

$$n'\hat{\mathbf{r}}'\times\hat{\mathbf{n}} = n\hat{\mathbf{r}}\times\hat{\mathbf{n}}$$

or:

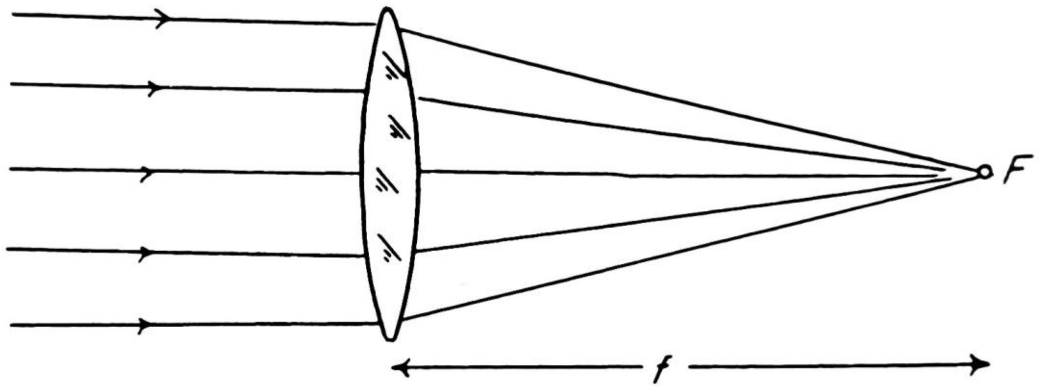
$$n'\hat{\mathbf{r}}' = n\hat{\mathbf{r}} + \left(\underbrace{n'\hat{\mathbf{r}}'\cdot\hat{\mathbf{n}} - n\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}}_{n'\cos I' - n\cos I}\right)\hat{\mathbf{n}}$$



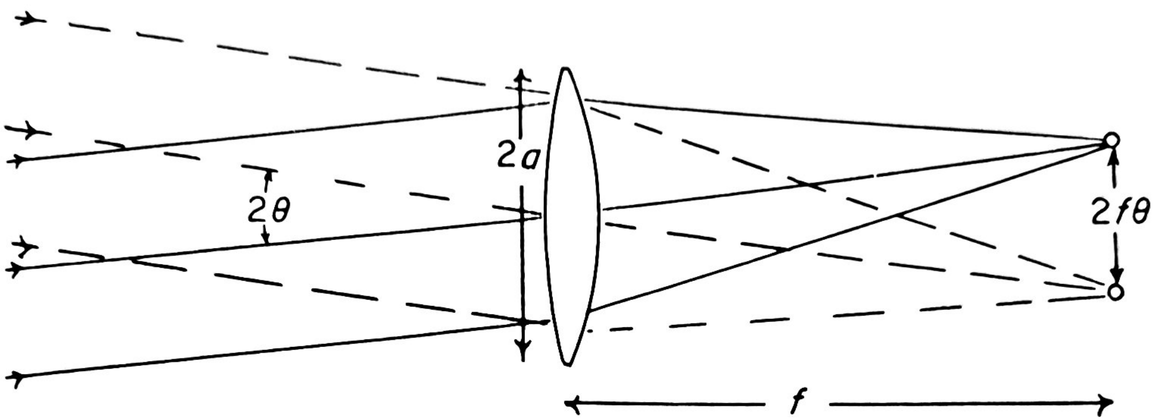
- Ray tracing procedure for reflection:
  - Find point of incidence
  - Find normal of surface
  - Find direction of refraction
- If $n'<n$, $I'>1$, i.e. total reflection

Basics of geometrical optics

- Image forming optical systems:
  - Focal length and focal point:



  - We get focal image if rays come from an object with finite size at great distance

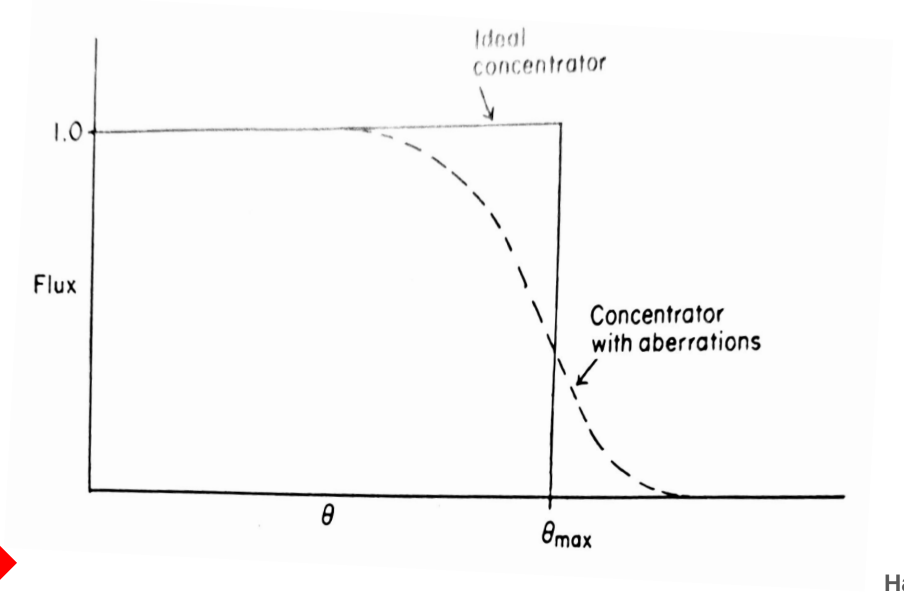


ideally:

$$C = \frac{(2a)^2}{(2f\theta)^2}$$

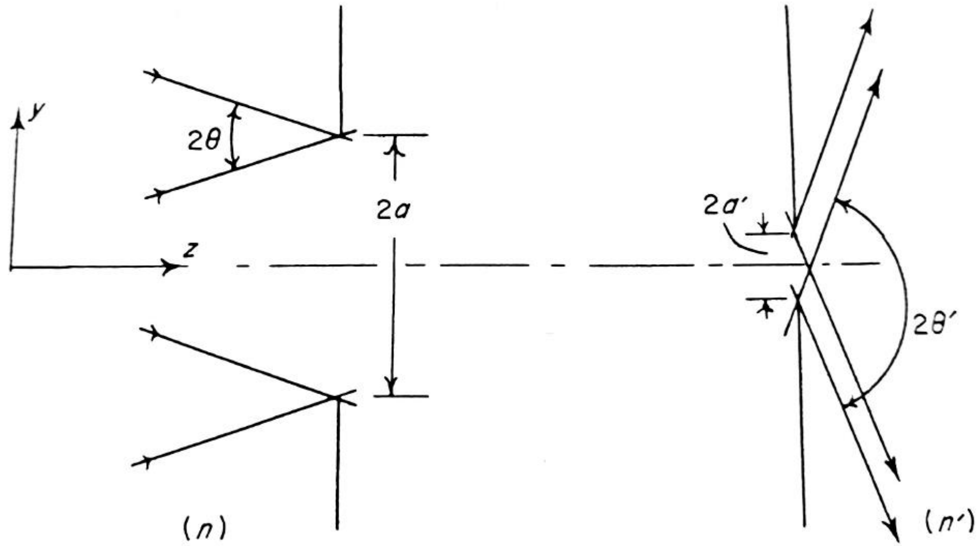
Basics of geometrical optics

- Image forming optical systems:
  - Aberrations in image-forming optical systems indicate that maximum concentration cannot be infinity
    - Lens has finite thickness (rays further out will focus closer to lens, so called spherical aberration)
    - Object points away from axis do not form image (off axis aberrations)
    - Refractive index is dependent on wavelength (so called chromatic aberrations)



Basics of geometrical optics

- Image forming optical systems:
  - Maximum concentration ratio



$$C_{\max,2D} = \frac{a}{a'} = \frac{n'\sin\theta'}{n\sin\theta} = \frac{n'}{n\sin\theta}$$

$$C_{\max,3D} = \left(\frac{a}{a'}\right)^2 = \left(\frac{n'\sin\theta'}{n\sin\theta}\right)^2 = \left(\frac{n'}{n\sin\theta}\right)^2$$

Recap: Monte Carlo method

Frequency, probability density, cumulative distribution functions

Frequency function: $f(x)$

Probability density function: $$g(x) = \frac{f(x)}{\int_{x=x_{min}}^{x_{max}} f(x)\,dx}$$

Probability that X belongs to dx around x: $$dP(x \le X < x + dx) = g(x)\,dx$$

Cumulative distribution function: $$F(x) = P(X < x) = \int_{x^*=x_{min}}^{x} g(x^*)\,dx^*$$

Frequency, probability density, cumulative distribution functions

Random number from the uniform distribution (0,1):

$$R$$

$$F(x) = R$$

$$x = F^{-1}(R)$$

For a large number m of random numbers $R_1, R_2, ..., R_m$:

$$x_1 = F^{-1}(R_1)$$

$$x_2 = F^{-1}(R_2)$$

$$\cdots$$

$$x_m = F^{-1}(R_m)$$

$\rightarrow$ $x_1, x_2, ..., x_m$ satisfy g(x) and hence f(x)

Monte Carlo method for thermal radiation

- Radiative energy/power = Sum of discrete amounts of energy/power bundles
- Each bundle consists of a group of photons and is launched with the same amount of energy/power
- Each bundle has an associated direction and wavelength
- Radiative flux = Number of bundles crossing a control surface area per unit area
- Radiative exchange $\rightarrow$ bundle paths are followed
- Monte-Carlo:

  1) Define generic **random** bundle
  2) Follow its path and history
  3) Repeat (2) for large number of randomly generated bundles

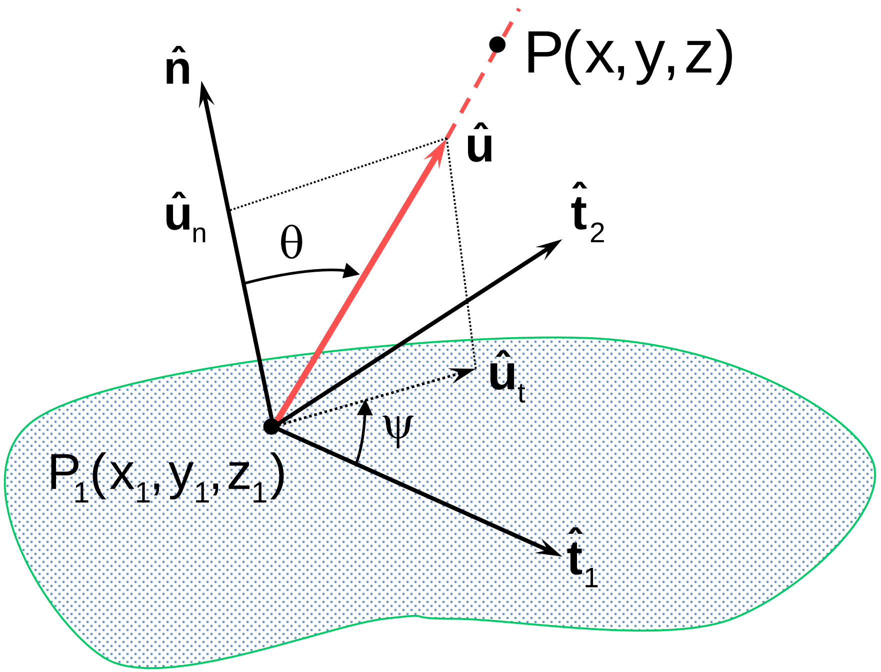
Monte Carlo method for thermal radiation

Assumptions:

1) Radiative variables are independent of each other.

