# Solar Energy Conversion Devices and Plants

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# Outline

- Energy and exergy analysis of solar devices and systems, i.e. thermodynamics of thermal radiation
  - Recap of exergy analysis
  - Thermodynamic properties of thermal radiation
  - Reversible processes
  - Irreversible processes
  - Ideal conversion of enclosed blackbody radiation
  - Maximizing of power output per unit collector area
  - Exergy for gravitational processes

# Recap of exergy concept

- Exergy – definition:

$$Ex = U - U_0 + KE + PE - T_0\left(S - S_0\right) + p_0\left(V - V_0\right)$$

- Specific exergy:

$$ex = u - u_0 + ke + pe - T_0\left(s - s_0\right) + p_0\left(v - v_0\right)$$

- Exergy difference between two states:

$$Ex_2 - Ex_1 = (U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1) - T_0\left(S_2 - S_1\right) + p_0\left(V_2 - V_1\right)$$

- Specific exergy difference between two states:

$$ex_2 - ex_1 = (u_2 - u_1) + (ke_2 - ke_1) + (pe_2 - pe_1) - T_0\left(s_2 - s_1\right) + p_0\left(v_2 - v_1\right)$$

# Exergy balance - closed systems

- Closed systems:

$$Ex_2 - Ex_1 = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q}_{} - \underbrace{\left(W_{12} - p_0\left(V_2 - V_1\right)\right)}_{} - \underbrace{T_0\sigma}_{}$$

**Exergy transfer by heat transfer** - **Exergy transfer by work** - **Exergy destruction** by irreversibilities

- Rate:

$$\frac{dEx}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W}_{12} - p_0\frac{dV}{dt}\right) - T_0\dot{\sigma}$$

- Expressed alternatively:

$$Ex_2 - Ex_1 = Ex_q - Ex_w - Ex_d$$

# Exergy balance - open systems

- Open systems – Exergy:

$$\frac{dEx}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j - \left(\dot{W} - p_0 \frac{dV}{dt}\right) + \sum_i \dot{m}_i ex_{f,i} - \sum_e \dot{m}_e ex_{f,e} - T_0\dot{\sigma}$$

**Change in exergy** within the volume = **Exergy transfer via heat transfer** - **Exergy transfer** via work + **Convective exergy transfer at the inlets and outlest** - **Exergy destruction due to irreversibilities**

- With flow exergy:

$$ex_f = u - u_0 + ke + pe - T_0\left(s - s_0\right) + p_0\left(v - v_0\right) + (p - p_0)v$$

$$ex_f = ex + (p - p_0)v$$

# Exergetic efficiency

- Exergy efficiency describes the effectiveness of energy resource utilization

$$\varepsilon_{ex} = \frac{\text{used exergy}}{\text{provided exergy}} \qquad \eta = \frac{\text{used energy}}{\text{provided energy}}$$

energy efficiency ($\eta$)

- Components:
  - Turbine:

$$\varepsilon_{ex} = \frac{\left(\dot{W} / \dot{m}\right)}{ex_{f,i} - ex_{f,e}}$$

  - Compressor/pump:

$$\varepsilon_{ex} = \frac{ex_{f,e} - ex_{f,\mathrm{i}}}{\left(-\dot{W}_{\mathrm{cv}} / \dot{m}\right)}$$

  - Heat exchanger: (non/mixing)

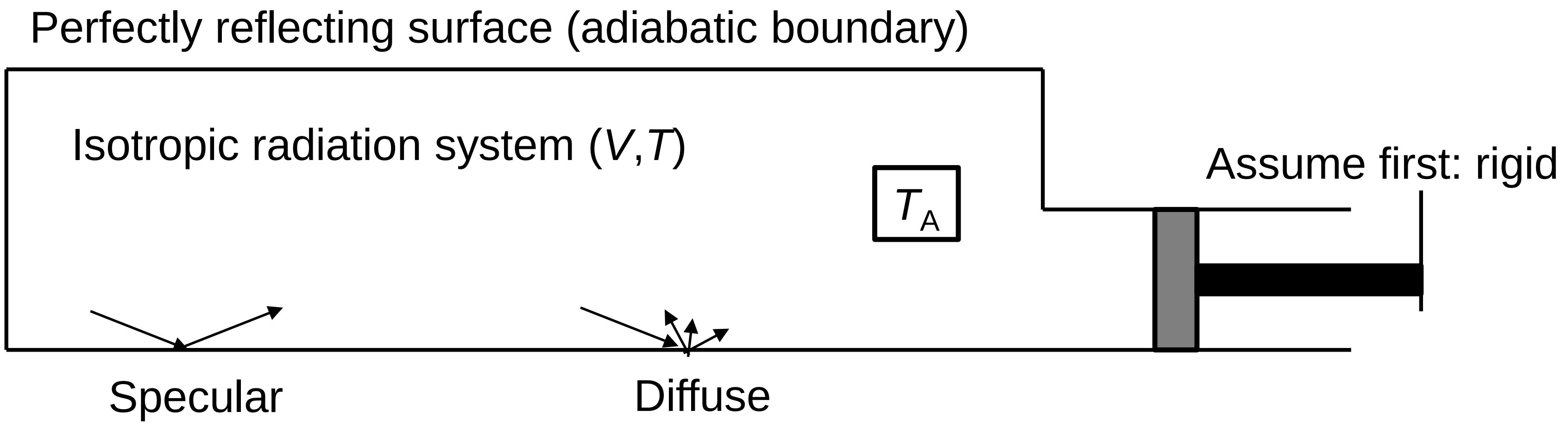
$$\varepsilon_{ex} = \frac{m_c\left(ex_{f,e,c} - ex_{f,i,c}\right)}{m_h\left(ex_{f,i,h} - ex_{f,e,h}\right)} \qquad \varepsilon_{ex} = \frac{m_2\left(ex_{f,3} - ex_{f,2}\right)}{m_1\left(ex_{f,1} - ex_{f,3}\right)}$$

# Extension of exergy concept

- Extension of exergy definition to radiation:
  - Assumptions:
    - Thermal equilibrium
    - Unpolarized radiation
    - Thermal radiation is described through discrete particles (photons) and not electromagnetic waves
  - Energy of photon: $E = \frac{hc}{\lambda} = h\nu$ [J]
  - Momentum of photon: $p = \frac{h}{\lambda} = \frac{h\nu}{c}$ [Js/m=kg·m/s]

# Extension of exergy concept

- Extension of exergy definition to radiation: photon gas




  - Volumetric density of black-body photons (Planck):

$$n_\nu = \frac{8\pi\nu^2 c^{-3}}{e^{h\nu/(kT)} - 1} \quad [\#_{photons}/m^3 \cdot s]$$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Energy
  - Energy per unit volume of black-body photon gas for a given frequency:

$$\left.\frac{U}{V}\right|_{\nu} = n_{\nu} h\nu = \frac{8\pi h\nu^{3} c^{-3}}{e^{h\nu/(kT)} - 1} \quad \text{[J/m}^3\cdot\text{s]}$$

  - Integrating over entire frequency domain:

$$\frac{U}{V} = \int_{0}^{\infty} \left.\frac{U}{V}\right|_{\nu} d\nu = \underbrace{\frac{8\pi^{5}}{15}\frac{k^{4}}{h^{3}c^{3}}}_{a=4\sigma/c} T^{4} \quad \text{[J/m}^3\text{]}$$

with radiation density constant: $a = \frac{4\sigma}{c} = 7.565 \cdot 10^{-16}\,\mathrm{J/(m^{3}K^{4})}$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Energy
  - Total energy of a volume V filled with black-body radiation is:

$$U = aT^4V = \frac{4\sigma}{c}T^4V \quad \text{[J]}$$

  - Pencil of ray of unit solid angle can be used to define energy that arrives per time, unit area, unit solid angle, and unit frequency from the direction that is the axis of the pencil:

$$i'_{\nu b} = \left.\frac{U}{V}\right|_{\nu} \frac{c}{4\pi} = \frac{2h\nu^3 c^{-2}}{e^{h\nu/(kT)} - 1} \quad \text{[W/(m}^2\cdot\text{s}\cdot\text{sr)]}$$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Energy
  - Spectral intensity of blackbody radiation (wavelength-dependent):

$$i'_{\lambda b} = \frac{2hc^2}{\lambda^5 e^{hc/(\lambda kT)} - 1}$$

  - Total intensity: $i'_b = \int_0^\infty i'_{\lambda b} d\lambda = \frac{\sigma}{\pi} T^4$

  - Temperature can not only be assigned to photon gas but also beam of radiation (such as solar radiation arriving from a very small solid angle from the sun)

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Pressure

Pressure: photon-wall collision is also exerting pressure

– Classical kinetic theory of monoatomic gases:

$$p = \frac{1}{3}\frac{N}{V} m W_{\mathrm{ave}}^2$$

– Applied to black-body photon gas: $p = \frac{1}{3}\frac{N}{V} h\nu = \frac{1}{3}\frac{U}{V}$

– Or: $p = \frac{4}{3}\frac{\sigma}{c} T^4$

– Note: If temperature of blackbody radiation is constant, so is the volume-specific energy, and the pressure

