SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

First Name: ____________ Last Name: ____________ Sciper: ____________

Solar Energy Conversion Devices and Plants: Exam Solutions

Date: January 29, 2024; Duration: 3 hours
Personal summary, calculator only. Answers on these sheets only.
Total: [90 points]

Section I: Multiple Choice Questions

1. Correct Answer **(B,C,D)**

   (a) In the quasi-neutral regions of a pn junction diode, the majority charge carriers (electrons in the n-region and holes in the p-region) do carry current. This is the drift component of the current arising from very small electric field residing in the quasi-neutral region.

   (b) In the quasi-neutral regions, the minority charge carriers (holes in the n-region and electrons in the p-region) primarily contribute to the diffusion current due to the concentration gradient. Since their concentration is small and also the existing electric field is small, drift component of the current is almost negligible.

   (c) The overall cell current is spatially constant across the entire length of the solar cell at equilibrium.

   (d) In the space charge region, the charge is constant, the electric field is linear and the electrostatic potential is therefore non-linear.

2. Correct Answer **(A)**

   (a) j_o determines the activation overpotential above the thermodynamic potential, at which we begin to observe a net current.

   (b) The value of j_o is ∞ for a non-polarizable electrode. An ideal reference electrode is a non-polarizable electrode.

   (c) If j_o is greater than limiting current, the reaction is mass-transfer limited.

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(d) The kinetic rate parameters (j_o and α) for a particular half cell reaction depends not only on the type of electrode on which the reaction is taking place, but also on the type of electrolyte in which the electroactive species are dissolved.

3. Correct Answer **(B,C)**

   (a) $AM = \frac{1}{cos(\theta)}$, where θ is measured from the vertical and is equal to the solar zenith angle. Solar altitude angle (α) and solar zenith angle are related as $\theta = 90 - \alpha$.

   (b) $DNI = \tau q_{solar}$.

   (c) $GHI = K_T * GHI + DNI * cos(\theta)$.

   (d) ESH depends on the total irradiance received on a particular day and not the total number of daylight hours.

4. Correct Answer **(A,B,C)**

   (a) The Fermi level (E_F) is given by:

$$E_F = E_i + \frac{1}{2}k_B T \ln\left(\frac{N_V}{N_C}\right)$$

   where E_i is the intrinsic energy level (mid-gap energy).
   The formulas for effective density of states in the conduction band (N_C) and the valence band (N_V) are as follows:

$$N_C = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{\frac{3}{2}}$$

$$N_V = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{\frac{3}{2}}$$

   Since, the effective mass of holes and electrons are not exactly the same, N_C and N_V turn out to be slightly different, and hence fermi energy level is not exactly at the middle of the band gap for an intrinsic semiconductor.

   (b) The Fermi level (E_F) is given by:

$$E_F = E_i + \frac{1}{2}k_B T \ln\left(\frac{N_V}{N_C}\right)$$

where E_i is the intrinsic energy level (mid-gap energy), and N_C and N_V are temperature-dependent.

The shift in the Fermi level when the temperature is doubled:

$$\Delta E_F = 0.5 k_B T \ln\left(\frac{N_V}{N_C}\right)$$

The percentage shift relative to half the bandgap energy:

$$\text{Percentage Shift} = \left(\frac{\Delta E_F}{E_F}\right) \times 100\%$$

Substituting numerical values:

$$\text{Percentage Shift} \approx 3.77\%$$

(c) To determine the doping concentration required to keep the Fermi level at the middle of the band gap in an n-doped semiconductor (c-Si) at 298 K, we use the following formula:

$$E_{\text{cond}} - E_F = k_B T \ln\left(\frac{N_C}{N_D}\right)$$

For the Fermi level to be at the mid-gap, $E_{\text{cond}} - E_F$ should be equal to half the band gap $E_g/2$:

$$\frac{E_g}{2} = k_B T \ln\left(\frac{N_C}{N_D}\right)$$

Rearranging to solve for N_D:

$$N_D = N_C e^{-\frac{E_g}{2k_B T}}$$

$$N_D \approx 1.92 \times 10^9\,\text{cm}^{-3}$$

(d) The intrinsic carrier concentration (n_i) of an intrinsic semiconductor is given by the formula:

$$n_i = \sqrt{N_C N_V}\, e^{-\frac{E_g}{2k_B T}}$$

N_C and N_V are the effective densities of states in the conduction and valence bands, respectively, and they vary with temperature approximately as $T^{3/2}$. Thus, the overall temperature dependence of n_i is not accurately described by just $T^{3/2}$ relationship.

