Most flows are unstable...

Vortex shedding

Saffman-Taylor

Flow separation

Tollmien-Schlichting

Rayleigh-Taylor

Lift-up and Streaks

Traffic waves

Meandering instability

Gravito-capillary waves

Taylor-Couette

Rayleigh-Plateau

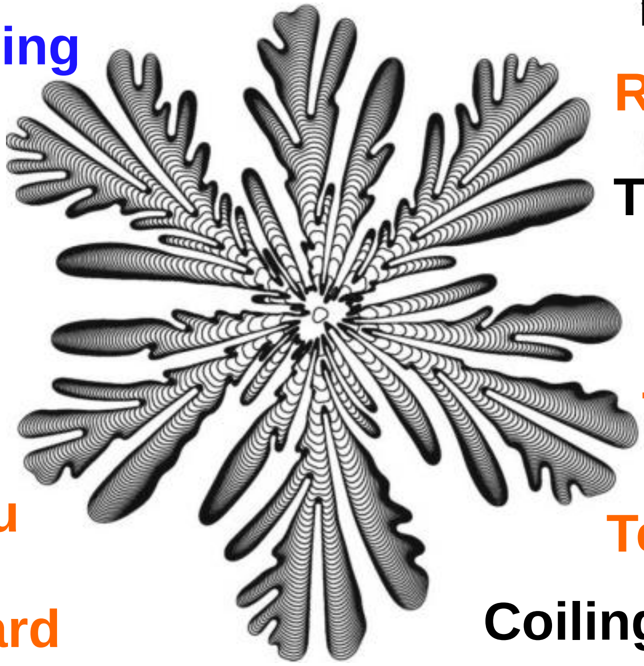
Tearing instability

Coiling instability

Rayleigh-Benard

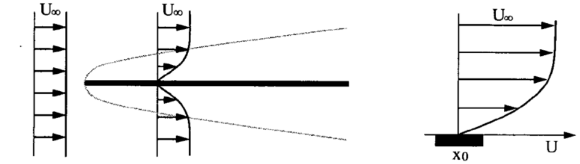
Benard-Marangoni

Kelvin-Helmholtz

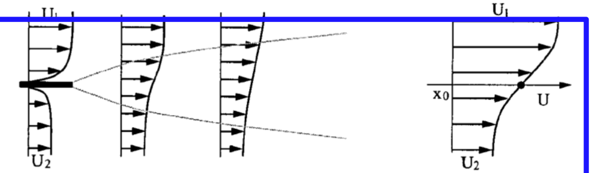


SPATIALLY DEVELOPING SHEAR FLOWS

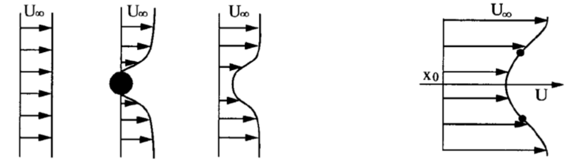
Flat plate boundary layer



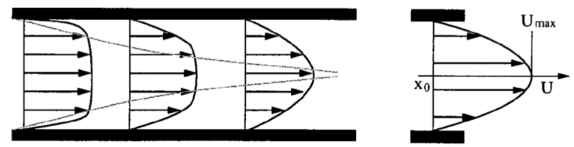
Mixing layer



Cylinder wake

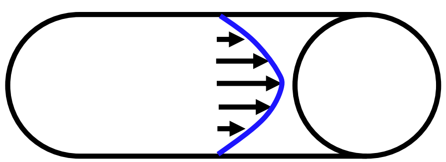


Plane channel flow

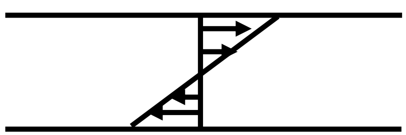


2D jet

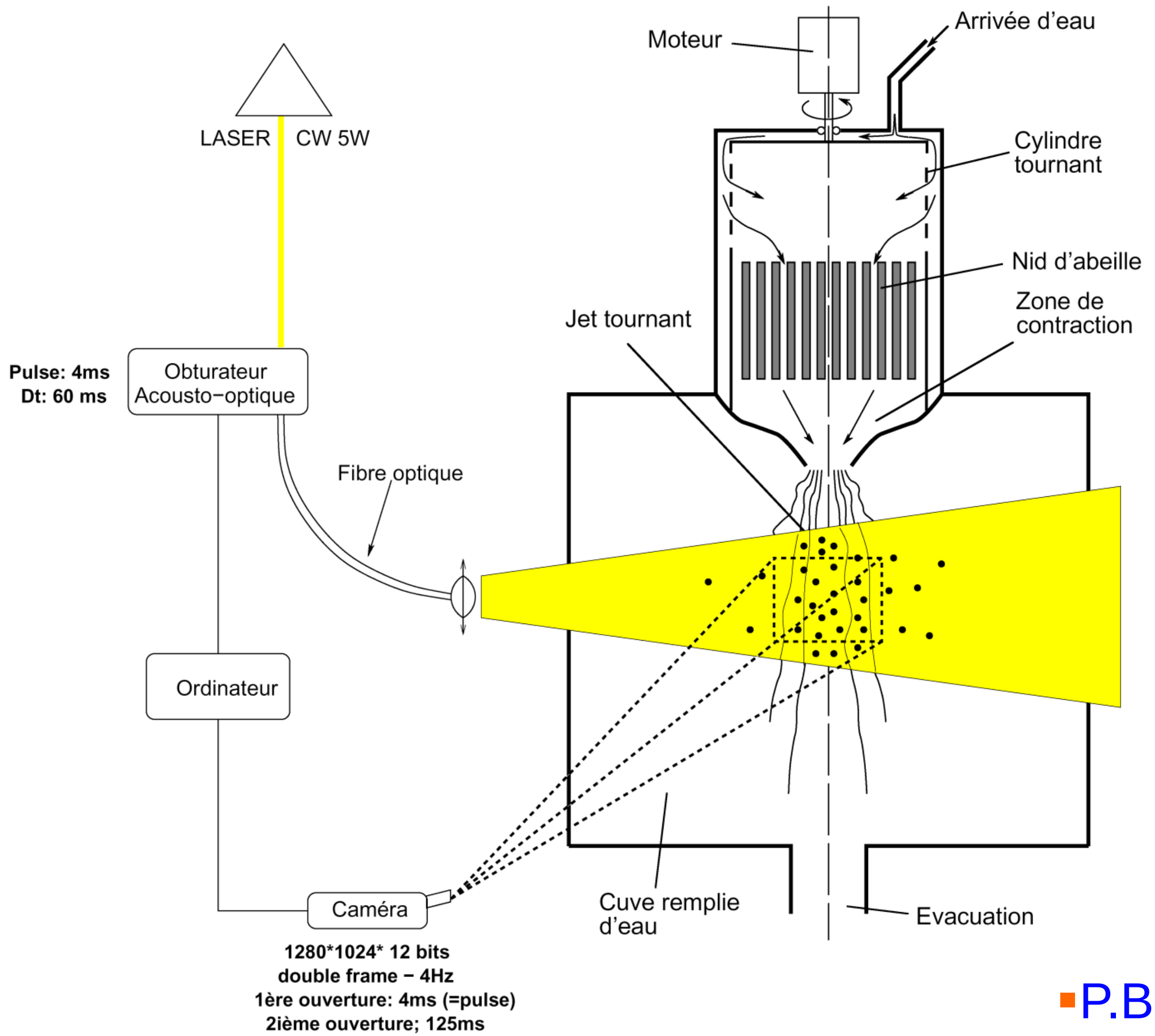
Hagen Poiseuille



Couette



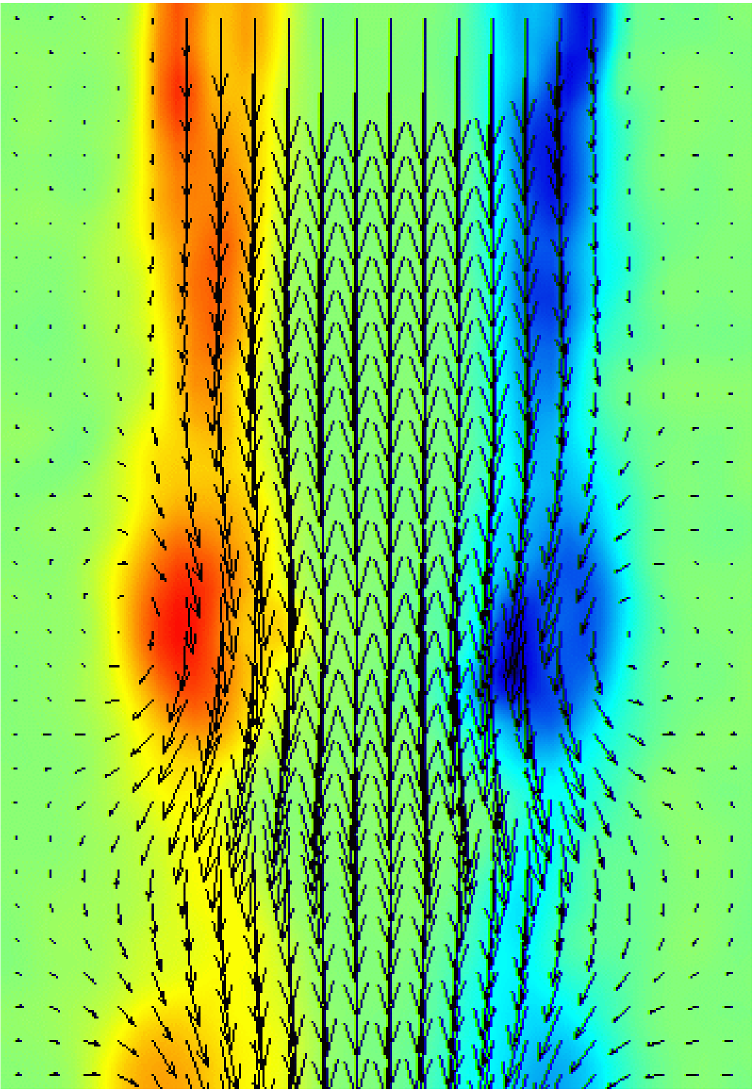
Round jet



- P.Billant
- T.Loiseleux

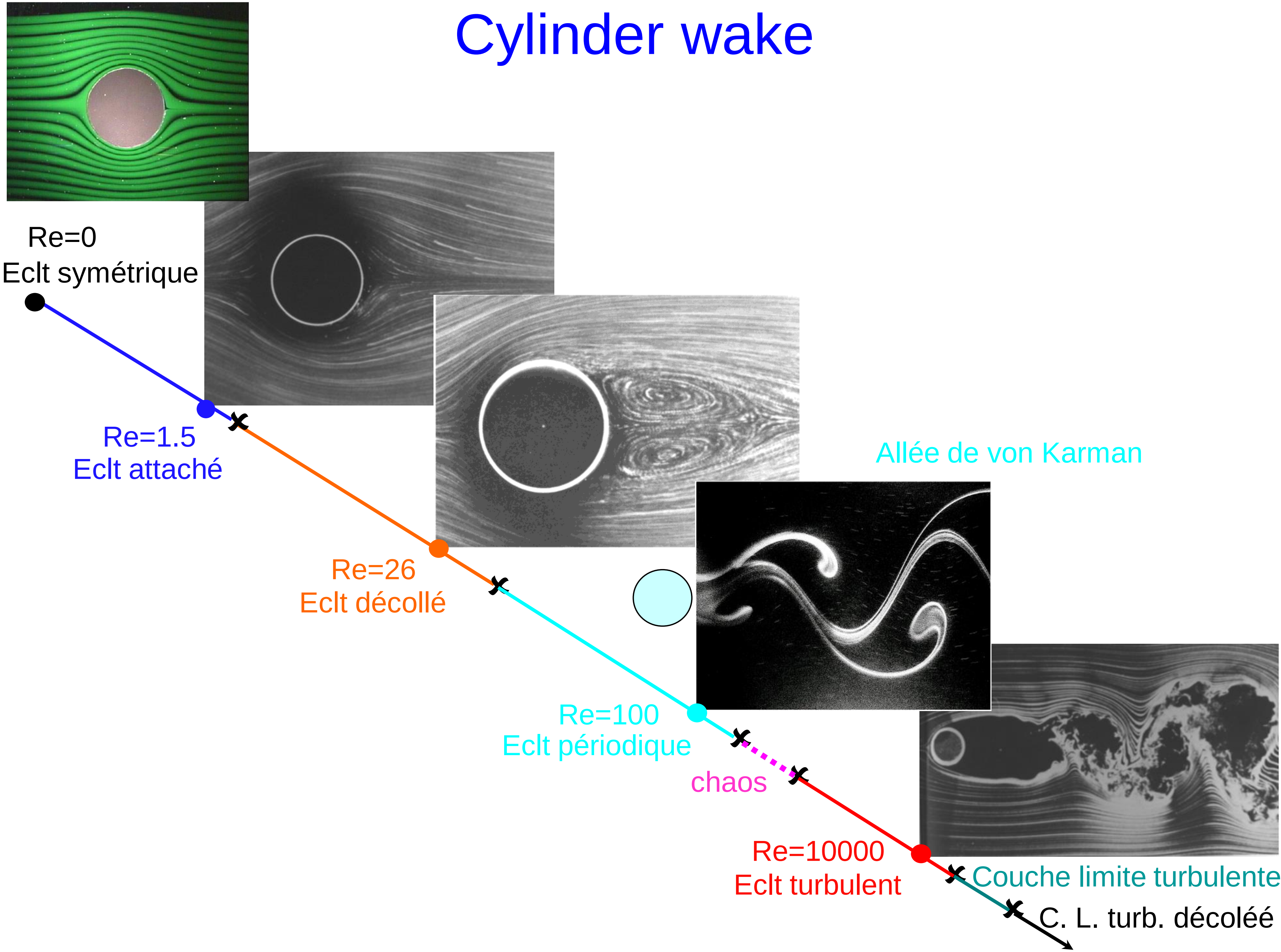
Toroidal vortices – vortex rings

Vorticity field measured by PIV

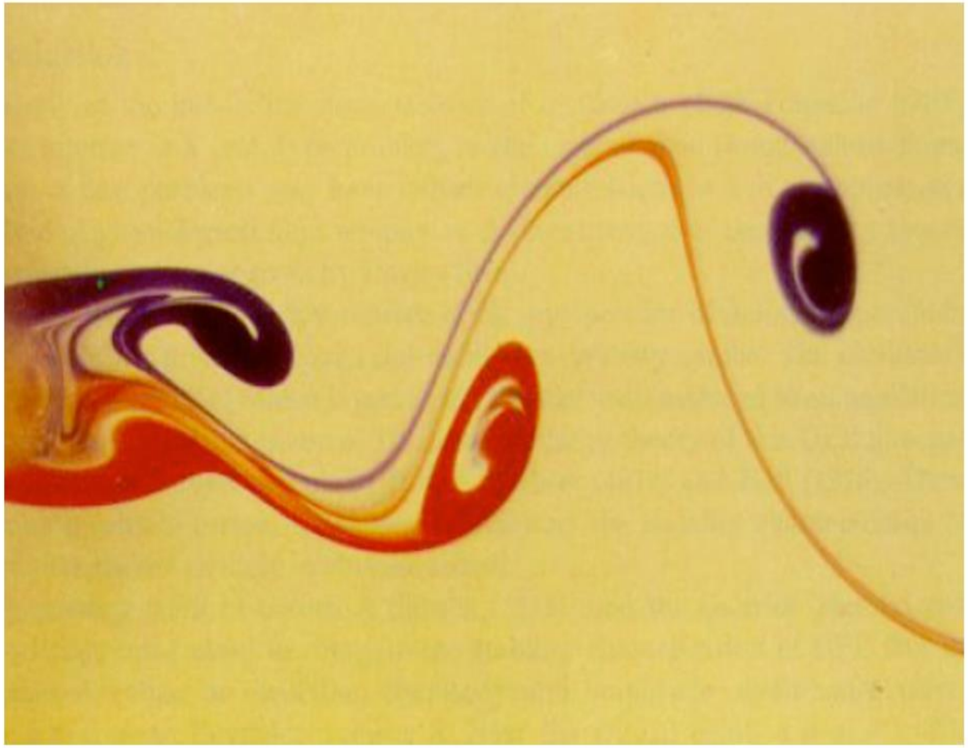


1 point de vitesse sur 4 est représenté

Sinuous mode


Cylinder wake
Re=0
Eclt symétrique
Re=1.5
Eclt attaché
Re=26
Eclt décollé
Allée de von Karman
Re=100
Eclt périodique
chaos
Re=10000
Eclt turbulent
Couche limite turbulente
C. L. turb. décoléé

Allée de von Karman



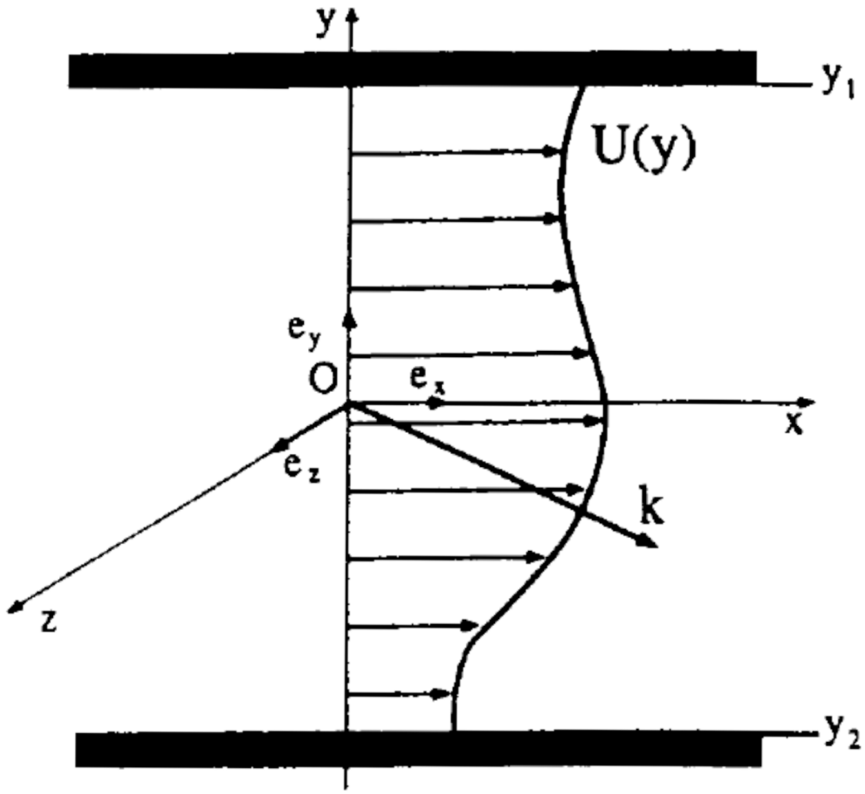
(Perry, Chong & Lim (1982)

Sinuous mode

PARALLEL FLOW CONCEPTS

Inviscid 3D instabilities

Basic flow



PARALLEL FLOW CONCEPTS

Inviscid 3D instabilities

3D Euler equations

$$\nabla \cdot \mathrm{U} = 0$$

$$\frac{\partial \mathrm{U}}{\partial t} + (\mathrm{U} \cdot \nabla)\mathrm{U} = -\nabla P$$

Basic flow + perturbation

$$\mathbf{U}(\mathbf{x}, t) = U(y)\mathbf{e}_x + \mathbf{u}(\mathbf{x}, t)$$

$$P(\mathbf{x}, t) = P_0 + p(\mathbf{x}, t)$$

Linear Euler equations

$$\nabla \cdot \mathbf{u} = 0 \qquad \left(\frac{\partial}{\partial t} + U(y)\frac{\partial}{\partial x}\right)\mathbf{u} + U'(y)\, v\, \mathbf{e}_x = -\nabla p$$

PARALLEL FLOW CONCEPTS

Inviscid 3D instabilities

Normal mode decomposition

$$\mathbf{u}(\mathbf{x},t) = \mathcal{R}e\left\{\hat{\mathbf{u}}(y)\exp[i(k_x x + k_z z - \omega t)]\right\}$$

$$p(\mathbf{x},t) = \mathcal{R}e\left\{\hat{p}(y)\exp[i(k_x x + k_z z - \omega t)]\right\}$$

$$\mathbf{k} = k_x \mathbf{e}_x + k_z \mathbf{e}_z \qquad\qquad c = \omega / k_x$$

Linear o.d.e.'s

$$ik_x \hat{u} + ik_z \hat{w} + \frac{d\hat{v}}{dy} = 0$$

$$ik_x\left[U(y) - c\right]\hat{u} + U'(y)\hat{v} = -ik_x\hat{p}$$

$$ik_x\left[U(y) - c\right]\hat{v} = -\frac{d\hat{p}}{dy},$$

$$ik_x\left[U(y) - c\right]\hat{w} = -ik_z\hat{p}$$

$$\hat{v}(y_1) = \hat{v}(y_2) = 0$$

3D dispersion relation

$$D(\mathbf{k},\omega) = 0$$

PARALLEL FLOW CONCEPTS

Inviscid 3D instabilities

Squire's transformation

$$\tilde{k}^2 = k_x^2 + k_z^2 \,, \qquad \tilde{c} = c \,,$$

$$\tilde{k}\tilde{u} = k_x \hat{u} + k_z \hat{w} \,, \qquad \tilde{v} = \hat{v} \,, \qquad \tilde{p}/\tilde{k} = \hat{p}/k_x \,.$$

Linear o.d.e.'s

$$i\tilde{k}\tilde{u} + \frac{d\tilde{v}}{dy} = 0 \,,$$

$$i\tilde{k}\left[U(y) - \tilde{c}\right]\tilde{u} + U'(y)\tilde{v} = -i\tilde{k}\tilde{p} \,,$$

$$i\tilde{k}\left[U(y) - \tilde{c}\right]\tilde{v} = -\frac{d\tilde{p}}{dy} \,,$$

$$\tilde{v}(y_1) = \tilde{v}(y_2) = 0 \,,$$

2D dispersion relation

$$\tilde{D}(\tilde{k}, \tilde{\omega}) = 0$$

PARALLEL FLOW CONCEPTS

Inviscid 3D instabilities

Squire's transformation

$$D(\mathbf{k},\omega) \equiv \tilde{D}\left[\left(k_x^2+k_z^2\right)^{1/2}, \left(\left(k_x^2+k_z^2\right)^{1/2}/k_x\right)\omega\right] = 0$$

to each oblique mode $(\mathbf{k},\omega)$ of temporal growth rate ω_i is associated a two-dimensional mode $(\tilde{k},\tilde{\omega})$ of larger temporal growth rate $\tilde{\omega}_i = \sqrt{k_x^2+k_z^2}\,\omega_i/k_x > \omega_i$. *The wave of maximum growth rate is therefore two-dimensional*