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Entropy
- *TdS*-equation:

$$TdS = pdV + dU$$

- We use volume-specific energy: $\frac{U}{V} = u$

- Therefore:

$$dS = \frac{u}{3T}dV + \frac{1}{T}\left(udV + Vdu\right)$$

$$dS = \frac{4aT^3}{3}dV + 4aT^2VdT$$

$$dS = \frac{4a}{3}d\left(VT^3\right)$$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Entropy
- Assuming $S$=0 at $T$=0K:

$$S = \frac{4a}{3} V T^3 = \frac{4}{3}\frac{U}{T}$$

- Volume specific, $s$:

$$s = \frac{4}{3} a T^3 = \frac{4}{3}\frac{u}{T}$$

- Note, also $s$ (like $u$) is only a function of $T$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Entropy
- Alternatively, use:

$$S = \frac{4a}{3} V T^3 = \frac{4}{3} \frac{U}{T}$$

- Which leads to:

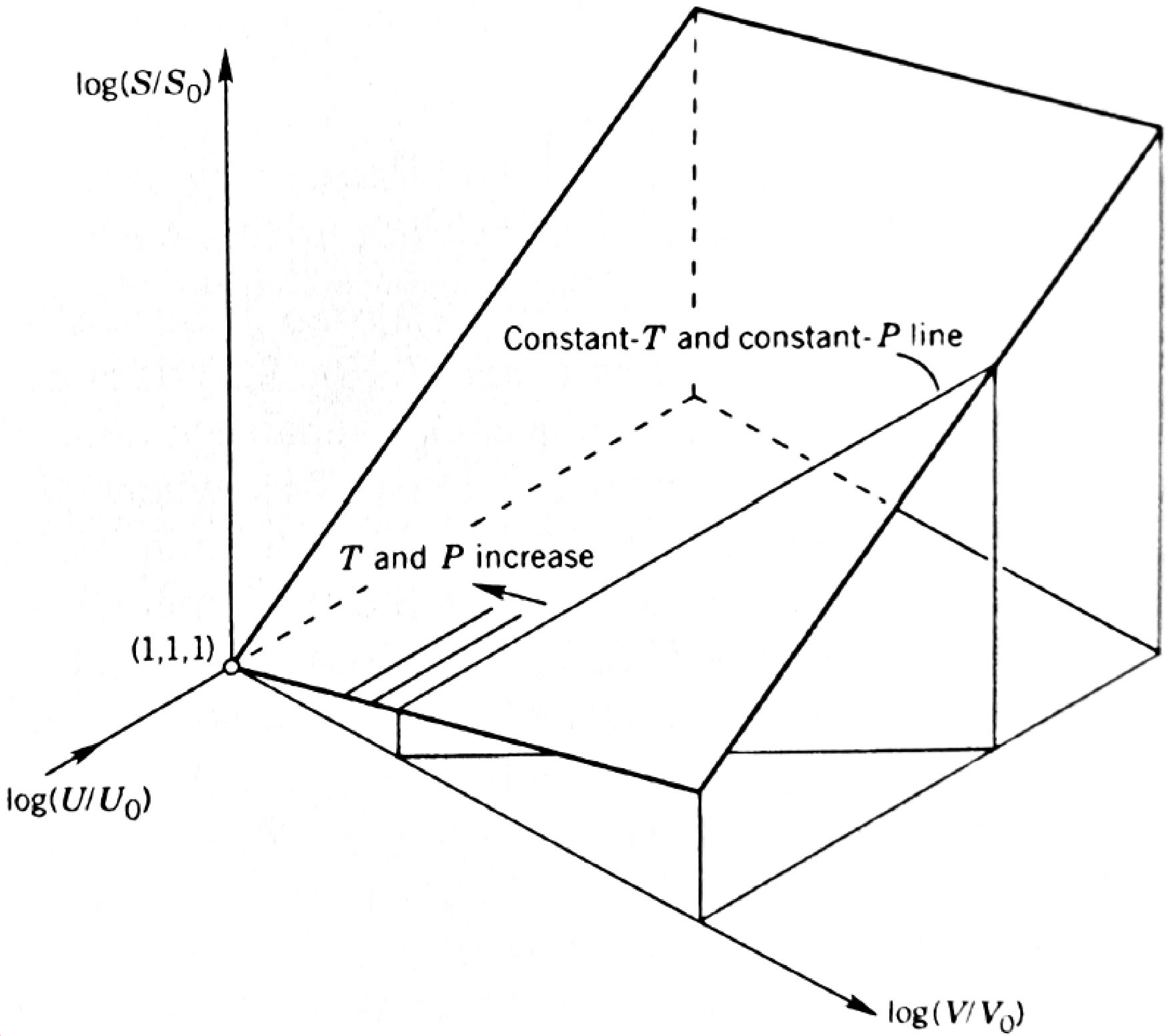
$$S(U,V) = \frac{4}{3} a^{1/4} V^{1/4} U^{3/4}$$

- Or:

$$U(S,V) = \left( \frac{3}{4} \right)^{4/3} a^{-1/3} V^{-1/3} S^{4/3}$$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation - Entropy
- Graphical equation of state:



# Extension of exergy concept

- Thermodynamic properties of thermal radiation – Specific heat
- Specific heat capacity at constant volume:

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$C_V = 4aVT^3$$

- Specific heat capacity at constant pressure:

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

$$C_p \rightarrow \infty$$

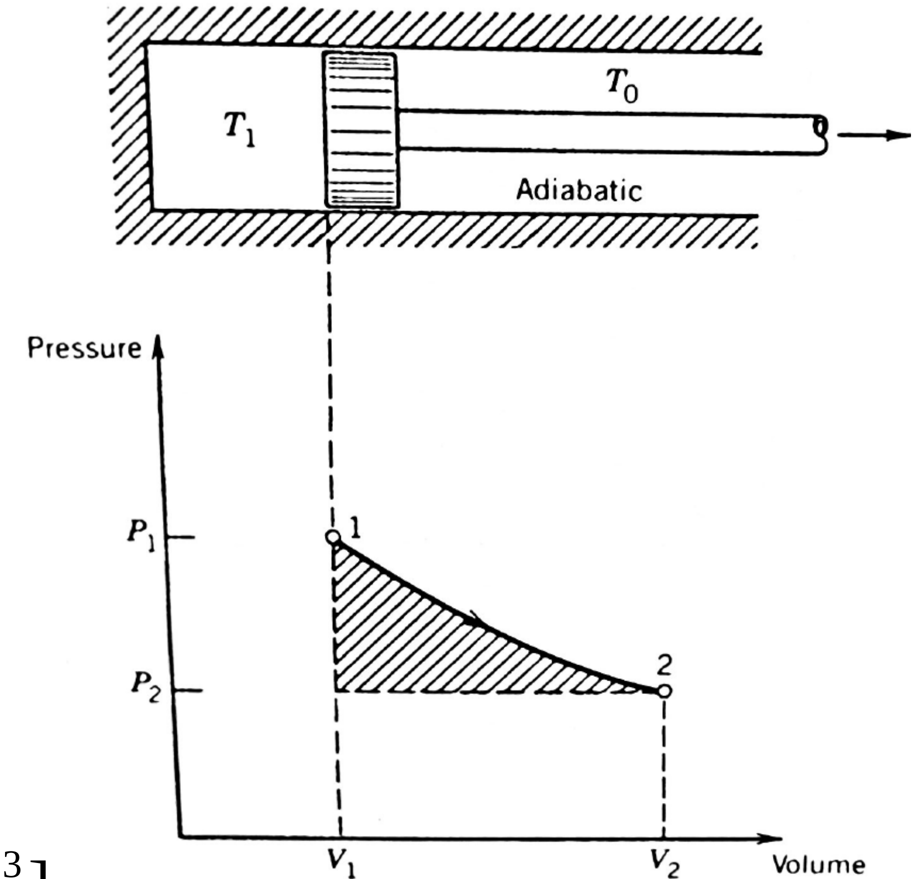
as temperature change is 0 for a given entropy and constant pressure

# Extension of exergy concept

- Thermodynamic properties of thermal radiation – Exergy

- Exergy balance:

$$Ex_2 - Ex_1 = \int_1^{2=0} p dV - p_2 (V_2 - V_1)$$




- State 1: $V_1$, $T_1$
- State 2: $V_2$, $T_2$=$T_0$ $\rightarrow p_2 = \frac{a}{3} T_0^4$

$$p = \text{const} \cdot V^{-4/3}$$

- Exergy of photon gas:

$$\frac{Ex}{V_1} = \frac{a}{3}\left(3T_1^4 + T_0^4 - 4T_0 T_1^3\right) \quad [\text{J/m}^3]$$

# Extension of exergy concept

- Thermodynamic properties of thermal radiation – Exergy of emission

- Exergy of emission of gray surface [W/m$^2$]:

$$ex = \varepsilon \frac{\sigma}{3}\left(3T^4 + T_0^4 - 4T_0 T^3\right)$$

- Exergy of emission of black surface [W/m$^2$]:

$$ex = \frac{\sigma}{3}\left(3T^4 + T_0^4 - 4T_0 T^3\right)$$

# Extension of exergy concept

- Reversible processes – reversible and adiabatic expansion or compression:
  - Reversible volume change form $V_1$ to $V_2$ in the absence of heat transfer → entropy stays constant

$$S = \frac{4a}{3} V T^3 = \text{const} \rightarrow V T^3 = \text{const}$$
$$\rightarrow V p^{3/4} = \text{const}$$

  - Work transfer: $$W_{12,rev} = \int_1^2 p dV = 3 p_1 V_1 \left(1 - \left(V_1 / V_2\right)^{1/3}\right)$$
  - Work is greatly influenced by initial pressure (or temperature) of black-body radiation

