SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Solar Energy Conversion Devices and Plants: Exercise 4

Analysis of 3-D Parabolic Dish – CPC – Receiver System

A Stirling engine system consists of a parabolic dish concentrator, coupled to a 3D CPC, and a cavity receiver. The cavity receiver is assumed to be a perfectly insulated blackbody, and the system is aligned such that all rays reflected from the dish are accepted by the inlet of the CPC and delivered to the receiver aperture. Figure 1 depicts the dish-CPC-receiver system. The system tracks the sun on two axes throughout the day, and useful work is extracted from the cavity receiver as it heats up.

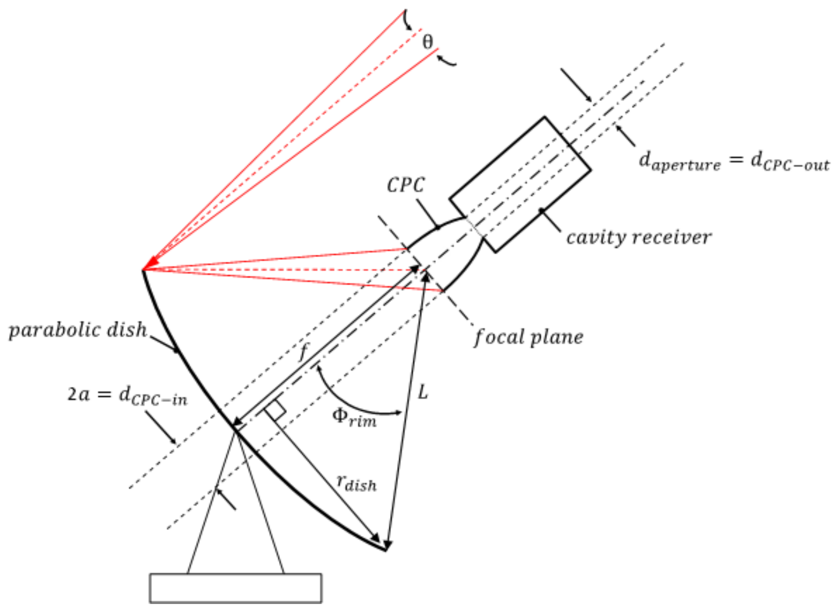

Figure 1: Cross-section schematic of a parabolic dish-CPC-receiver system.

Assumptions:

- Dish surface's reflectivity is $\rho_{\text{dish}} = 95\%$ (independent of wavelength and direction).
- CPC surface's reflectivity is $\rho_{\text{CPC}} = 90\%$ (independent of wavelength and direction).
- Beam normal irradiance is $G_{\text{bn}} = 890 \text{ W/m}^2$.
- Shading losses on the dish by the receiver, η_{shading}, remain constant during the day.
- Half acceptance angle, including tracking errors, is $\theta = 0.5°$.
- The parabolic dish has rim angle $\phi_{\text{rim}} = 40°$ and focal length $f = 4.5$ m.

Solution:

a)
- Acceptance angle of CPC should equal rim angle of dish

$$\theta_{max} = 40°$$

- All rays from dish should enter CPC, thus

$$b = r_{in,CPC} = \frac{d}{2} = \frac{r \sin\theta}{\cos\phi}$$
$$r = \frac{2f}{1+\cos\phi_{rim}} = \frac{2 \cdot 4.5}{1+\cos 40°} = 5.1m$$
$$b = r_{in,CPC} = \frac{r \sin\theta}{\cos\phi} = \frac{5.1 \cdot \sin 0.5°}{\cos 40°} = 0.058m$$
$$A_{CPC,in} = \pi r^2_{CPC,in} = \pi \cdot 0.058m^2 = 0.011m^2$$
$$C_{CPC,3D} = \frac{A_{CPC,in}}{A_{CPC,out}} = \frac{1}{\sin\phi^2}$$
$$A_{CPC,out} = 0.011m^2 \cdot \sin 40°^2 = 0.0045m^2$$

b)

$$\frac{r_{cavity}}{r_{CPC,out}} = 10$$
$$r_{CPC,out} = \sqrt{\frac{A_{CPC,out}}{\pi}} = \sqrt{\frac{0.0045m^2}{3.14}} = 0.038m$$
$$r_{cavity} = 0.38m$$
$$\frac{D_{dish}}{d} = \frac{D_{dish}}{2 \cdot r_{CPC,in}} = \frac{1}{2}\frac{\sin{(2\phi)}}{\sin{(\theta)}}$$
$$D_{dish} = r_{CPC,in} \cdot \frac{\sin{(2\phi)}}{\sin{(\theta)}} = 0.058m \cdot$$
$$\frac{\sin{(2 \cdot 40°)}}{\sin{(0.5°)}} = 6.5m$$
$$\eta_{shading} = 1 - \frac{A_{receiver}}{A_{dish}} = 1 - \frac{\pi \cdot r_{cavity^2}}{\pi \cdot r_{dish}^2} = 1 - \frac{0.38^2}{\frac{6.5m}{2}^2} = 98.6\%$$