5. Correct Answer **(A,D)**
   - (a) For enclosed black body radiation, $P \propto T^4$, and $U \propto T^4 V$. Hence $U \propto PV$ and is constant for an adiabatic free expansion process.
   - (b) The final temperature $T_2 = T_1 \frac{P_2}{P_1}^{1/4}$.
   - (c) Since the process is irreversible and also adiabatic, the entropy change of the system has to be greater than zero. It is given by $S_1(\frac{T_1}{T_2} - 1)$.
   - (d) Exergy destruction is given by $T_o(S_2 - S_1)$, where $(S_2 - S_1) = S_1(\frac{T_1}{T_2} - 1)$ and $S_1 = \frac{4}{3} a T_1^3 V_1$.

6. Correct Answer **(C)**
   - (a) The Fermi level is a hypothetical energy level
   - (b) Temperature affects the position
   - (c) Yes, the Fermi level can be directly measured
   - (d) n-doping of a semiconductor will move the Fermi level towards the conduction band

7. Correct Answer **(C)**
   - (a) Energy efficiency is given by $\frac{E}{Ac\sigma T_s^4}$
   - (b) Useful heat flux is given by $c\sigma T_s^4 - \sigma T_c^4 + \sigma T_o^4 - \frac{E}{A}$
   - (c) Exergy associated with useful heat is given by $q_c(1 - \frac{T_o}{T_c})$
   - (d) Exergy associated with radiation energy is given by $\frac{\sigma}{3}(3T_c^4 + T_o^4 - 4T_o T_c^3)$

8. Correct Answer **(C)**

   The energy levels of the anodic and cathodic reactions are: $E_{\mathrm{OH^-/O_2}} = -4.8$ eV and $E_{\mathrm{H_2O/H_2}} = -3.6$ eV. While the position of the conduction levels are: $E_{\mathrm{c}} = -4.05$ eV and $E_{\mathrm{v}} = -5.17$ eV. This means that the electrode could only be used as a photoanode.

9. Correct Answer **(C,D)**

   The maximum wavelength for electron transfer in a c-Si semiconductor with a 1.12 eV band gap is approximately 1108 nm, calculated using the equation

   $$\lambda_{\text{max}} = \frac{hc}{E_{\text{gap}}} \tag{1}$$

   where h is Planck's constant, c is the speed of light, and E_{gap} is the band gap energy.

10. Correct Answer **(C)**

Step 1: Computing the Enthalpy Change for Water Vapor Splitting

Given:

- Molar flow rate of water: $0.59\,\text{mol/s}$
- Enthalpy change for water vapor splitting ΔH_{react}: $285.8\,\text{kJ/mol}$

Calculation:

$$\Delta H = \text{Molar flow rate} \times \Delta H_{\text{react}}$$
$$\Delta H = 0.59\,\text{mol/s} \times 285.8\,\text{kJ/mol} = 168.622\,\text{kW}$$

This gives the total enthalpy change required for the reaction.

Step 2: Computing the Solar Heat Input Q_{solar}

Given:

- Reactor temperature: $1200\,\text{K}$
- Solar irradiation: $1000\,\text{W/m}^2$
- Concentration factor: 200
- Stefan-Boltzmann constant σ: $5.67 \times 10^{-8}\,\text{W/m}^2\text{K}^4$

Calculation:

$$Q_{\text{solar}} = \frac{\Delta H}{1 - \frac{\sigma T^4}{\text{concentration}\times\text{solar radiation}}}$$
$$Q_{\text{solar}} = \frac{168.622\,\text{kW}}{1 - \frac{5.67\times10^{-8}\times(1200)^4}{200\times1000}} = 409.182\,\text{kW}$$

Thus, the required solar heat input for the reactor is **409.182 kW**.

11. Correct Answer **(D)**

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Calculation of Reradiating Heat by the Reactor

Given Data

- Solar heat input (Q_{solar}): 409.18 kW
- Enthalpy change for the reaction (ΔH_{react}): 285.8 kJ/mol
- Molar flow rate of water: 0.59 mol/s

Calculation

First, calculate the total enthalpy change (ΔH) for the reaction:

$$\Delta H = \text{Molar flow rate} \times \Delta H_{\text{react}}$$

$$\Delta H = 0.59\,\text{mol/s} \times 285.8\,\text{kJ/mol} = 168.622\,\text{kW}$$

Then, calculate the reradiating heat ($Q_{\text{reradiate}}$) by the reactor:

$$Q_{\text{reradiate}} = Q_{\text{solar}} - \Delta H$$

Using the given values:

$$\begin{aligned} \Delta H &= 0.59\,\text{mol/s} \times 285.8\,\text{kJ/mol} = 168.622\,\text{kW} \\ Q_{\text{reradiate}} &= 409.18\,\text{kW} - 168.622\,\text{kW} \\ &= 240.558\,\text{kW} \end{aligned}$$

Thus, the reradiating heat by the reactor is **240.558 kW**.