2) Radiative variables are independent of position on the emitting surface element → isothermal surface element with uniform properties.

$$\left.\begin{array}{l} x_1 \rightarrow \lambda \\ x_2 \rightarrow \theta \\ x_3 \rightarrow \psi \end{array}\right\} f(\lambda,\theta,\psi) = f(\lambda)f(\theta)f(\psi)$$



Relations for surface emission

Radiative power emitted from a surface element dA:

$$d\dot{Q}_e\left(T,\lambda,\hat{\mathbf{s}}\right) = E'_\lambda\left(T,\lambda,\hat{\mathbf{s}}\right) d\Omega\, d\lambda\, dA$$

$$= \varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) E'_{b\lambda}\left(T,\lambda,\theta,\psi\right) \sin\theta\, d\theta\, d\psi\, d\lambda\, dA$$

$$= \varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta\, d\theta\, d\psi\, d\lambda\, dA$$

Frequency function for surface emission:

$$f_e\left(T,\lambda,\theta,\psi\right) = \frac{d\dot{Q}_e\left(T,\lambda,\theta,\psi\right)}{d\lambda\, d\theta\, d\psi}$$

$$= \varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta\, dA$$

Probability density function for surface emission:

$$g_e\left(\lambda,\theta,\psi\right) = \frac{\varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta}{\int\limits_{\lambda=0}^{\infty} \int\limits_{\psi=0}^{2\pi} \int\limits_{\theta=0}^{\pi/2} \varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta\, d\theta\, d\psi\, d\lambda}$$

$$= \frac{\varepsilon'_\lambda\left(T,\lambda,\theta,\psi\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta}{\varepsilon\left(T\right) \sigma T^4}$$

Relations for surface emission

Probability of emission into $d\lambda d\psi d\theta$:

$$dP_e\left(\lambda \le \Lambda < \lambda + d\lambda, \theta \le \Theta < \theta + d\theta, \psi \le \Psi < \psi + d\psi\right) = g_e\left(\lambda, \theta, \psi\right) d\lambda\, d\theta\, d\psi$$

Cumulative distribution functions:

Variable λ $\rightarrow$ $$R_\lambda = F_{e,\lambda}\left(\lambda\right) = \int_{\lambda^*=0}^{\lambda}\left(\int_{\psi=0}^{2\pi}\int_{\theta=0}^{\pi/2} g_e\left(\lambda^*, \theta, \psi\right) d\theta\, d\psi\right) d\lambda^* \rightarrow \lambda = F_{e,\lambda}^{-1}\left(R_\lambda\right)$$

Variable θ $\rightarrow$ $$R_\theta = F_{e,\theta}\left(\theta\right) = \int_{\theta^*=0}^{\theta}\left(\int_{\psi=0}^{2\pi}\int_{\lambda=0}^{\infty} g_e\left(\lambda, \theta^*, \psi\right) d\lambda\, d\psi\right) d\theta^* \rightarrow \theta = F_{e,\theta}^{-1}\left(R_\theta\right)$$

Variable ψ $\rightarrow$ $$R_\psi = F_{e,\psi}\left(\psi\right) = \int_{\psi^*=0}^{\psi}\left(\int_{\theta=0}^{\pi/2}\int_{\lambda=0}^{\infty} g_e\left(\lambda, \theta, \psi^*\right) d\lambda\, d\theta\right) d\psi^* \rightarrow \psi = F_{e,\psi}^{-1}\left(R_\psi\right)$$

Relations for surface emission

Variable λ

$$R_\lambda = \frac{\int_{\lambda^*=0}^{\lambda}\left(\int_{\psi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\varepsilon_\lambda^{'}\left(T,\lambda^*,\theta,\psi\right) I_{b\lambda}\left(T,\lambda^*\right)\cos\theta\sin\theta d\theta d\psi\right)d\lambda^*}{\varepsilon\left(T\right)\sigma T^4}$$

For emission independent of azimuthal angle ψ

For emission from diffuse non-gray surface:

$$R_\lambda = \frac{\int_{\lambda^*=0}^{\lambda}\left(\int_{\psi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\varepsilon_\lambda^{'}\left(T,\lambda^*,\theta,\psi\right) I_{b\lambda}\left(T,\lambda^*\right)\cos\theta\sin\theta d\theta d\psi\right)d\lambda^*}{\varepsilon\left(T\right)\sigma T^4}$$

For emission from **diffuse-gray** surface:

$$R_\lambda = \frac{\int_{\lambda^*=0}^{\lambda}\left(\int_{\psi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\varepsilon_\lambda^{'}\left(T,\lambda^*,\theta,\psi\right) I_{b\lambda}\left(T,\lambda^*\right)\cos\theta\sin\theta d\theta d\psi\right)d\lambda^*}{\varepsilon\left(T\right)\sigma T^4}$$

Relations for surface emission

Variable θ

$$R_\theta = \frac{\int_{\theta^*=0}^{\theta} \left(\int_{\psi=0}^{2\pi} \int_{\lambda=0}^{\infty} \varepsilon'_\lambda\left(T,\lambda,\theta^*,\psi\right) I_{b\lambda}\left(T,\lambda\right)\, d\lambda \cos\theta^* \sin\theta^*\, d\psi \right) d\theta^*}{\varepsilon\left(T\right)\sigma T^4}$$

For emission independent of azimuthal angle ψ

For emission from gray surface, and independent of azimuthal angle ψ

$$R_\theta = \frac{2\pi \int_{\theta^*=0}^{\theta} \left(\int_{\lambda=0}^{\infty} \varepsilon'_\lambda\left(T,\lambda,\theta^*,\psi\right) I_{b\lambda}\left(T,\lambda^*\right) d\lambda \cos\theta^* \sin\theta^* \right) d\theta^*}{\varepsilon\left(T\right)\sigma T^4}$$

For emission from **diffuse-gray** surface:

$$R_\theta = \frac{2 \int_{\theta^*=0}^{\theta} \varepsilon'_\lambda\left(T\right) \cos\theta^* \sin\theta^* d\theta^*}{\varepsilon\left(T\right)}$$

Relations for surface emission

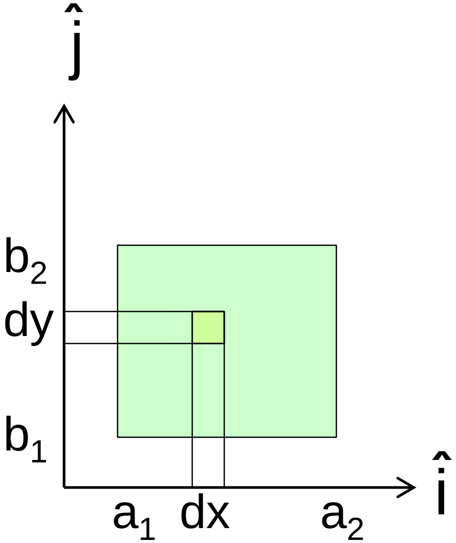
Variable ψ

$$R_\psi = \frac{\int\limits_{\psi^*=0}^{\psi} \left(\int\limits_{\theta=0}^{\pi/2} \int\limits_{\lambda=0}^{\infty} \varepsilon'_\lambda\left(T,\lambda,\theta,\psi^*\right) I_{b\lambda}\left(T,\lambda\right) \cos\theta \sin\theta d\theta\, d\lambda \right) d\psi^*}{\varepsilon\left(T\right)\sigma T^4}$$

For emission from diffuse non-gray surface:

Relations for surface emission

Point of emission from a rectangular surface element:



Uniform emission

Variable **x** $$R_x = F_x(x) = \frac{\int_{x^*=a_1}^{x} \int_{y=b_1}^{b_2} \varepsilon(T)\sigma T^4 \, dx^* \, dy}{\int_{x=a_1}^{a_2} \int_{y=b_1}^{b_2} \varepsilon(T)\sigma T^4 \, dx \, dy}$$

Variable **y** $$R_y = F_y(y) = \frac{\int_{x=a_1}^{a_2} \int_{y^*=b_1}^{y} \varepsilon(T)\sigma T^4 \, dx \, dy^*}{\int_{x=a_1}^{a_2} \int_{y=b_1}^{b_2} \varepsilon(T)\sigma T^4 \, dx \, dy}$$

Relations for surface absorption/reflection

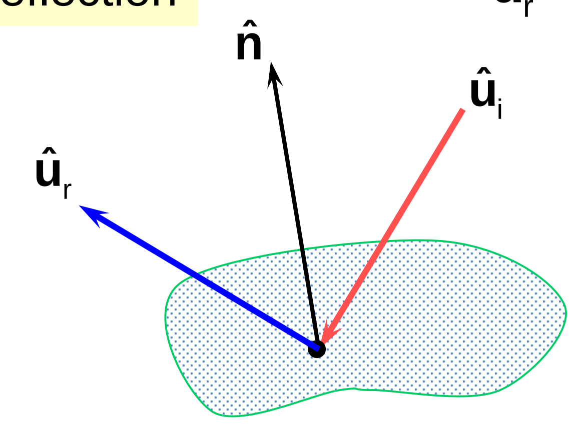
$R_a = \text{random}(0,1)$

- $R_a \leq \alpha$ → bundle absorbed
- $R_a > \alpha$ → bundle reflected

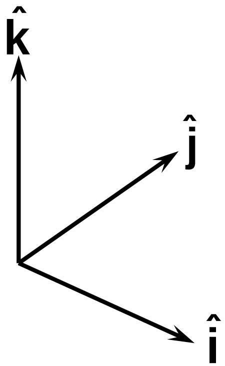
Reflection

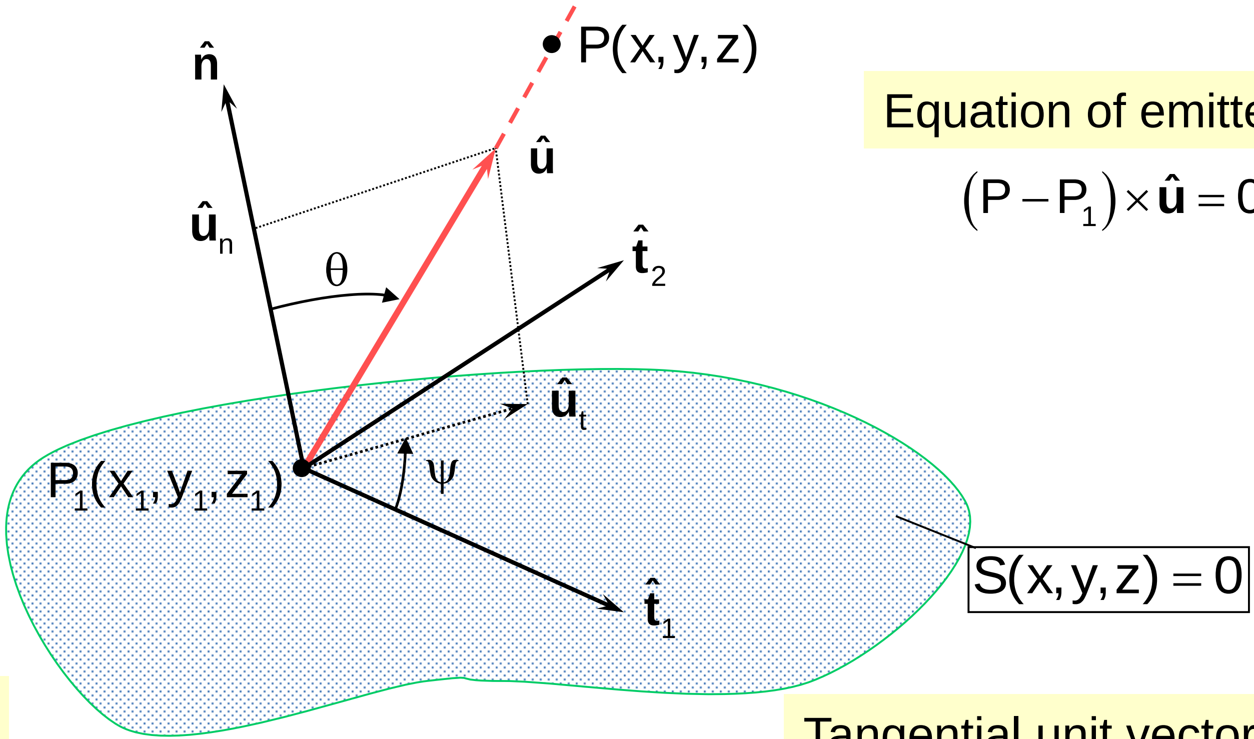
$R_r > \dfrac{\rho_s}{\rho_s + \rho_d}$ → diffuse reflection —Equivalent→ diffuse emission