2D PARALLEL FLOW CONCEPTS

Dispersion relation

2D vorticity equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}\right)\nabla^2\Psi = 0$$

Basic flow + perturbation

$$\Psi(x,t) = \int U(y)dy + \psi(x,y,t)$$

$$u = \partial_y \psi \text{ et } v = -\partial_x \psi$$

Linear vorticity equation

$$\left(\frac{\partial}{\partial t} + U(y)\frac{\partial}{\partial x}\right)\nabla^2\psi - U''(y)\frac{\partial \psi}{\partial x} = 0$$

Rayleigh equation

Dispersion relation

Normal mode decomposition

$$\psi(x, y, t) = \mathcal{R}e\left\{\phi(y)\, e^{i(kx-\omega t)}\right\}$$

$$[U(y) - c]\,[\phi'' - k^2\phi] - U''(y)\,\phi = 0$$

$\phi(y)$ ⇨0 at $y = \pm\infty$

Dispersion relation

$$D(k, \omega) = 0$$

PARALLEL FLOW CONCEPTS

Inviscid instabilities

Rayleigh's inflection point criterion

In order for the basic flow U(y) to be unstable, it should have an inflection point, say at $y = y_s$, such that $U''(y_s) = 0$

or, in other words $\Omega(y)$ has an extremum

Rayleigh theorem (1916)

$$(U-c)(\frac{d^2\psi}{dy^2}-k^2\psi)-U''(y)\psi=0$$

$$\int_{y_1}^{y_2}\left((\frac{d^2\psi}{dy^2}-k^2\psi)\psi^*-\frac{U''(y)\psi\psi^*}{U-c}\right)dy=0$$

$$\int_{y_1}^{y_2}\left((\frac{d^2\psi}{dy^2}-k^2\psi)\psi^*-\frac{U''(y)\psi\psi^*}{|U-c|^2}(U-c^*)\right)dy=0$$

$$[\frac{d\psi}{dy}\psi^*]_{y_1}^{y_2}+\int_{y_1}^{y_2}\left(-\frac{d\psi}{dy}\frac{d\psi^*}{dy}-k^2\psi\psi^*-\frac{U''(y)\psi\psi^*}{|U-c|^2}(U-c^*)\right)dy=0$$

Rayleigh theorem

$$\left[\frac{d\psi}{dy}\psi^*\right]_{y_1}^{y_2} + \int_{y_1}^{y_2}\left(-\frac{d\psi}{dy}\frac{d\psi^*}{dy} - k^2\psi\psi^* - \frac{U''(y)\psi\psi^*}{|U-c|^2}(U-c^*)\right)dy = 0$$

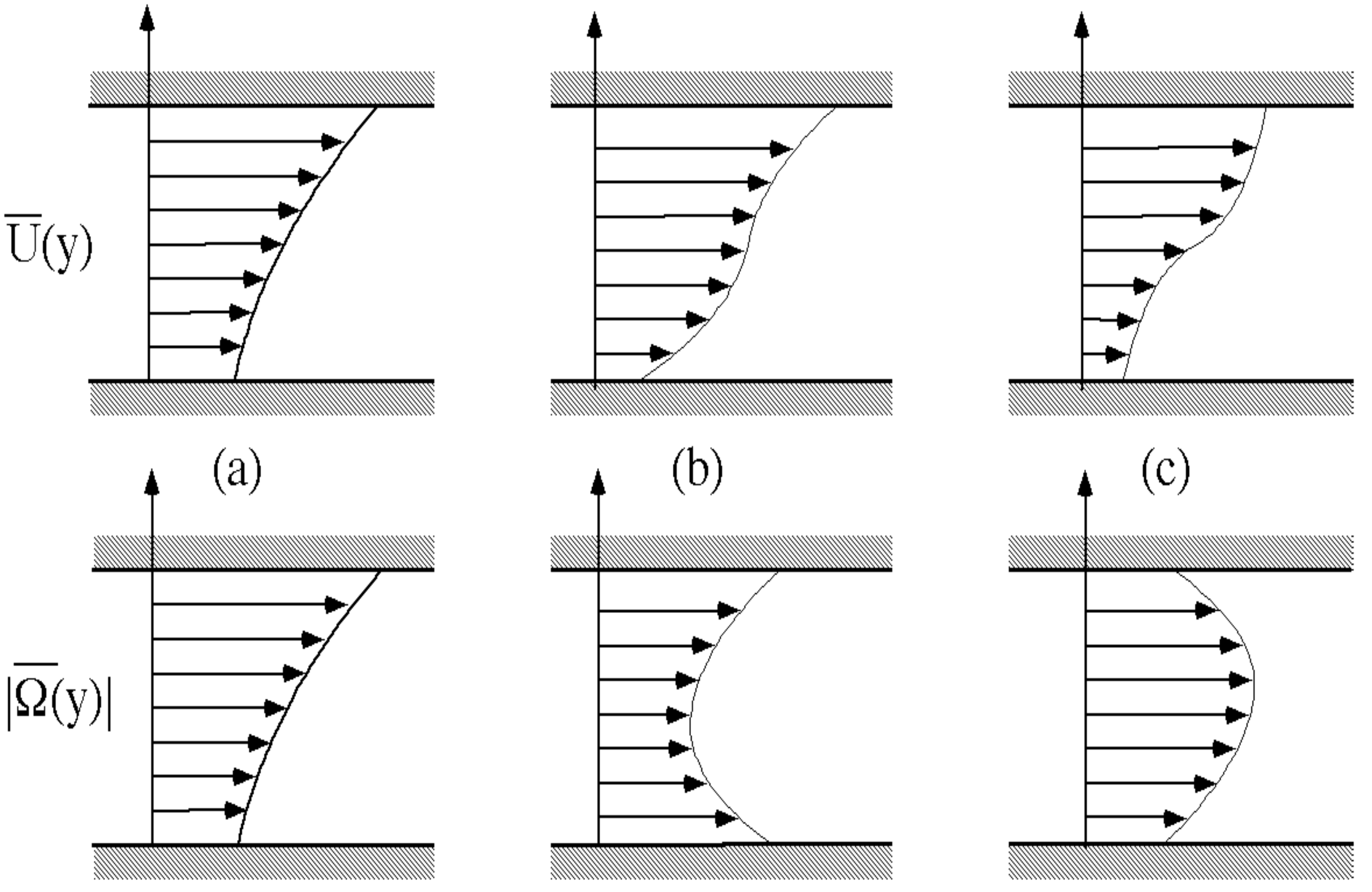
0

real real real

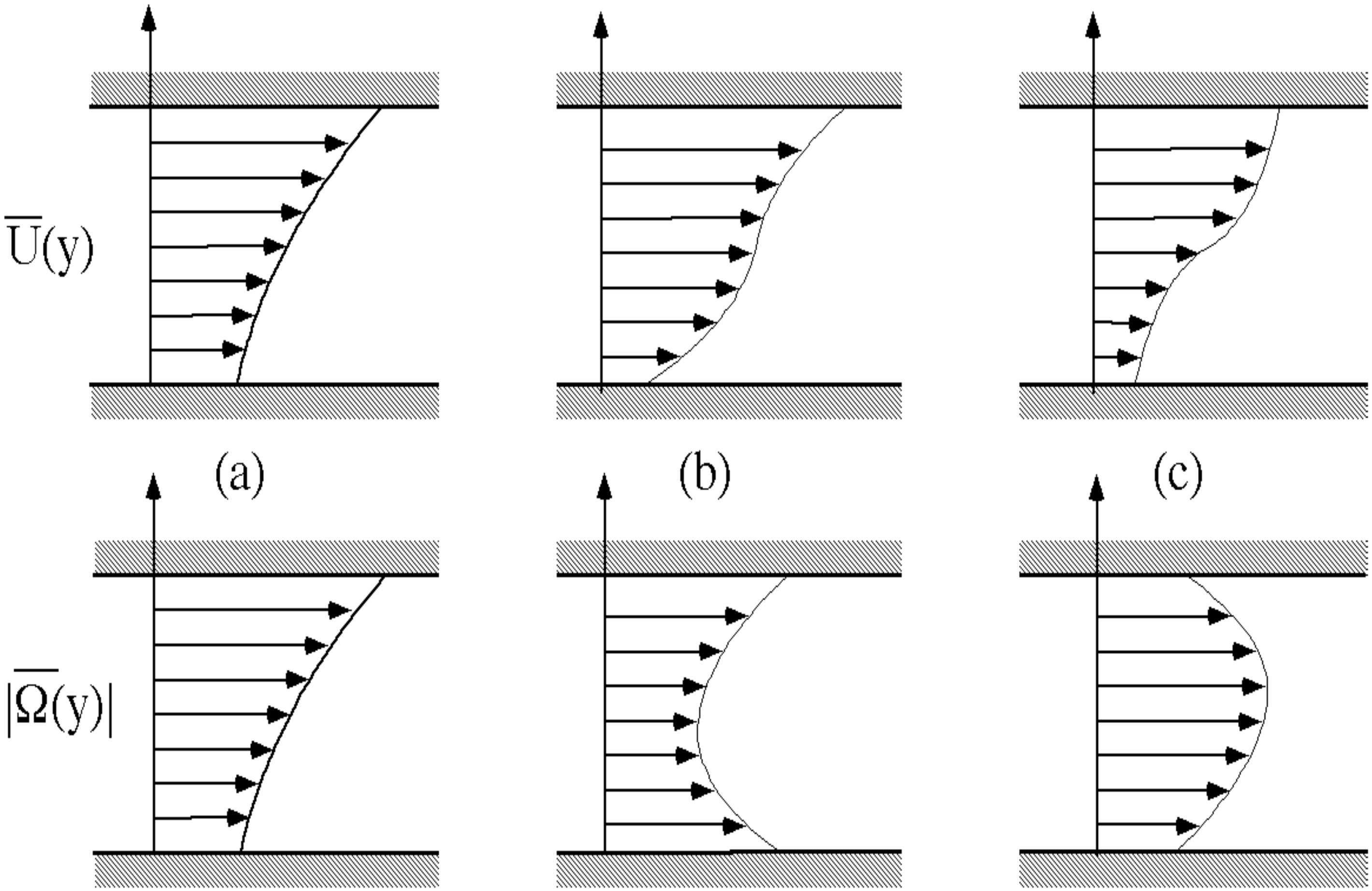
take the imaginary part

$$\int_{y_1}^{y_2}\frac{U''(y)|\psi|^2}{|U-c|^2}c_i dy = 0$$

Quiz



Quiz



stable

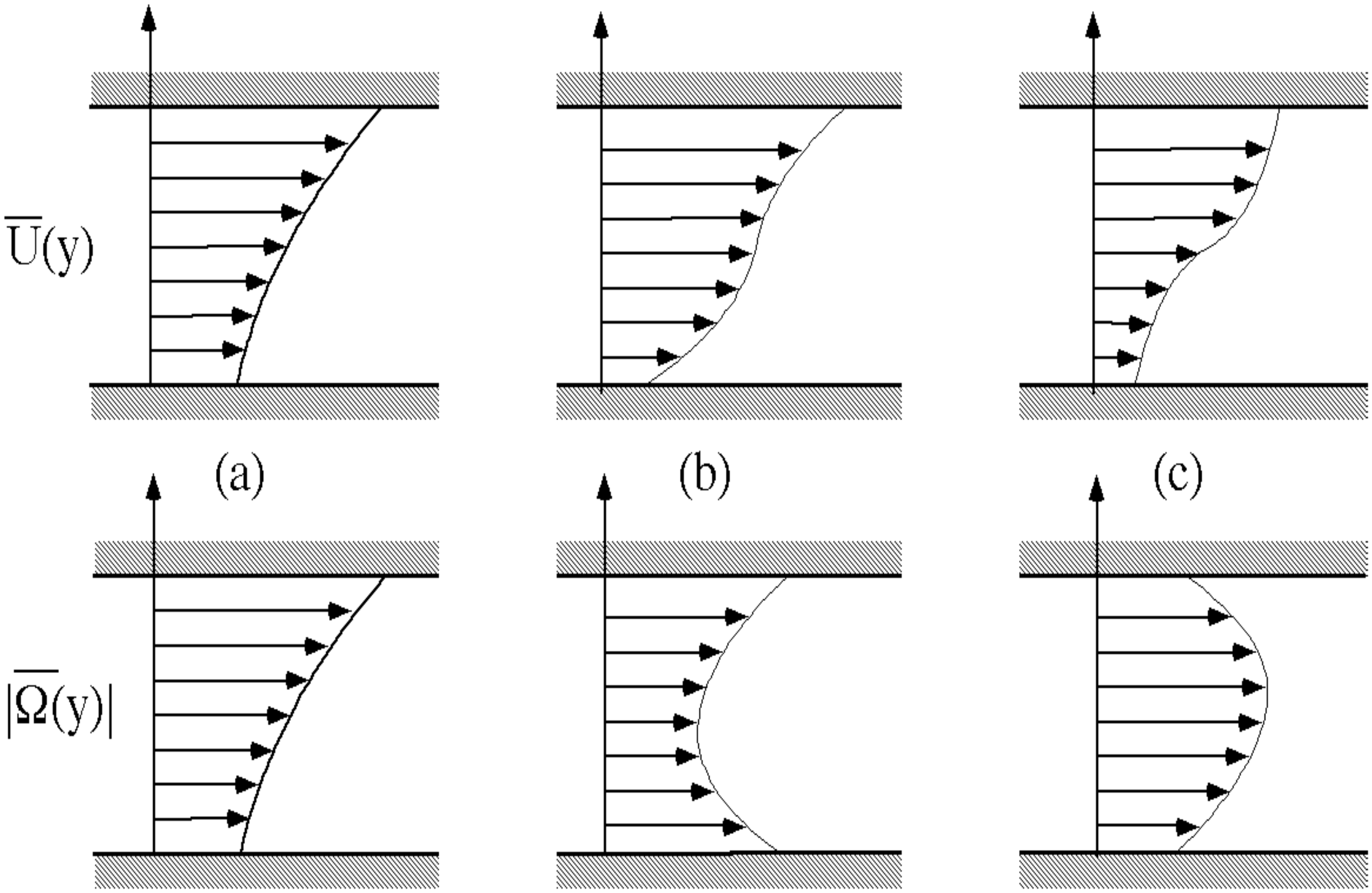
Unstable?

Unstable?

Fjorthoft criterion (1950)

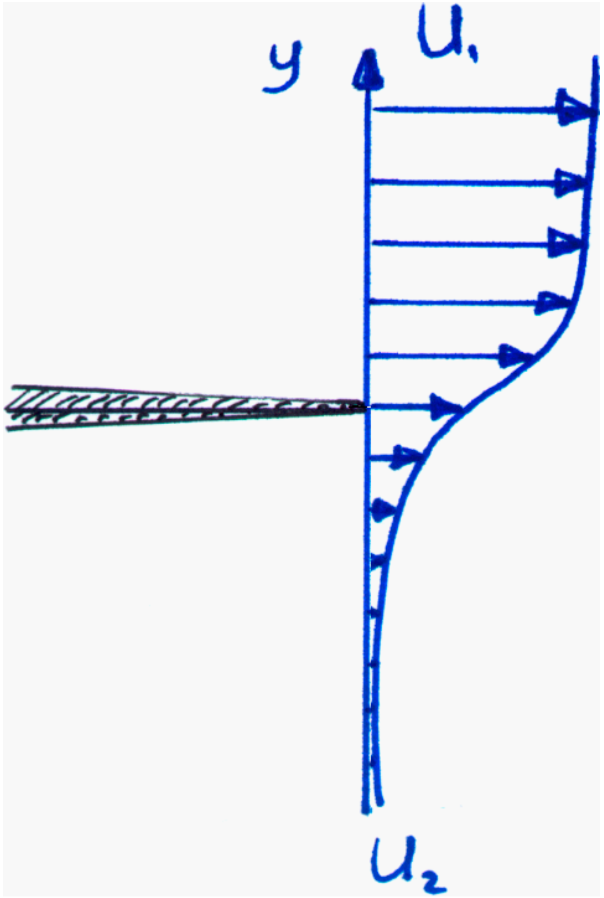
For monotonous velocity profiles with only one inflection point, ¦Ω(y)¦ should have a maximum for instability to be possible

Quiz

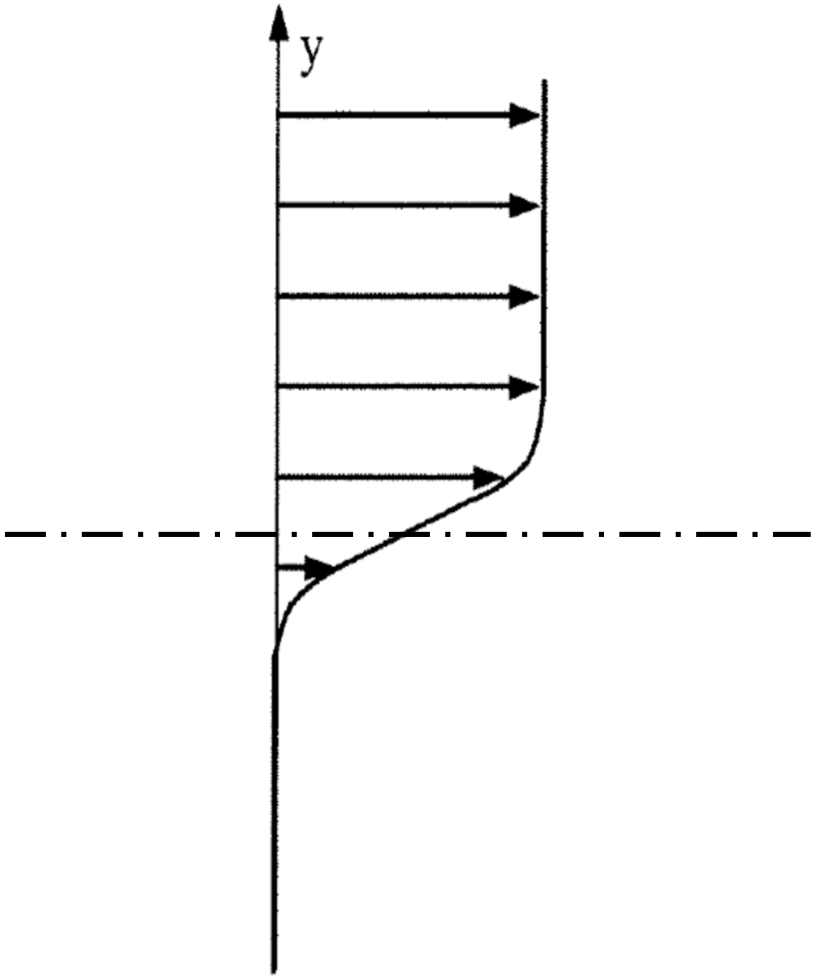


stable Stable Unstable?