12. Correct Answer **(B)**

Calculation of Process Efficiency

Given:

- Gibbs free energy change (ΔG): 87.28 kJ/mol
- Solar heat input (Q_{solar}): 409.18 kW
- Molar flow rate of water: 0.59 mol/s

Calculation

First, convert ΔG into kW considering the molar flow rate:

$$\Delta G_{\text{total}} = \text{Molar flow rate} \times \Delta G$$

$$\Delta G_{\text{total}} = 0.59\,\text{mol/s} \times 87.28\,\text{kJ/mol} = 51.4952\,\text{kW}$$

Then, calculate the efficiency (η):

$$\eta = \frac{\Delta G_{\text{total}}}{Q_{\text{solar}}}$$

$$\eta = \frac{51.4952\,\text{kW}}{409.18\,\text{kW}} \approx 0.126 \text{ or } 12.6\%$$

Thus, the efficiency of the process is approximately **12.6%**.

13. Correct Answer **(C)**

Calculation of Maximum Theoretical Efficiency

Given Data

- Temperature of the reactor (T_{reactor}): 1200 K
- Temperature of the environment (T_{sink}): 298 K (equivalent to 25°C)
- Concentration factor: 200
- Solar irradiation (I): $1000\,\text{W/m}^2$
- Stefan-Boltzmann constant (σ): $5.67 \times 10^{-8}\,\text{W/m}^2\text{K}^4$

Carnot Efficiency

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

$$\eta_{\text{Carnot}} = 1 - \frac{298\,\text{K}}{1200\,\text{K}} \approx 0.7517 \text{ or } 75.17\%$$

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Reactor Efficiency

$$\eta_{\text{reactor}} = 1 - \frac{\sigma T_{\text{reactor}}^4}{\text{concentration} \times I}$$

$$\eta_{\text{reactor}} = 1 - \frac{5.67 \times 10^{-8} \times (1200)^4}{200 \times 1000} \approx 0.4121 \text{ or } 41.21\%$$

Maximum Theoretical Efficiency

$$\eta_{\text{max}} = \eta_{\text{Carnot}} \times \eta_{\text{reactor}}$$

$$\eta_{\text{max}} = 0.7517 \times 0.4121 \approx 0.3100 \text{ or } 31.00\%$$

Thus, the maximum theoretical efficiency of the process is approximately **31.00%**.

14. Correct Answer **(A,D)**

From the I-V curves, we can see that the magnitude of cathodic and anodic current density is equal at $E = 0.1V$ for reaction 1 and $E = -0.2V$ for reaction 2. Hence, $E_{eq,1} = 0.1V$ and $E_{eq,2} = -0.2V$.

15. Correct Answer **(A,B,C)**

   (a) At equilibrium, cathodic and anodic current densities are equal in magnitude and are equal to the exchange current density (j_o). From the I-V curve, we see $j_o = 10$ mA/cm^2 for reaction 1 and $j_o = 17.5$ mA/cm^2 for reaction 2.

   (b) E^o can be calculated from the Nernst equation given by Eq. 2

$$E_{eq} = E^o - \frac{RT}{nF} ln \frac{C_R^*}{C_O^*} \tag{2}$$

   (c) For a fuel cell reaction 2 would be occurring at the anode since it has more negative E_{eq}.

   (d) If the value of exchange current density is known, the number of moles of electrons transferred (n) can be calculated from the expression given in Eq. 3

$$R_{ct} = \frac{RT}{nFj_oA} \tag{3}$$

   For reaction 1 and 2, 2 and 1 moles of electrons are exchanged, respectively.

16. Correct Answer **(C,D)**
    (a) Incorrect, typical efficiencies are below 20%
    (b) Incorrect, for solar tower systems heat transfer fluids are liquid salts.
    (c) Correct, the largest optical losses are cosine losses.
    (d) Correct, 24h operation is enable and it is done with sensible heat storage (large vessels with molten salts).