$R_r \leq \dfrac{\rho_s}{\rho_s + \rho_d}$ → specular reflection → $\hat{\mathbf{u}}_r = \hat{\mathbf{u}}_i - 2(\hat{\mathbf{u}}_i \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$



Surface description





Equation of emitted ray:

$$(P-P_1)\times\hat{\mathbf{u}}=0$$

Normal unit vector:

$$\hat{\mathbf{n}}=\frac{\nabla S}{|\nabla S|}=\frac{\left(\frac{\partial S}{\partial x}\hat{\mathbf{i}}+\frac{\partial S}{\partial y}\hat{\mathbf{j}}+\frac{\partial S}{\partial z}\hat{\mathbf{k}}\right)}{\sqrt{\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2}}=n_x\hat{\mathbf{i}}+n_y\hat{\mathbf{j}}+n_z\hat{\mathbf{k}}$$

Tangential unit vectors:

$$\hat{\mathbf{t}}_1\cdot\hat{\mathbf{n}}=t_{1,x}\cdot n_x+t_{1,y}\cdot n_y+t_{1,z}\cdot n_z=0$$

$$\hat{\mathbf{t}}_2=\hat{\mathbf{n}}\times\hat{\mathbf{t}}_1=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ n_x & n_y & n_z\\ t_{1,x} & t_{1,y} & t_{1,z}\end{vmatrix}$$

$$\hat{\mathbf{u}}=u_x\hat{\mathbf{i}}+u_y\hat{\mathbf{j}}+u_z\hat{\mathbf{k}}=\hat{\mathbf{u}}_n+\hat{\mathbf{u}}_t$$

$$\hat{\mathbf{u}}_n=\cos\theta\cdot\hat{\mathbf{n}}$$

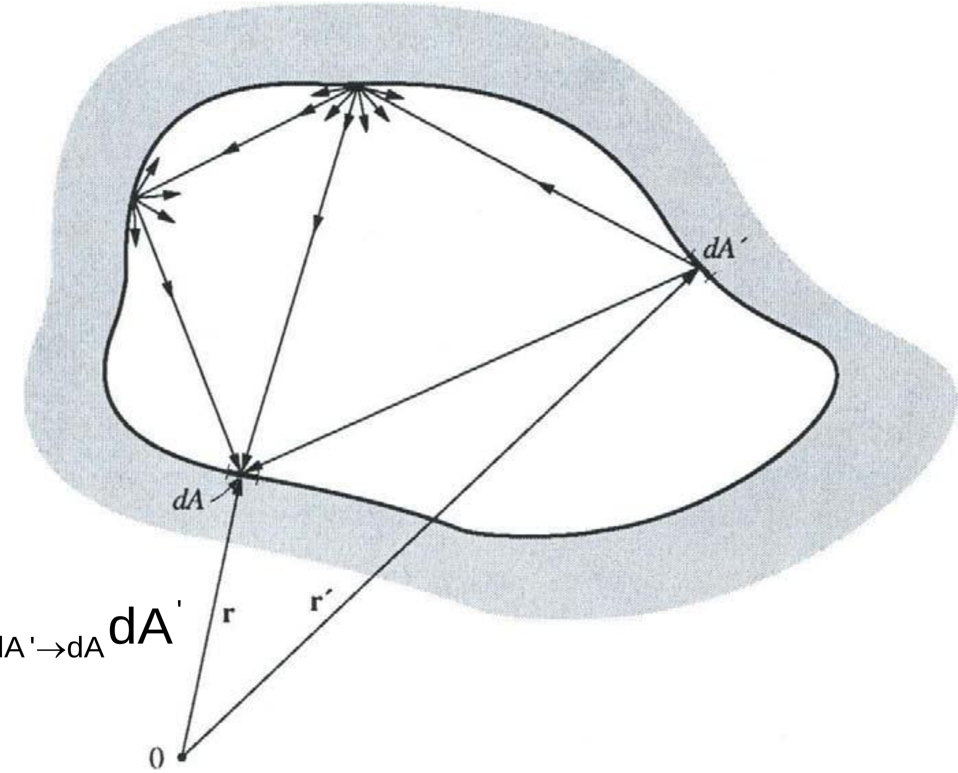
$$\hat{\mathbf{u}}_t=\sin\theta\cdot\left(\cos\psi\cdot\hat{\mathbf{t}}_1+\sin\psi\cdot\hat{\mathbf{t}}_2\right)$$

Computing radiative exchange

Net heat flux from a surface:

$$\dot{q}(\mathbf{r}) = \varepsilon(\mathbf{r})E_b(\mathbf{r}) - \alpha(\mathbf{r})H(\mathbf{r}) = J(\mathbf{r}) - H(\mathbf{r})$$

$$\dot{q}(\mathbf{r}) = \varepsilon(\mathbf{r})\sigma T^4(\mathbf{r}) - \frac{1}{dA}\int_A \varepsilon(\mathbf{r}')\sigma T^4(\mathbf{r}')\, dX_{dA'\to dA}\, dA'$$



$dX_{dA'\to dA}$ = generalized radiation exchange factor between surface elements dA' and dA

≡ fraction of the total energy emitted by dA' that is absorbed by dA, either directly or after any number and type of reflections

Computing radiative exchange

$$\dot{Q}_i = \int_{A_i} \dot{q}_i \, dA_i$$

$$= \varepsilon_i \sigma T_i^4 A_i - \sum_{j=1}^{J} \varepsilon_j \sigma T_j^4 A_j X_{j\rightarrow i}$$

$$\approx \varepsilon_i \sigma T_i^4 A_i - \sum_{j=1}^{J} \varepsilon_j \sigma T_j^4 A_j \frac{N_{a,j-i}}{N_{e,j}}$$

$$\approx \varepsilon_i \sigma T_i^4 A_i - \sum_{j=1}^{J} \dot{Q}_{ray} N_{a,j-i}$$

$$\approx \varepsilon_i \sigma T_i^4 A_i - \dot{Q}_{ray} \sum_{j=1}^{J} N_{a,j-i}$$

$$\approx \varepsilon_i \sigma T_i^4 A_i - \dot{Q}_{ray} N_{a,i}$$

$$\approx \left(N_{e,i} - N_{a,i}\right) \dot{Q}_{ray}, \qquad i = 1, 2, \ldots, J$$

$$X_{j-i} = \lim_{N_j \to \infty} \left(\frac{N_{a,j-i}}{N_{e,j}} \right) \approx \left(\frac{N_{a,j-i}}{N_{e,j}} \right)_{N_j \gg 1}$$

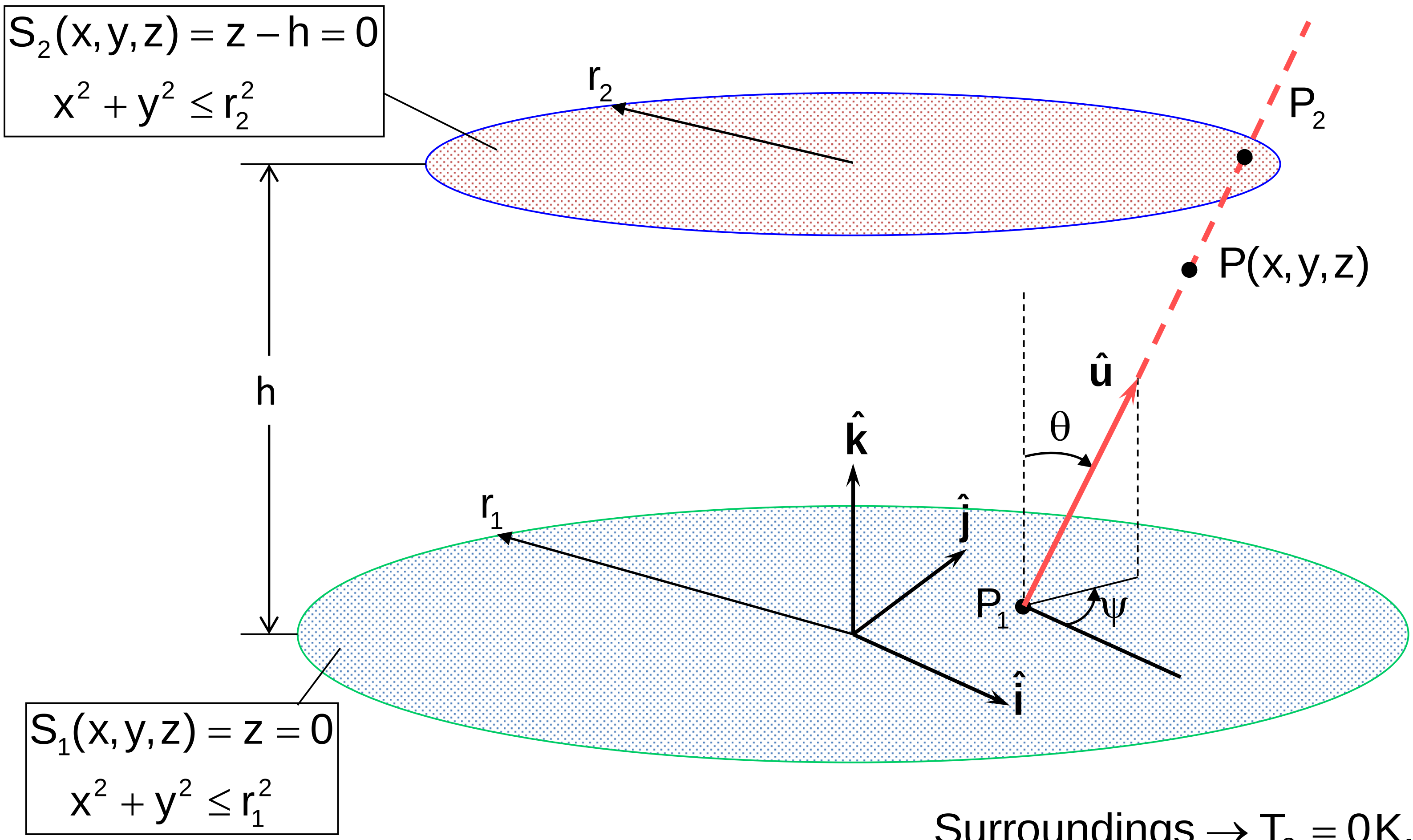
$$N_{e,j} = \frac{\varepsilon_j \sigma T_j^4 A_j}{\dot{Q}_{ray}}$$

$$\dot{Q}_{ray} = \frac{\sum_{j=1}^{J} \varepsilon_j \sigma T_j^4 A_j}{N_{rays}}$$