Tangent hyperbolic mixing layer

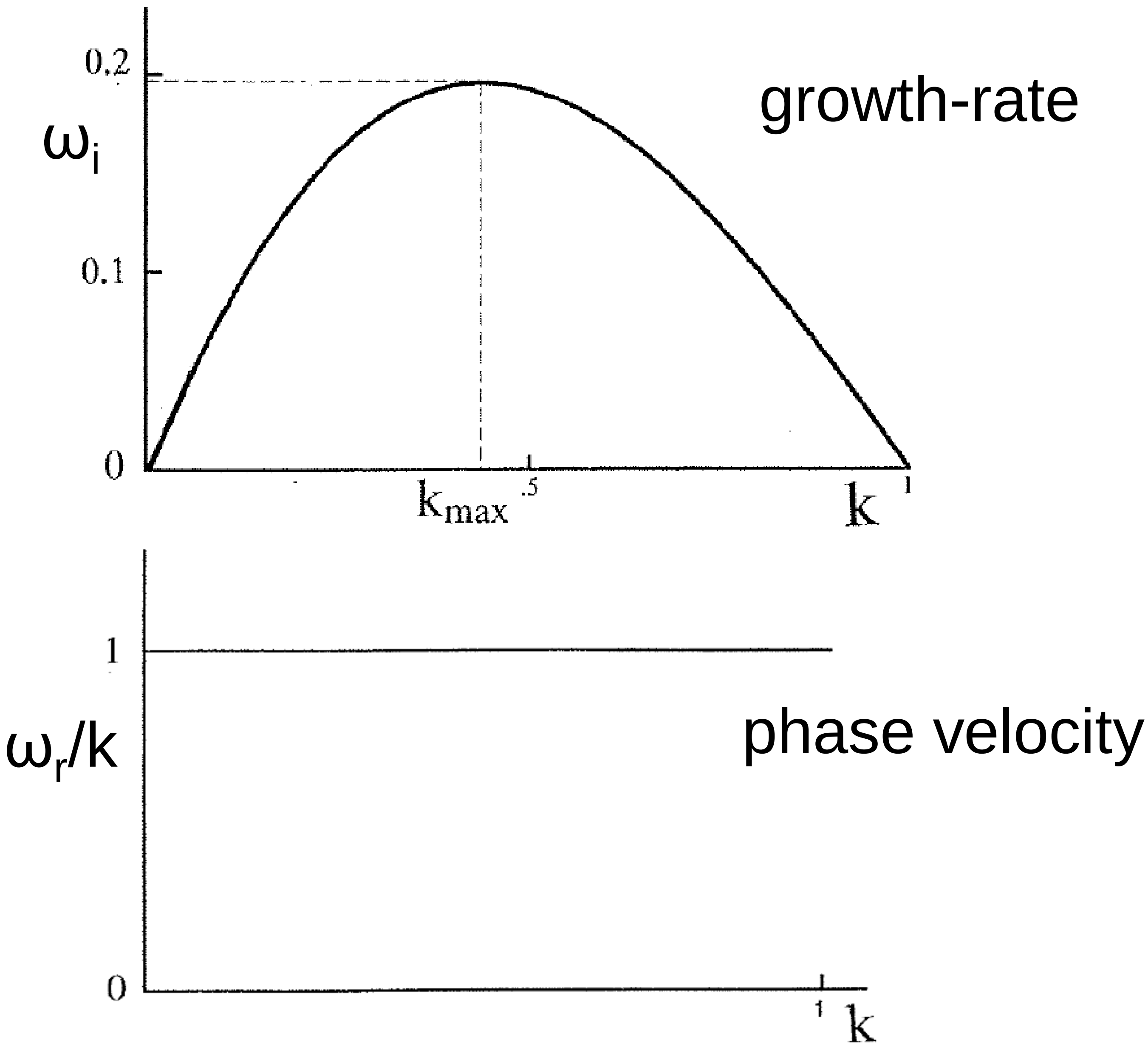


Tangent hyperbolic shear layer



$$U_B(y) = \tanh y$$

Tangent hyperbolic shear layer

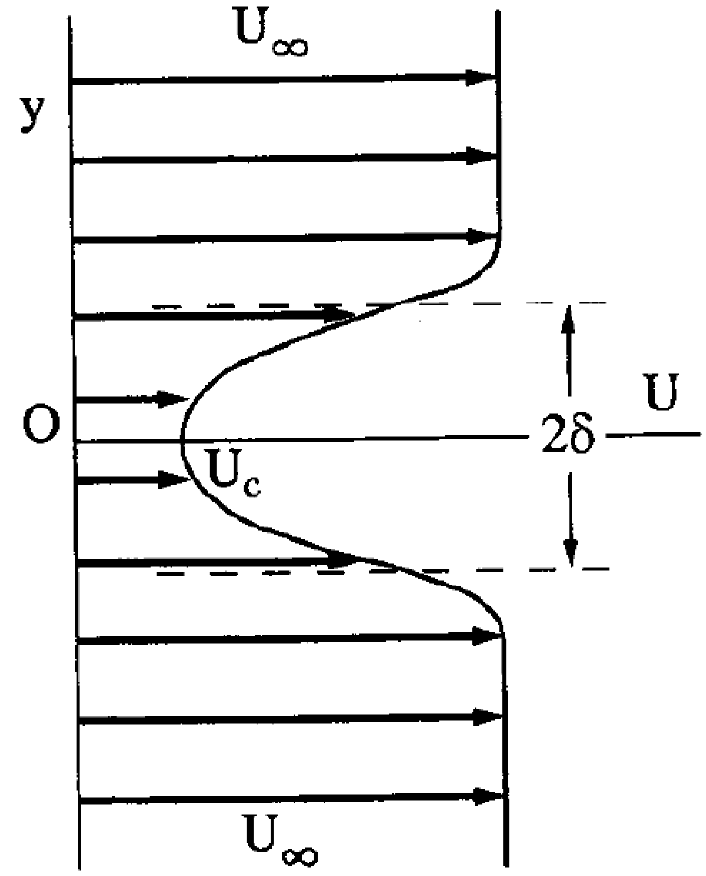


Wake instability

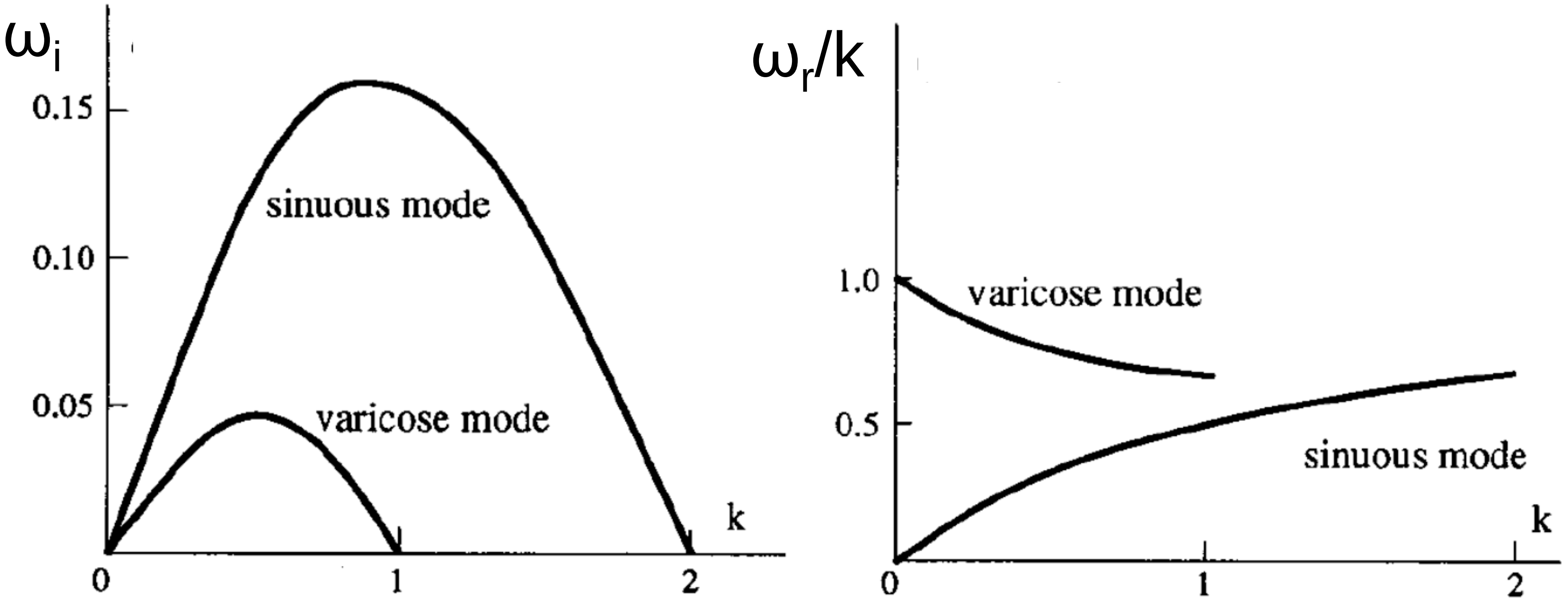
$$U(y) = U_\infty + (U_c - U_\infty)\,\mathrm{sech}^2\,\frac{y}{\delta}$$

Velocity ratio

$$R \equiv (U_c - U_\infty)/(U_c + U_\infty)$$



Wake instability



Betchov & Criminale (1966)

How to solve Rayleigh equation?

We fix k, we need to find all c and ψ such that

$$\left(U\left(\frac{d^2}{dy^2}-k^2\right)-U''(y)\right)\psi = c\left(\frac{d^2}{dy^2}-k^2\right)\psi$$

$$\psi(-L)=\psi(L)=0$$

Formally,

$$\mathcal{A}\psi = c\,\mathcal{E}\psi$$

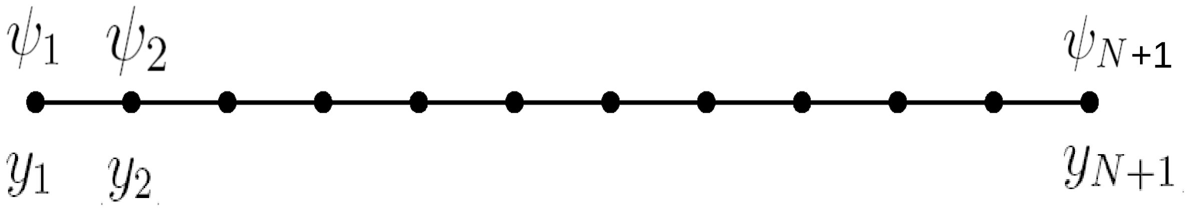
Discretize

$$\mathbf{A}\Psi = c\mathbf{E}\Psi$$

Generalized eigenvalue problem

How to solve Rayleigh equation?

Method 1: Finite differences of order 1.



$$\Psi = \begin{pmatrix} \psi(y_1) \\ \psi(y_2) \\ \vdots \\ \psi(y_N) \\ \psi(y_{N+1}) \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \\ \psi_{N+1} \end{pmatrix} \qquad \Psi'' = \begin{pmatrix} \psi''(y_1) \\ \psi''(y_2) \\ \vdots \\ \psi''(y_N) \\ \psi''(y_{N+1}) \end{pmatrix}$$

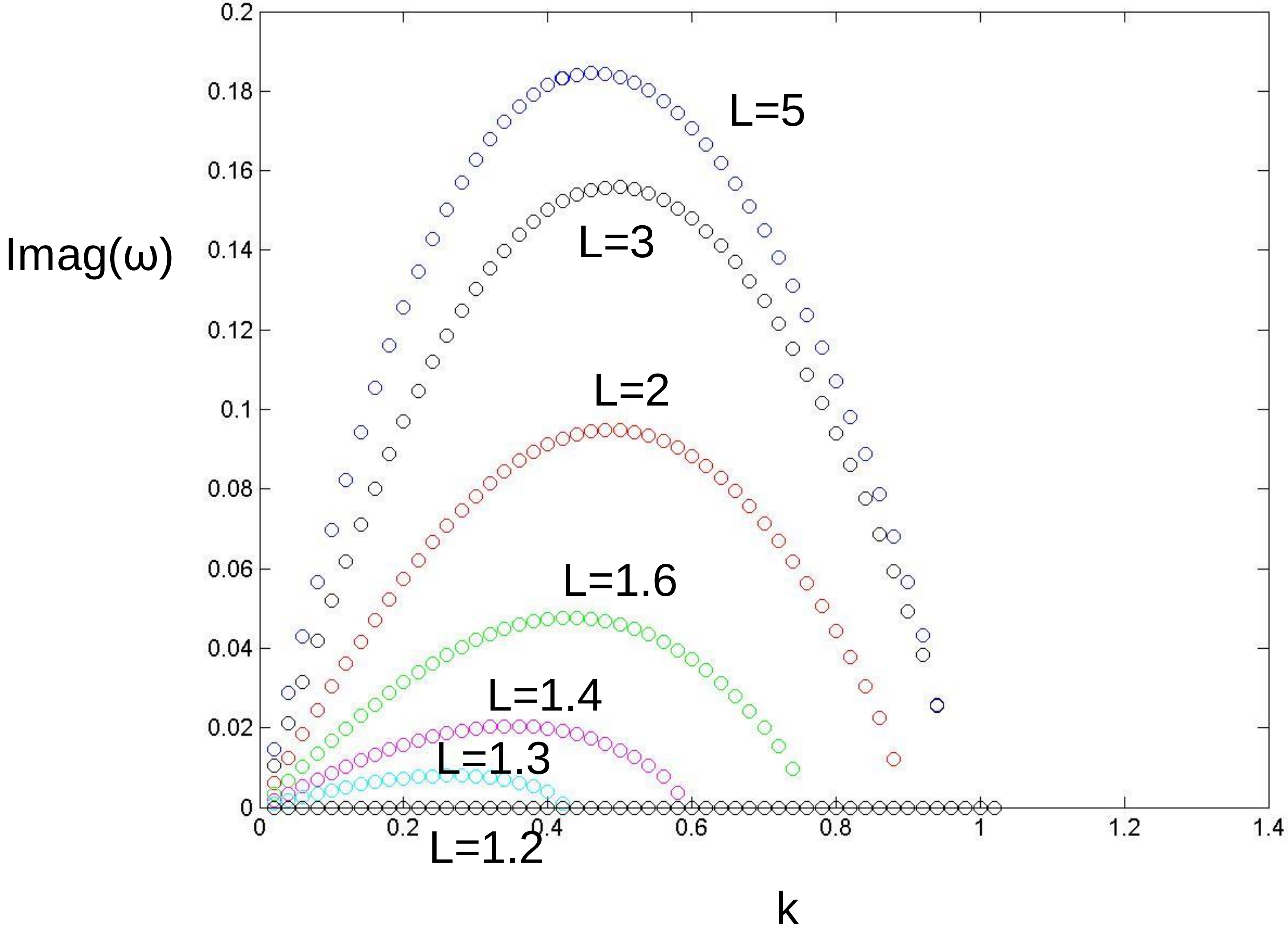
How to solve Rayleigh equation?

Method 1: Finite differences

$$\begin{pmatrix} \psi_2'' \\ \psi_3'' \\ \psi_4'' \\ \vdots \\ \psi_{N-3}'' \\ \psi_{N-2}'' \\ \psi_{N-1}'' \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & -2 & 1 & 0 \\ 0 & \dots & \dots & 0 & 1 & -2 & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_3 \\ \psi_4 \\ \vdots \\ \psi_{N-3} \\ \psi_{N-2} \\ \psi_{N-1} \end{pmatrix}$$

Sparse matrix but low order!

Influence of confinement



PARALLEL FLOW CONCEPTS

Viscous 3D instabilities

3D Navier - Stokes equations

$$\nabla \cdot \mathbf{U} = 0\,,$$

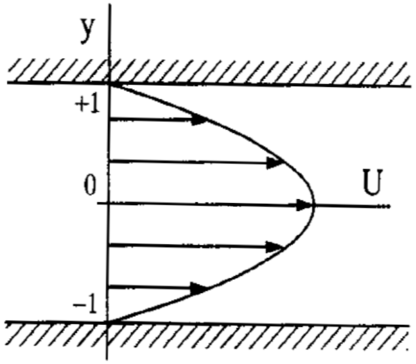
$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{U} = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{U}$$

Basic flow + perturbation

$$\mathbf{U}(\mathbf{x}, t) = U(y)\mathbf{e}_x + \mathbf{u}(\mathbf{x}, t)$$

$$P(\mathbf{x}, t) = P_0(x) + p(\mathbf{x}, t)$$

Basic flow



PARALLEL FLOW CONCEPTS

Viscous 3D instabilities

Squire's transformation

$$\tilde{k}^2 = k_x^2 + k_z^2\,, \qquad \tilde{c} = c\,,$$

$$\tilde{k}\tilde{u} = k_x\hat{u} + k_z\hat{w}\,, \qquad \tilde{v} = \hat{v}\,, \qquad \tilde{p}/\tilde{k} = \hat{p}/k_x\,.$$

$$\tilde{k}\widetilde{\mathrm{Re}} = k_x\,\mathrm{Re}$$

3D dispersion relation

$$D(\mathbf{k},\omega;\mathrm{Re}) \equiv \tilde{D}\left[(k_x^2+k_z^2)^{1/2}, \frac{(k_x^2+k_z^2)^{1/2}}{k_x}\omega; \frac{k_x}{(k_x^2+k_z^2)^{1/2}}\mathrm{Re}\right] = 0$$

To each oblique mode $(\mathbf{k},\omega)$ of temporal growth rate ω_i, at Reynolds number Re, corresponds a two-dimensional mode $(\tilde{k},\tilde{\omega})$ of larger growth rate $\tilde{\omega}_i = \omega_i\sqrt{k_x^2+k_z^2}/k_x$, at a *lower* Reynolds number $\widetilde{\mathrm{Re}} = \mathrm{Re}\,k_x/\sqrt{k_x^2+k_z^2}$.

2D PARALLEL FLOW CONCEPTS

Dispersion relation

2D vorticity equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}\right)\nabla^2\Psi = \frac{1}{Re}\nabla^4\Psi$$

Basic flow + perturbation

$$\Psi(x,t) = \int U(y)dy + \psi(x,y,t)$$

Linear vorticity equation

$$\left(\frac{\partial}{\partial t} + U(y)\frac{\partial}{\partial x}\right)\nabla^2\psi - U''(y)\frac{\partial \psi}{\partial x} = \frac{1}{Re}\nabla^4\psi$$

2D PARALLEL FLOW CONCEPTS

Dispersion relation

Normal mode decomposition

$$\psi(x,y,t) = \mathcal{R}e\left\{\phi(y)\,e^{i(kx-\omega t)}\right\}$$

Orr-Sommerfeld equation

$$[U(y)-c][\phi''-k^2\phi] - U''(y)\,\phi = \frac{1}{i\,k\,Re}\left(\frac{d^2}{dy^2}-k^2\right)^2\phi$$

$\phi(y)$ ⇨0 at $y=\pm\infty$

Dispersion relation

$$D(k,\omega;Re)=0$$

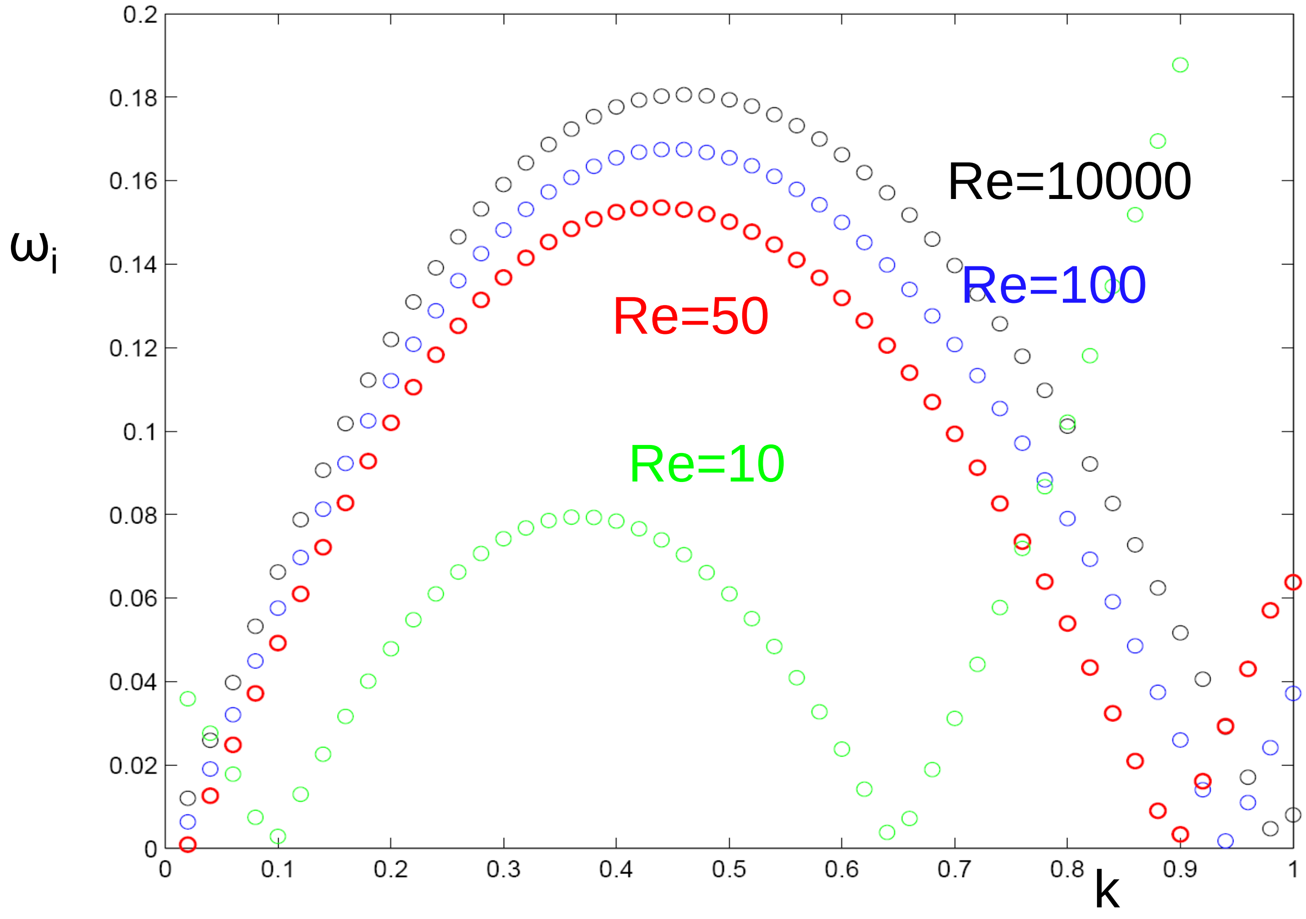
Viscosity has limited stabilizing influence on K-H instability

$$Re := \Delta U \, \delta / \nu$$

$$\omega_{i,max} \frac{\delta}{\Delta U} \approx \sqrt{\frac{0,2}{1 + a/0,2Re}}$$

constant

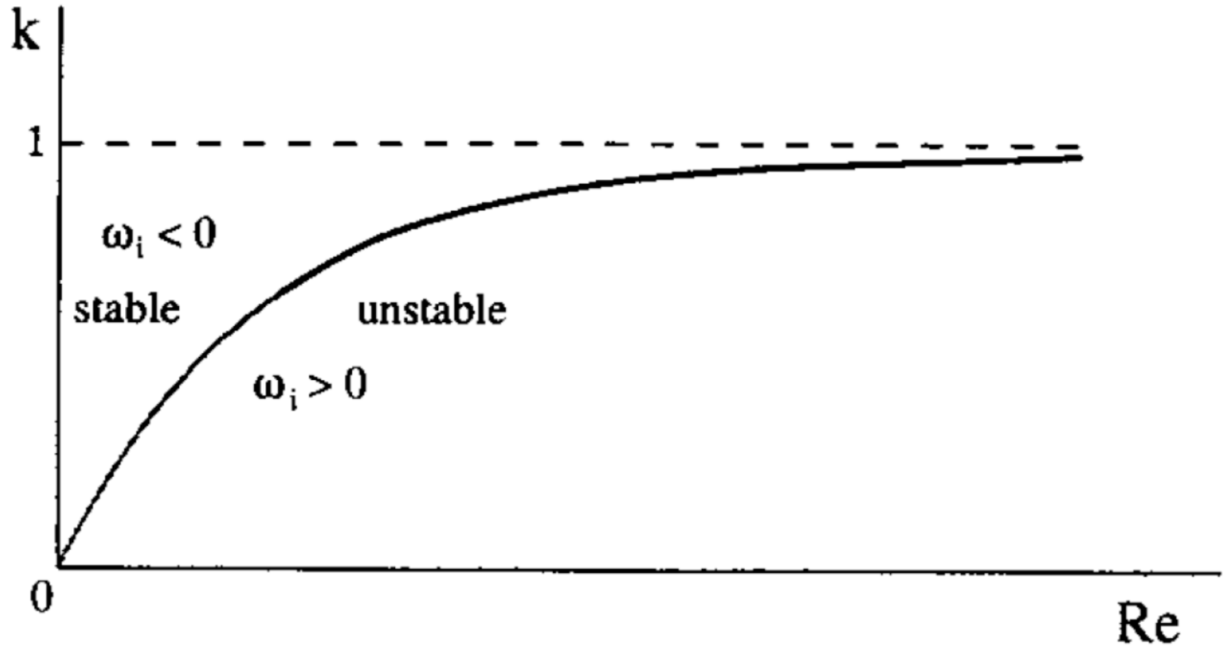
Viscosity has stabilizing influence on K-H instability