c)

$$\dot{Q}_{in} = G_{bn} \cdot A_{dish} \cdot \rho_{dish} \cdot \rho_{CPC} \cdot \eta_{shading}$$
$$\dot{Q}_{in} = 890\frac{W}{m^2} \cdot \pi\frac{6.5m}{2}^2 \cdot 0.95 \cdot 0.9 \cdot 0.986$$
$$\dot{Q}_{in} = 24897.1W$$
$$\dot{Q}_{out} = A_{CPC,out}\sigma T_{receiver}^4 = 5.68 \times 10^{-8} \times 2000K^4 \cdot \pi 0.038m^2 = 4122.73W$$
$$\dot{Q}_{useful} = \dot{Q}_{in} - \dot{Q}_{out} = 24897.1W - 4122.73W = 20774.37W$$

d)

Fig. 2 shows the flow chart used for performing Monte-Carlo ray tracing simulation on the given Stirling engine system. Using 10^5 rays hitting the dish surface, we get the following results for peak and average concentration at the CPC inlet and CPC outlet, along with the CPC concentration ratio.

At CPC inlet:

$$C_{\text{peak}} = 5620$$
$$C_{\text{average}} = 3024$$

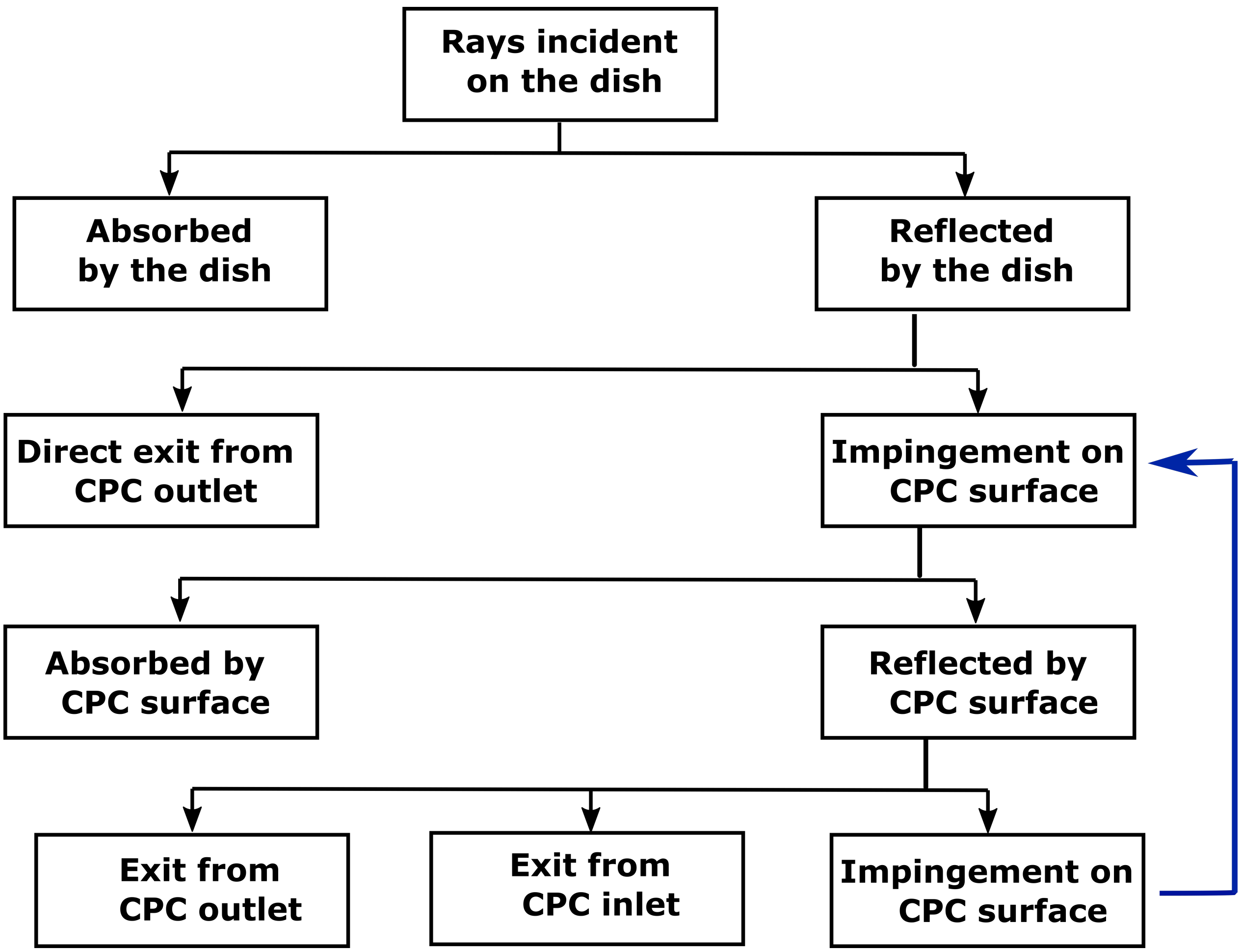

Figure 2: Flow Chart for Monte Carlo ray tracing simulation

At CPC outlet:

$$C_{\text{peak}} = 8966$$
$$C_{\text{average}} = 6843$$

CPC Concentration Ratio:

$$C_{\text{CPC}} = 2.26$$

The flux distributions at CPC inlet and CPC outlet are shown below:

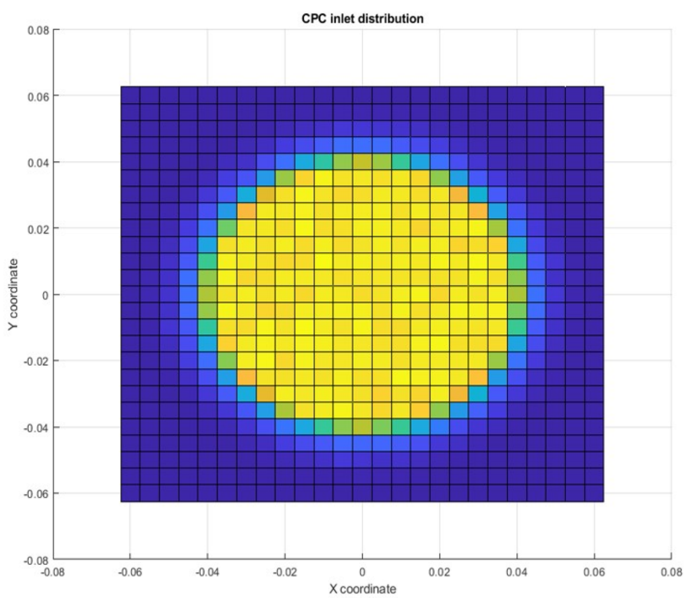

Figure 3: CPC inlet surface.

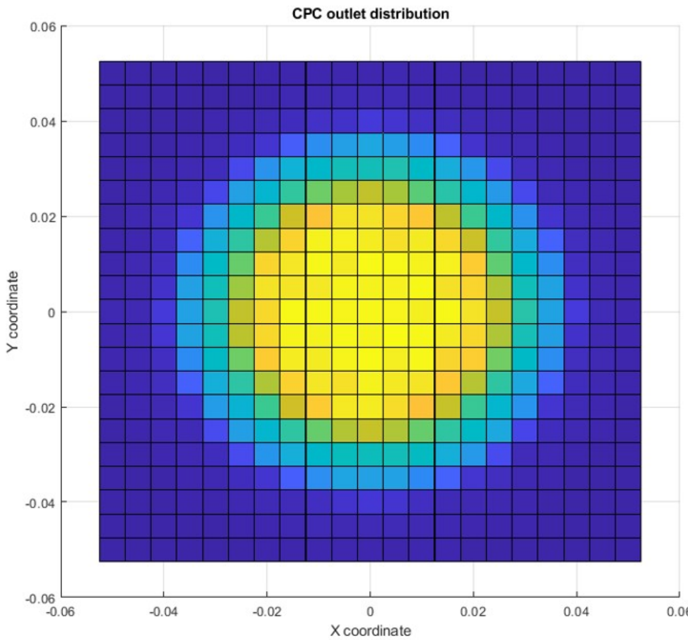

Figure 4: CPC outlet surface.

e)
Theoretical prediction, in the focal plane of a dish:

$$C_{\text{peak}} = \frac{\sin^2 \Phi_{\text{rim}}}{\theta^2} \times \rho_{\text{dish}} = 5154.2$$
$$C_{\text{average}} = \frac{\sin^2 \Phi_{\text{rim}} \cos^2 \Phi_{\text{rim}}}{\sin^2 \theta} \times \rho_{\text{dish}} = 3024.7$$

Theoretical prediction, for 3D-CPC:

$$C_{\text{3D-CPC}} = \frac{1}{\sin^2 (\Phi)} \times \rho_{\text{CPC}} = 2.18$$

No. of rays	Conc. Ratio	Error (%)	Run Time (s)
10^2	2.29	5.1	0.04
10^3	2.28	4.8	0.08
10^4	2.26	3.7	0.49
10^5	2.26	3.7	41.69

Table 1: Comparison of MC simulation results with theoretical predictions for the CPC concentration ratio.

No. of rays	Peak Conc. at CPC inlet	Error (%)	Run Time (s)
10^2	5914	14.74	0.04
10^3	5670	10.02	0.08
10^4	5630	9.24	0.49
10^5	5620	9	41.69

Table 2: Comparison of MC simulation results with theoretical predictions for the peak concentration at the CPC inlet.

No. of rays	Avg Conc. at CPC inlet	Error (%)	Run Time (s)
10^2	3088	2.10	0.04
10^3	3008	0.52	0.08
10^4	3029	0.15	0.49
10^5	3024	0.02	41.69

Table 3: Comparison of MC simulation results with theoretical predictions for the average concentration at the CPC inlet.