Computing radiative exchange

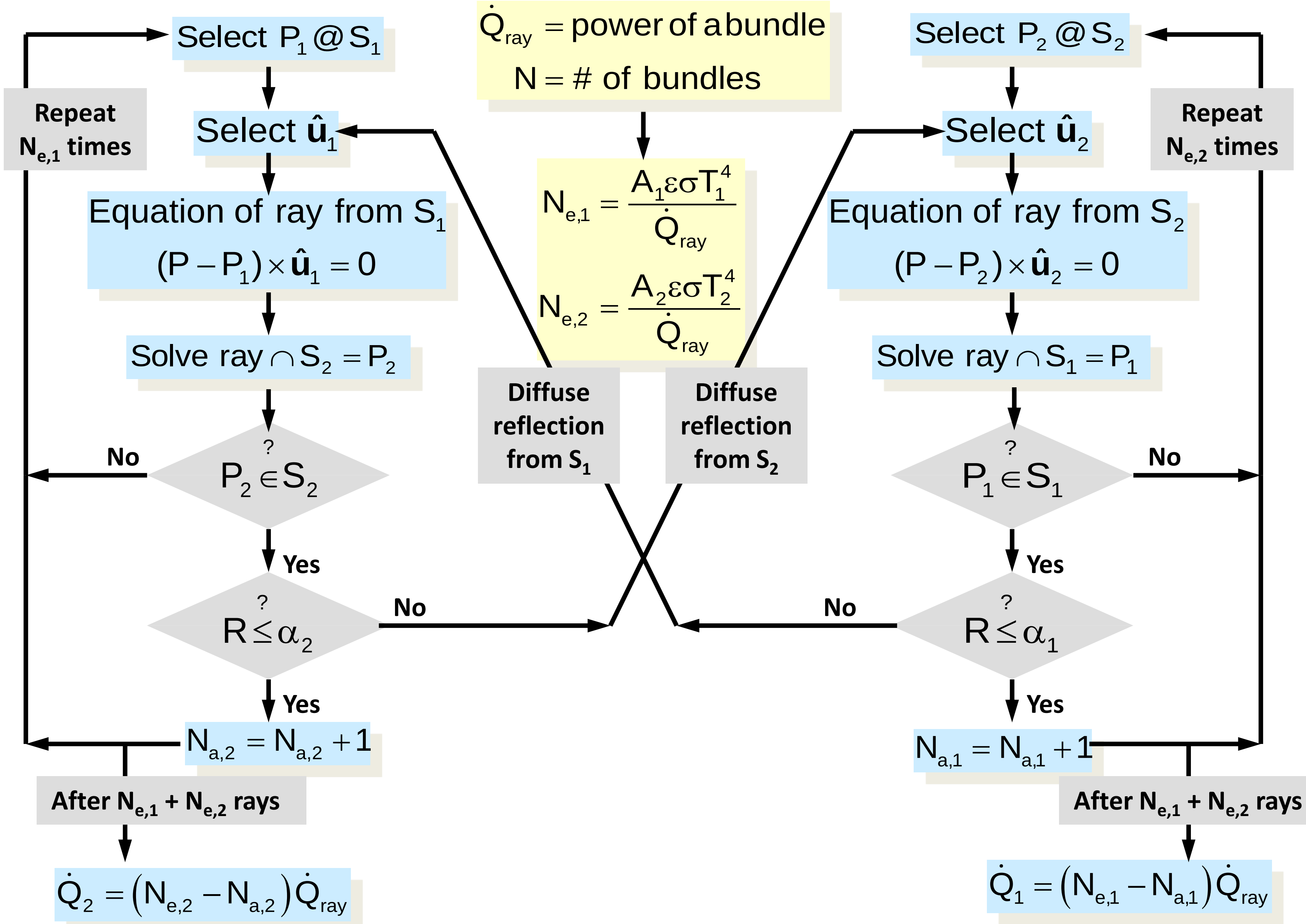
Example:

Compute net radiative fluxes leaving parallel coaxial gray and diffuse disks 1 and 2.



Surroundings $\rightarrow T_3 = 0K, \varepsilon_3 = 1$

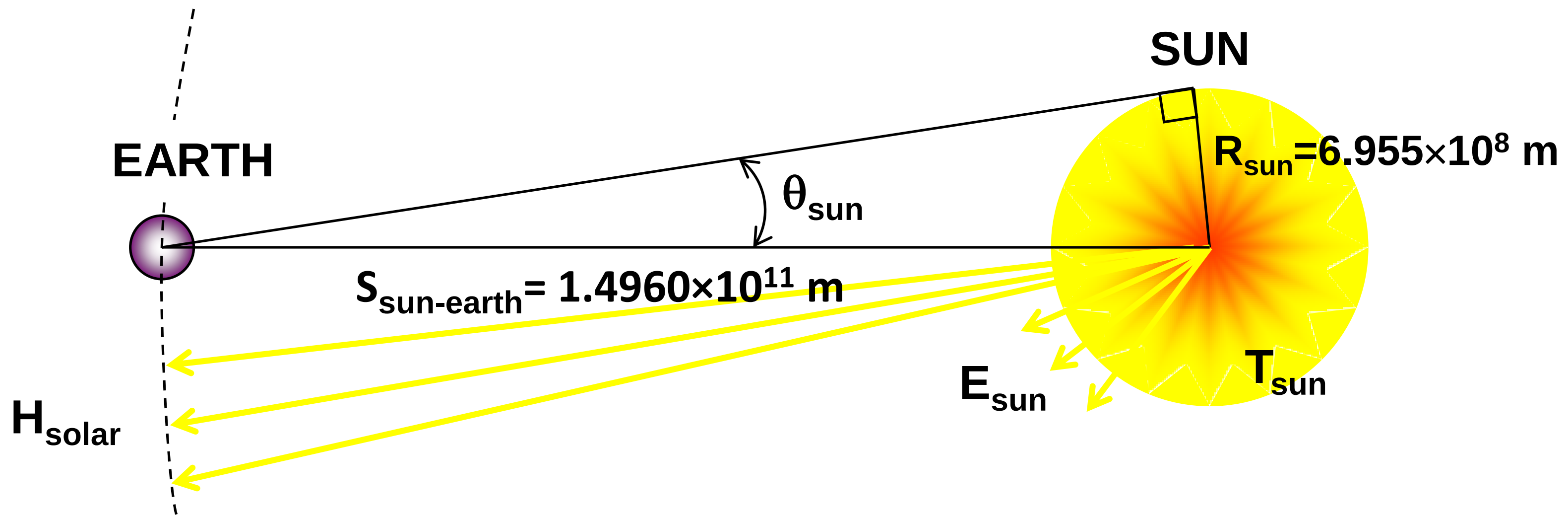
Computing radiative exchange



Nonimaging optical systems

Nonimaging optical systems

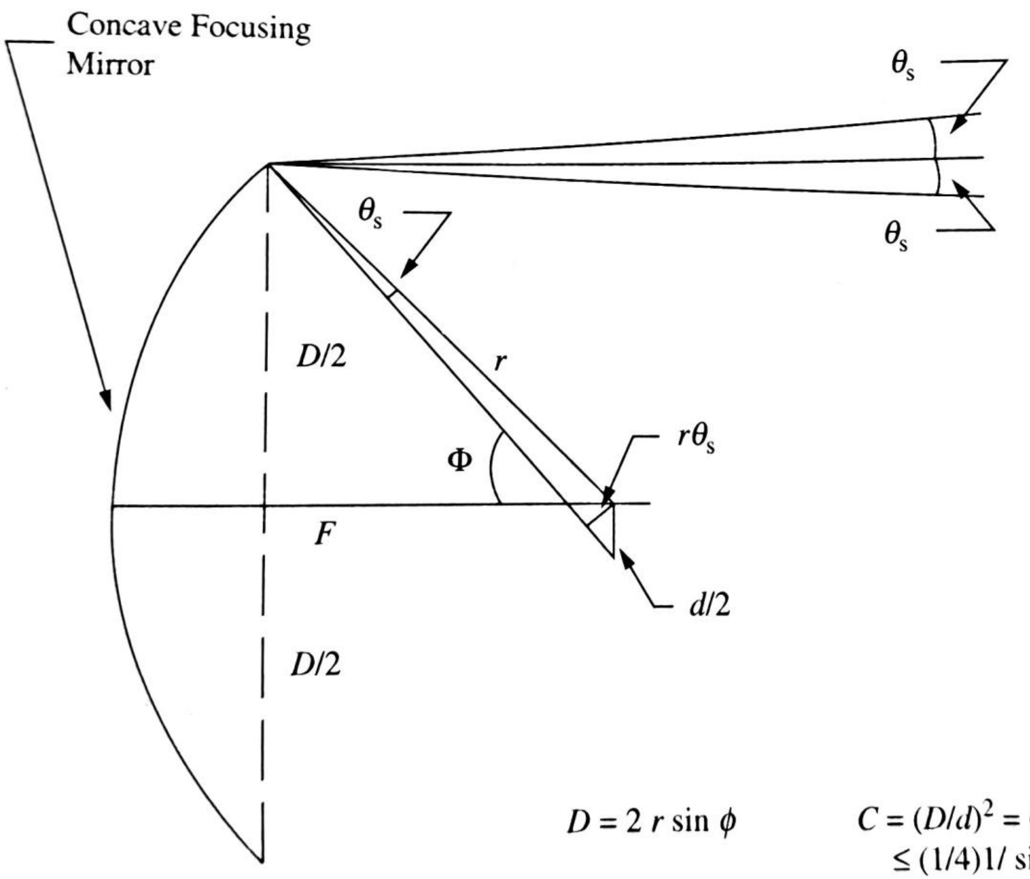
- 2nd-law limit:



$$\sin\theta_s = \frac{R_{sun}}{S_{sun-earth}} \xrightarrow{\max concentration} C = \left(\frac{S_{sun-earth}}{R_{sun}}\right)^2 = \frac{1}{\sin^2\theta_s}$$

Nonimaging optical systems

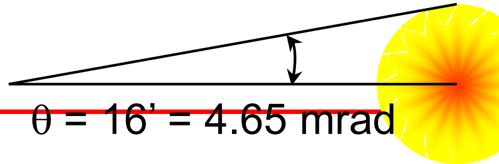
- Limits – Example image forming mirror systems:

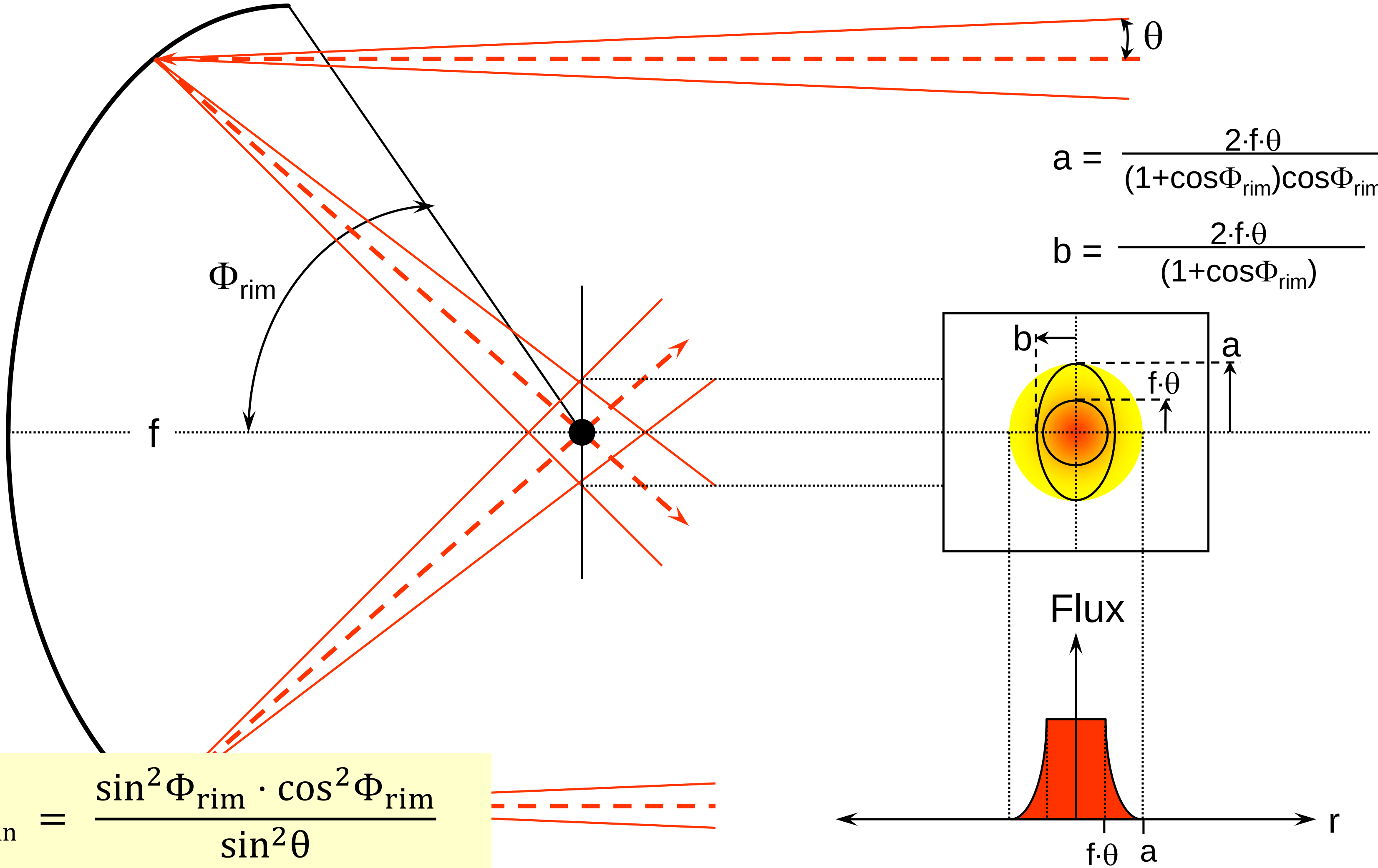


$$D = 2\,r \sin\phi \qquad C = (D/d)^2 = (1/4)\sin^2 2\phi / \sin^2\theta_s$$
$$\le (1/4) 1/\sin^2\theta_s \le (1/4)\, C_{max}$$

$$d = 2\,r\sin\theta / \cos\phi$$
$$D/d = \sin\phi\cos\phi / \sin\theta$$
$$= \sin 2\phi / 2\sin\theta$$