PARALLEL FLOW CONCEPTS

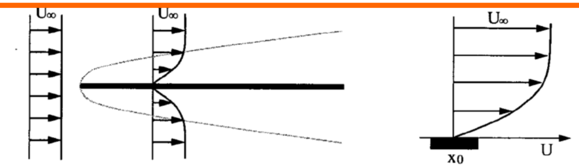
Viscous instabilities

Hyperbolic tangent mixing layer

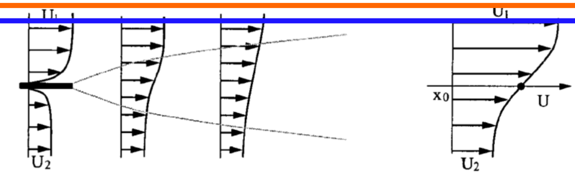


SPATIALLY DEVELOPING SHEAR FLOWS

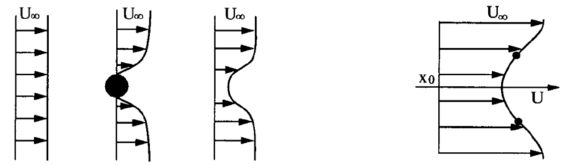
Flat plate boundary layer



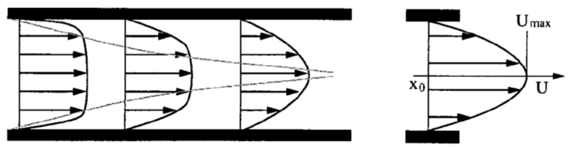
Mixing layer



Cylinder wake

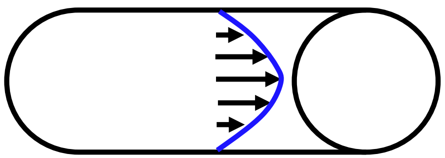


Plane channel flow

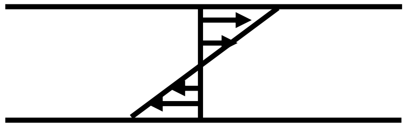


2D jet

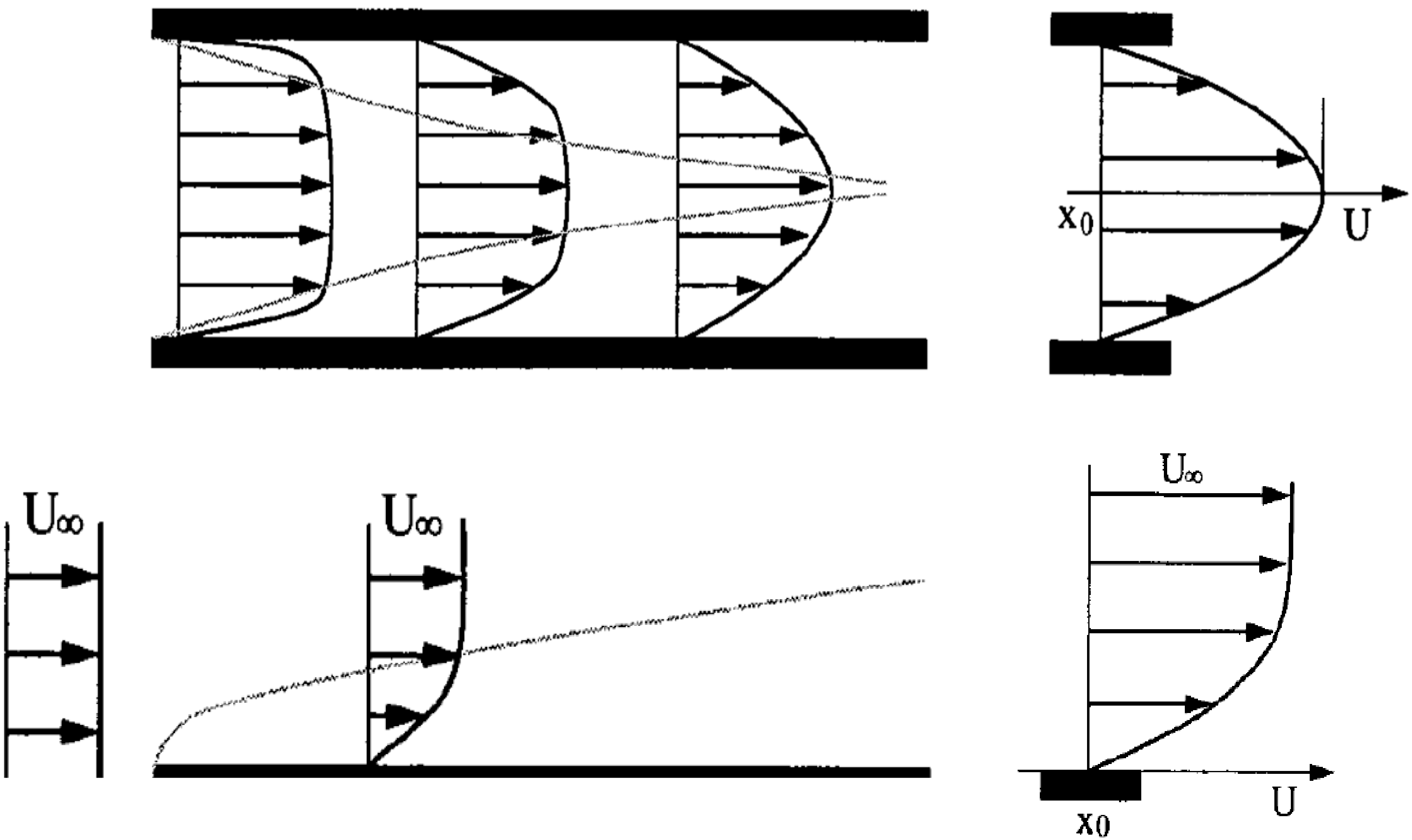
Hagen Poiseuille



Couette

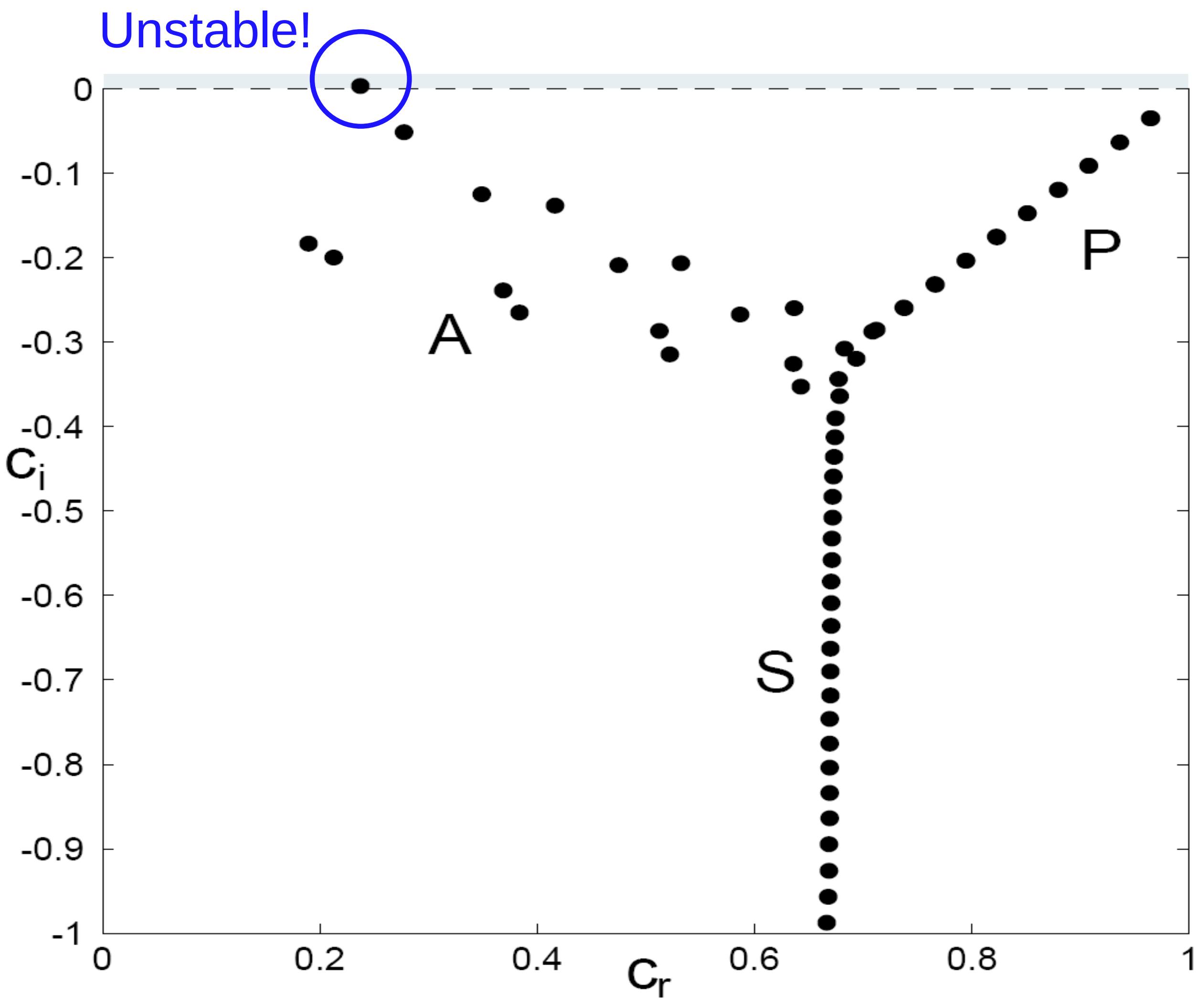


What about inviscidly stable flows (no inflexion point)?

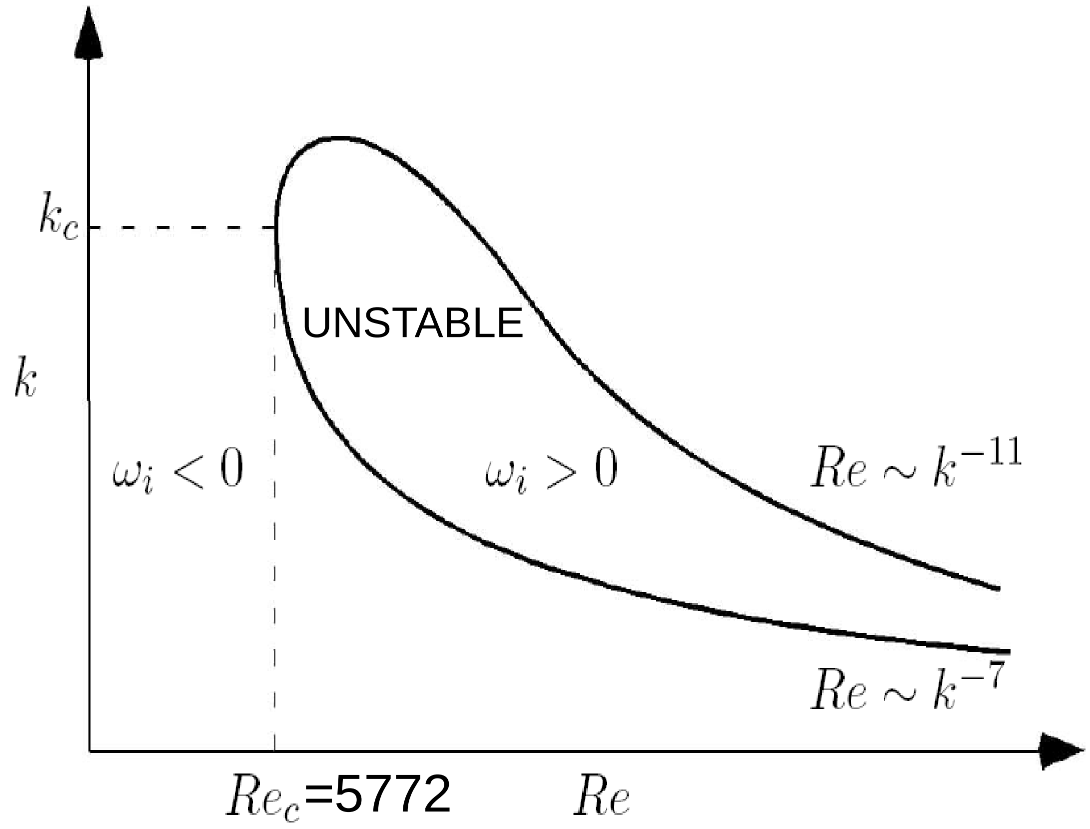


They can be unstable with finite viscosity

Plane poiseuille flow; Re=10000; α=1, β=0

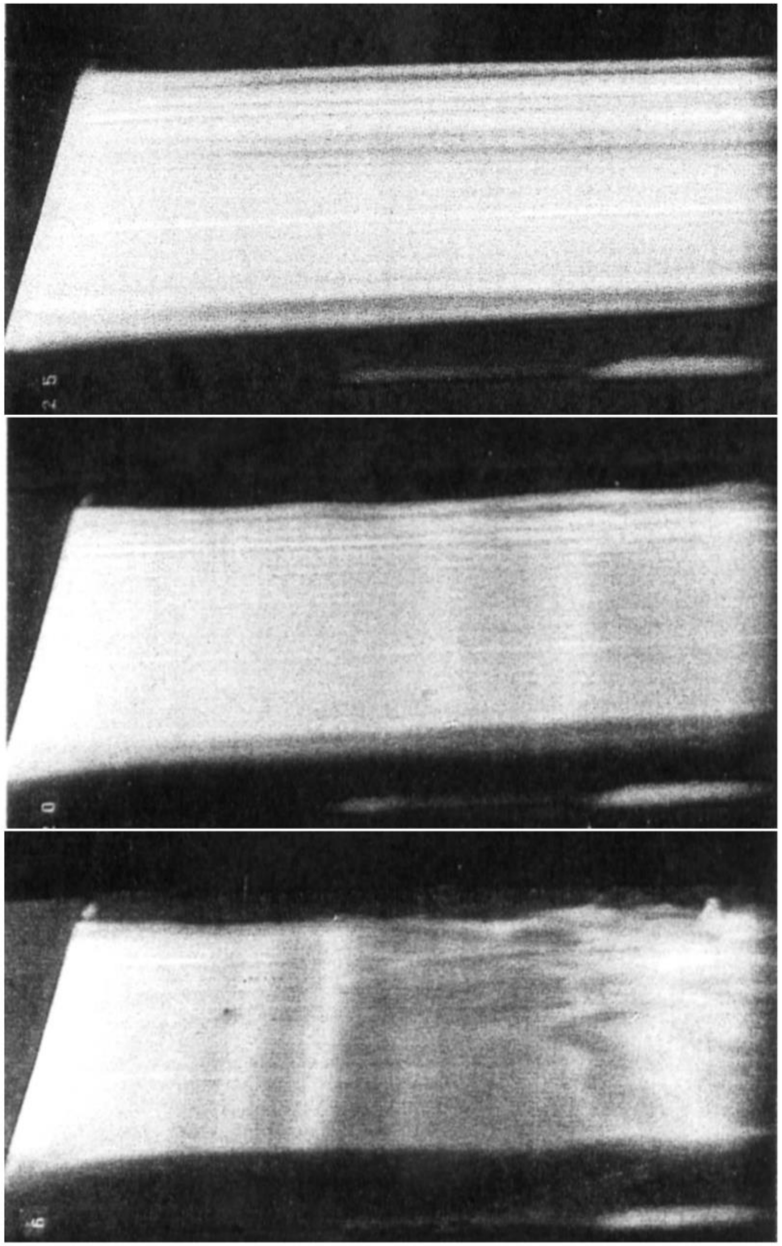


Neutral curve for plane poiseuille flow



– *Allure de la courbe de stabilité marginale dans le plan $Re - k$ pour l'écoulement de Poiseuille plan.*

Boundary layer at increasing Reynolds number



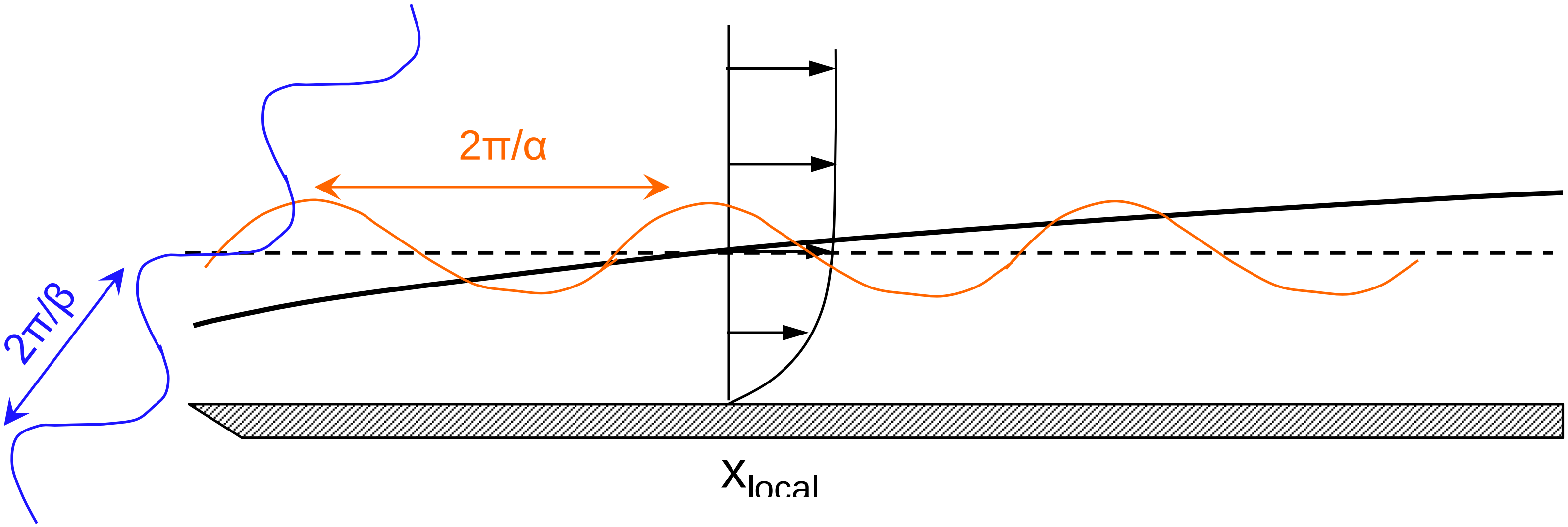
Boundary layers are destabilized by viscosity
Reynolds Orr equation

Perturbation kinetic energy: $e_c = \frac{1}{2}(u^2 + v^2 + w^2)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{y_1}^{y_2} \langle e_c \rangle \mathrm{d}y = \int_{y_1}^{y_2} \partial_y \overline{U} \tau_{xy} \mathrm{d}y - \frac{1}{Re}\int_{y_1}^{y_2} \langle \boldsymbol{\omega}.\boldsymbol{\omega} \rangle \mathrm{d}y$$

Production term Dissipation term

Local parallel flow approximation



$$(\mathbf{u}, p) = (\mathbf{u}(y), p(y))\, \mathrm{e}^{\sigma t + i(\alpha x + \beta z)}$$

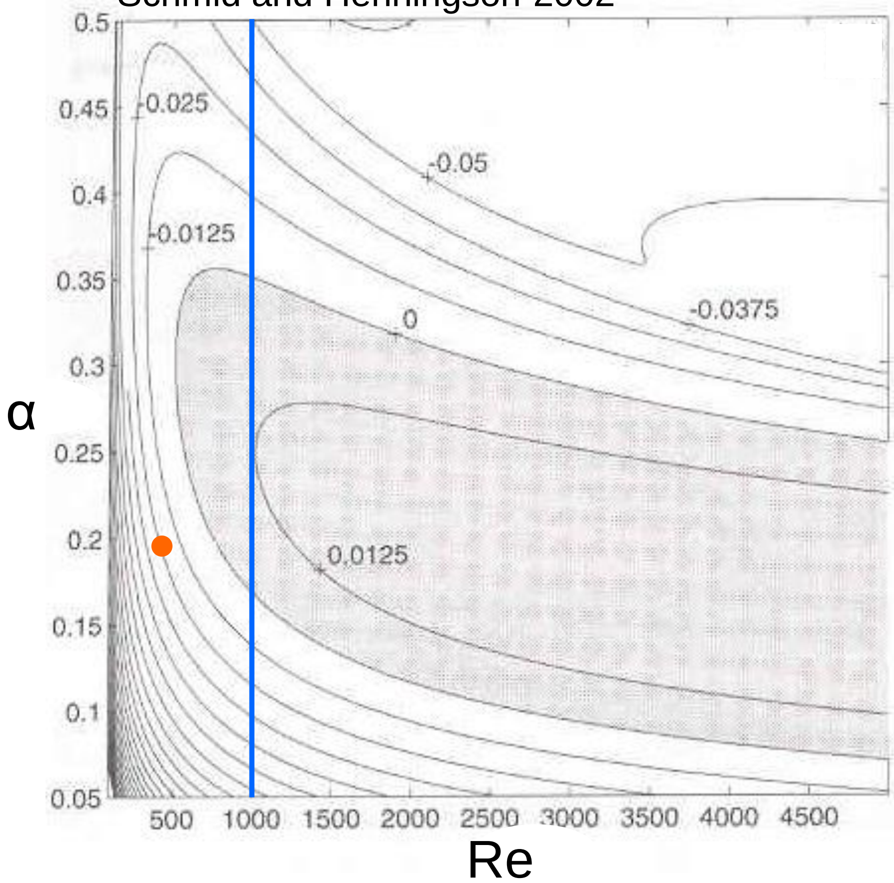
⇨Orr-Sommerfeld-Squire equation

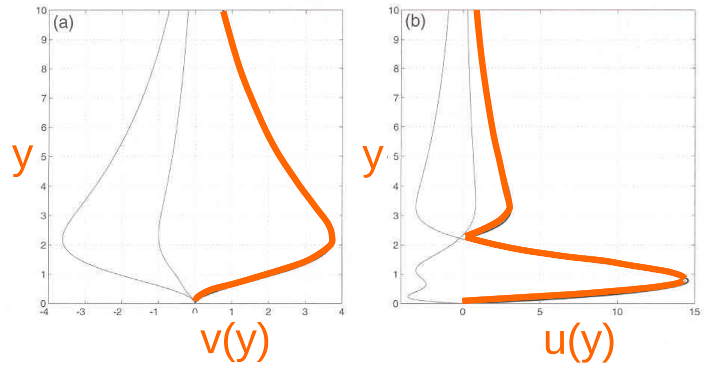
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{U}(y)\nabla\mathbf{u} + \mathbf{u}\nabla\mathbf{U}(y) = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{u}$$

