SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Solar Energy Conversion Devices and Plants: Exercise 2

1 Photovoltaic cell

Consider a 1 m^2 surface of polycrystalline silicon photovoltaic cell that generates 170 W of electrical power, see figure 1. The cell has temperature $T_c = 330$ K, emissivity $\epsilon_C = 0.9$, and reflectivity $\rho_C = 1 - \epsilon_C$. Environment temperature is $T_0 = 300$ K. The temperature of the sun is assumed as $T_S = 5800$ K. The convective heat transfer coefficient is $h_k = 3$ W/(m^2K), and the heat transfer coefficient of useful heat is $h_C = 10$ W/(m^2K). Compute the energy and exergy efficiency, plus the exergy loss of this photovoltaic cell.

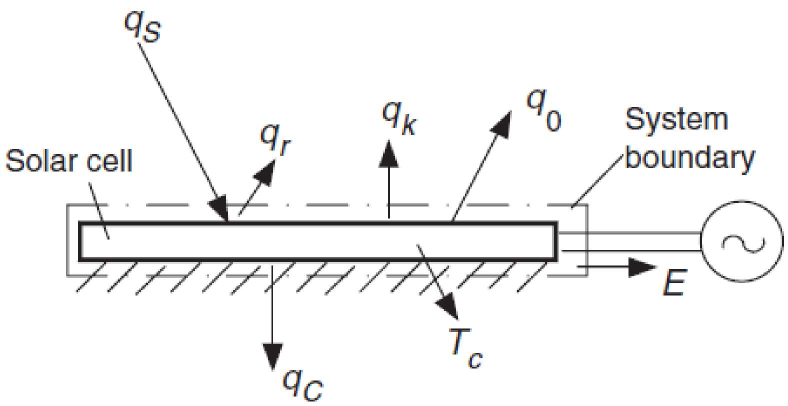

Figure 1: Scheme of the energy streams of a solar cell.

We start by computing the solar energy q_S:

$$q_S = C \cdot \sigma \cdot T_S^4 = 1385.8 \text{ W/m}^2$$

where $C = (\frac{R_{sun}}{S_{earth-sun}})^2 = 2.17 * 10^{-5}$
Then the energetic efficiency is:

$$\eta_e = \frac{E}{q_S} \cdot 100 = 12.27\%$$

The exergy efficiency for the solar cell is the ratio between electrical energy and the solar exergy. The solar exergy is computed as follows:

$$b_S = C \cdot \frac{\sigma}{3} \cdot (3T_S^4 + T_0^4 - 4T_0 \cdot T_S^3) = 1290.2 \text{ W/m}^2$$

Then the exergy efficiency is:

$$\eta_{ex} = \frac{E}{b_S} = 13.18\%$$

The cell temperature T_c can be calculated with the energy balance of the solar cell:

$$q_\mathrm{S} = E + q_\mathrm{C} + q_\mathrm{r} + q_\mathrm{k} + q_0$$

Where:

$$\begin{aligned} q_\mathrm{r} &= \rho_\mathrm{C} \cdot q_\mathrm{S} \\ q_\mathrm{k} &= h_\mathrm{k} \cdot (T_\mathrm{C} - T_0) \\ q_\mathrm{C} &= h_C \cdot (T_\mathrm{C} - T_0) \\ q_0 &= \epsilon_\mathrm{C} \cdot \sigma \cdot (T_\mathrm{C}^4 - T_0^4) \end{aligned}$$

Solving this fourth-order polynomial equation results in $T_C = 353.4\mathrm{K}$.
The exergy loss of the solar cell can be computed from the following exergy balance:

$$b_\mathrm{S} = b_\mathrm{r} + b_\mathrm{k} + b_0 + b_\mathrm{C} + E + \delta b$$

Where:

$$\begin{aligned} b_\mathrm{r} &= \rho_\mathrm{C} \cdot b_\mathrm{S} \\ b_\mathrm{k} &= q_\mathrm{k} \cdot \left(1 - \frac{T_0}{T_\mathrm{C}}\right) \\ b_\mathrm{c} &= q_\mathrm{c} \cdot \left(1 - \frac{T_0}{T_\mathrm{C}}\right) \\ b_0 &= \epsilon_\mathrm{C} \cdot \frac{\sigma}{3} \cdot (3T_\mathrm{C}^4 + T_0^4 - 4T_0 \cdot T_\mathrm{C}^3) \end{aligned}$$

Calculating the numerical values with the given parameters gives:

Term	Energy ($\mathrm{W/m^2}$)	Exergy ($\mathrm{W/m^2}$)
Reflection	138.6	129
Convection	160.3	24.22
Radiation	382.8	32.85
Useful heat	534.2	80.74
Loss	0	909.9

2 Solar Chimney Power Plant

A chimney solar power plant (fig 2) is made of a transparent deck, a turbine, and a chimney. Air that is located in the collector (below the deck) is heated by the floor and deck, and, since hot air is lighter than cold air, it starts to move inside the plant where mechanical power can be extracted thanks to the use of a turbine, and finally the air is extracted to the environment through the chimney.