# Extension of exergy concept

- Reversible processes – reversible and isothermal expansion or compression:
  - Temperature (i.e. pressure) stays also constant. Work:

$$W_{12,rev} = \int_1^2 p dV = p\left(V_2 - V_1\right) = \frac{a}{3} T^4 \left(V_2 - V_1\right)$$

  - Work depends strongly on temperature of process
  - Heat transfer:

$$Q_{12,rev} = W_{12,rev} + U_2 - U_1 = \frac{4}{3} a T^4 \left(V_2 - V_1\right)$$

  - During expansion heat input is 4x larger than work output

# Extension of exergy concept

- Reversible processes – Carnot cycle (two adiabatic and two isothermal processes, all reversible):
  - Isothermal processes bound by $T_H$ and $T_L$
  - or by $p_H=(a/3)T_H^3$ and $p_L=(a/3)T_L^3$
  - and by $S_1=(4/3)aV_1T_H^3$ and $S_2=(4/3)aV_2T_H^3$
  - Net work and heat:

$$\oint \delta W_{rev} = \oint \delta Q_{rev} = \oint T dS = \frac{4}{3} a T_H^3 (V_2 - V_1)(T_H - T_L)$$

  - Carnot:

$$\eta_C = \frac{\oint \delta W_{rev}}{Q_H} = \frac{\frac{4}{3} a T_H^3 (V_2 - V_1)(T_H - T_L)}{\frac{4}{3} a T_H^4 (V_2 - V_1)} = 1 - \frac{T_L}{T_H}$$

# Extension of exergy concept

- Irreversible processes – Adiabatic free expansion:
  - Isolated system, free expansion: initial and final states

$$U_1 = aT_1^4 V_1 \qquad U_2 = aT_2^4 V_2$$

$$S_1 = \frac{4a}{3} T_1^3 V_1 \qquad S_2 = \frac{4a}{3} T_2^3 V_2$$

  - As isolated:

$$Q_{12} = 0 = W_{12} \rightarrow U_1 = U_2$$

  - Therefore:

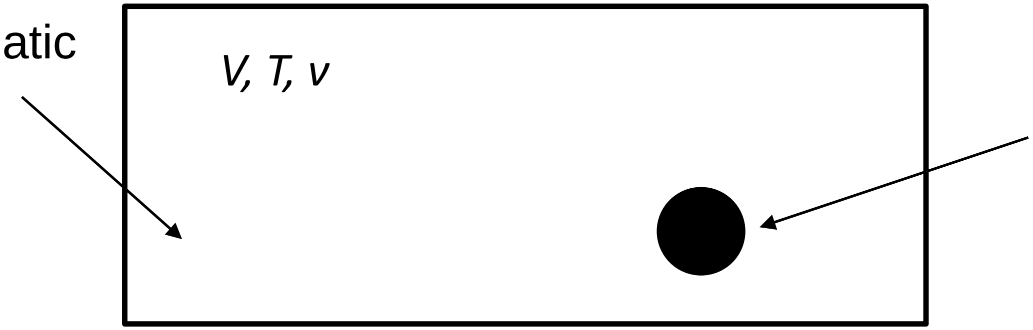
$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{1/4} \rightarrow T_2 < T_1$$

  - And:

$$S_2 - S_1 = S_1 \left( \left( V_2 / V_1 \right)^{1/4} - 1 \right) = S_1 \left( \left( T_1 / T_2 \right) - 1 \right) > 0$$

# Extension of exergy concept

- Irreversible processes – Transformation of monochromatic radiation into black-body radiation:




- Initial energy: $U = \dfrac{8\pi h \nu^3 c^{-3}}{e^{h\nu/(kT)} - 1} V \Delta\nu$
- Final energy: $U = aVT_b^4$

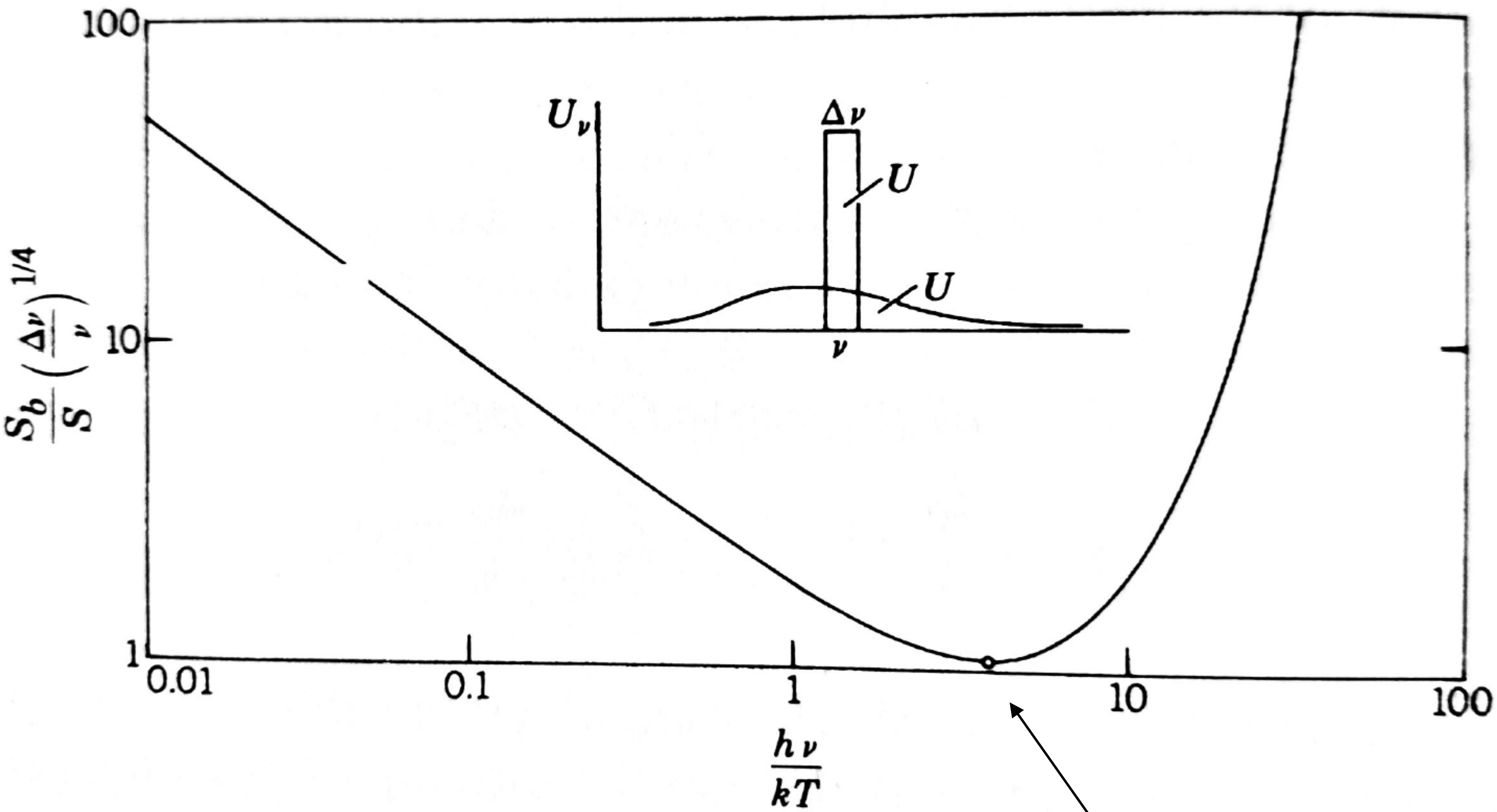
$$\frac{kT_b}{h\nu} = \frac{15^{1/4}}{\pi}\left(\frac{\Delta\nu/\nu}{e^{h\nu/(kT)}-1}\right)^{1/4}$$

- Generated entropy:

$$\frac{S_b}{S} = \frac{4/3 \cdot U/T_b}{4/3 \cdot U/T} = \frac{T}{T_b} = \frac{\pi}{15^{1/4}}\left(\frac{\Delta\nu}{\nu}\right)^{-1/4} \frac{kT}{h\nu}\left(e^{h\nu/(kT)}-1\right)^{1/4}$$