Solar parabolic dish





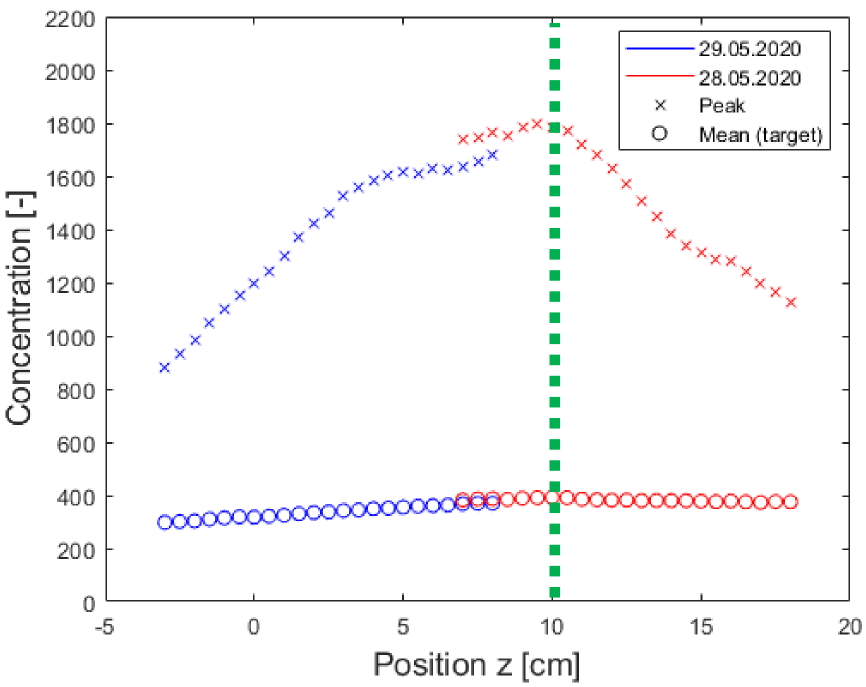
$$C_{mean} = \frac{\sin^2\Phi_{rim} \cdot \cos^2\Phi_{rim}}{\sin^2\theta}$$

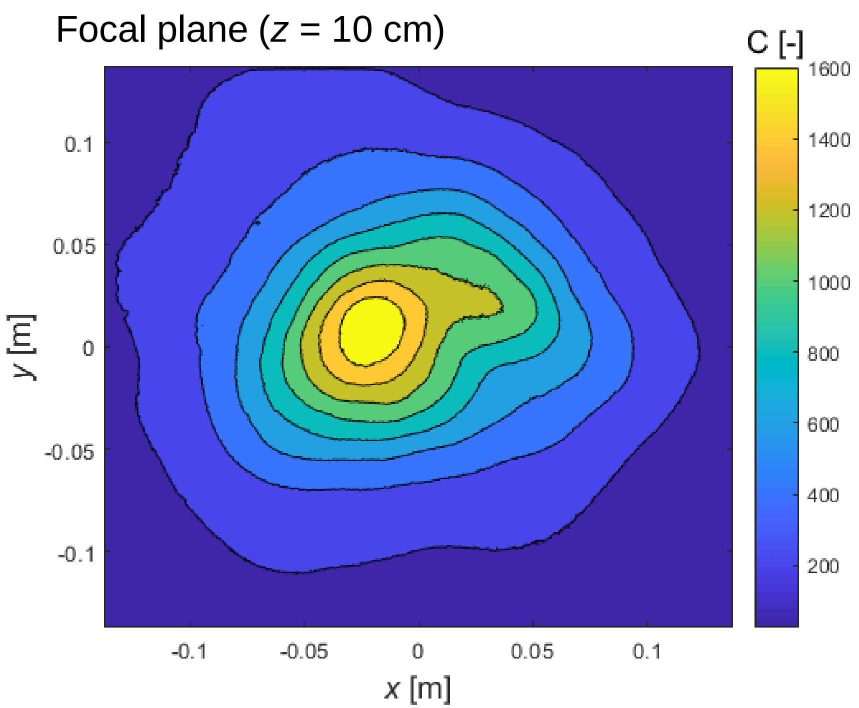
$$C_{peak} = \frac{\sin^2\Phi_{rim}}{\theta^2}$$

$\theta = 4.65$ mrad, $\Phi_{rim} = 45°$ → $C_{mean} \approx 11{,}562$, $C_{peak} \approx 23{,}124$

Identification of focal plane

- Identification of focal plane





- Characteristics:
  - Peak concentration 1765
  - Intercepted power on target: 29.6 kW @ DNI=1000 W/m^2
  - Optical efficiency:

$$\eta_{\text{opt}} = \frac{\dot{Q}_{\text{target}}}{A_{\text{dish}} \cdot E_{\text{bn}}} = 77\%$$

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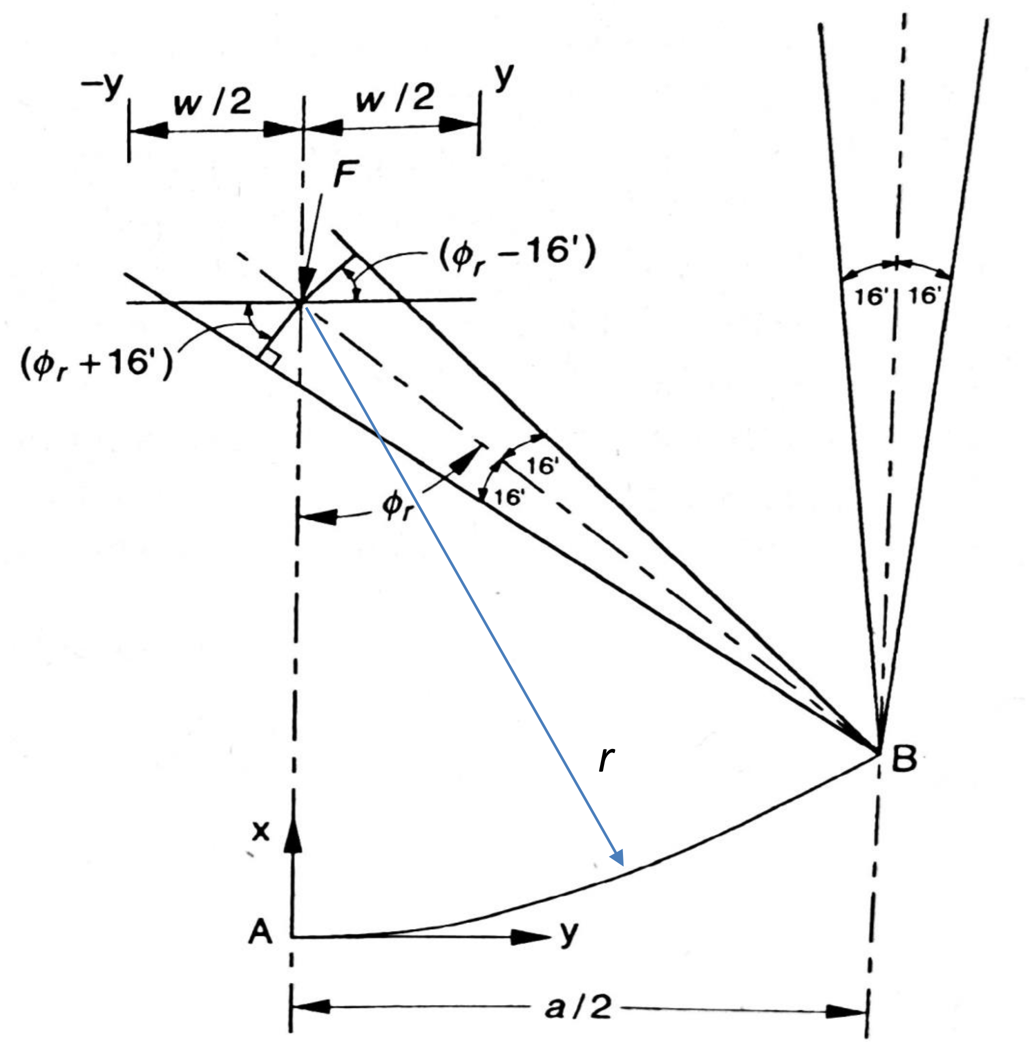
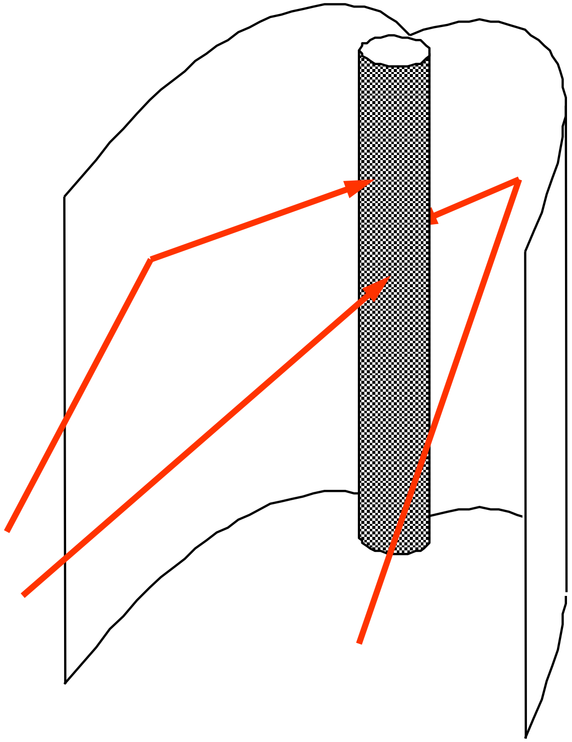
Linear concentrator

- As for dish:

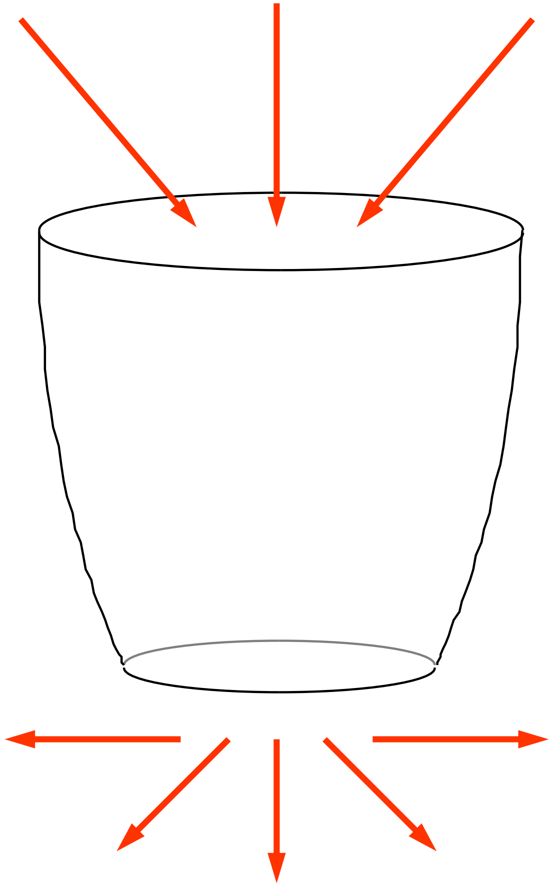


$$r = \frac{2f}{1+\cos\phi}$$

CPC – Compound parabolic concentrator

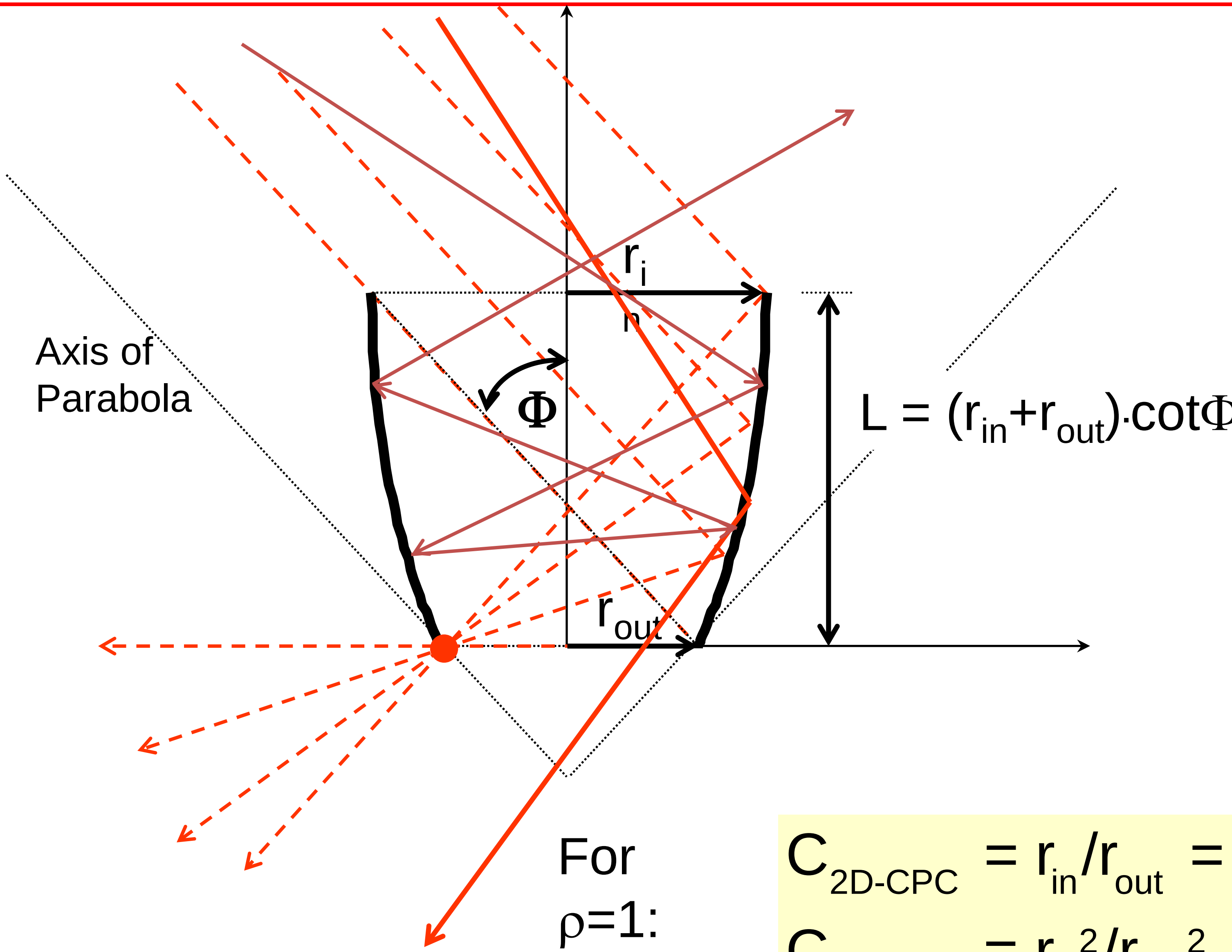


2D - CPC



3D - CPC

CPC



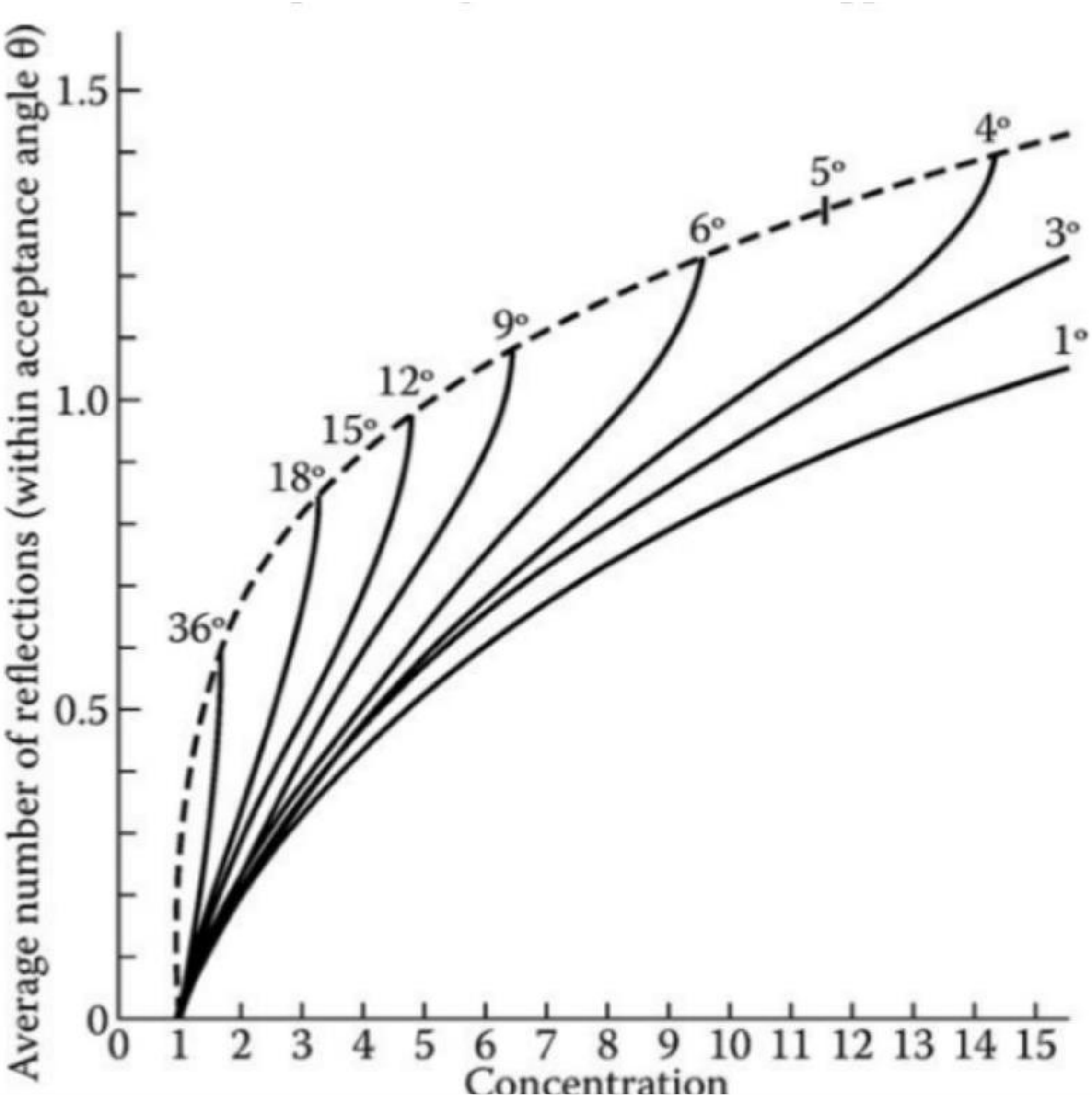
For $\rho=1$:

$$C_{2D\text{-}CPC} = r_{in}/r_{out} = 1/\sin\Phi$$

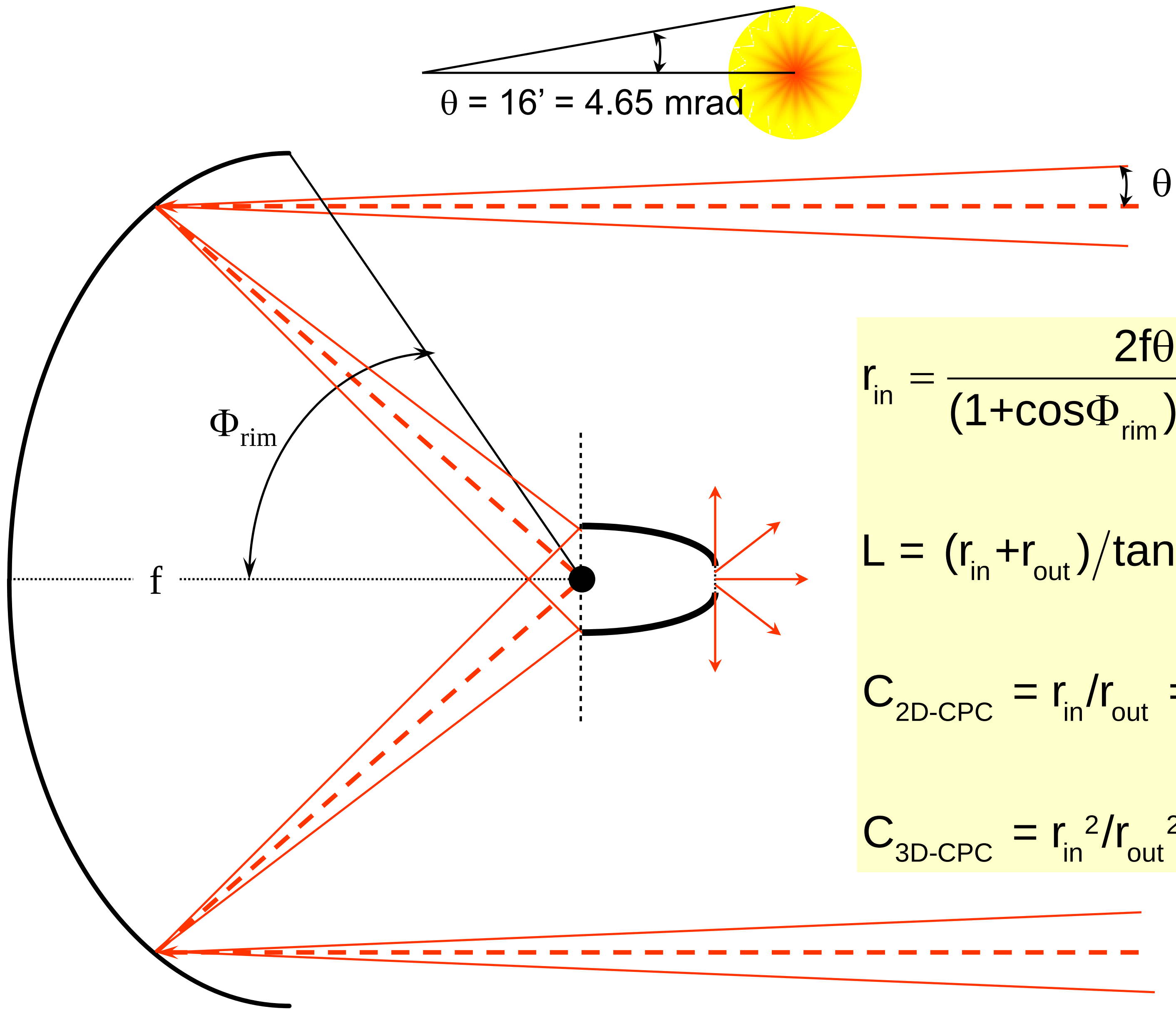
$$C_{3D\text{-}CPC} = r_{in}^2/r_{out}^2 = 1/\sin^2\Phi$$

