SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Solar Energy Conversion Devices and Plants: Exercise 3

Consider a solar collector with plate-to-cover spacing of 20mm, a plate emittance of 0.90, glass emittance of 0.85, and tilted 30°. According to measurements the mean temperature of the plate is 110°C, the ambient temperature is 10°C, and the wind heat transfer coefficient is 10 W/m^{2}K. Assuming that the sky temperature is the same as the ambient temperature, calculate the overall heat loss coefficient, U_t.

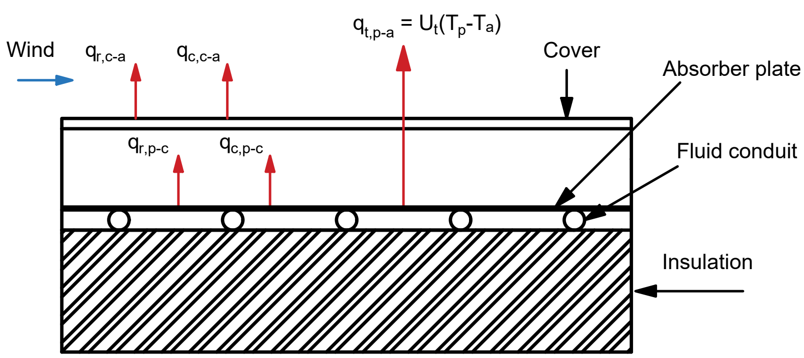

Figure 1: Scheme of the solar collector.

The overall heat transfer loss coefficient U_t, can be computed with the following expression:

$$U_\mathrm{t} = \left(\frac{1}{h_\mathrm{c,p-c} + h_\mathrm{r,p-c}} + \frac{1}{h_\mathrm{w} + h_\mathrm{r,c-a}}\right)^{-1}$$

The convection coefficient, $h_\mathrm{c,p-c}$ is determined with the following expression:

$$Nu = 1 + 1.44\left[1 - \frac{1708(\sin 1.8\beta)^{1.6}}{Ra\cos\beta}\right]\left[1 - \frac{1708}{Ra\cos\beta}\right]^{+} + \left[\left(\frac{Ra\cos\beta}{5830}\right)^{1/3} - 1\right]^{+}$$

$$Ra = \frac{g\beta'\Delta T L^3 Pr}{\nu^2}$$

Since the air properties have to be evaluated with an average temperature between the cover and the plate, $T_\mathrm{m} = (T_\mathrm{p} + T_\mathrm{c})/2$, it will be necessary to assume an initial value for T_c and proceed in an iterative process until a certain tolerance is achieved.
The radiation coefficient between the plate and cover $h_\mathrm{r,p-c}$ is

$$h_\mathrm{r,p-c} = \frac{\sigma(T_\mathrm{p}^2 + T_\mathrm{c}^2)(T_\mathrm{p} + T_\mathrm{c})}{1/\epsilon_\mathrm{p} + 1/\epsilon_\mathrm{c} - 1}$$

The radiation coefficient for the cover to air $h_{\mathrm{r,c-a}}$ is

$$h_{\mathrm{r,c-a}} = \epsilon_c \sigma (T_{\mathrm{c}}^2 + T_{\mathrm{s}}^2)(T_{\mathrm{c}} + T_{\mathrm{s}})$$

Since the heat loss from the plate to the cover is equal to the heat loss from the plate to the external air, we can use that expression to verify the assumed temperature for the cover.

$$(h_{\mathrm{c,p-c}} + h_{\mathrm{r,p-c}})(T_{\mathrm{p}} - T_{\mathrm{c}}) = U_{\mathrm{t}}(T_{\mathrm{p}} - T_{\mathrm{a}})$$
$$T_{\mathrm{c}} = T_{\mathrm{p}} - \frac{U_{\mathrm{t}}(T_{\mathrm{p}} - T_{\mathrm{a}})}{h_{\mathrm{c,p-c}} + h_{\mathrm{r,p-c}}}$$

Assuming $T_{\mathrm{c}} = 308$ K, the following results are obtained:

ν, $\mathrm{m^2/s}$	2.023e-5	K, W/mK	0.03
Pr	0.702	Ra	2.92e4
Nu	2.91	$h_{\mathrm{c,p-c}}$, $\mathrm{W/m^2K}$	4.32
$h_{\mathrm{r,p-c}}$, $\mathrm{W/m^2K}$	7.35	$h_{\mathrm{r,c-a}}$, $\mathrm{W/m^2K}$	4.98
U_{t}, $\mathrm{W/m^2K}$	6.56	$T_{\mathrm{c,new}}$, K	326.78

After some iterations, the final temperature of the cover and the overall heat lost are $T_{\mathrm{c}} = 326.46$ K and $U_{\mathrm{t}} = 6.73$ $\mathrm{W/m^2K}$.

Exercise 2

An array of 12 solar collector modules installed in parallel, at a slope of 60° and a surface azimuth of 0°. The hourly radiation on the plane of the collector I_{T}, the hourly radiation absorbed by the absorber plate S, and the hourly ambient temperature T_{a} are given in table 1. For the collector assume the overall loss coefficient U_{L} to be 7.0 W/m^{2}K and the plate efficiency factor $F^{'}$ to be 0.8. The water flow rate through each 1 × 2-m collector panel is 0.02 kg/s and the inlet water temperature remains constant at 40°C. Assume a controller turns off the water flow whenever the outlet temperature is less than the inlet temperature and $c_{\mathrm{p}} = 4190$ J/kgK.

(i) Calculate the daily useful gain and efficiency of the array.

Replacing the right values, the removal heat factor is

$$f_1 = \frac{\dot{m}c_{\mathrm{p}}}{A_{\mathrm{c}}U_{\mathrm{L}}F^{'}} = 7.48$$

$$F^{''} = f_1\left[1 - \exp\left(-\frac{1}{f_1}\right)\right] = 0.94$$

$$F_{\mathrm{R}} = F^{'}F^{''} = 0.75$$

Therefore the average useful energy gain per unit collector area for the time period 7-8 is

$$q_{\mathrm{u}} = F_{\mathrm{R}}[S - U_{\mathrm{L}}(T_{\mathrm{i}} - T_{\mathrm{a}})] = 0.75 \cdot [0.01 - 7(40 + 11) \cdot 3600/1e6] = -0.95 \text{ MJ/m}^2\text{h}$$

Since the value is negative this means that the outlet temperature of water will be lower than the inlet water temperature, therefore the solar collector doesn't operate during this period of time. Repeating the same process but for the period 10-11, gives $q_{\mathrm{u}} = 1.75$ MJ/m^{2}h.

The efficiency for this period of time is

$$\eta = \frac{q_{\mathrm{u}}}{I_{\mathrm{T}}} = 0.446$$

While the daily efficiency is

$$\eta = \frac{\sum(q_{\mathrm{u}} \cdot \Delta t)}{\sum(I_{\mathrm{T}} \cdot \Delta t)} = 0.385$$

And the daily useful gain for 12 collector modules is

$$\sum Q_{\mathrm{u}} = N \cdot A \cdot \sum(q_{\mathrm{u}} \cdot \Delta t) = 183.03 \text{ MJ}$$

More details are shown in table 1

Time	T_a (°C)	I_T (MJ/m^{2}h)	S (MJ/m^{2}h)	q_u (MJ/m^{2}h)	η -
7-8	-11	0.02	0.01	0	0
8-9	-8	0.43	0.35	0	0
9-10	-2	0.99	0.82	0	0
10-11	2	3.92	3.29	1.75	0.446
11-12	3	3.36	2.84	1.43	0.425
12-1	6	4.01	3.39	1.90	0.473
1-2	7	3.84	3.21	1.78	0.464
2-3	8	1.96	1.63	0.62	0.315
3-4	9	1.21	0.99	0.16	0.129
4-5	7	0.05	0.04	0	0
sum (MJ/m^2)		19.79		7.63	

Table 1: Solar collector operational conditions.

(ii) Reevaluate the daily performance considering the effect and shading.