# Extension of exergy concept

- Irreversible processes – Transformation of monochromatic radiation into black-body radiation:




$$\text{minimal @ } \frac{kT}{h\nu} = 3.921 \rightarrow \lambda T = 0.367\text{cmK}$$

$$\text{close to Wien:} \lambda T = 0.29\text{cmK}$$

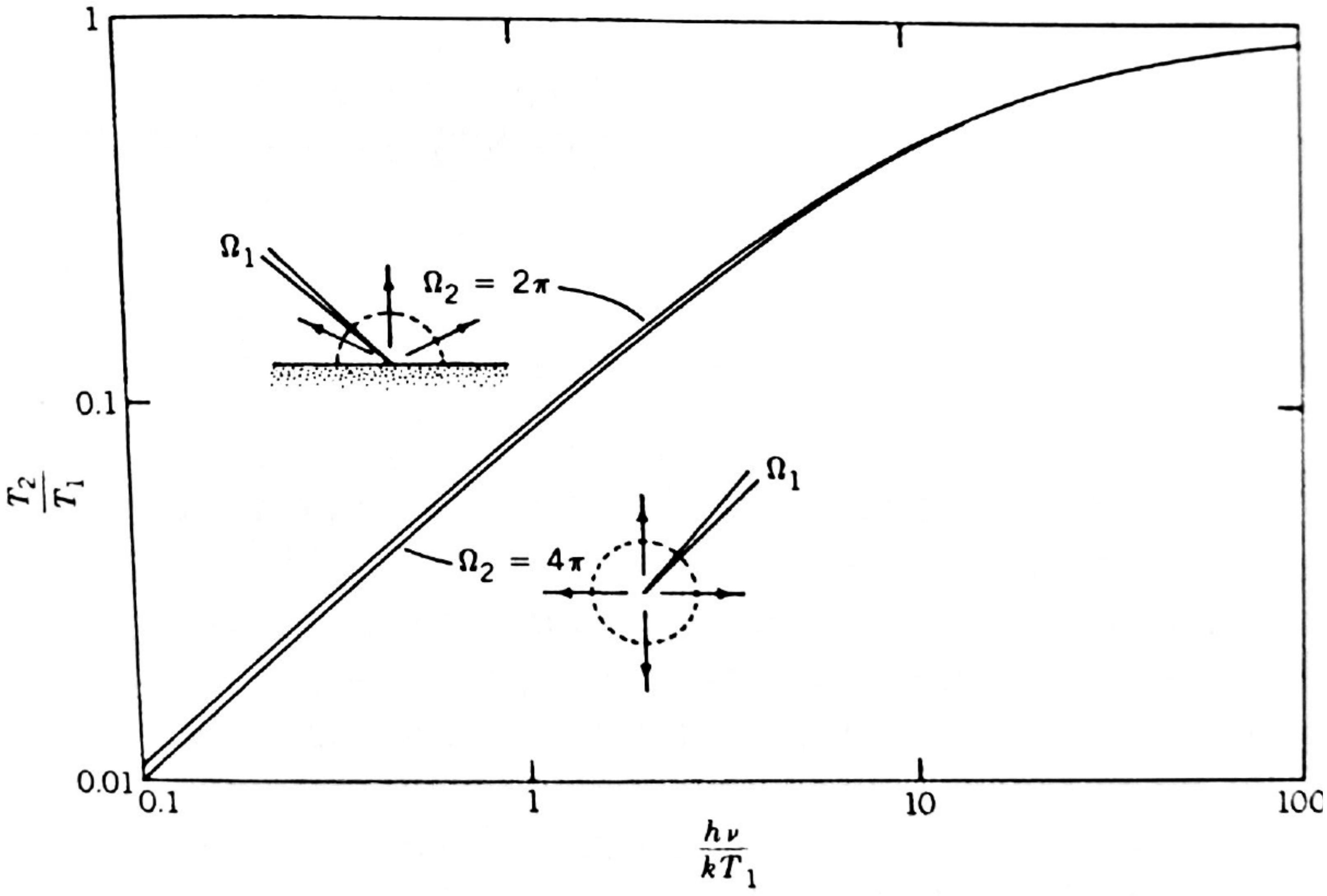
# Extension of exergy concept

- Irreversible processes – Scattering:
- Incident radiation: $\frac{i_{\nu b}}{\Omega_1} = \frac{2h\nu^3 c^{-2}}{e^{h\nu/(kT_1)} - 1}$ with $\Omega_1 = 2\theta_{sun}$
- Scattered: $\frac{i_{\nu b}}{\Omega_2} = \frac{2h\nu^3 c^{-2}}{e^{h\nu/(kT_2)} - 1}$ with $\Omega_2 = 4\pi$ or $2\pi$
- Divide, yields: $\frac{T_1}{T_2} = \frac{1}{h\nu/(kT_1)} \ln\left[\frac{\Omega_2}{\Omega_1}\left(e^{h\nu/(kT_1)} - 1\right) + 1\right]$
- Temperature drop leads to entropy generation:

$$S_{gen} = \frac{4}{3} i_{\nu b} \left(\frac{1}{T_2} - \frac{1}{T_1}\right) \geq 0$$

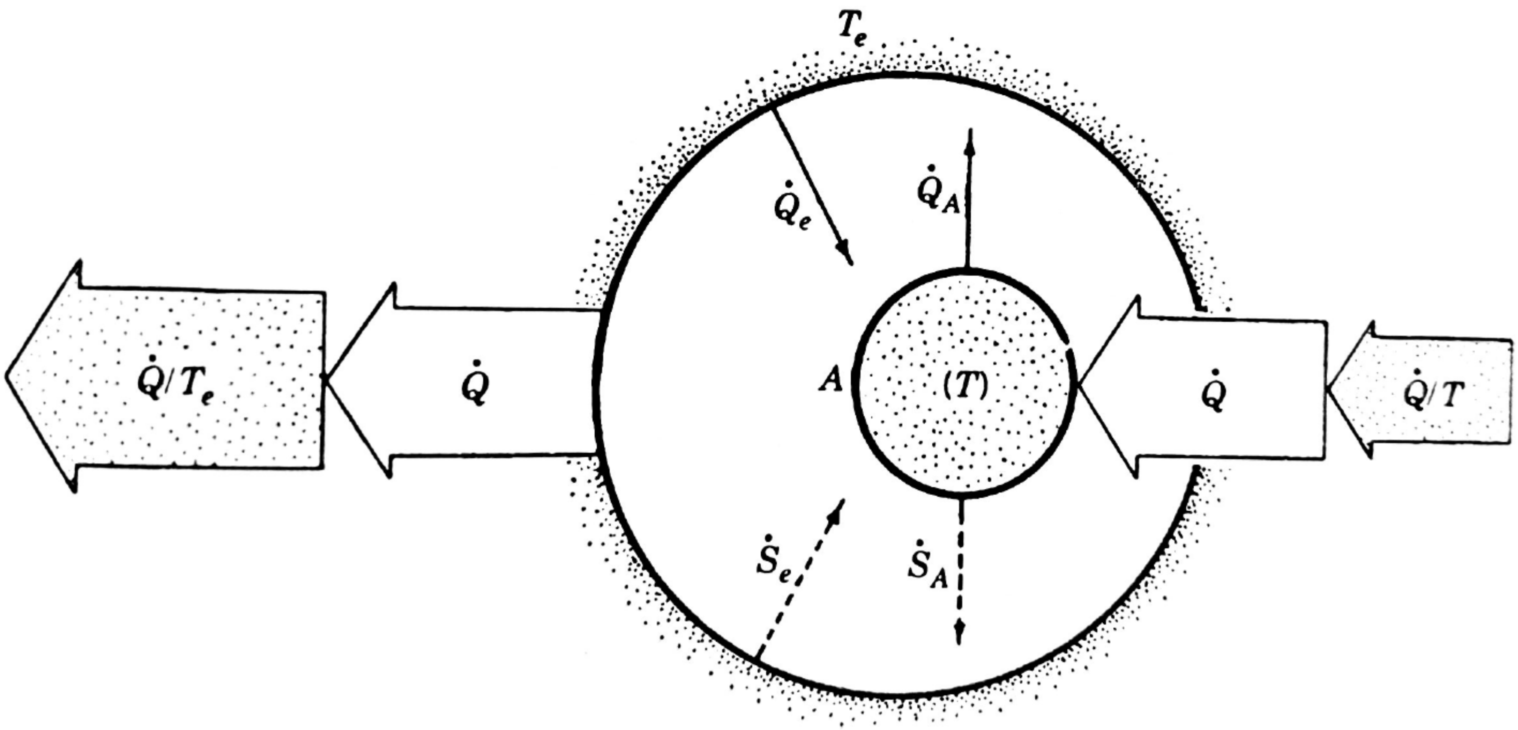
# Extension of exergy concept

- Irreversible processes – Scattering:



# Extension of exergy concept

- Irreversible processes – Net radiative heat transfer:
- Two black bodies (inner and outer enclosures) in thermal equilibrium and each in contact with thermal reservoirs:



# Extension of exergy concept

- Irreversible processes – Net radiative heat transfer:
- Inner enclosure:
  - Emitted rate of energy: $\dot{Q}_A = A\sigma T^4$
  - Corresponding entropy: $\dot{S}_A = \frac{4}{3T}\dot{Q}_A = \frac{4}{3}A\sigma T^3$
- Energy arriving to inner enclosure: $\dot{Q}_e = A\sigma T_e^{\,4}$
  - And entropy: $\dot{S}_e = \frac{4}{3T}\dot{Q}_e = \frac{4}{3}A\sigma T_e^{\,3}$
- Heat from reservoir, fulfilling energy conservation (for surface A):

$$\dot{Q} = \dot{Q}_A - \dot{Q}_e = A\sigma(T^4 - T_e^{\,4})$$

- Entropy generation of body A:

$$\dot{S}_{gen,A} = \dot{S}_A - \dot{S}_e - \dot{Q}/T = \frac{\sigma A}{3T}(T - T_e)^2(T^2 + 3T_e^2 - 2TT_e) \geq 0$$

# Extension of exergy concept

- Irreversible processes – Net radiative heat transfer:
  - As soon as temperature differences between two black bodies exists, there are irreversibilities
  - Surface *A* origin of irreversibility
  - Important as any solar conversion device can be seen as blackbody
- If $\dot{S}_e$ and $\dot{Q}_e$ are negligible (e.g. $T_e \approx$ 0K): $\dot{S}_{gen,A} = \frac{1}{3} A\sigma T^3 > 0$
  - Emission in the absence of absorption is a definitive source of irreversibility
- Other extreme: absorption in absence of emission: $\dot{Q}_A = 0, T_e \rightarrow T$
  - First law says: $\dot{Q} = -\dot{Q}_e < 0$
  - Second law violated: $\dot{S}_{gen,A} = -\dot{S}_e - \dot{Q}/T = -\frac{4}{3T}\dot{Q}_e + \frac{\dot{Q}_e}{T} < 0$

