SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Solar Energy Conversion Devices and Plants: Exercise 2

In this exercise, you will evaluate energy and exergy efficiency, plus the exergy losses of a Photovoltaic cell and a Solar Chimney Power Plant.

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1. Photovoltaic cell

Consider a 1 m^2 surface of polycrystalline silicon photovoltaic cell that generates 170 W of electrical power, see figure 1. The cell has emissivity $\epsilon_C = 0.9$, and reflectivity $\rho_C = 1 - \epsilon_C$. Environment temperature is $T_0 = 300$ K. The temperature of the sun is assumed as $T_S = 5800$ K. The convective heat transfer coefficient is $h_k = 3$ W/(m^{2}K) and the heat transfer coefficient of useful heat is $h_C = 10$ W/(m^{2}K). Compute the energy and exergy efficiency, the cell temperature, and the exergy loss of this photovoltaic cell.

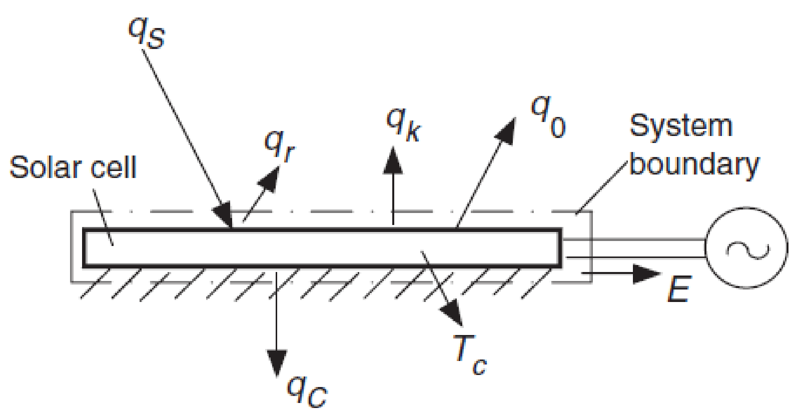

Figure 1: Scheme of the energy streams of a solar cell.

q_S is heat transferred from the sun, q_r is the reflected solar radiation, q_k is the convection heat, q_0 is the radiation energy ($T_{sky} = T_0$), and q_C is the useful heat.
Note that $q_S = 2.16\text{e-}5\sigma T_S^4$ W/m^2.

Hints: Some useful equations for exergy computation are shown at the end of the second exercise.

2. Solar Chimney Power Plant
A chimney solar power plant (fig 2) is made of a transparent deck, a turbine, and a chimney. Air that is located in the collector (below the deck) is heated by the floor and deck, and, since hot air is lighter than cold air, it starts to move inside the plant where mechanical power can be extracted thanks to the use of a turbine, and finally the air is extracted to the environment through the chimney.

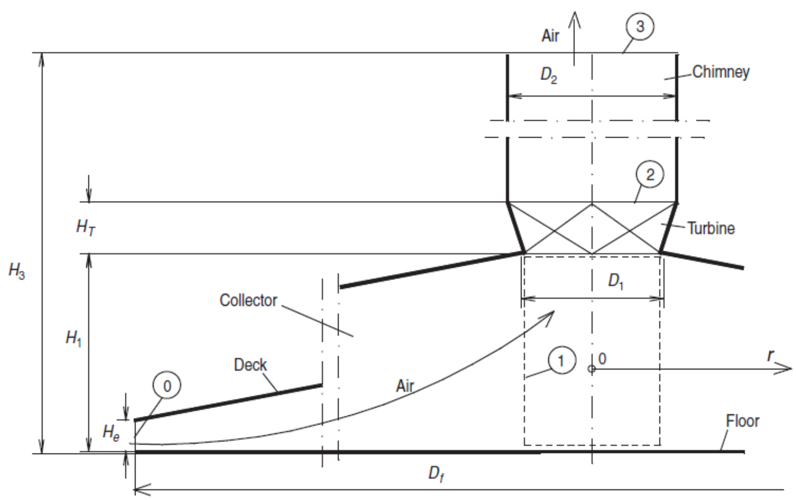

Figure 2: Scheme of the solar chimney power plant.

Determine the energy and exergy efficiency of this power plant, plus the exergy losses in the floor, deck, air in the collector, turbine and chimney, considering the following information:

External environment – The plant is located at a site with an ambient temperature , T_0, an ambient pressure, p_0, and a solar irradiation, S, of 288.14 K, 101'235 Pa, and 850 W/m^2, respectively. Also, consider that the sky, T_{sky}, has the same temperature as the environment.
Turbine – Consider an isentropic coefficient of 1.4 and and isentropic efficiency of 70%. The turbine has a height, H_T, of 1 m and the ratio of diameters, D_1/D_2, is 0.95. For maximum fluid power, use a relative pressure drop ratio, r_T, of 2/3.