17. Correct Answer **(A,B,C,D)**
    (a) Using the formula for the concentration ratio C of a CPC,

$$C = \frac{1}{\sin(\phi)^2},$$

with $C = 2.42$, we aim to find the acceptance angle ϕ.
Given the concentration ratio, we can rearrange the formula to solve for ϕ as follows:

$$\sin(\phi) = \frac{1}{\sqrt{C}}.$$

Substituting $C = 2.42$ into the equation yields:

$$\sin(\phi) = \frac{1}{\sqrt{2.42}}.$$

Therefore, the acceptance angle ϕ can be calculated using the arcsin function:

$$\phi = \arcsin\left(\frac{1}{\sqrt{2.42}}\right).$$

Upon calculation, the acceptance angle ϕ is found to be approximately 40.00°.

(b) Given the formula for the inner radius of a CPC,

$$r_{\text{in}} = \frac{L \cdot \tan(\phi)}{1 + \sin(\phi)},$$

where L is CPC length parameter and ϕ is the CPC acceptance angle, the calculation proceeds as follows.
With $\phi \approx 40.00°$ and a specified value for L, the inner radius r_{in} is calculated to be approximately 0.0581 meters.

(c) To find the outer radius r_{out} of a Compound Parabolic Concentrator (CPC) from the concentration ratio C, we use the formula:

$$r_{\text{out}} = \frac{r_{\text{in}}}{\sqrt{C}}.$$

Given $C = 2.42$ and a previously calculated r_{in}, the outer radius r_{out} is determined to be approximately 0.0373 meters.

(d) Given the formula for r_{in}:

$$r_{\text{in}} = \frac{2f\sin(\theta)}{(1+\cos(\phi))\cos(\phi)},$$

we solve for θ and find it in milliradians (mrad) as:

$$\theta = \arcsin\left(\frac{r_{\text{in}}(1+\cos(\phi))\cos(\phi)}{2f}\right) \times 1000.$$

With the specified values, θ is approximately 8.73 mrad.

18. Correct Answer **(A,B,C)**

(a) The formula for calculating C_{dish} is given by:

$$C_{\text{dish}} = \frac{\sin(2\phi)^2}{4\sin(\theta)^2}.$$

Using the previously determined values for ϕ and θ (converted from mrad to radians), C_{dish} is calculated to be approximately 3183.90.

(b) The aperture area A of the dish is calculated using the formula:

$$A = \pi r_{\text{dish}}^2.$$

With the value of r_{dish} determined previously, the aperture area A is found to be approximately 33.71 square meters.

(c) The minimum possible half acceptance angle θ_s for a parabolic dish in milliradians is calculated using the formula

$$\sin(\theta_s) = \frac{\text{Radius of sun}}{\text{Distance between sun and earth}}$$

resulting in

$$\theta_s \approx \arcsin\left(\frac{695700000\text{ m}}{149600000000\text{ m}}\right)$$

which gives $\theta_s \approx 4.65$ mrad.

(d) The maximum theoretical concentration ratio of a parabolic dish is given by the formula $C_{\max} = 1/(4 * sin(\theta_s)^2)$, θ_s is given by 4.65 mrad, which we convert to radians before plugging it into the formula. The resulting approximate value is 11562.11.

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Section II: Solar cell analysis

Intrinsic Carrier Concentration (n_i)

The intrinsic carrier concentration is calculated using the formula:

$$n_i = \sqrt{N_c N_v}\, e^{-\frac{E_g}{2kT}} \,\textbf{[1.0 points]} \tag{4}$$

where:

- $N_c = 4.91 \times 10^{18}\,\text{cm}^{-3}$ (effective density of states in the conduction band).
- $N_v = 2.7 \times 10^{19}\,\text{cm}^{-3}$ (effective density of states in the valence band).
- $E_g = 1.12\,\text{eV}$ (bandgap energy of silicon).
- $k = 8.617 \times 10^{-5}\,\text{eV/K}$ (Boltzmann constant).
- $T = 300\,\text{K}$ (temperature).

Using these values, the recalculated n_i is:

$$n_i \approx 4.50 \times 10^{9}\,\text{cm}^{-3}\textbf{[0.5 points]} \tag{5}$$

Built-in Potential (V_{bi})

The built-in potential is calculated using the formula:

$$V_{\text{bi}} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) \,\textbf{[1.5 points]} \tag{6}$$

where:

- $N_A = N_D = 10^{17}\,\text{cm}^{-3}$ (doping concentrations).
- $q = 1.6 \times 10^{-19}\,\text{C}$ (charge of an electron).