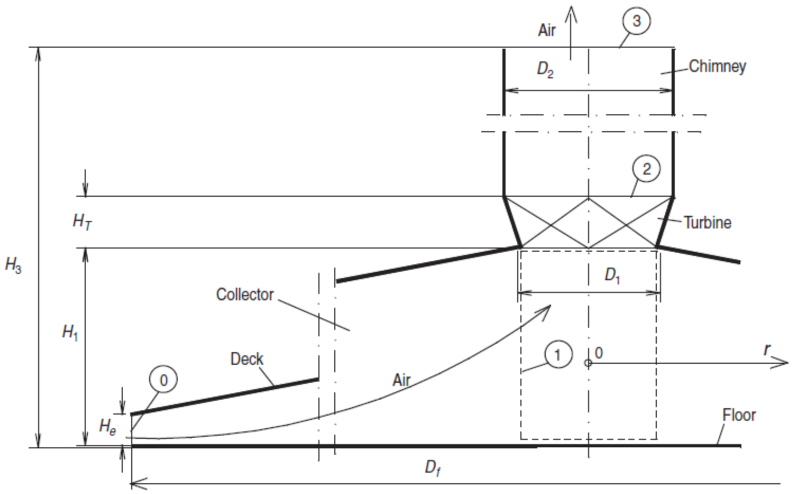

Figure 2: Scheme of the solar chimney power plant.

Determine the energy and exergy efficiency of this power plant, plus the exergy losses.

The energy balance equations for the floor, air in the collector, collector (including floor, air, deck, external air), turbine, chimney, and chimney surface are the following (see fig 3):

$$Q_{\mathrm{s-f}} = Q_{\mathrm{f-a}} + Q_{\mathrm{f-d}} \tag{1a}$$

$$Q_{\mathrm{f-a}} + Q_{\mathrm{d-a}} = H_{\mathrm{a1}} + E_{\mathrm{w1}} + E_{\mathrm{p1}} \tag{1b}$$

$$Q_{\mathrm{s-f}} = H_{\mathrm{a1}} + E_{\mathrm{w1}} + E_{\mathrm{p1}} + Q_{\mathrm{d-sky}} + Q_{\mathrm{d-0}} + Q_{\mathrm{d-ch}} \tag{1c}$$

$$H_{\mathrm{a1}} + E_{\mathrm{w1}} + E_{\mathrm{p1}} = H_{\mathrm{a2}} + E_{\mathrm{w2}} + E_{\mathrm{p2}} + W_{\mathrm{t}} \tag{1d}$$

$$H_{\mathrm{a2}} + E_{\mathrm{w2}} + E_{\mathrm{p2}} + Q_{\mathrm{d-ch}} = H_{\mathrm{a3}} + E_{\mathrm{w3}} + E_{\mathrm{p3}} + Q_{\mathrm{ch-0}} + Q_{\mathrm{ch-sky}} + Q_{\mathrm{ch-gr}} \tag{1e}$$

$$Q_{\mathrm{a-ch}} + Q_{\mathrm{d-ch}} = Q_{\mathrm{ch-0}} + Q_{\mathrm{ch-sky}} + Q_{\mathrm{ch-gr}} \tag{1f}$$

These energies have the following scripts:
s-f: solar radiation arriving at the floor.
f-a: convection heat from floor to air.
f-d: heat exchanged by radiation between floor and deck.

d-a: convection heat from deck to air.
d-sky: heat exchanged by radiation between deck and sky.
d-0: convection heat from deck to atmosphere.
d-ch: heat exchanged by radiation between deck and chimney.
ch-0: convection heat from chimney to atmosphere.
ch-sky: heat exchanged by radiation between chimney and sky.
ch-gr: heat exchanged by radiation between chimney and ground.
a-ch: heat transferred from chimney air to the chimney surface.
a1,a2,a3: enthalpy of air at point 1, 2, and 3.
w1,w2,w3: kinetic energy of air at point 1, 2, and 3.
p1,p2,p3: potential energy of air at point 1, 2, and 3.
t: turbine power.