It is impossible for a black body to absorb radiation of the same temperature without emitting radiation

# Extension of exergy concept

- Irreversible processes – Net radiative heat transfer:

- Entropy generation of enclosure:

$$\dot{S}_{gen,e} = \dot{S}_e - \dot{S}_A - \dot{Q}/T_e = \frac{\sigma A}{3T_e}(T - T_e)^2(T_e^{\,2} + 3T^2 - 2T_e T) \geq 0$$

- Entropy generation of complete system:

$$\dot{S}_{gen} = \dot{S}_{gen,A} + \dot{S}_{gen,e} = \frac{\sigma A}{TT_e}(T - T_e)^2(T + T_e)(T^2 + T_e^{\,2}) \geq 0$$

Regardless of relative size of $T$ and $T_e$ (i.e. regardless of physical direction of $\dot{Q}$), the occurrence of net heat transfer is accompanied by the generation of entropy

# Extension of exergy concept

- Irreversible processes – Kirchhoff law:
  - Same example as slide 28, but assume non-black surfaces:
    - Energy and entropy emitted by surface A, at *T:*

$$\dot{Q}_A = \iiint \varepsilon_\nu' i_{\nu b}' \cos\theta dA d\Omega d\nu$$

$$\dot{S}_A = \iiint \varepsilon_\nu' \frac{4}{3T} i_{\nu b}' \cos\theta dA d\Omega d\nu$$

- Energy and entropy arriving at A and absorbed, at $T_e$:

$$\dot{Q}_e = \iiint \alpha_\nu' i_{\nu b}' \cos\theta dA d\Omega d\nu$$

$$\dot{S}_A = \iiint \alpha' \frac{4}{3T} i_{\nu b}' \cos\theta dA d\Omega d\nu$$

# Extension of exergy concept

- Irreversible processes – Kirchhoff law:
  - Same example as slide 28, but assume non-black surfaces, assume system is insulated:
    - Energy balance:

$$\dot{Q}_A = \dot{Q}_e \rightarrow \varepsilon'_\nu(\nu, T, \theta, \varphi) i'_{\nu b}(\nu, T) = \alpha'_\nu(\nu, T_e, \theta, \varphi) i'_{\nu b}(\nu, T_e)$$

    - Entropy balance at internal equilibrium:

$$\dot{S}_A = \dot{S}_e \rightarrow \varepsilon'_\nu(\nu, T, \theta, \varphi) \frac{4}{3T} i'_{\nu b}(\nu, T) = \alpha'_\nu(\nu, T_e, \theta, \varphi) \frac{4}{3T_e} i'_{\nu b}(\nu, T_e)$$

    - We get: $T = T_e$

$$\varepsilon'_\nu(\nu, T, \theta, \varphi) = \alpha'_\nu(\nu, T, \theta, \varphi)$$

Spectral directional emissivity of a surface of *T* is exactly equal to the spectral directional absorptivity, when the surface is in equilibrium with the radiation that surrounds it

# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- «Petela's theory»:

  Consider deformable, reflective enclosure. What is the maximal work we can extract when it settles to dead state?

- State 1: $V_1$, $T_1$
- State 2: $V_2$, $T_2$=$T_0$ $\rightarrow p_2 = \frac{a}{3} T_0^4$
- Max. Work in reversible process:

$$W_{12} = \int_1^2 p dV - p_2 (V_2 - V_1)$$

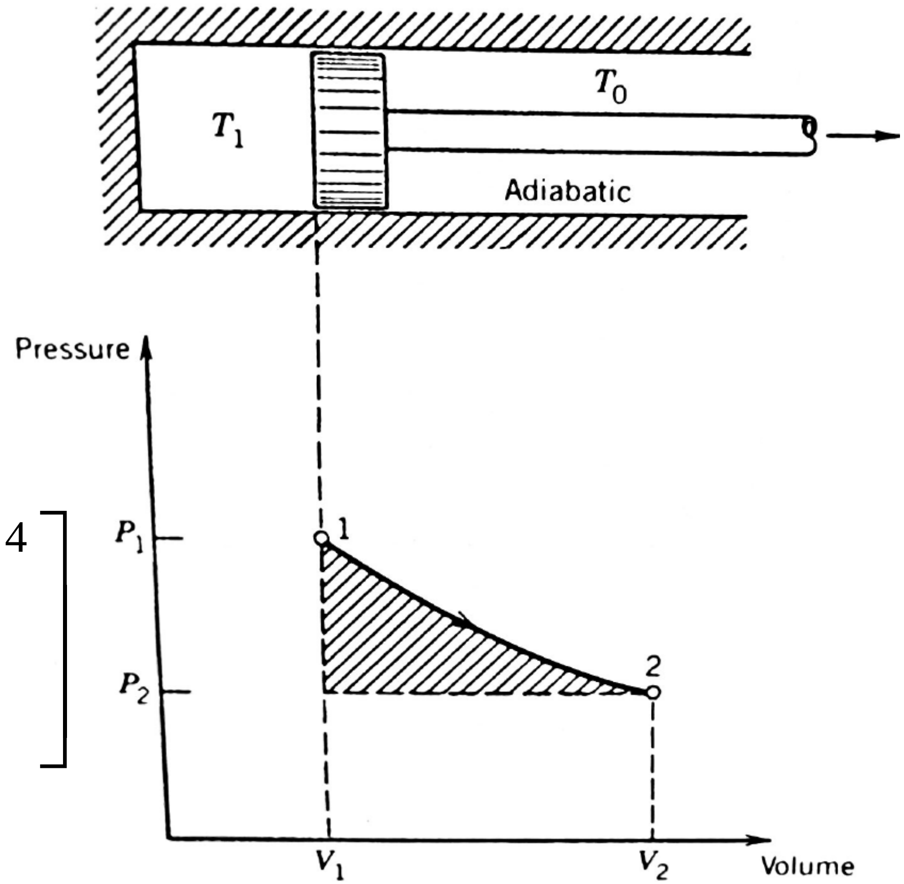
with

$$p = \text{const} \cdot V^{-4/3}$$

$$\rightarrow W_{12} = U_1 \left[ 1 - \frac{4}{3} \frac{T_2}{T_1} + \frac{1}{3} \left( \frac{T_2}{T_1} \right)^4 \right]$$

with

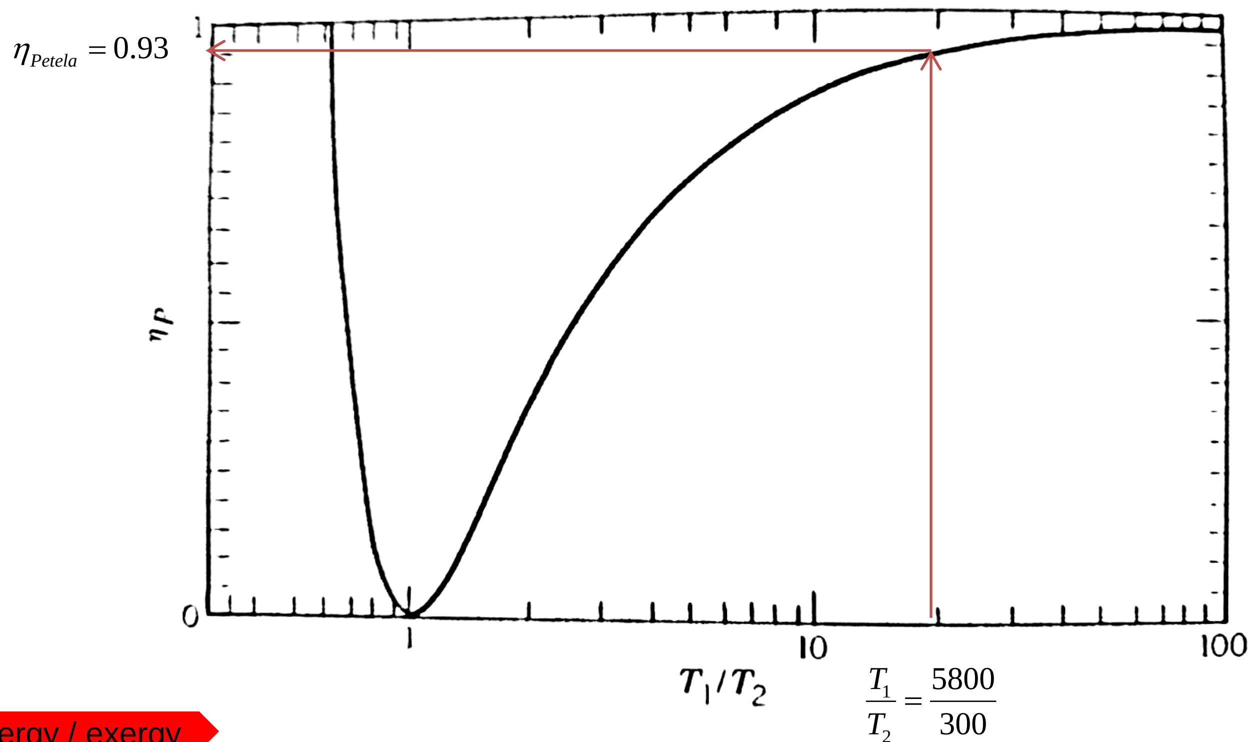
$$U_1 = a T_1^4 V_1$$



# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- «Petela's theory»:
- Efficiency to convert $U_1$ into work:

$$\eta_{Petela} = \frac{W_{12}}{U_1} = \left[1 - \frac{4}{3}\frac{T_2}{T_1} + \frac{1}{3}\left(\frac{T_2}{T_1}\right)^4\right]$$



# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- «Petela's theory»:

- Alternative derivations:
  - Exergy balance for closed system, max. possible:

$$Ex_2 - Ex_1 = \int_1^2 \cancel{\left(1 - \frac{T_0}{T}\right)} \delta Q - \left(W_{12} - p_0\left(V_2 - V_1\right)\right) - \cancel{T_0 \sigma}$$

$$Ex_2 - Ex_1 = U_2 - U_1 - T_0(S_2 - S_1) + p_0(V_2 - V_1)$$

with $T_2 = T_0, p_0 = \frac{a}{3} T_0^4 = 2.02 \cdot 10^{-11} \text{atm} \neq p_{atm}$

$$U = aVT^4, S = \frac{4}{3} aVT^3 \quad \rightarrow \eta_{Petela}$$

# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- Two alternative efficiencies given in literature:
  - Spanner:

$$\eta_{Spanner} = 1 - \frac{4}{3}\frac{T_2}{T_1}$$

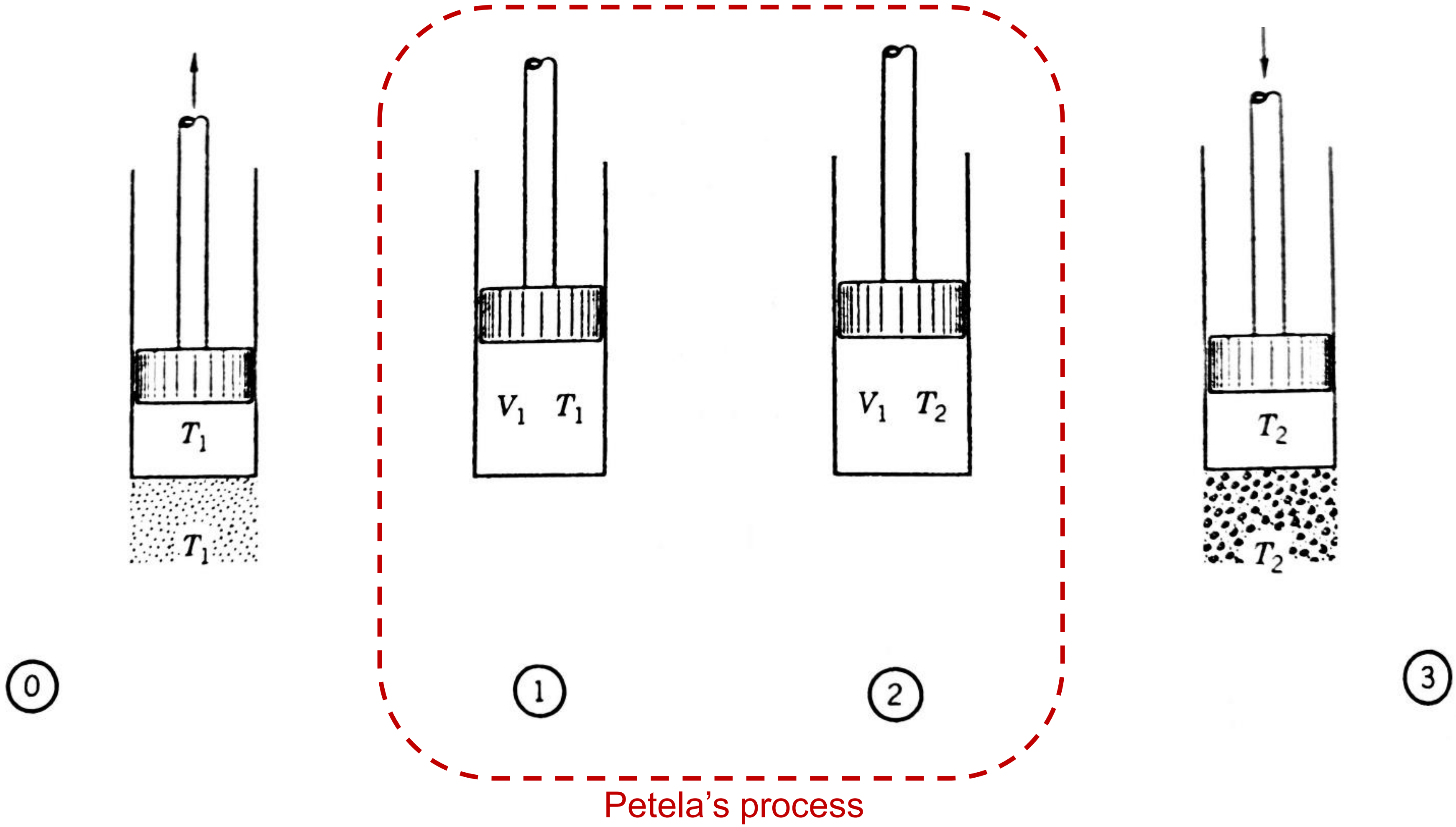
  - Jeter:

$$\eta_{Jeter} = 1 - \frac{T_2}{T_1}$$

- How do the three approaches complement each other?
  - Origin of equilibrium blackbody radiation postulated at the beginning of the process?
  - What is the ultimate fate of the blackbod radiation of $T_2$ left when the system reaches dead end?

# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- Bejan:



# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- By Bejan:
  - Process 0-1: Reversible manufacturing of ($V_1$,$T_1$) radiation system, while in contact with thermal reservoir at $T_1$. Volume increased from 0 to $V_1$ while isothermally heating:

$$W_{01} = \int p dV$$

$$S_{gen,01} = S_1 - \frac{Q_{01}}{T_1} = 0$$

  - Process 2-3: Reverse of process 0-1

$$W_{23} = -\frac{1}{3} U_2 , Q_{23} = -\frac{4}{3} U_2$$

$$S_{gen,23} = -S_2 - \frac{Q_{23}}{T_2} = 0$$

# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- By Bejan:
  - Overall work:

$$\oint \delta W = W_{01} + W_{12} + W_{23} = \frac{4}{3} U_1 \left[ 1 - \frac{T_2}{T_1} \right]$$

  - Max. efficiency:

$$\eta_C = \frac{\oint \delta W}{Q_{01}} = \frac{\frac{4}{3} U_1 \left[ 1 - \frac{T_2}{T_1} \right]}{\frac{4}{3} U_1} = 1 - \frac{T_2}{T_1} = \eta_{Jeter}$$

    - Intuitively we have heat exchange with two reservoirs, Carnot efficiency

# Extension of exergy concept

- Ideal conversion of enclosed blackbody radiation:
- By Bejan:
  - Adapted process 0-1: spontaneous (irreversible) no work process:

$$W_{01} = 0, Q_{01} = U_1$$

$$S_{gen,01} = S_1 - \frac{Q_{01}}{T_1} > 0$$

  - Therefore: $$\oint \delta W = \cancel{W_{01}} + W_{12} + W_{23} = aT_1^4 V_1 \left[ 1 - \frac{4T_2}{3T_1} \right]$$

  - And max. efficiency: $$\eta = \frac{\left( \oint \delta W \right)_{\text{irrev}}}{Q_{01,\text{no-work}}} = \left[ 1 - \frac{4T_2}{3T_1} \right] = \eta_{Spanner}$$

# Recap and extension of exergy concept

- Comparing Bejan/Jeter, Petela, and Spanner:

$$\eta_{Jeter} = 1 - \frac{T_2}{T_1} \quad \Bigg| \quad \eta_{Petela} = \left[1 - \frac{4}{3}\frac{T_2}{T_1} + \frac{1}{3}\left(\frac{T_2}{T_1}\right)^4\right] \quad \Bigg| \quad \eta_{Spanner} = 1 - \frac{4}{3}\frac{T_2}{T_1}$$