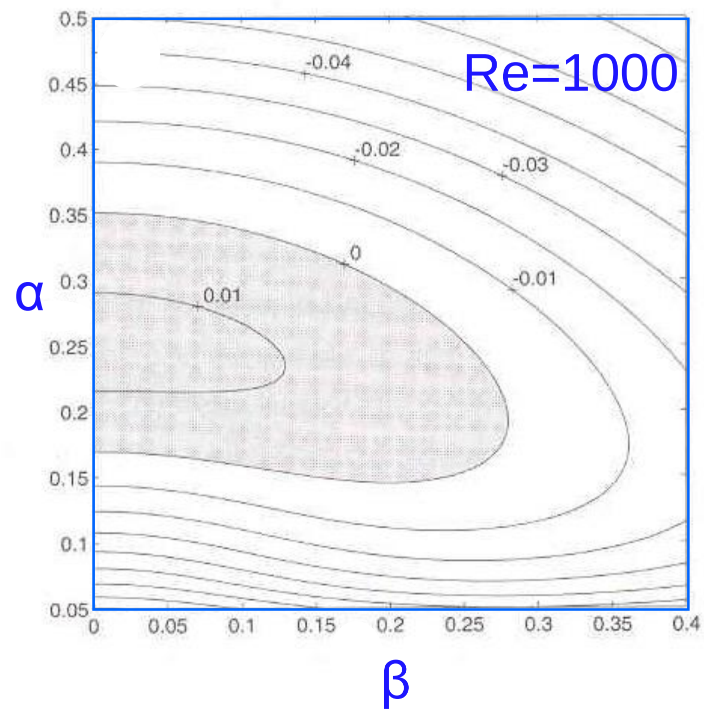
$$\nabla . \mathbf{u} = 0$$

Tollmien Schlichting waves

Schmid and Henningson 2002



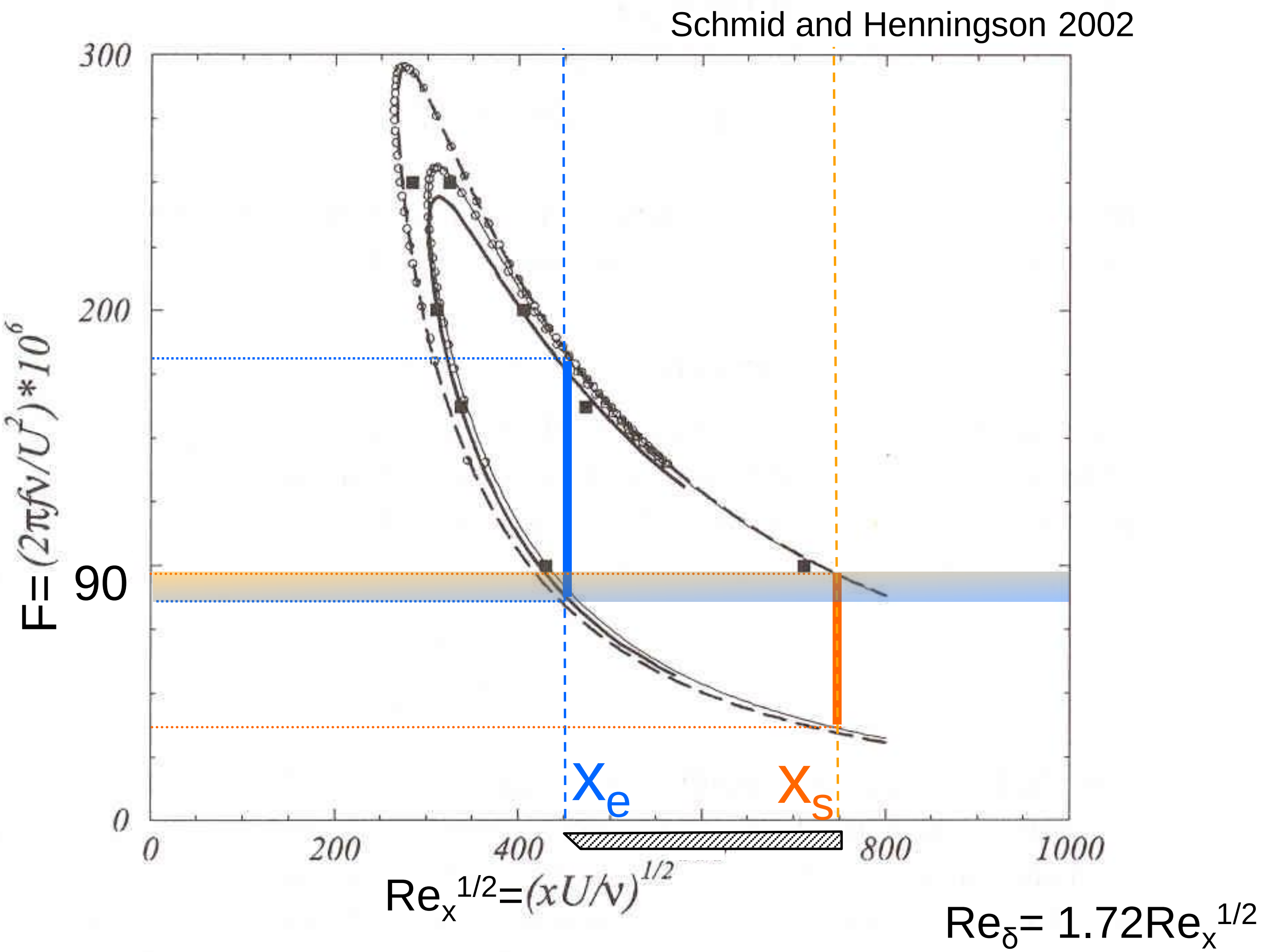




α: axial wavenumber

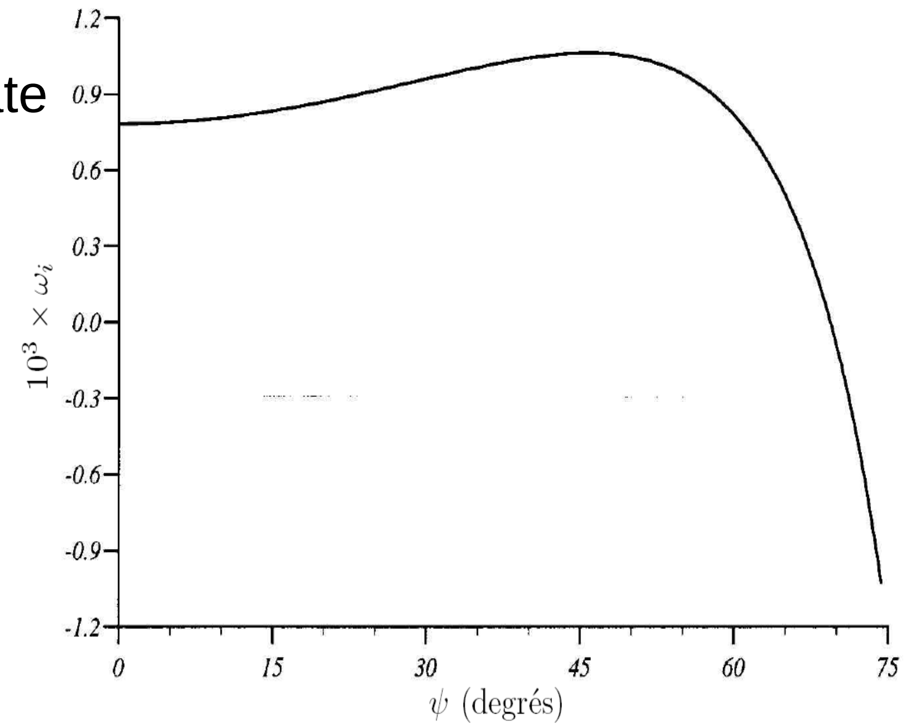
β: transverse wavenumber

Neutral curve



Boundary layer at Re_δ =1500

Growth-rate



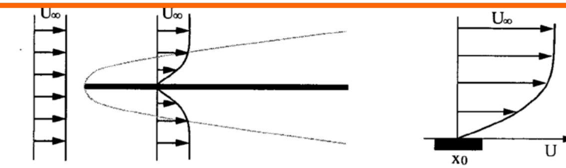
ψ= tan(β/α)

Taux de croissance temporel (s^{-1}) de l'instabilité d'une couche limite, en fonction de l'angle ψ de la perturbation avec la direction de l'écoulement de base. $R_{\delta 1} = 1500$, $\omega\nu/U_\infty^2 = 0,3 \times 10^{-4}$ (calcul G. Casalis, ONERA).

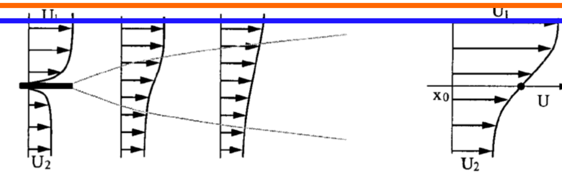
The most unstable perturbation is oblique!

SPATIALLY DEVELOPING SHEAR FLOWS

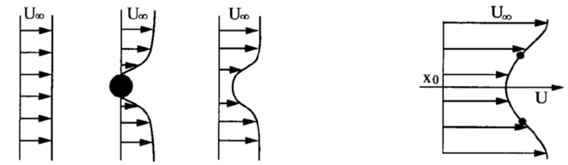
Flat plate boundary layer



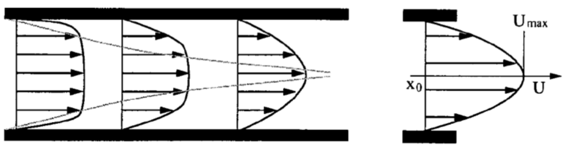
Mixing layer



Cylinder wake

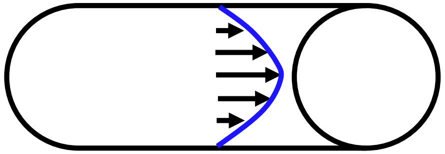


Plane channel flow

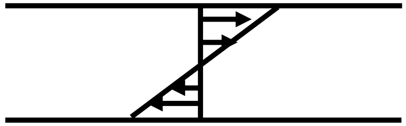


2D jet

Hagen Poiseuille

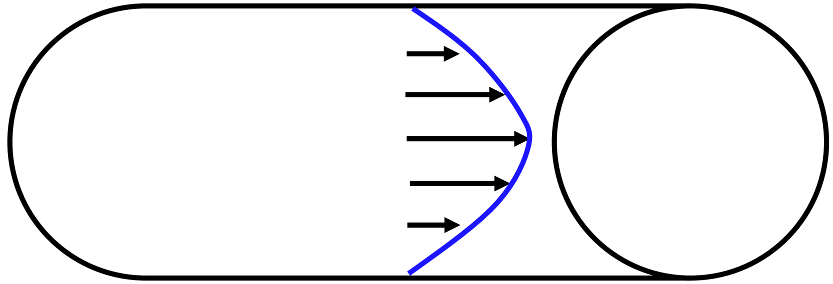


Couette

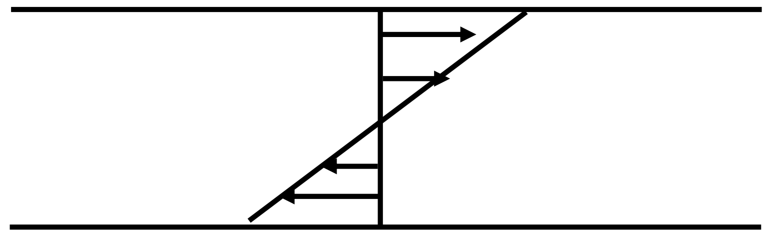


What about inviscidly stable flows (no inflexion point)?

Hagen Poiseuille



Couette



Flow type	Re_{exp}	Re_{lin}
Pipe flow	≈ 2000	∞
Plane Poiseuille flow	≈ 1000	5772
Plane Couette flow	≈ 360	∞

They can also remain stable with finite viscosity

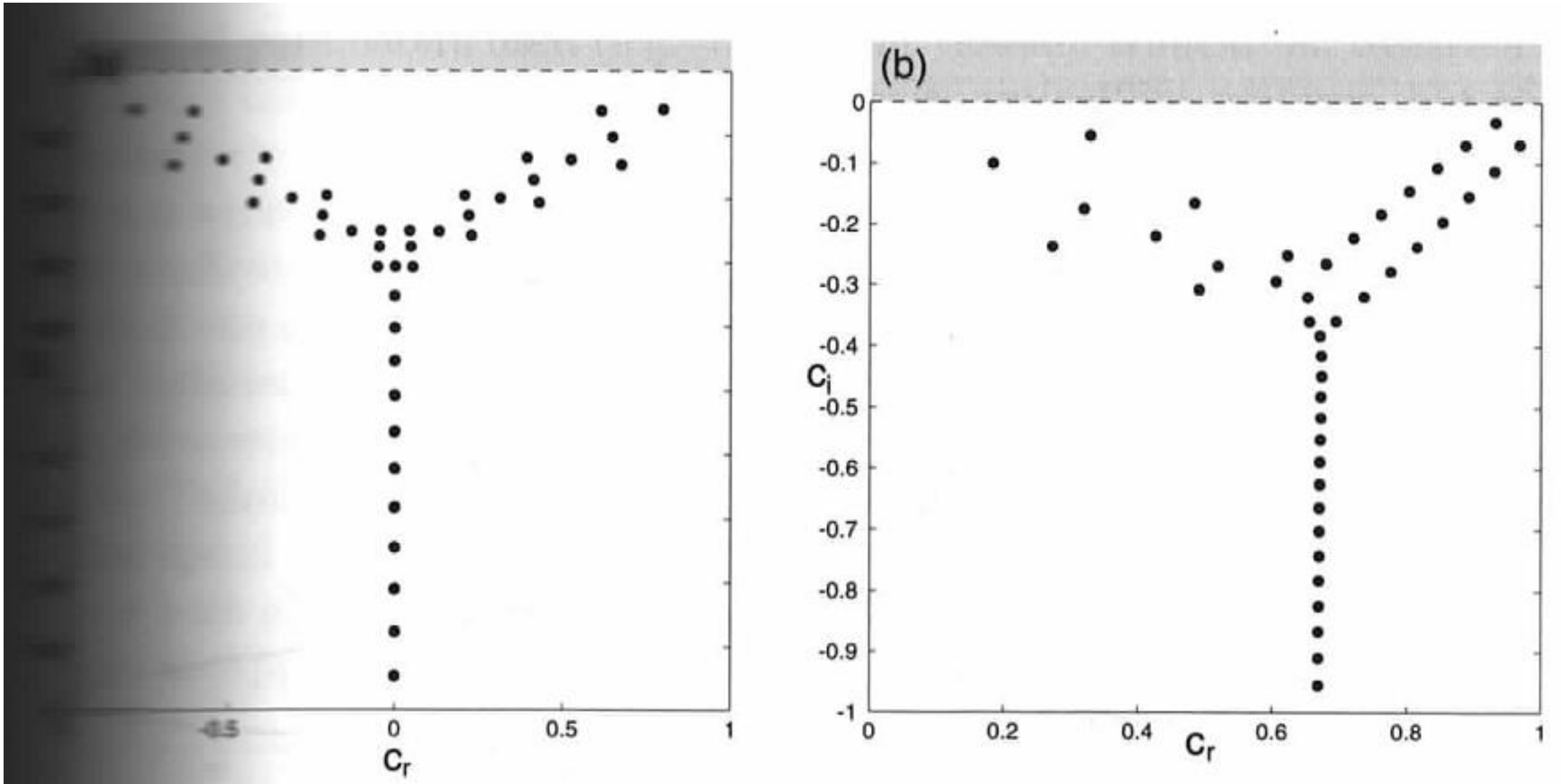


FIGURE 3.3. Spectrum of plane Couette and pipe flow. (a) Plane Couette flow for $\alpha = 1, \beta = 1, \mathrm{Re} = 1000$; (b) Pipe flow for $\alpha = 1, n = 1, \mathrm{Re} = 5000$.

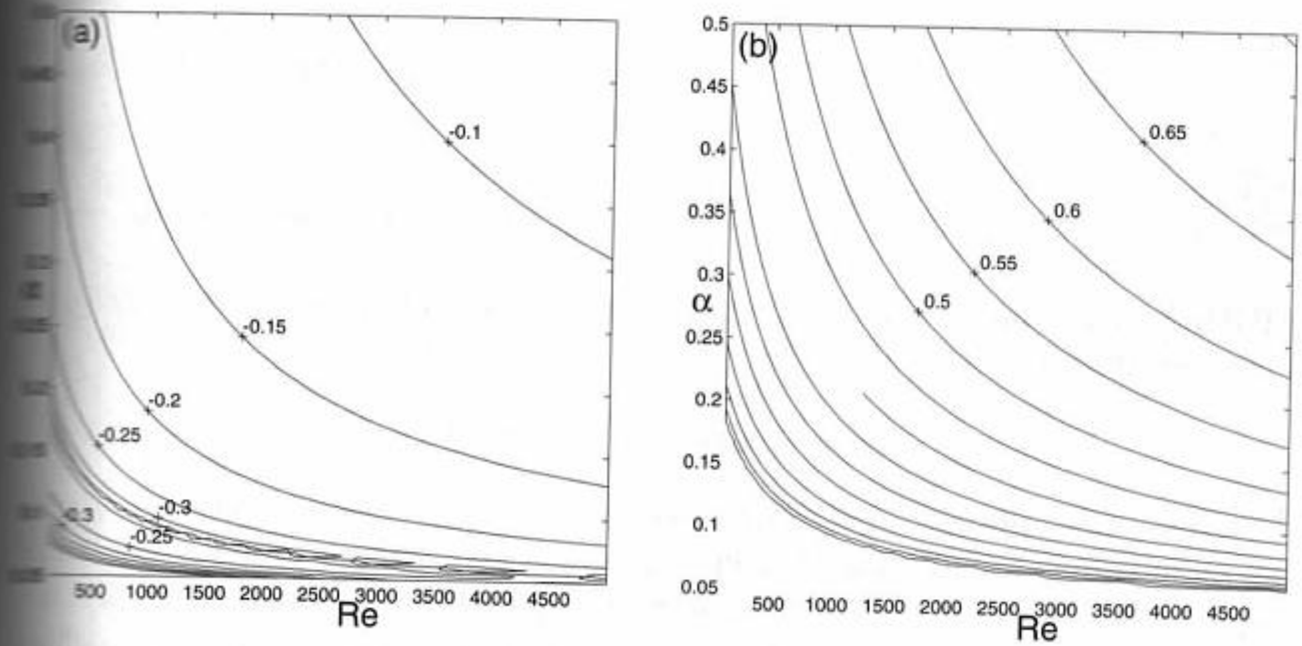


FIGURE 3.11. Contours of constant growth rate (a) and phase speed (b) for plane Couette flow.