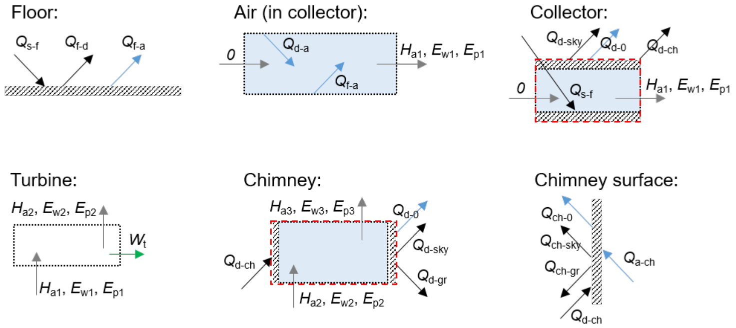

Figure 3: Schematic of energy balance for the 6 sub-systems.

These 6 equations have 6 unknowns: T_{a1}, T_{f}, T_{d}, T_{ch}, W_{t}, and $h_{\mathrm{a-ch}}$. In order to compute the temperature after the turbine, polytropic process and isentropic turbine efficiency are used:

$$T_{\mathrm{a2,s}} = T_{\mathrm{a1}} \cdot \left(\frac{p_2}{p_1}\right)^{(k_{\mathrm{s}}-1)/k_{\mathrm{s}}} \tag{2a}$$

$$T_{\mathrm{a2}} = T_{\mathrm{a1}} + \eta_{\mathrm{s,t}} \cdot (T_{\mathrm{a2,s}} - T_{\mathrm{a1}}) \tag{2b}$$

We can setup exergy balances for similar sub-systems to compute the exergy losses in the floor, deck, air in the collector, turbine, and chimney:

$$B_{\mathrm{s-f}} = B_{\mathrm{f-a}} + B_{\mathrm{f-d}} + \Delta B_{\mathrm{f}} \tag{3a}$$

$$B_{\mathrm{f-d}} = B_{\mathrm{d-a}} + B_{\mathrm{d-sky}} + B_{\mathrm{d-0}} + B_{\mathrm{d-ch}} + \Delta B_{\mathrm{d}} \tag{3b}$$

$$B_{\mathrm{f-a}} + B_{\mathrm{d-a}} = B_{\mathrm{a1}} + B_{\mathrm{w1}} + B_{\mathrm{p1}} + \Delta B_{\mathrm{a}} \tag{3c}$$

$$B_{\mathrm{a1}} + B_{\mathrm{w1}} + B_{\mathrm{p1}} = B_{\mathrm{a2}} + B_{\mathrm{w2}} + B_{\mathrm{p2}} + B_{\mathrm{t}} + \Delta B_{\mathrm{t}} \tag{3d}$$

$$B_{\mathrm{a2}} + B_{\mathrm{w2}} + B_{\mathrm{p2}} + B_{\mathrm{d-ch}} = B_{\mathrm{a3}} + B_{\mathrm{w3}} + B_{\mathrm{p3}} + B_{\mathrm{ch-0}} + B_{\mathrm{ch-sky}} + B_{\mathrm{ch-gr}} + \Delta B_{\mathrm{ch}} \tag{3e}$$

Solving these equations and the intermediate equations stated in the problem give:

Parameter	Value	Parameter	Value
D_1, m	16.97	H_1, m	4.24
p_1, Pa	101233.93	T_{a1}, K	325.65
p_2, Pa	99686.11	T_{a2}, K	324.64
p_3, Pa	98912.2	T_{a3}, K	322.44
w_1, m/s	0.99	m, kg/s	243.5
$\phi_{\mathrm{ch-d}}$, -	0.17	$\phi_{\mathrm{d-ch}}$, -	0.04
$\phi_{\mathrm{d-sky}}$, -	0.95	$\phi_{\mathrm{ch-gr}}$, -	0.32
T_{f}, K	396.6	T_{d}, K	338.6
T_{ch}, K	298.7	$h_{\mathrm{a-ch}}$, W/(m^{2}K)	2.93
W_{t}, kW	199.73	η_{e}, %	0.52
$\Delta B_{\mathrm{f}}/B_{\mathrm{S}}$, %	70.66	$\Delta B_{\mathrm{d}}/B_{\mathrm{S}}$, %	9.05
$\Delta B_{\mathrm{a}}/B_{\mathrm{S}}$, %	4.35	$\Delta B_{\mathrm{t}}/B_{\mathrm{S}}$, %	0.27
$\Delta B_{\mathrm{ch}}/B_{\mathrm{S}}$, %	1.37	η_{ex}, %	0.58