With the updated n_i, the built-in potential V_{bi} is:

$$V_{\text{bi}} \approx 0.875\,\text{V}\textbf{[0.5 points]} \tag{7}$$

Computation of Depletion Width

The depletion width W was calculated using the formula:

$$W = \sqrt{\frac{2\epsilon_{\mathrm{si}}(V_{\mathrm{bi}} - V)\left(\frac{N_{\mathrm{A}}+N_{\mathrm{D}}}{N_{\mathrm{A}}N_{\mathrm{D}}}\right)}{q}} \quad (8)$$

[1.5 points]

where ϵ_{si} is the permittivity of silicon, V_{bi} is the built-in potential, V is the applied potential, q is the charge of an electron, and N_{A}, N_{D} are the doping concentrations. The depletion width was calculated to be approximately 150.5 nanometers. **[0.5 points]**

Calculation of Absorption Coefficient

The absorption coefficient α was calculated using:

$$\alpha = \frac{4\pi k}{\lambda} \quad (9)$$

[1.5 points]

where λ is the wavelength of the incident light (500 nm). The absorption coefficient was determined to be approximately $1.256 \times 10^{6}\,\mathrm{m}^{-1}$. **[0.5 points]**

Generation Term Calculation

[1 points] The generation term G was defined as:

$$G = \alpha \Phi_{x=0}(1 - R)\exp(-\alpha x) \quad (10)$$

where $\Phi_{x=0}$ is the photon flux at the surface, and R is the reflectance (assumed to be zero). G represents the rate of generation of electron-hole pairs at different depths.

Calculation of hole current density at the end of p-side depletion region

The hole current density at the end of the p-side depletion region was calculated using:

$$J_{\mathrm{h,P}}|_{x=W_{\mathrm{N}}+x_{\mathrm{P}}} = J_{\mathrm{h,N}}|_{x=W_{\mathrm{N}}-x_{\mathrm{N}}} + q\int_{W_{\mathrm{N}}-x_{\mathrm{N}}}^{W_{\mathrm{N}}+x_{\mathrm{P}}} G(x)\mathrm{d}x \quad (11)$$

[1.5 points]

The hole current density at the end of the n-side depletion region was given as 25 mA/cm^2. The calculated hole current density at the end of p-side depletion region is calculated as **697.25** mA/cm^2. **[0.5 points]**

Short-circuit current density or photocurrent density calculation

The total photocurrent density J_{tot} in the solar cell is calculated as:

$$J_{\text{tot}} = J_{\text{h,P}}|_{x=W_{\text{N}}+x_{\text{P}}} + J_{\text{e,P}}|_{x=W_{\text{N}}+x_{\text{P}}} \textbf{[1.5 points]} \tag{12}$$

The total photocurrent density is found to be **722.25** mA/cm^2. **[0.5 points]**

Calculation of Solar Cell Efficiency

Given Data

- Photocurrent density (i_{ph}): $722.25\,\text{mA/cm}^2$
- Dark saturation current (i_{dark}): $10^{-10}\,\text{mA/cm}^2$
- Temperature of the solar cell: $300\,\text{K}$
- Exponential factor in load current density equation: 38.64

Calculation

1. **Determine the Operating Point:**

The operating point is found by equating the current density of the solar cell to the current density of the load:

$$722.25 - 10^{-10} \cdot \left(e^{\frac{1.6\times10^{-19}\times V_{\text{load}}}{1\times1.38\times10^{-23}\times300}} - 1 \right) = 3.19 \times 10^{-5} \cdot e^{38.65\times V_{\text{load}}} \textbf{[1.5 points]}$$

Solving for V_{load}, we get:

$$V_{\text{load}} \approx \mathbf{0.4383}\,\text{V} \textbf{[0.5 points]}$$

2. **Calculate Power Output:**

The power output per unit area (P_{out}) is given by:

$$P_{\text{out}} = V_{\text{load}} \times i_{\text{cell}} \textbf{[1 points]}$$

where i_{cell} is the operating current density:

$$i_{\text{cell}} \approx 722.25\,\text{mA/cm}^2 \textbf{[0.5 points]}$$

Thus,

$$P_{\text{out}} \approx \mathbf{316.55}\,\mathrm{W/m^2}\textbf{[0.5 points]}$$

3. **Calculate Efficiency:**

The efficiency (η) is the ratio of the power output to the power input ($1\,\mathrm{KW/m^2}$):

$$\textbf{[1.5 points]}\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \approx \frac{316.55\,\mathrm{W/m^2}}{1000\,\mathrm{W/m^2}} \approx \mathbf{31.65}\%\textbf{[0.5 points]}$$