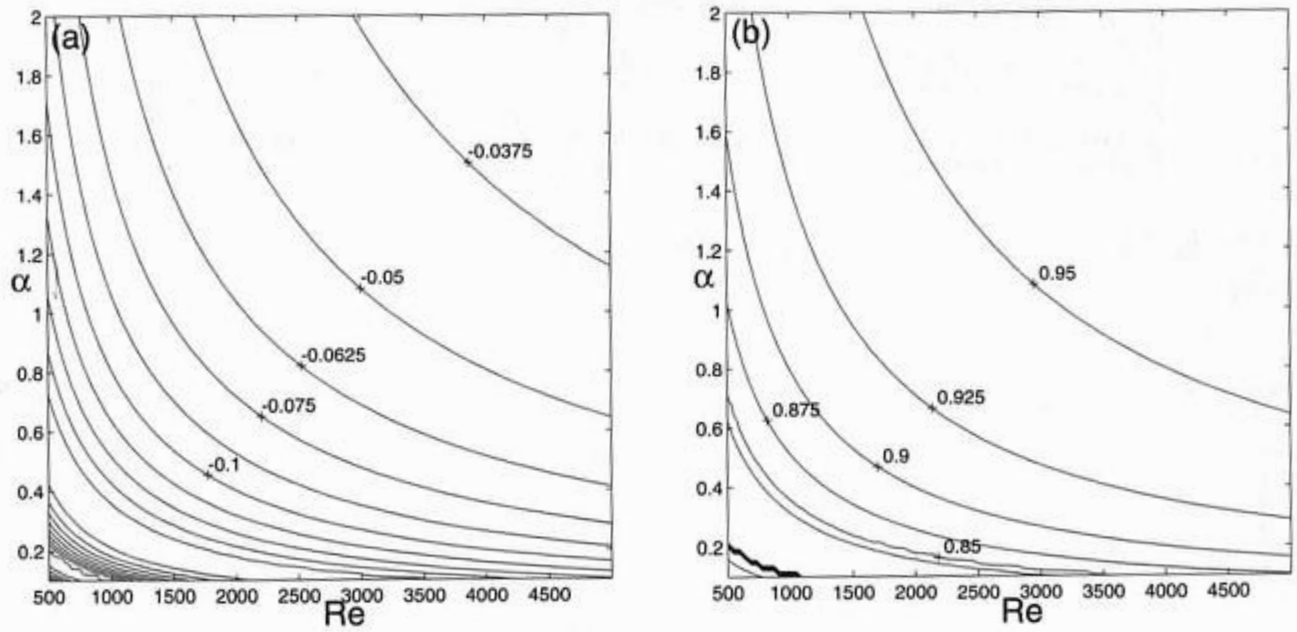
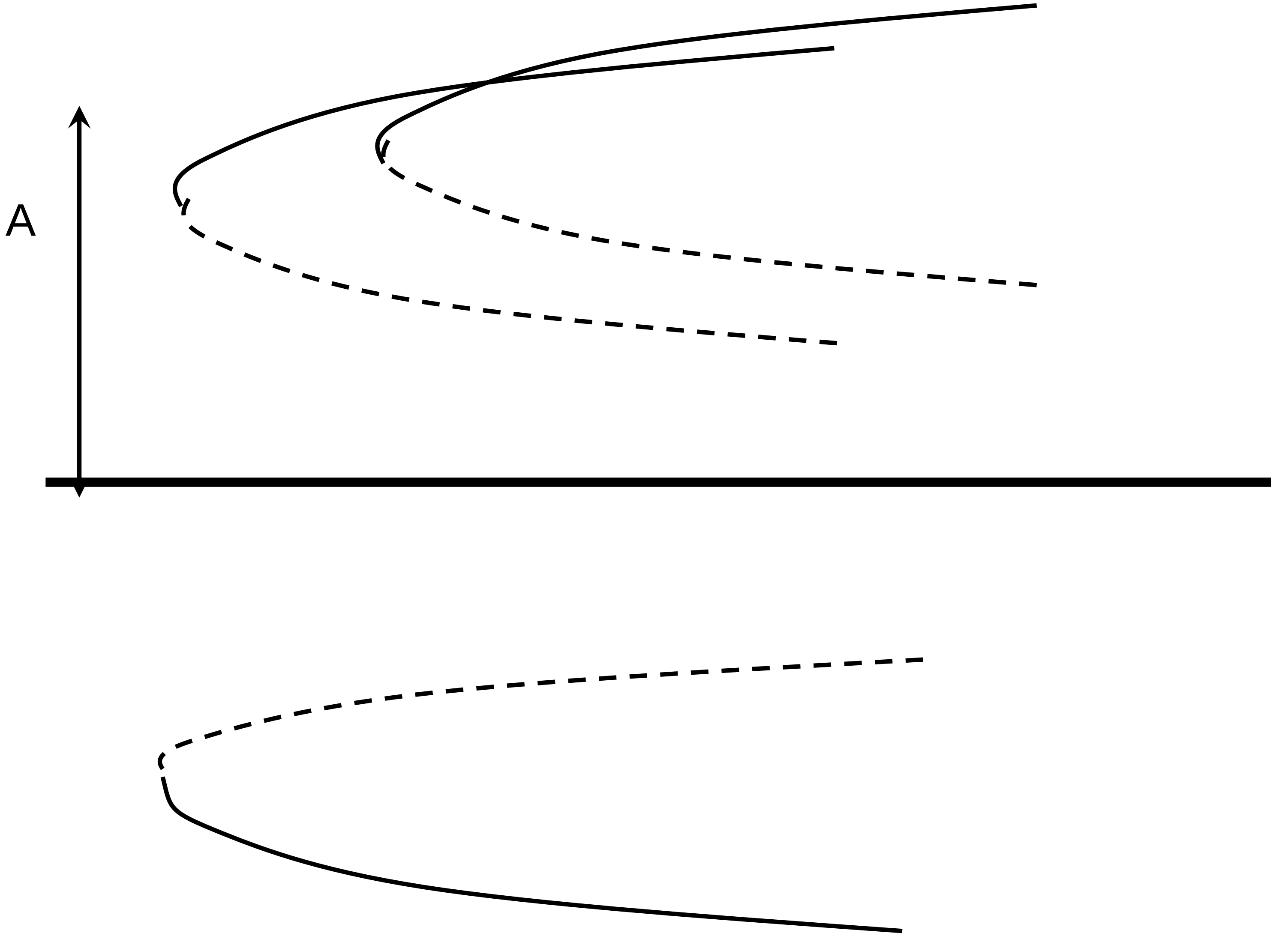


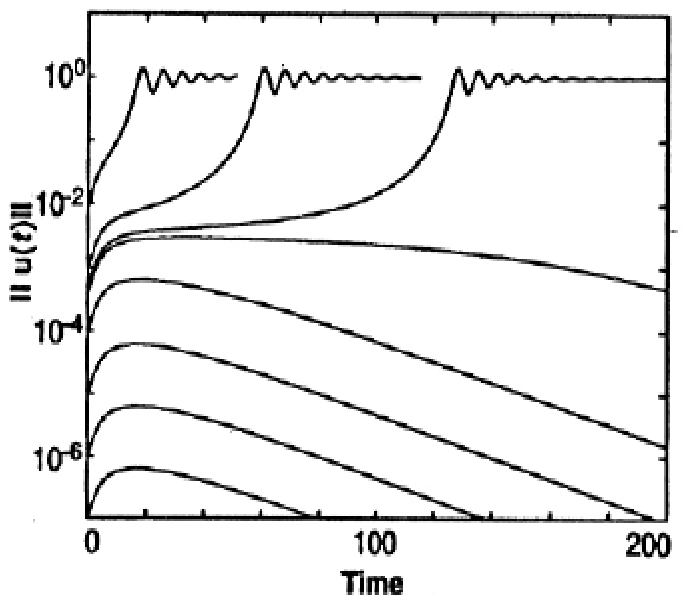
FIGURE 3.12. Contours of constant growth rate (a) and phase speed (b) for pipe Poiseuille flow.

What about nonlinearities?



Edge states

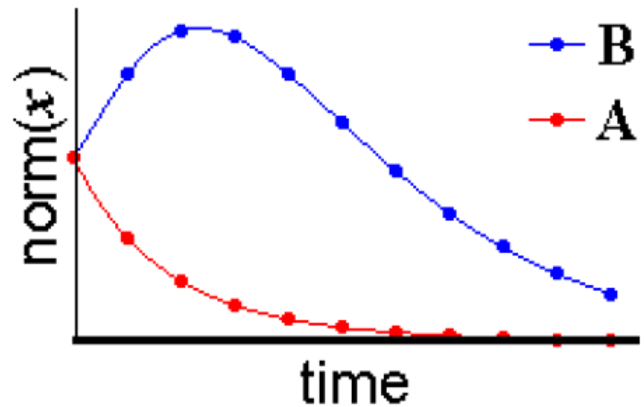
Transient growth and bye pass transition

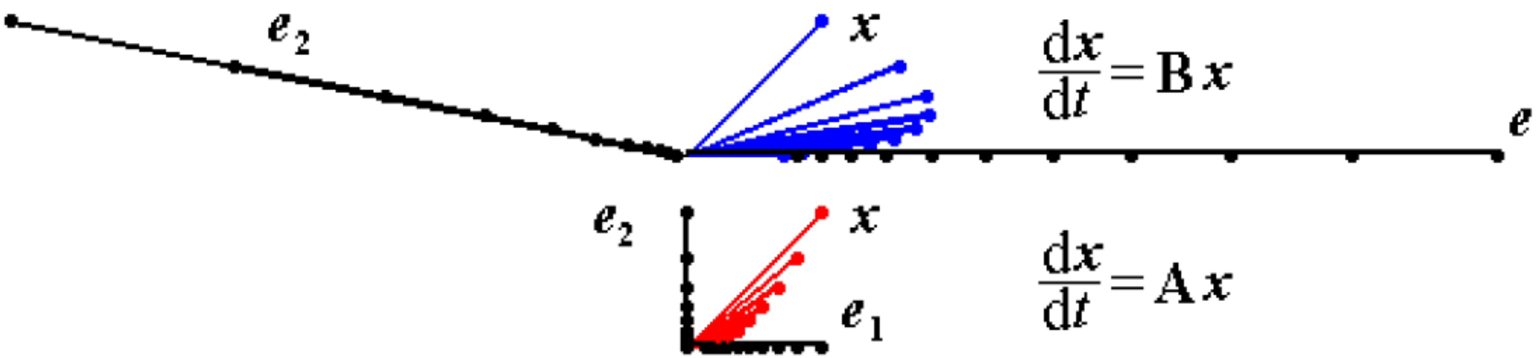


Conditional stability – subcritical bifurcation
Trefethen 1993

$$\mathbf{B} = \begin{bmatrix} -1 & 5 \\ 0 & -2 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$





$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \mathbf{B}\,\boldsymbol{x}$$

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \mathbf{A}\,\boldsymbol{x}$$

Definition of optimal gain of a linear system

Linear system $$\frac{dq}{dt} = \mathbf{L}q$$

Initial condition $$q(t=0) = q_0$$

Optimal gain $$G(t) = \max_{q_0} \frac{||q||}{||q_0||}\ ?$$

norm

Definition of optimal gain of a linear system

$$\frac{dq}{dt} = \mathbf{L}q$$

$$q(t=0) = q_0$$

Matrix exponential

$$q(t) = \exp(\mathbf{L}t)q_0$$

Definition of optimal gain of a linear system

Optimal gain $G(t)$

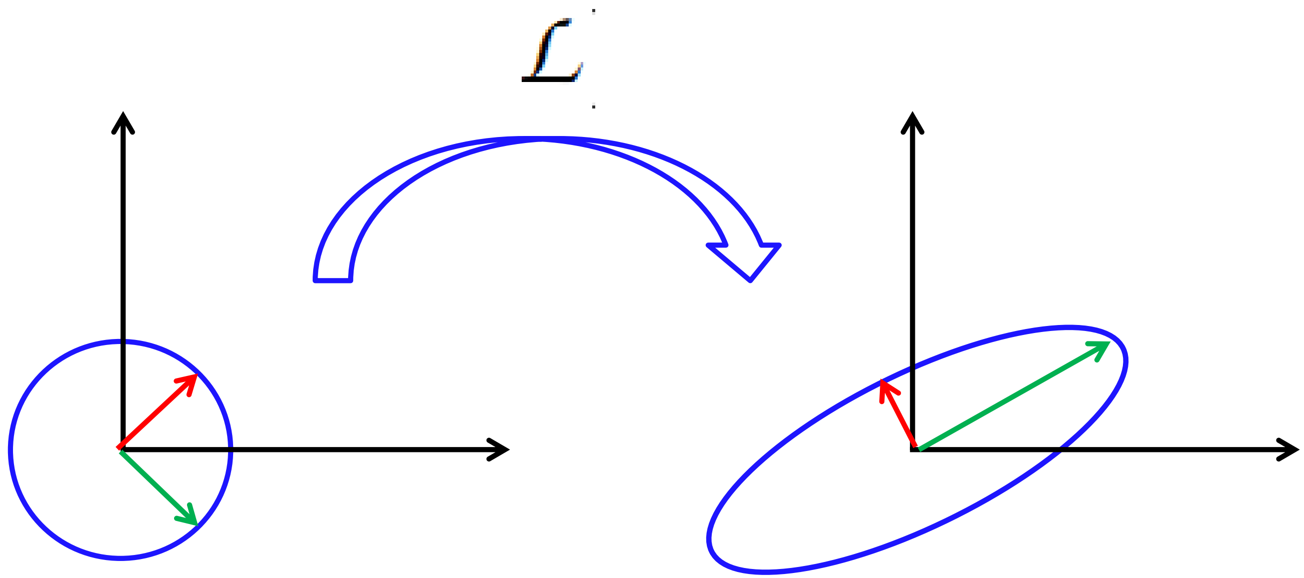
$$G(t) = \max_{q_0} \frac{||q||}{||q_0||}$$

$$= \max_{q_0} \frac{||\exp(t\mathcal{L})q_0||}{||q_0||}$$

$$= ||\exp(t\mathcal{L})||$$

How to compute a matrix 2-norm?

Optimal gain $$G(t) = \|\exp(t\mathcal{L})\|$$



unitary sphere

rotated ellipsoid

Optimal gain is associated to a norm

Kinetic energy written in v,η form

iαu=Dv-iβw and η=iβu-iαw

$$E(t) = \frac{1}{2k^2}\int_{\Omega}\left[|\mathcal{D}v|^2 + k^2|v|^2 + |\eta|^2\right]\, d\Omega$$

Optimal gain is associated to a norm

Kinetic energy written in v,η form

iαu=Dv-iβw and η=iβu-iαw

$$E(t) = \frac{1}{2k^2}\int_\Omega \left[|\mathcal{D}v|^2 + k^2|v|^2 + |\eta|^2\right]\, d\Omega$$

$$= \|q\|^2 = \frac{1}{2k^2}\int_\Omega \begin{pmatrix} v \\ \eta \end{pmatrix}^H \begin{pmatrix} -\mathcal{D}^2+k^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \eta \end{pmatrix}\, d\Omega$$

$$= \frac{1}{2k^2}\int_\Omega q^H M q\, d\Omega$$

energy matrix

Definition of optimal gain of a linear system

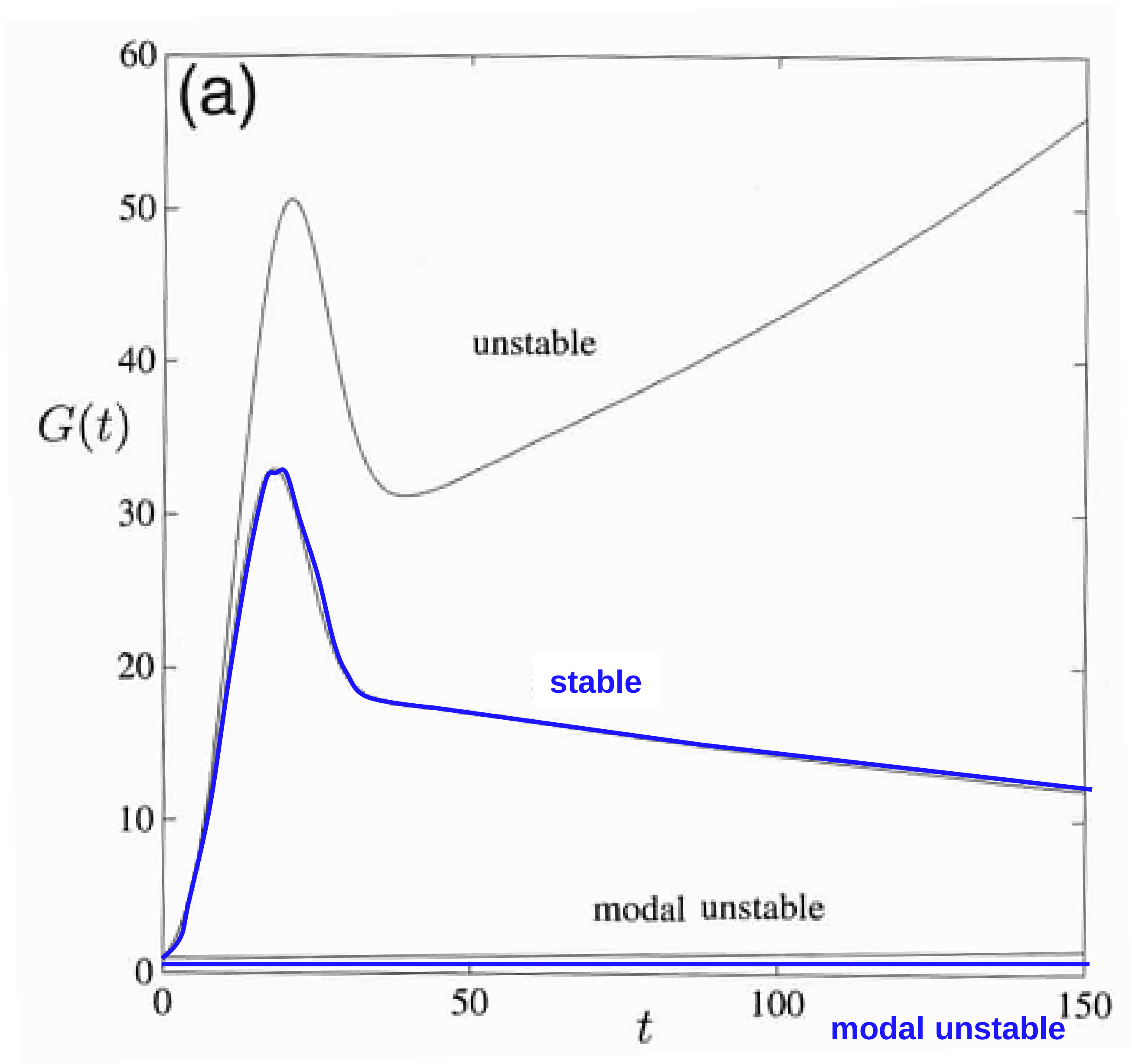
Cholevski dec. $M = F^H F$

$$||q||^2 = \frac{1}{2k^2}\int_\Omega q^H F^H F q \, d\Omega = \frac{1}{2k^2}\int_\Omega (Fq)^H F q \, d\Omega$$

Optimal gain

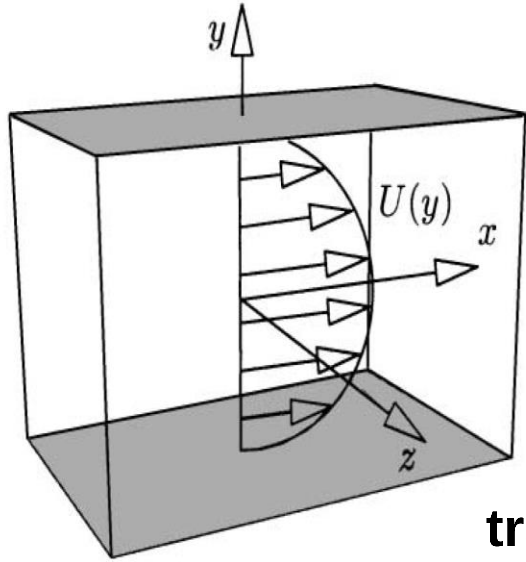
$$\begin{aligned} G(t) &= \max_{q_0} \frac{||q||_E}{||q_0||_E} = \max_{q_0} \frac{||Fq||_2}{||Fq_0||_2} = \max_{q_0} \frac{||F\exp(tL)q_0||_2}{||Fq_0||_2} \\ &= \max_{q_0} \frac{||F\exp(tL)F^{-1}Fq_0||_2}{||Fq_0||_2} = ||F\exp(tL)F^{-1}||_2 \end{aligned}$$

$= q'_0$


(a)
60
50
40
30
20
10
0
$G(t)$
unstable
stable
modal unstable
0
50
100
150
t
modal unstable

Plane Poiseuille flow

normal

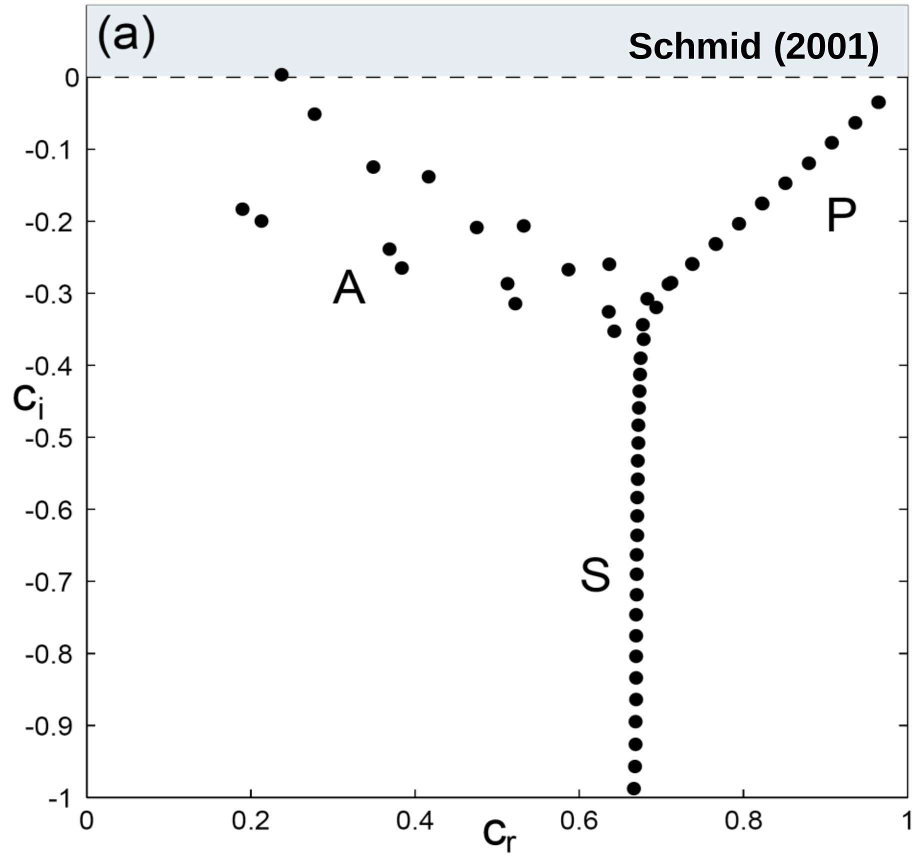


longitudinal, $e^{i\alpha x}$

transverse, $e^{i\beta z}$

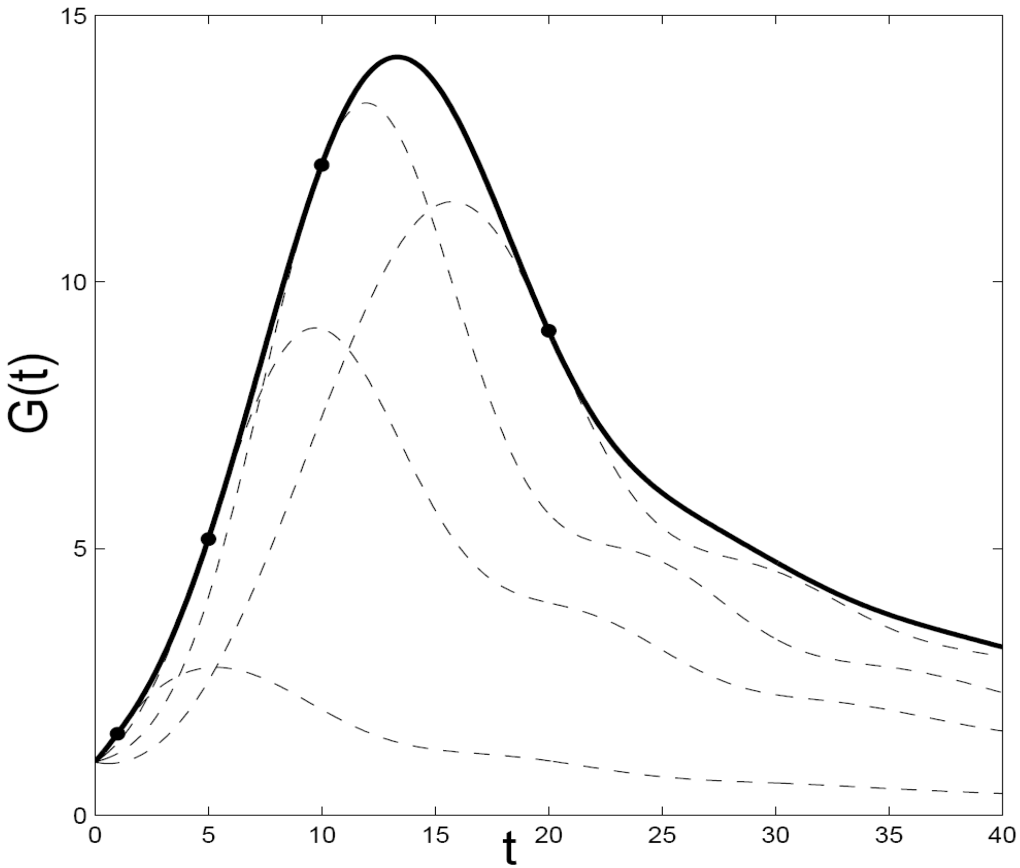
Linearly stable until Re=5772, but the transition is observed experimentally close to Re=1000-2000!