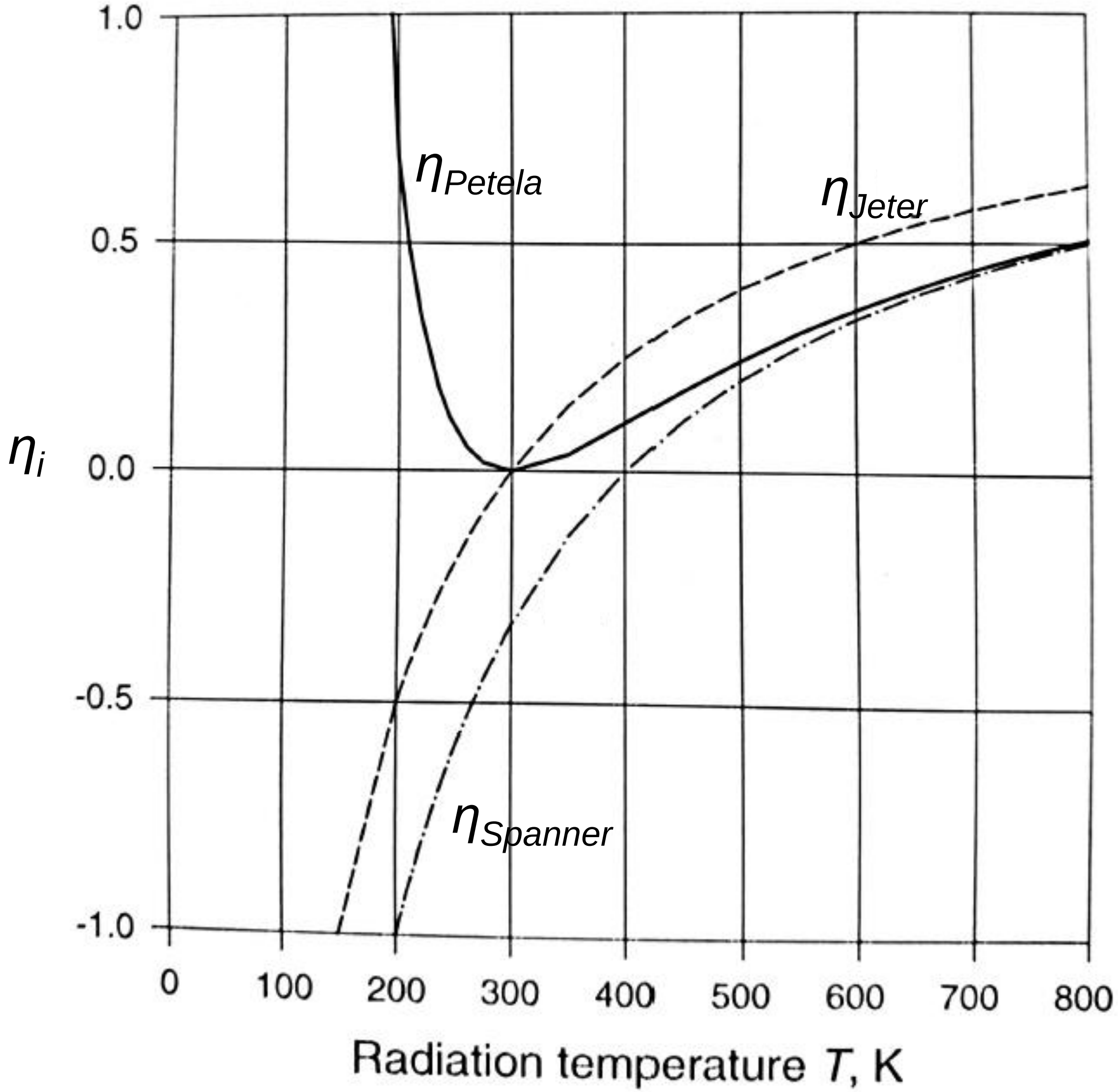
  - $W_{01} > 0$, $W_{23} < 0$:

$$W_{0-3,Bejan/Jeter} > W_{1-2,Petela} > W_{1-3,Spanner}$$

  - $\eta_{\mathrm{Spanner}}$ can drop below 0, when work delivered ($W_{12}$) cannot compensate work required to eliminate radiation ($W_{23}$)

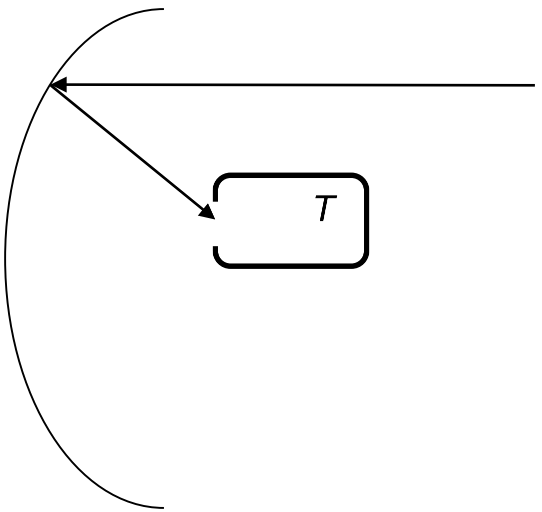
# Recap and extension of exergy concept

- Comparing Bejan/Jeter, Petela, and Spanner:



# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Ideal concentrators:




- Insulated enclosure filled by isentropic blackbody radiation at *T*
- In limiting case:

$$\dot{Q}_{in} = \sigma A T_s^4 = \sigma A T^4 = \dot{Q}_{out}$$

$$\rightarrow T = T_s$$

- However, this case is not interesting as no heat is actually available

# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Ideal concentrators:

- Instead use heat: $\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out} = \sigma A\left(T_s^4 - T^4\right)$

  in Carnot engine: $\dot{W}_C = \dot{Q}\left(1 - T_0 / T\right)$

  to $T$=$T_s$ limit, Carnot efficiency is maximized but $\dot{Q} = 0 = \dot{W}$

  only way to have finite work and heat, $A \rightarrow \infty$

- Why not instead maximize $\dfrac{\dot{W}_C}{A}$ ?

- Maximize

$$\eta_A = \frac{\dot{W}_C}{\sigma A T_s^4} = \left[1 - \left(\frac{\theta}{\theta_s}\right)^4\right]\left(1 - \frac{1}{\theta}\right) \quad \text{with} \quad \theta = \frac{T}{T_0} \text{ and } \theta_s = \frac{T_s}{T_0}$$

# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Ideal concentrators:

$$\frac{d\eta_A}{dA} \overset{!}{=} 0 \rightarrow 4\theta^5 - 3\theta^4 - \theta_s^4 = 0$$

- Optimal temperature and efficiency, approximated:

$$\theta_{opt} \cong 4^{-1/5} \theta_s^{4/5} \qquad \eta_A \cong 1 - 5\left(4\theta_s\right)^{-4/5}$$

- For example for $T_s$=5800K and $T_0$=300K: $\theta_{opt}$=8.22 and $\eta_{A,\max}$=0.849

# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Omnicolor series of ideal concentrators
  - Assume intercepted radiation is monochromatic:

$$\dot{W}_{C,\nu}(T,\nu) = \dot{Q}_{\nu}\left(1 - T_0 / T\right)$$

$$\dot{Q}_{\nu} = \pi A\left(i'_{\nu b}(T_s,\nu) - i'_{\nu b}(T,\nu)\right)$$

  - As before: find temperature at which efficiency is optimized

$$\dot{W}_{C,\nu,\max} = \dot{W}_{C,\nu}(T_{opt},\nu)$$

  - And power of series of omnicolor collectors:

$$\dot{W}_{C,\max,omnicolor} = \int_0^{\infty} \dot{W}_{C,\nu,\max}(\nu)\,d\nu$$

# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Omnicolor series of ideal concentrators

- Maximization with respect to area, gives:

$$\eta_{\mathrm{A,max,omnicolor}} = \frac{\dot{W}_{\mathrm{C,max,omincolor}}}{\sigma A T_s^4} \cong 1 - \left(1.567 + 0.37 \ln \theta_s\right) \theta_s^{-1}$$

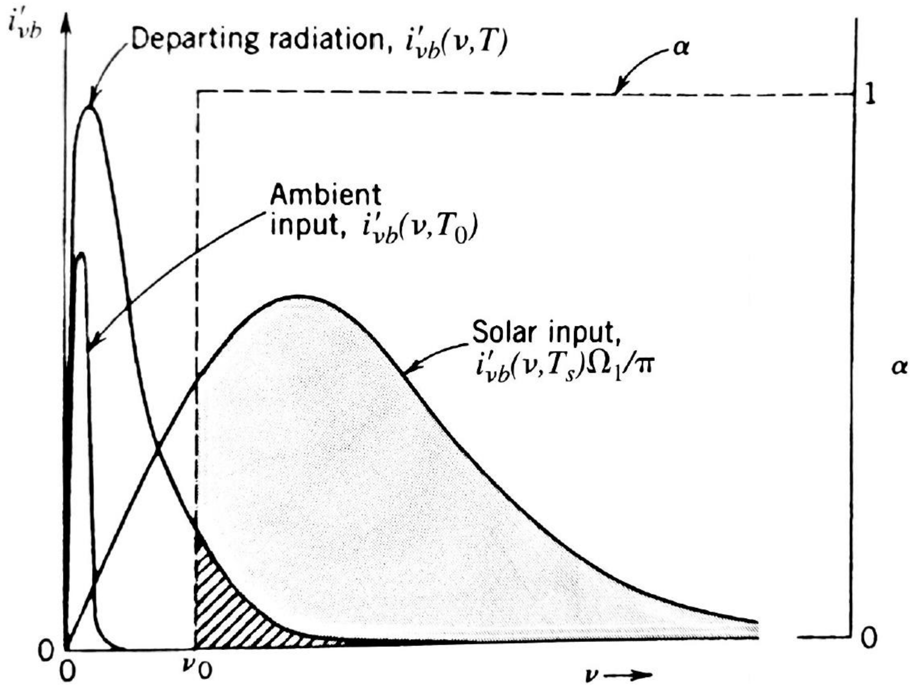
- For example for $T_s$=5800K and $T_0$=300K: $\eta_{A,\mathrm{max,omnicolor}}$=0.861

# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Unconcentrated solar radiation:
  - Radiation from sun is seen by small solid angle ($\Omega_1$), and environment also contributes. Additionally we assume absorptivity