Modern collectors use only one cover, and the module areas are quite large, both of which reduce shading effects. A reduction of S of 1% may be an appropriate correction. By re-doing the previous computations with the appropriate reduction on S, the daily useful gain is reduced to $Q_u = 180.27$ MJ and the daily efficiency to $\eta = 0.38$.

Exercise 3

Estimate the reduction in useful energy gain due to heat capacity effects. The plate and tubes are cooper. The collector has the following specifications:

Plate thickness	0.5 mm
Tube thickness	0.5 mm
Internal diameter of the tube	12 mm
Tube spacing	125 mm
Glass cover thickness	3.5 mm
Back-insulation thickness	70 mm

	c, kJ/kgK	ρ, kg/m^3
Copper	0.48	8800
Glass	0.80	2500
Insulation	0.80	50

The effective heat capacitance, $(mc)_\mathrm{e}$ is computed as follows

$$(mc)_\mathrm{e} = (mc)_\mathrm{p} + \sum_{i=1}^{n} a_i (mc)_{\mathrm{c},i}$$

$$(mc)_\mathrm{plate} = t_\mathrm{p} \cdot A_\mathrm{c} \cdot \rho_\mathrm{cu} \cdot c_\mathrm{p,cu} = 4224 \text{ J/K}$$

$$(mc)_\mathrm{tubes} = \frac{\pi}{4}(d_\mathrm{e} - d_\mathrm{i})^2 \cdot N_\mathrm{t} \cdot L \cdot \rho_\mathrm{cu} \cdot c_\mathrm{p,cu} = 1327 \text{ J/K}$$

$$(mc)_\mathrm{water} = \frac{\pi}{4} d_\mathrm{i}^2 \cdot N_\mathrm{t} \cdot L \cdot \rho . c_\mathrm{p} = 7582 \text{ J/K}$$

$$(mc)_\mathrm{back} = t_\mathrm{b} \cdot A_\mathrm{c} \cdot \rho_\mathrm{b} \cdot c_\mathrm{p,b} = 5600 \text{ J/K}$$

$$(mc)_\mathrm{glass} = t_\mathrm{g} \cdot A_\mathrm{c} \cdot \rho_\mathrm{g} \cdot c_\mathrm{p,g} = 14000 \text{ J/K}$$

$$(mc)_\mathrm{e} = 4224 + 1327 + 7582 + 5600/2 + 0.27 \cdot 14000 = 19713 \text{ J/K}$$

Assuming the initial temperature of the plate assembly to be equal to the ambient temperature, then the temperature after the first period will be:

$$T_\mathrm{p} = T_\mathrm{a} + \frac{S}{U_\mathrm{L}} - \left[\frac{S}{U_\mathrm{L}} - (T_\mathrm{p,initial} - T_\mathrm{a})\right] \exp\left(-\frac{A_\mathrm{c} U_\mathrm{L} t}{(mc)_\mathrm{e}}\right)$$

$$T_\mathrm{p} = 262 + \frac{9900}{3600 \cdot 7} - \left[\frac{9900}{3600 \cdot 7} - (262 - 262)\right] \exp\left(-\frac{2 \cdot 7 \cdot 3600}{19713}\right)$$

$$T_\mathrm{p} = 262.36 \text{ K}$$

By repeating the same process, but updating $T_\mathrm{p,initial}$ with the previews result will result in

T_p equal to 277.48 and 301.22 K at the end of the second and third period, respectively. Then for the fourth period (10-11 am), q_u will be reduced by the amount of heat required to heat the plate assembly from $T_\mathrm{p} = 301.22$ K to the inlet water temperature at the collector (40°C)

$$S_{10-11,reduced} = S_{10-11} - \frac{(mc)_\mathrm{e} \cdot (T_\mathrm{p} - T_\mathrm{i})}{3600 \cdot A_\mathrm{c}}$$
$$S_{10-11,reduced} = 3.14 \text{ MJ/m}^2\text{h}$$

This will result in a reduction of the total useful gain to $Q_\mathrm{u} = 178.18$ MJ and the daily efficiency to $\eta = 0.375$.