Plane Poiseuille flow - stability



Spectrum for plane Poiseuille flow for $\alpha = 1, \beta = 0, Re = 10000$.

Orr mechanism (2D)

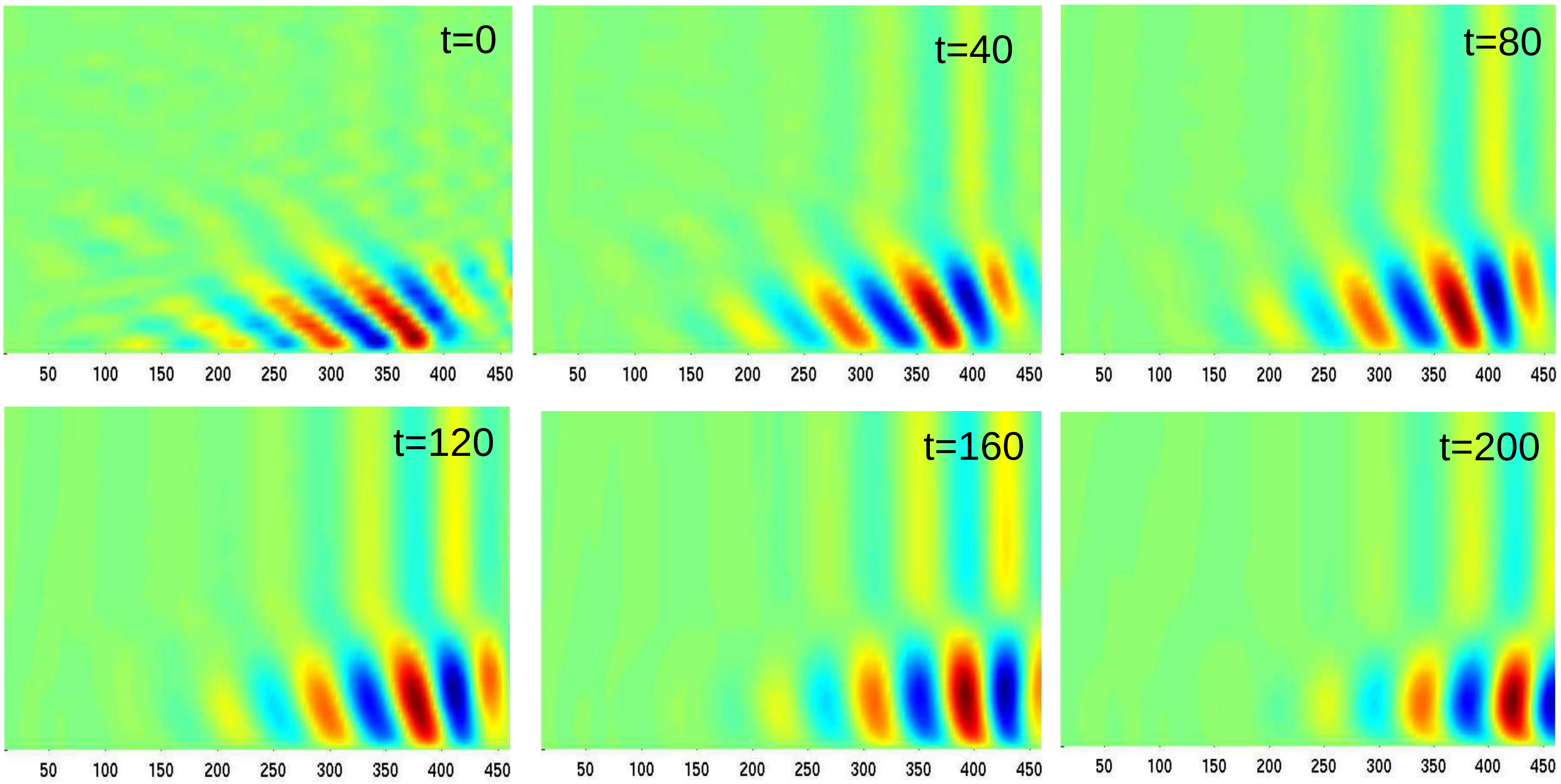


Amplification $G(t)$ for Poiseuille flow with $\mathrm{Re} = 1000, \alpha = 1$ (solid line)

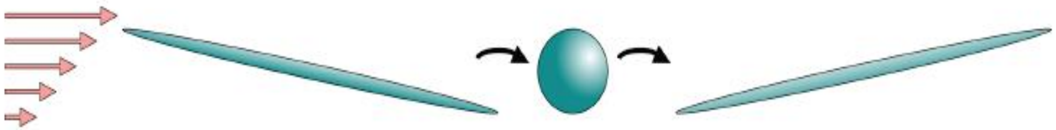
« By-pass transition »

Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

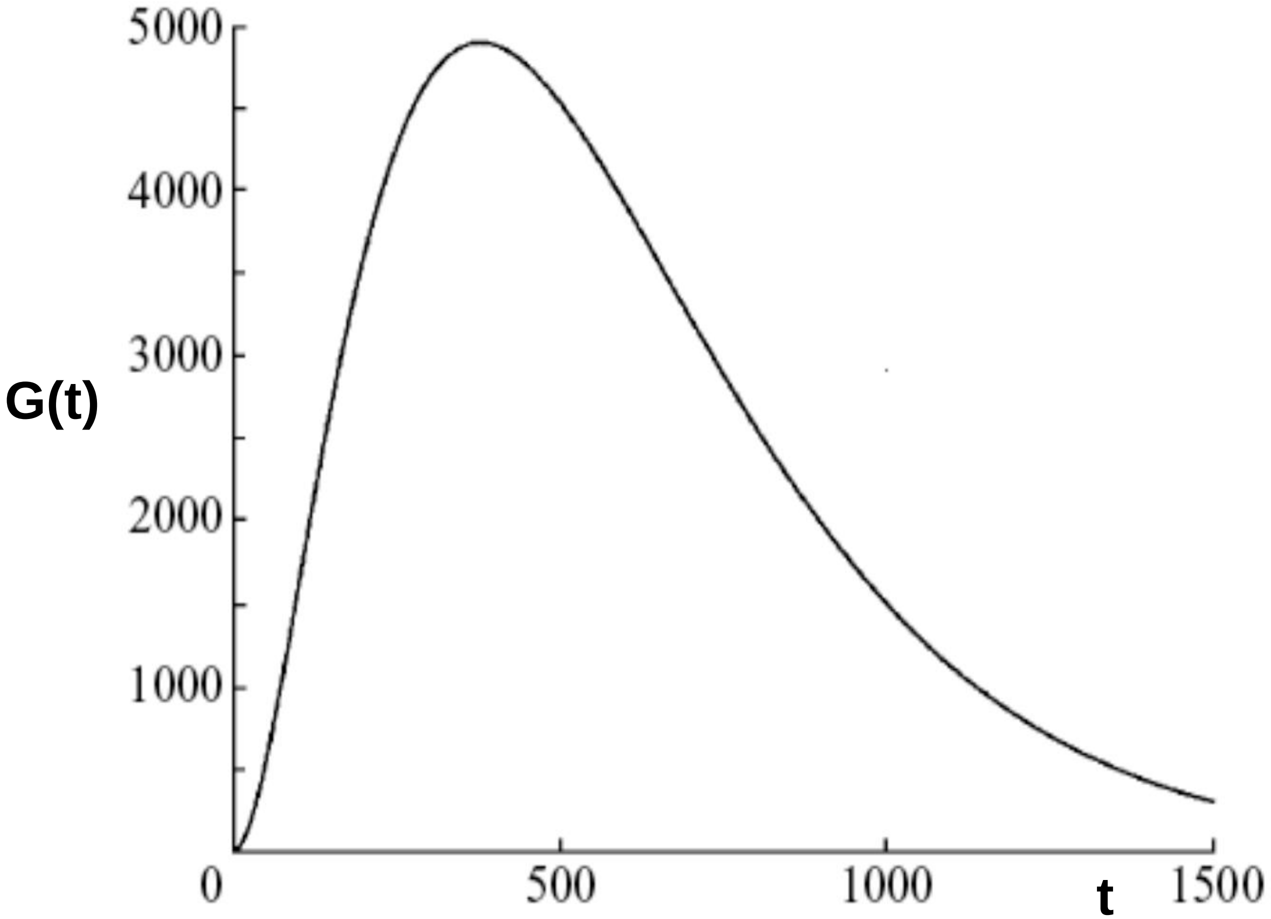
Optimal perturbation (here in boundary layer)



Orr mechanism



Plane Poiseuille flow
Lift-up mechanism (3D)



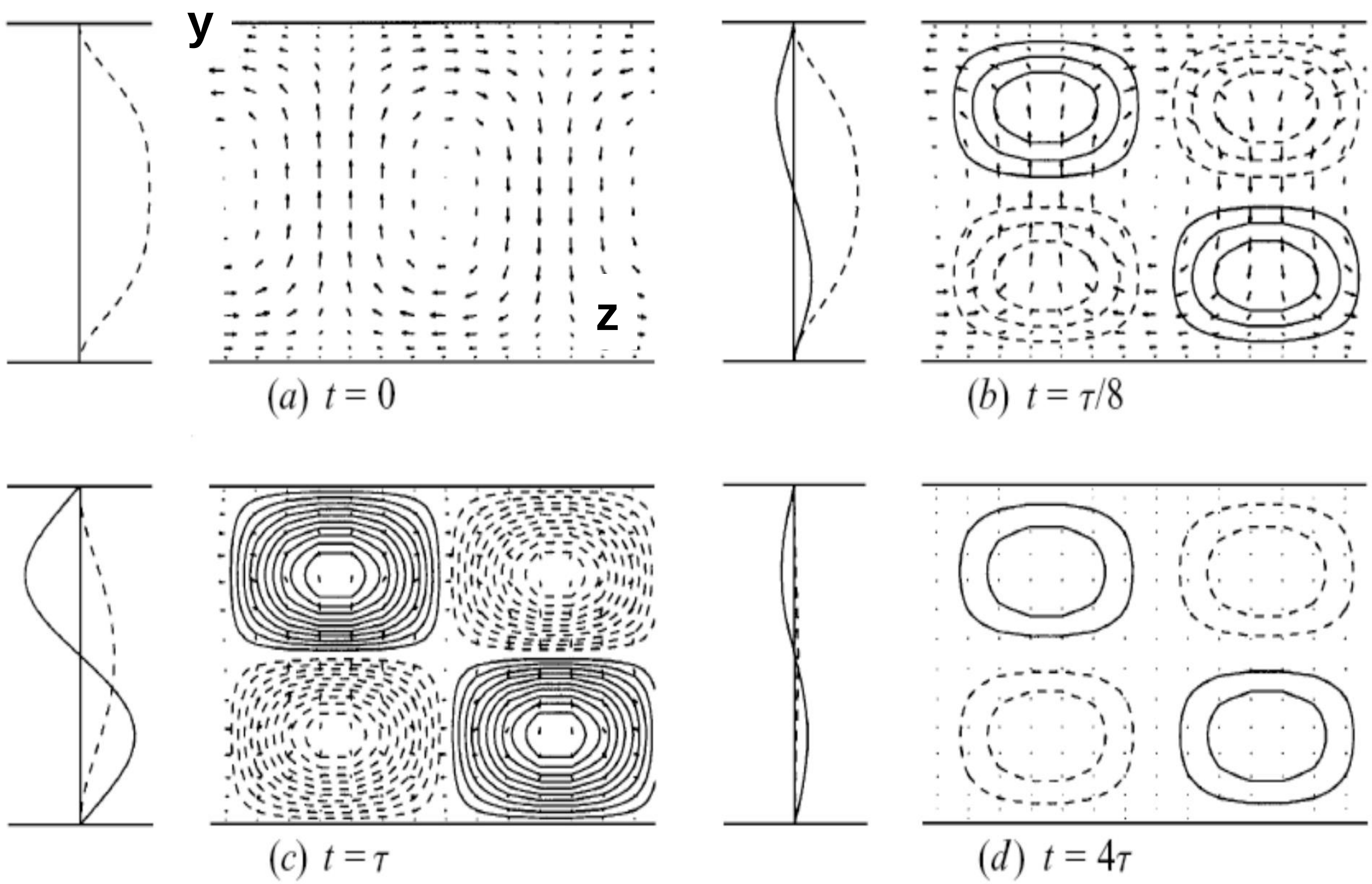
Re=5000, α=0, β=1

« By-pass transition »

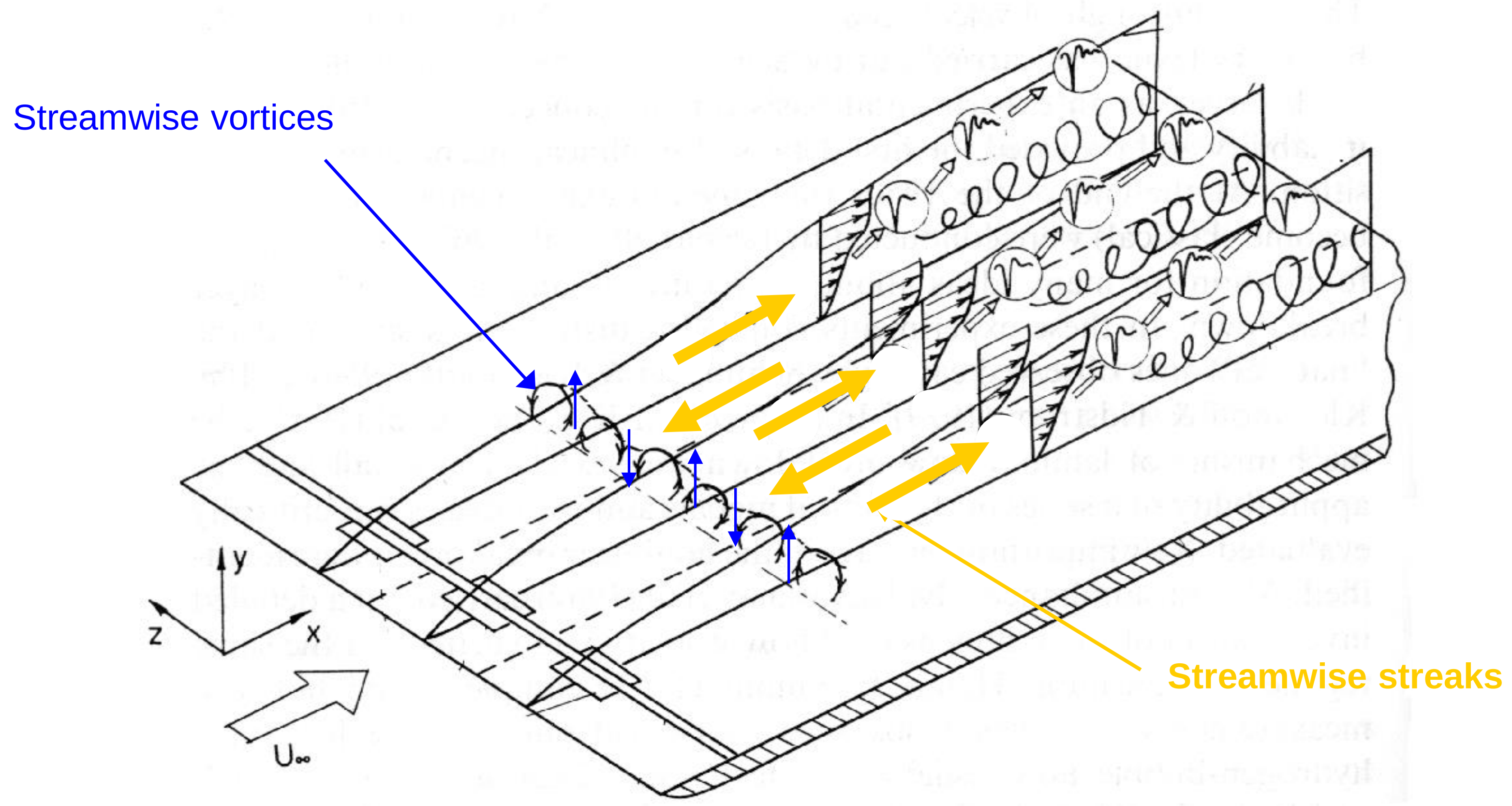
Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

Lift-up mechanisms

Optimal transformation of vortices into streaks



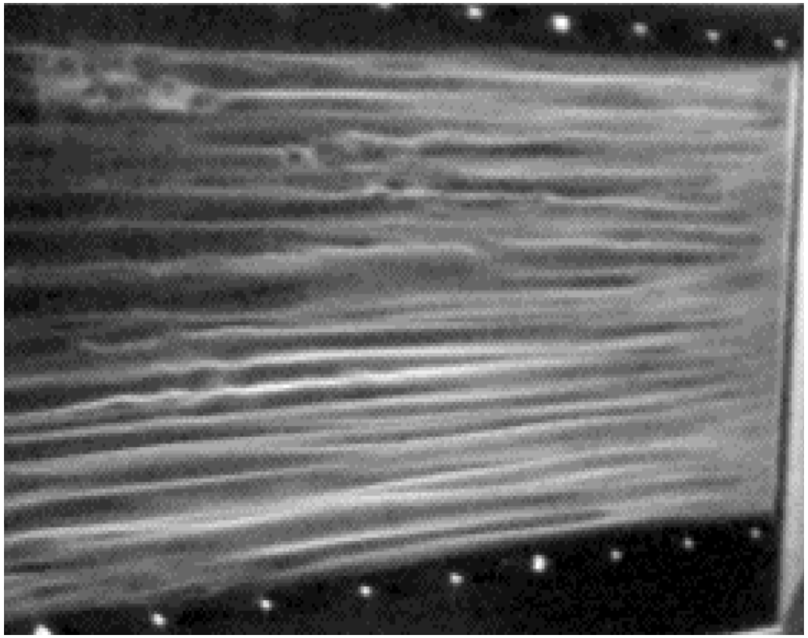
Lift-up mechanism



Schmid & Henningson.(2001)