Truncated CPC

- Average number of reflections:



CPC and solar dish



$$r_{in} = \frac{2f\theta}{(1+\cos\Phi_{rim}).\cos\Phi_{rim}}$$

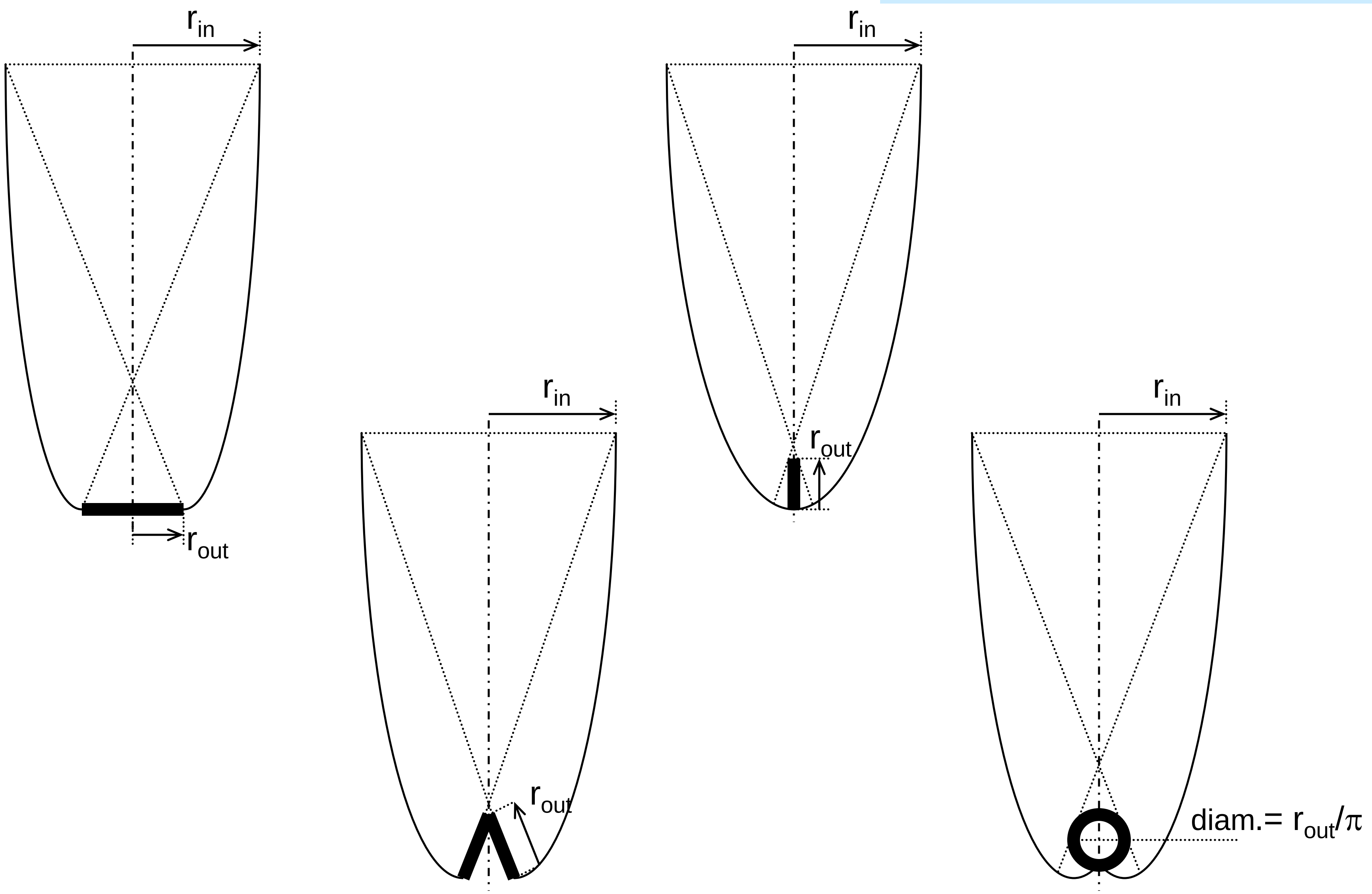
$$L = (r_{in}+r_{out})/\tan\Phi_{rim}$$

$$C_{2D\text{-}CPC} = r_{in}/r_{out} = 1/\sin\Phi$$

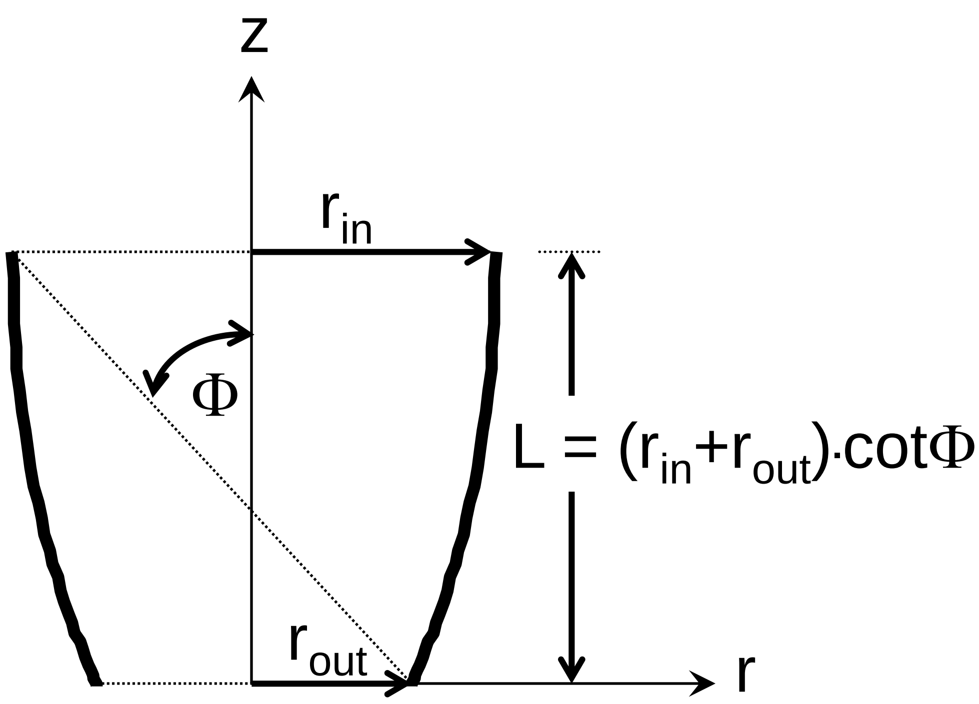
$$C_{3D\text{-}CPC} = r_{in}^{2}/r_{out}^{2} = 1/\sin^2\Phi$$

2D-CPC (trough) for various receivers

$C_{2D\text{-}CPC} = r_{in}/r_{out} = 1/\sin\Phi$



Equations of CPC



$$\begin{cases} r = \dfrac{2f\sin(\varphi-\Phi)}{1-\cos\varphi} - r_{out} \\ z = \dfrac{2f\cos(\varphi-\Phi)}{1-\cos\varphi} \end{cases}$$

where:

$$r_{out} = r_{in}\sin\Phi$$

$$f = r_{out}(1+\sin\Phi)$$

$$2\Phi \le \varphi \le \frac{\pi}{2} + \Phi$$

Example

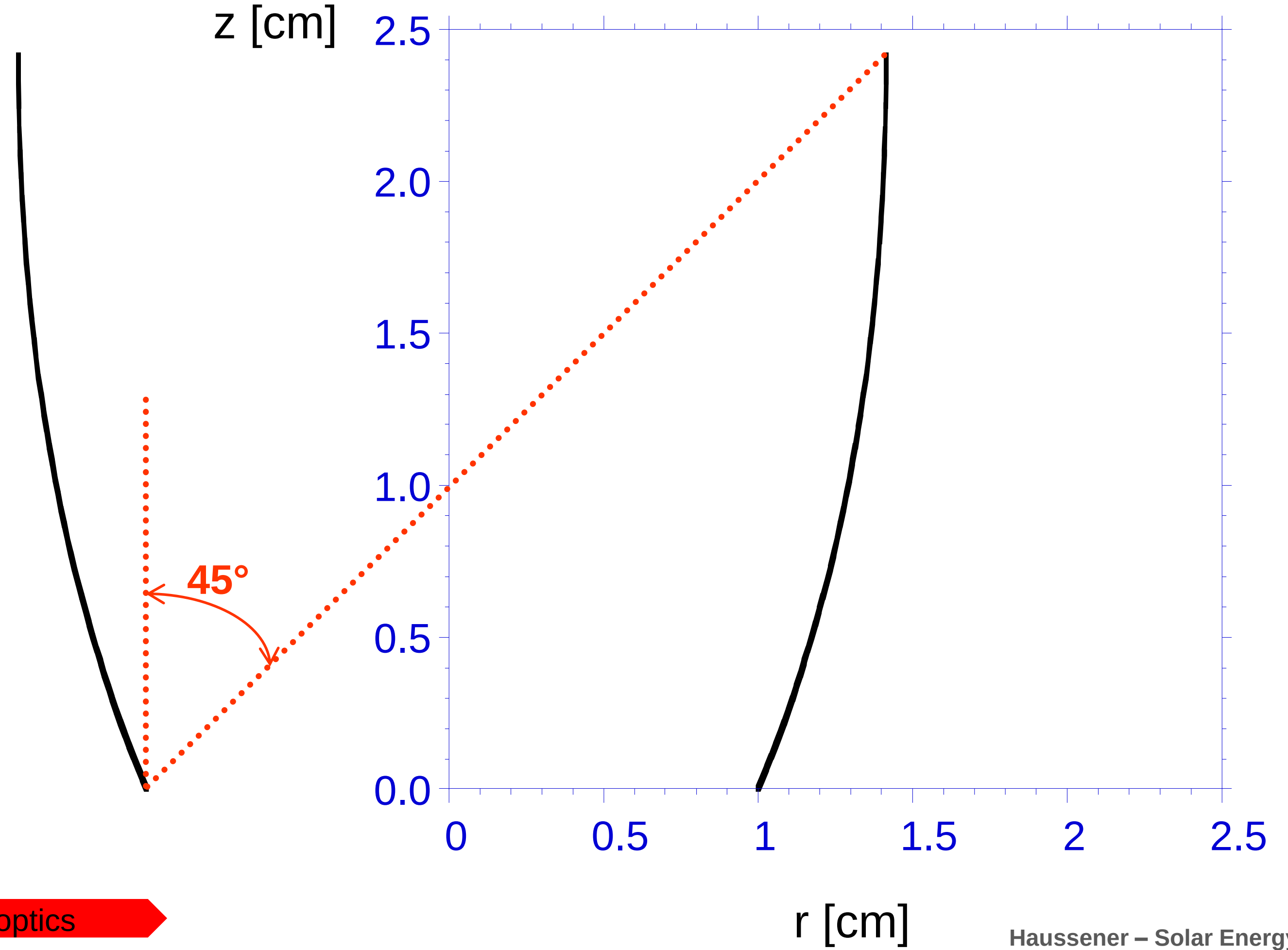
$\Phi = 45°$

$r_{out} = 1$ cm

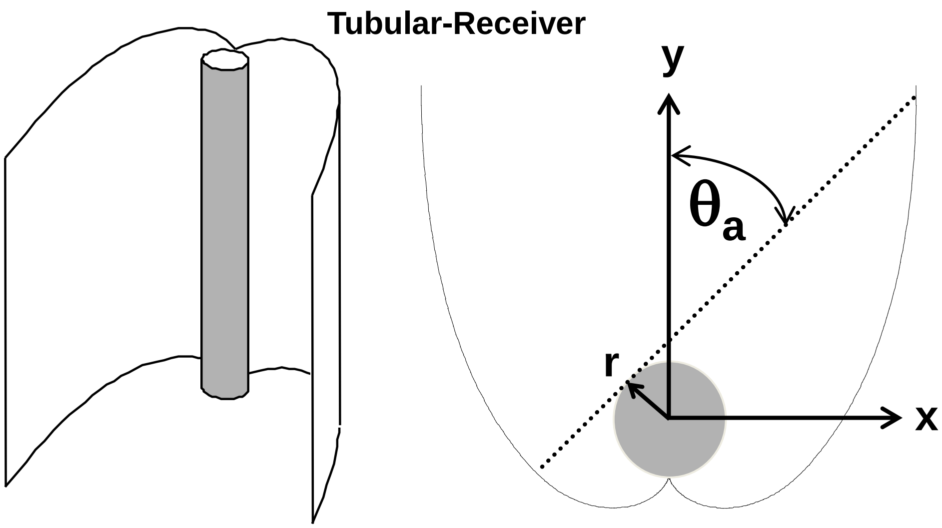
$r_{in} = r_{out}/\sin\Phi = 1.4141$

$L = (r_{in}+r_{out}) \cot \Phi = 2.4141$

$C = (r_{in}/r_{out})^2 = 2$



Equations of the 2-D CPC + involute



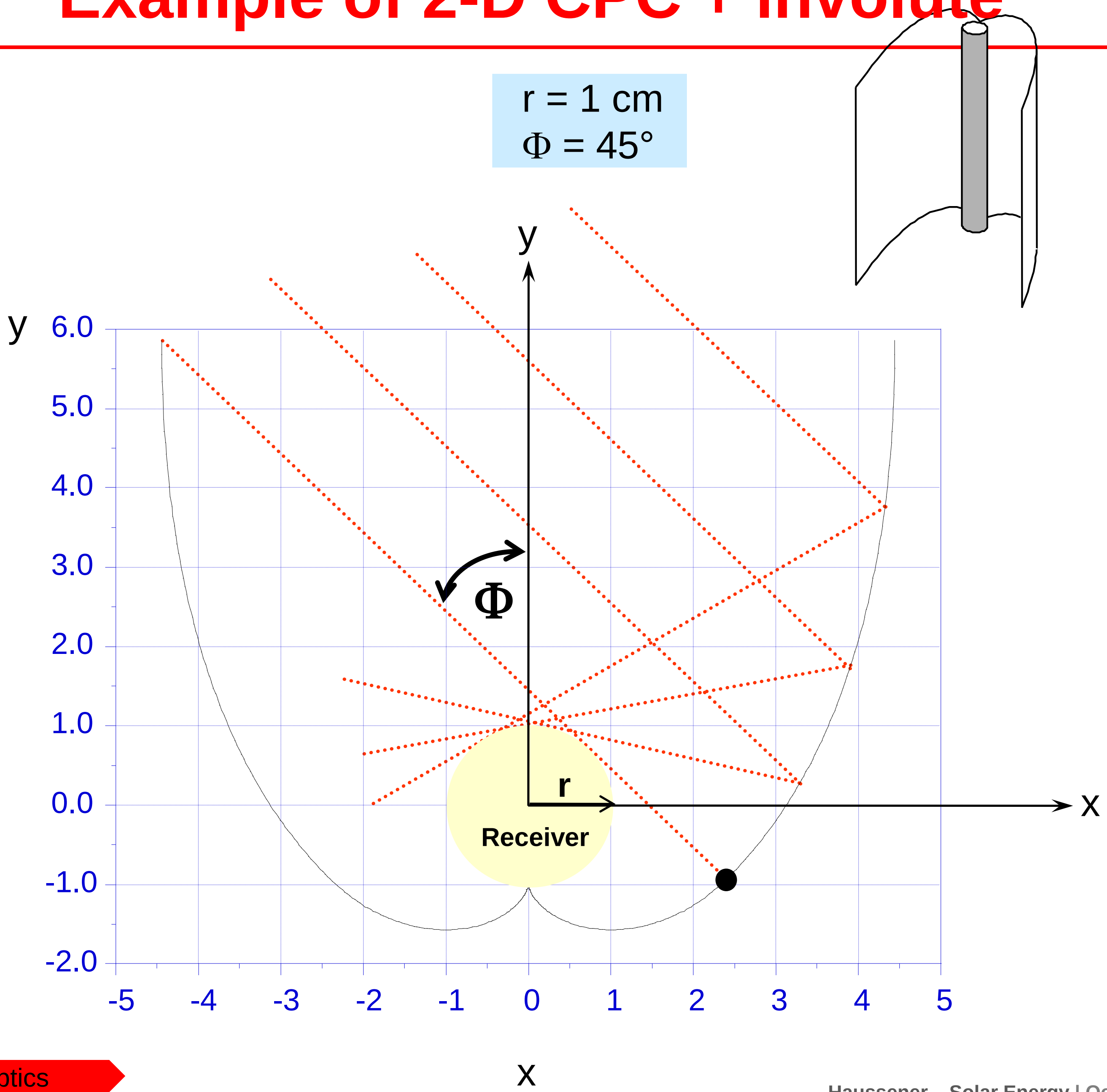
$$\begin{cases} x = r[\sin\theta - M(\theta)\cos\theta] \\ y = r[-\cos\theta - M(\theta)\sin\theta] \end{cases}$$

$$M(\theta) = \begin{cases} \theta & \text{for } 0 \le \theta \le \dfrac{\pi}{2} + \theta_a \\ \dfrac{\pi/2 + \theta_a + \theta - \cos(\theta - \theta_a)}{1 + \sin(\theta - \theta_a)} & \text{for } \dfrac{\pi}{2} + \theta_a \le \theta \le \dfrac{3\pi}{2} - \theta_a \end{cases}$$

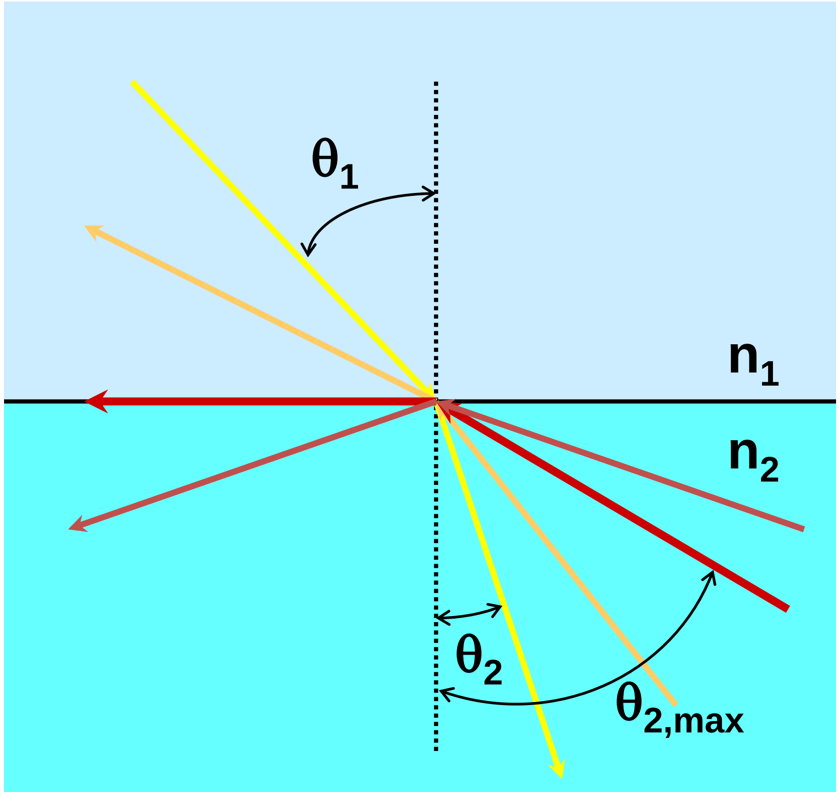
θ_a CPC's half acceptance angle and is taken equal to the rim angle of the primary parabolic concentrator.

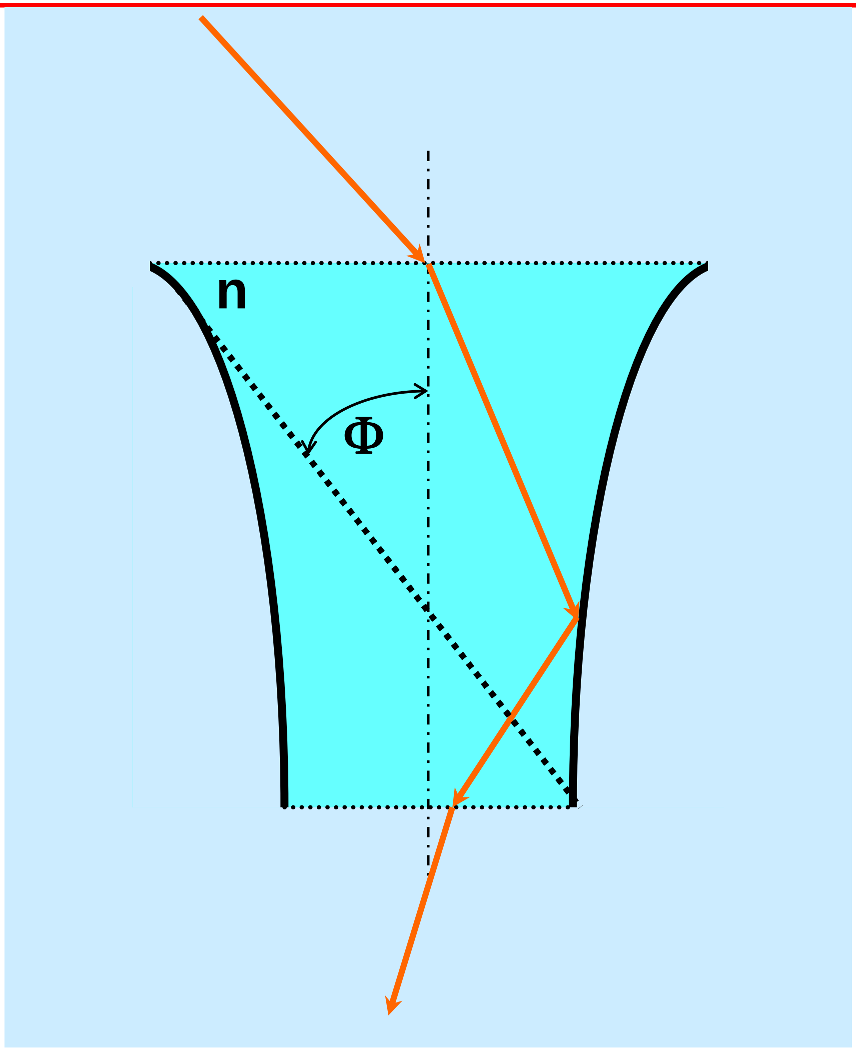
r radius tubular receiver.

Example of 2-D CPC + involute



Refractive CPC





$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_{1,max} = 90° \rightarrow \theta_{2,max} = \sin^{-1}(n_1/n_2)$$

$$C_{2D\text{-}CPC} = n/\sin\Phi$$

$$C_{3D\text{-}CPC} = n^2/\sin\Phi$$

Application to solar

- Requirements and considerations:
  - Cost and ease of manufacturing
  - Extend and guidance needed to follow the sun
  - Durability and maintenance
  - Required working temperature of absorber
  - Contamination and durability
  - Concentration level
  - Uniformity
  - Receiver temperature

Literature

- Winston et al., Nonimaging Optics, Elsevier, 2005