$$\frac{p_1 - p_2}{p_1 - p_3} = r_\text{T} \tag{1}$$

Chimney – The chimney has a height, H_3, of 195 m. Assume an external convective heat transfer coefficient, $h_{\text{ch}-0}$, of 7 W/(m^{2}K) and a thickness factor, c_d, of 1.015 ($D_{\text{ext},2} = c_\text{d} \cdot D_2$). The air temperature drop can be estimated as proportional to the chimney surface and inversely proportional to the air mass rate:

$$T_{\text{a},2} - T_{\text{a},3} = 0.154 \cdot D_2 \cdot H_3/\dot{m} \tag{2}$$

Floor and collector – Regarding the dimensions, the entrance has a height, H_e, of 0.3 m and the floor has a diameter, D_f, of 240 m. Regarding the thermal part, consider the floor as a black body, a convective heat transfer coefficient between the floor-air, h_{f-a}, and deck-air, h_{d-a}, equal to 1.7 W/(m^{2}K), and that the floor is thermally isolated from the ground ($T_{gr} = T_0$). Regarding the operational conditions, there is a pressure drop from the entrance to the inlet of the turbine of 1.07 bar, and the air temperature distribution in the collector is linear, thus the effective temperature is:

$$T_{a,eff} = (T_0 + T_{a,1})/2 \tag{3}$$

The momentum conservation equation for the air flow within the collector is derived as:

$$p_0 - p_1 = \rho_{a,1} w_1^2 \tag{4}$$

Where w and ρ are the velocity and density of air. The collector was designed with a constant cross-sectional area, meaning that:

$$\pi D_f H_e = \pi D_1 H_1 = \pi D_1^2/4 \tag{5}$$

Deck – The transparency of the deck, τ_d, is of 0.95, assume an external heat convection coefficient, h_{d-0}, of 5 W/(m^{2}K). Its area can be computed as follows:

$$A_d = \pi \cdot (D_f^2 - D_1^2)/4 \tag{6}$$

Air properties – Consider air as an ideal gas, a specific heat constant, c_p, of 1000 J/(kg K), and an individual gas constant, R, of 287.04 J/(kgK). The pressure, gravitational acceleration ($g_0 = 9.81$ m/s^2) and density of air at the exit of the chimney can be computed with the following expressions:

$$g_3 = g_0 - 3.086\text{e-}6 \cdot H_3 \tag{7a}$$

$$\rho_3 = \rho_0 - 9.973\text{e-}5 \cdot H_3 \tag{7b}$$

$$p_3 = p_0 + \frac{g_0 + g_3}{2} \cdot \frac{\rho_0 + \rho_3}{2} H_3 \tag{7c}$$

Emissivity and view factors – The emissivity of all the surfaces is equal to 1 for low temperature radiation. Assume a view factor from the chimney to the sky, ϕ_{ch-sky}, of 0.5.
The view factor deck-chimney can be calculated from the reciprocity relation:

$$\phi_{d-ch} \frac{\pi}{4} \left[D_f^2 - D_{ext,2}^2 \right] = \phi_{ch-d} \pi D_{ext,2} (H_3 - H_2) \tag{8}$$

The view factor chimney-deck is $\phi_{ch-d} = 0.5 \cdot (90 - \beta)/90$, where the angle β is determined by $\tan \beta = 2H_3/D_f$.
The view factors fulfil the following relations:

$$\phi_{d-sky} + \phi_{d-ch} = 1 \tag{9a}$$

$$\phi_{ch-sky} + \phi_{ch-d} + \phi_{ch-gr} = 1 \tag{9b}$$

Potential energy – Use the following expression for the potential energy:

$$E_{\mathrm{p}} = \dot{m}\left\{-\frac{1}{a_4\rho}\left[\frac{a_2}{6a_4}(\rho - a_3)^3 + \frac{a_1}{2}(\rho - a_3)^2\right]\right\} \tag{10}$$

Where $a_1 = 9.7807$ m/s^2, $a_2 = -3.086\text{e-}6$ 1/s^2, $a_3 = 1.217$ kg/m^3, and $a_4 = -9.973\text{e-}5$ kg/m^4.
Exergy equations – Exergy of solar irradiation can be estimated as, $B_{\mathrm{S}} = 0.9 \cdot E_{\mathrm{S}}$.
Radiation exergy between two different surfaces can be computed according to the following expression:

$$B = \phi_{x-y} A_x \epsilon_{x-y} \frac{\sigma}{3}\left[3(T_x^4 - T_y^4) - 4T_0(T_x^3 - T_y^3)\right] \tag{11}$$

The effective emissivity simplifies to $\epsilon_{x-y} = 1$ when the emissivity $\epsilon_x = \epsilon_y = 1$.
The exergy of air is calculated based on:

$$B_{\mathrm{a}} = \dot{m}\left[c_{\mathrm{p}}(T_{\mathrm{a}} - T_0) - T_0\left(c_{\mathrm{p}} \ln\frac{T_{\mathrm{a}}}{T_0} - R\ln\frac{p}{p_0}\right)\right] \tag{12}$$

Exergy B for convective heat transfer can be computed based on the heat Q:

$$B = Q\left(1 - \frac{T_0}{T}\right) \tag{13}$$

Electrical, kinetic, and potential exergies are equal to the respective energies.

Hints: Perform energy balances in different sections of the solar chimney, e.g. floor surface, air in collector, collector (including floor, air, deck, external air), turbine, chimney, and chimney surface. You will end with six unknown variables and six equations. The unknowns are the temperature of floor, deck, and chimney, air temperature at point 1, turbine power, and heat transfer coefficient between chimney and air. Proceed similarly for the exergy balance equations. The system of equations can be solved numerically using MATLAB or any other software you feel more comfortable to use.