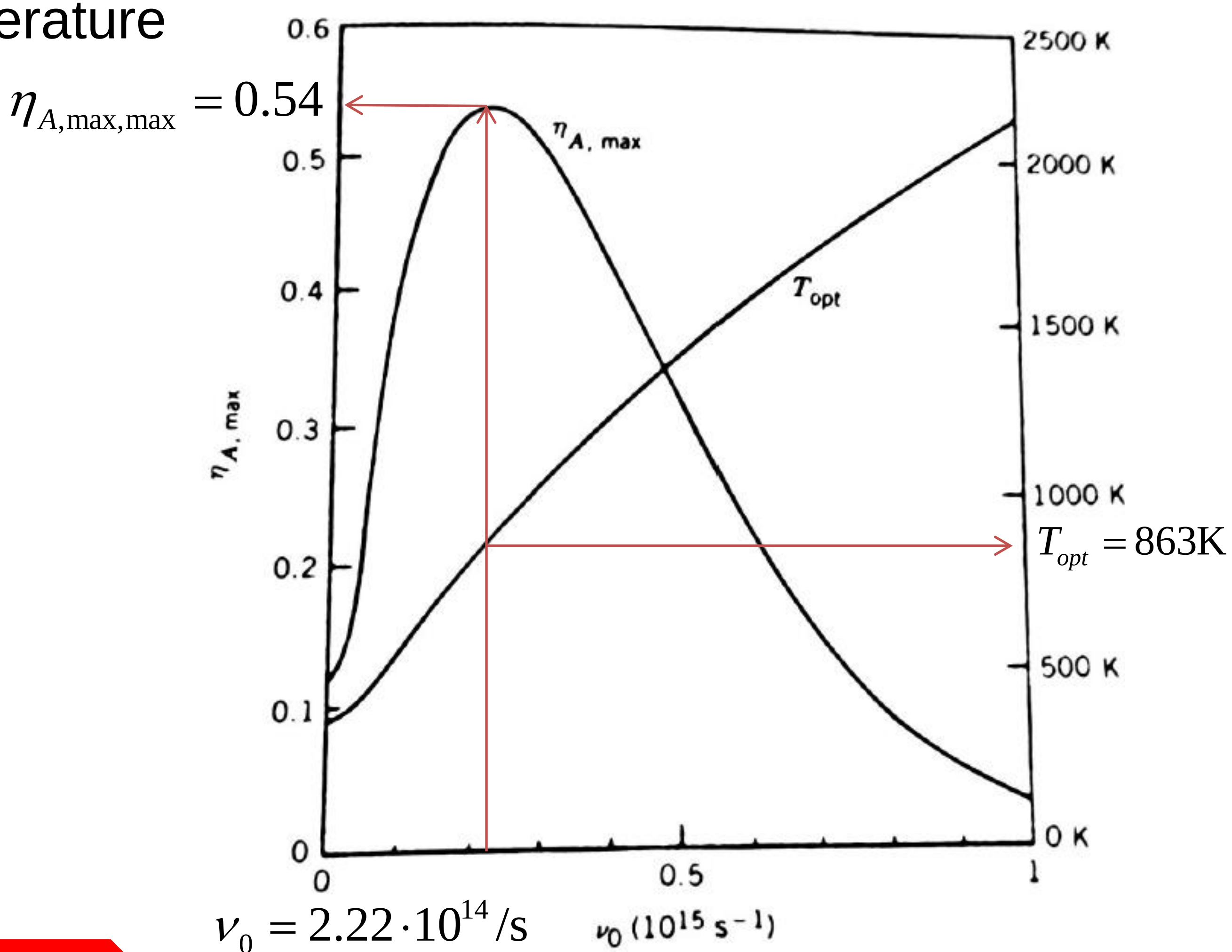
$$\dot{Q} = \pi A \int_0^\infty \left( i'_{\nu b}(T_s, \nu)\Omega_1 / \pi + i'_{\nu b}(T_0, \nu) - i'_{\nu b}(T, \nu) \right) \alpha(T, \nu) d\nu$$

  - Concept of selective absorber:
    - Optimal cutoff frequency expected



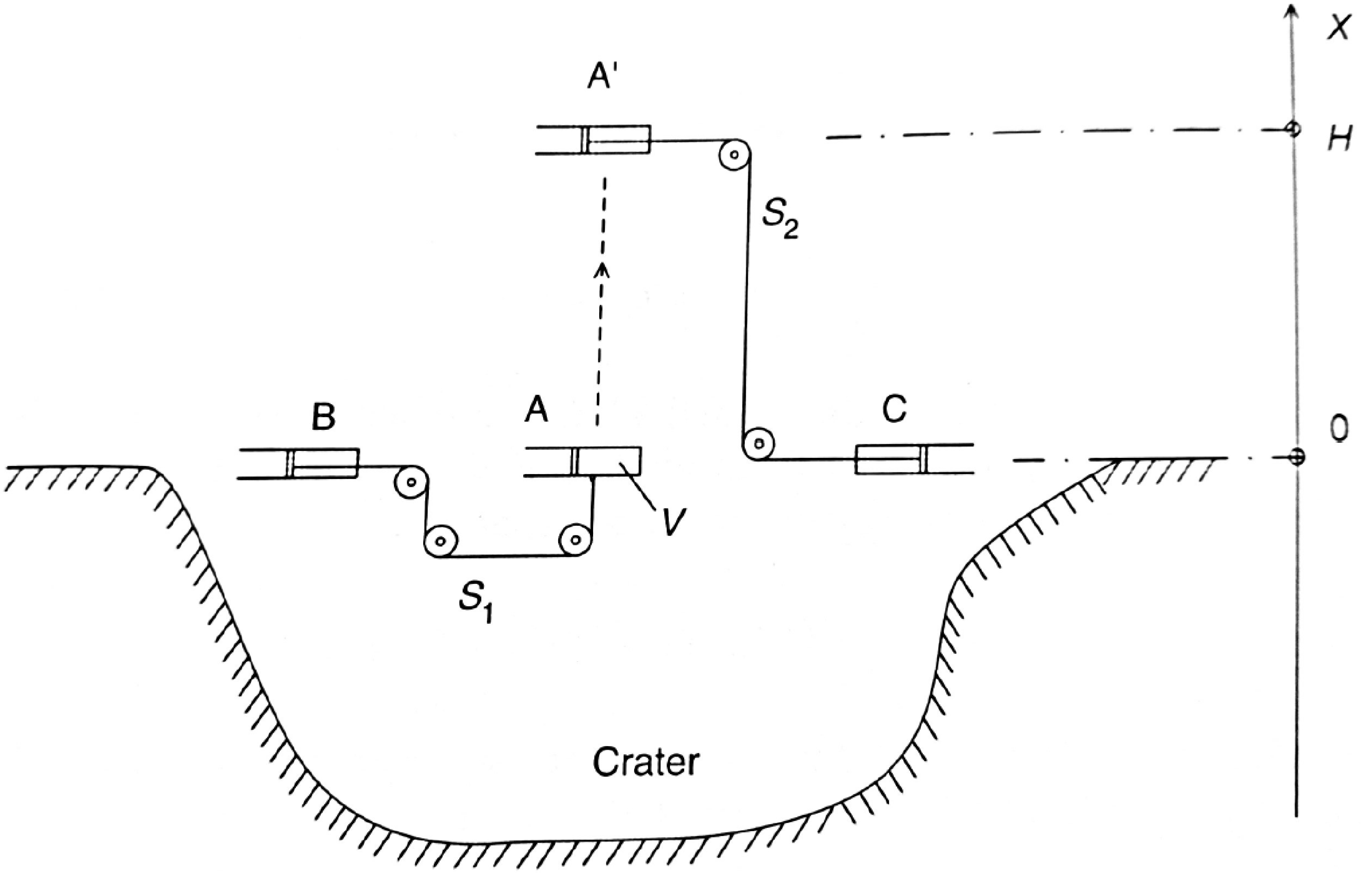
# Recap and extension of exergy concept

- Maximizing of power output per unit collector area – Unconcentrated solar radiation:
  - Optimal cutoff and corresponding maximal efficiency and optimal temperature



# Recap and extension of exergy concept

- Extension of exergy definition to gravitational systems:



# Recap and extension of exergy concept

- Extension of exergy definition to gravitational systems:
  - Buoyant exergy:

$$ex_b = \int_0^H g(x)\left(\frac{\rho_0(x)}{\rho} - 1\right)dx$$

Utilizing approximation of temperature and pressure dependence on height:

$$T = 288.16 - x \cdot 0.0065$$
$$p = 100399.5 - x \cdot 9.699$$
$$g = 9.7807 - x \cdot 3.068 \cdot 10^{-6}$$

$$\rightarrow ex_b \cong 164.186 - 357.258\rho + 253.398\rho^2 - 58.096\rho^3 \ \text{[kJ/kg]}$$

# Recap and extension of exergy concept

- Extension of exergy definition to gravitational systems:
  - Additionally equilibration with environment at *x*=*H*:

$$ex_H = c_p(T - T_{0,H})\left(c_p \ln\left(\frac{T}{T_{0,H}}\right) - R\ln\left(\frac{p}{p_{0,H}}\right)\right)$$

  - Plus gas at *x*=0 also has:

$$ex = c_p(T - T_0)\left(c_p \ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{p}{p_0}\right)\right)$$

  - Gravitational exergy is therefore: $ex_g = ex_b + \left(ex_H - ex\right)$
  - And max. Posible work is exergy: $ex_{work} = \max\left[\left(ex_g + ex\right), ex\right]$