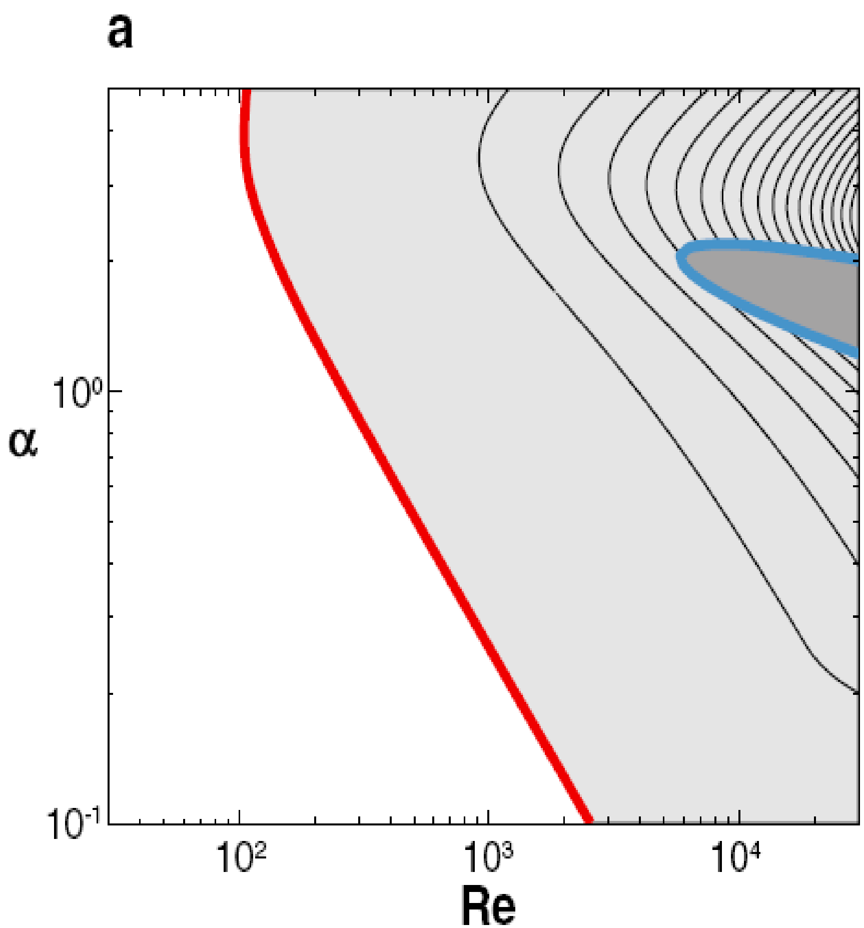
Lift-up mechanism

Optimal transformation of vortices into streaks



Alfredson & Matsubara (1996), streaky structures in the boudary layer

Isolines of maximum transient amplification
β=0



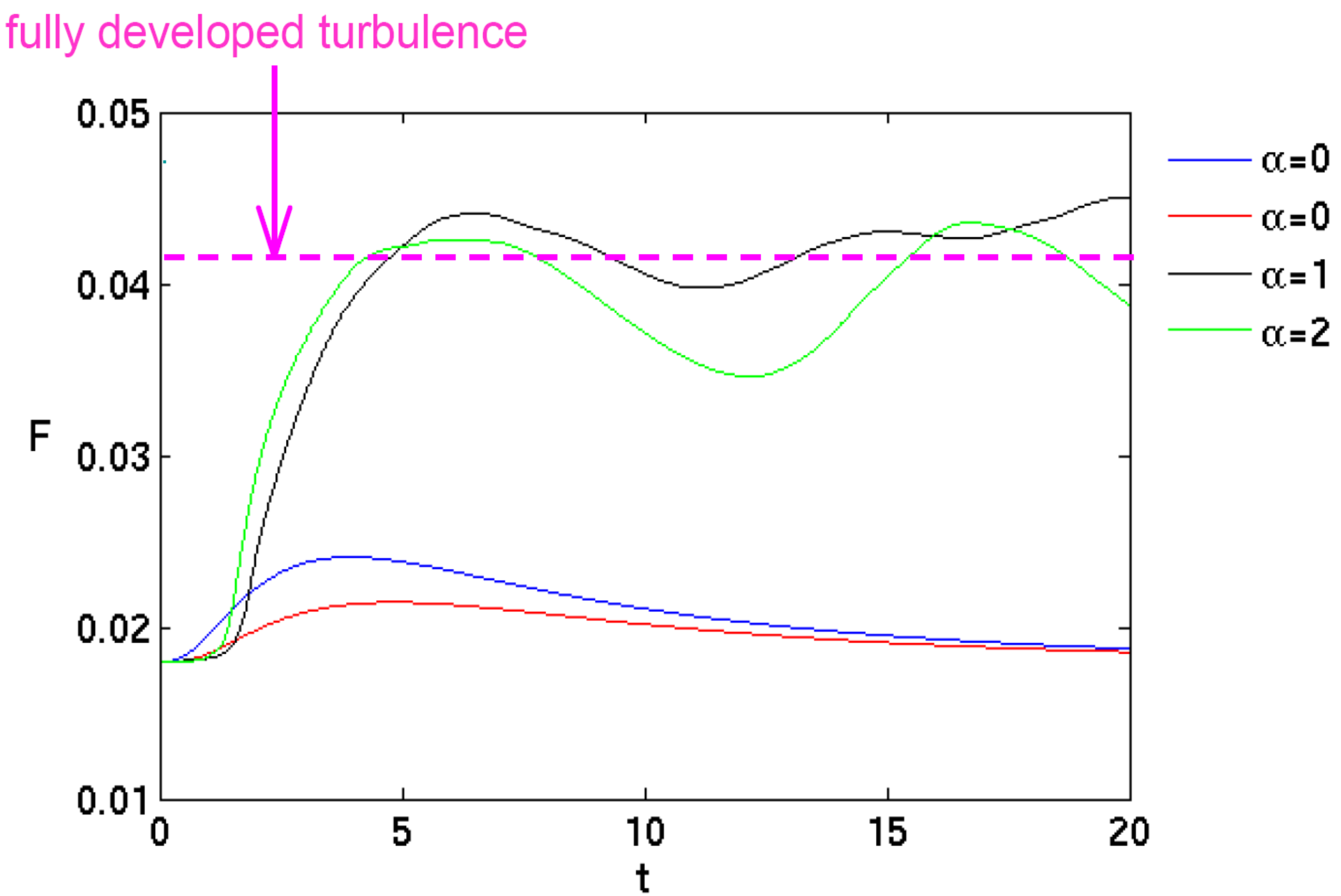
« By-pass transition »

Trefethen et al (1993), Buttler & Farrell (1993) , Schmid & Henningson (2001)

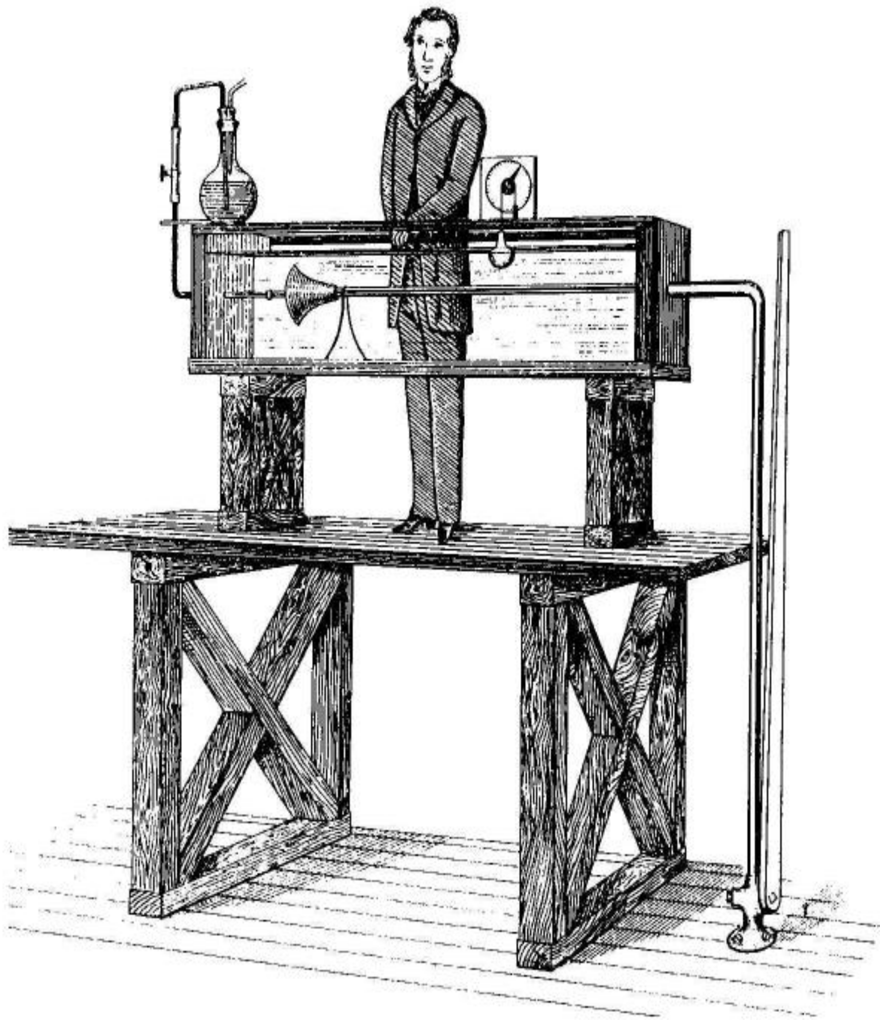
	$G_{\max}$ (10^{-3})	t_{max}	α	β
Plane Poiseuille	0.20 Re^2	0.076 Re	0	2.04
Couette	1.18 Re^2	0.117 Re	35/Re	1.6
Pipe	0.07 Re^2	0.048 Re	0	1
Boundary layer	1.50 Re^2	0.778 Re	0	0.65

Lift up mechanism in a square duct in the stable regime

Bye-pass transition to turbulence

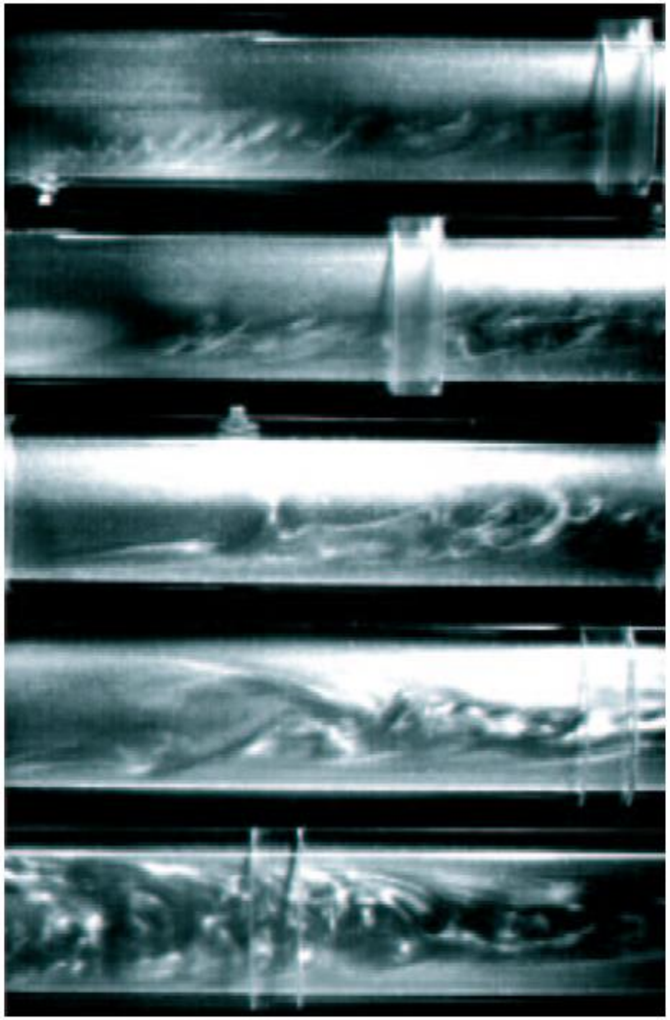


Bottaro et al. (2008)



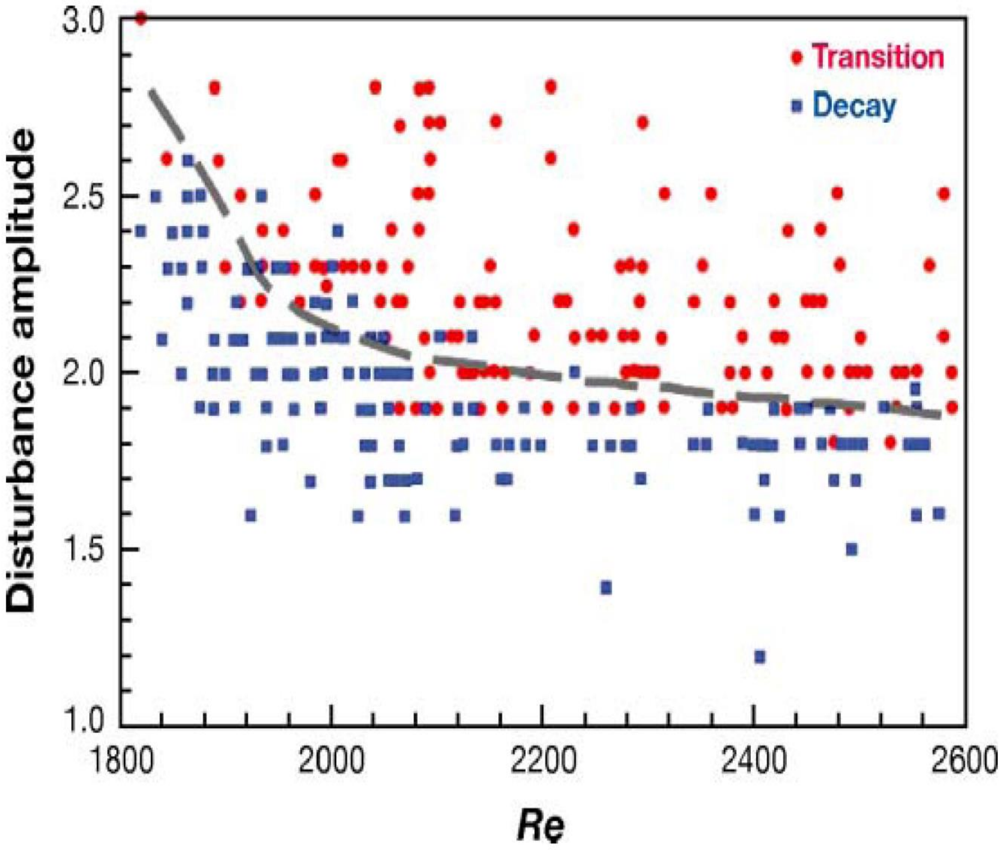
5.1 – *Osborne Reynolds en 1883 derrière son expérience à Manchester.*

Transition in Cylindrical Pipe Poiseuille flow



Mullin (2008)

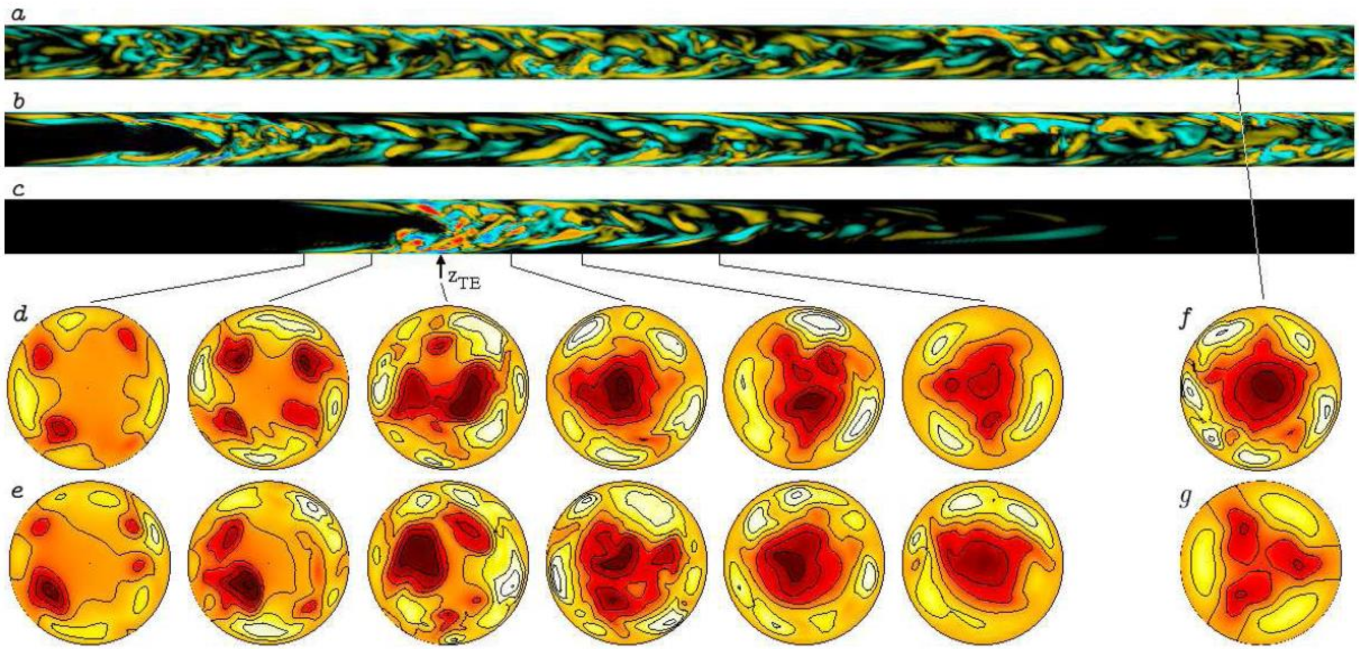
Transition in Cylindrical Pipe Poiseuille flow



Mullin (2008)

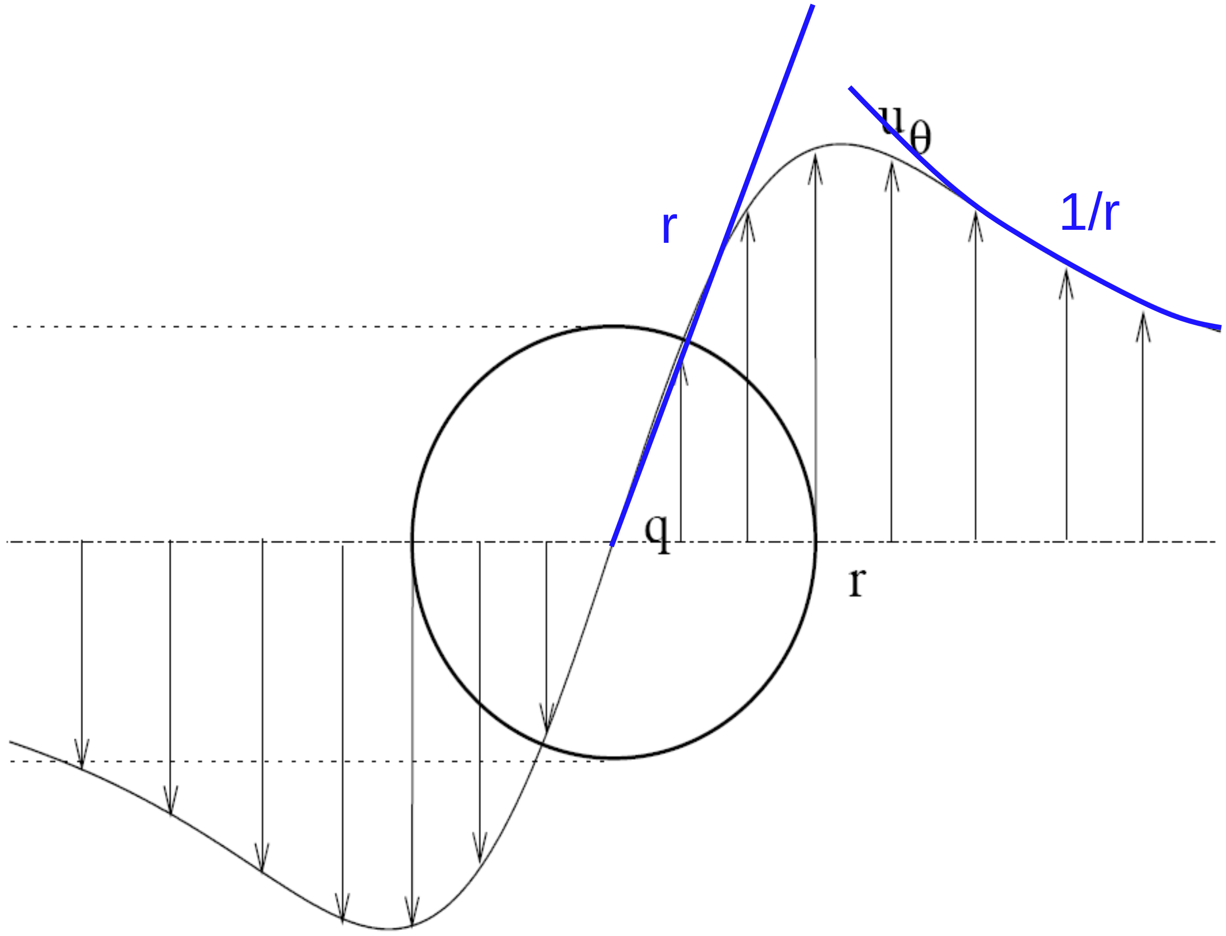
Transition in Cylindrical Pipe Poiseuille flow

Numerical study



Kerswell (2008)

Tourbillon de Lamb-Oseen



Stable lineairement!

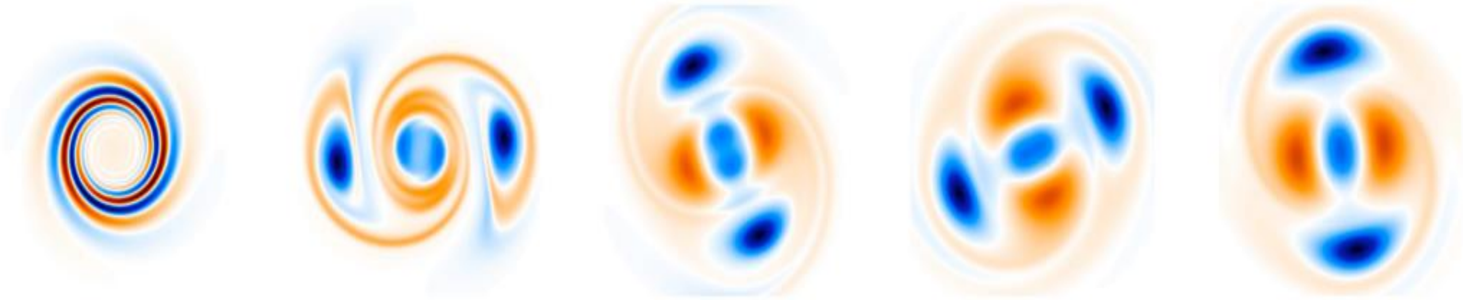
TOWARDS AN ELLIPTICAL STATE

Nonlinear evolution of the optimal perturbation with initial amplitude below a given threshold...

Re = 1000

TOWARDS AN ELLIPTICAL STATE

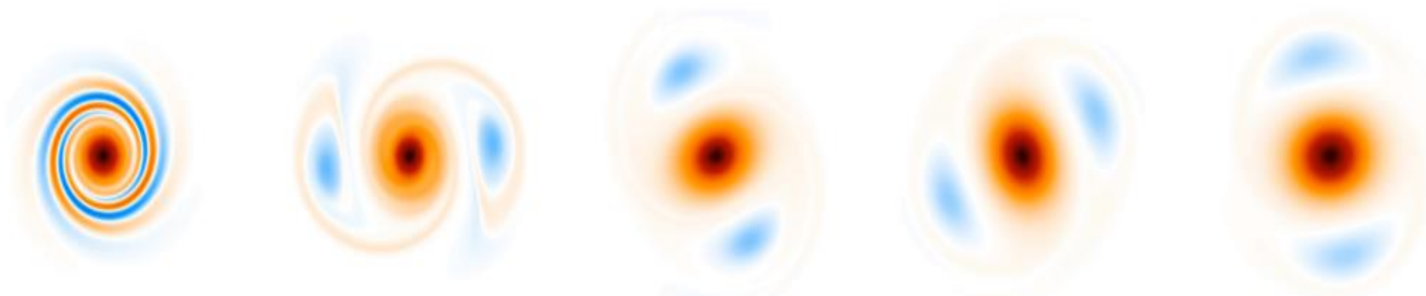
... and now ***ABOVE*** *the threshold*



Re = 1000

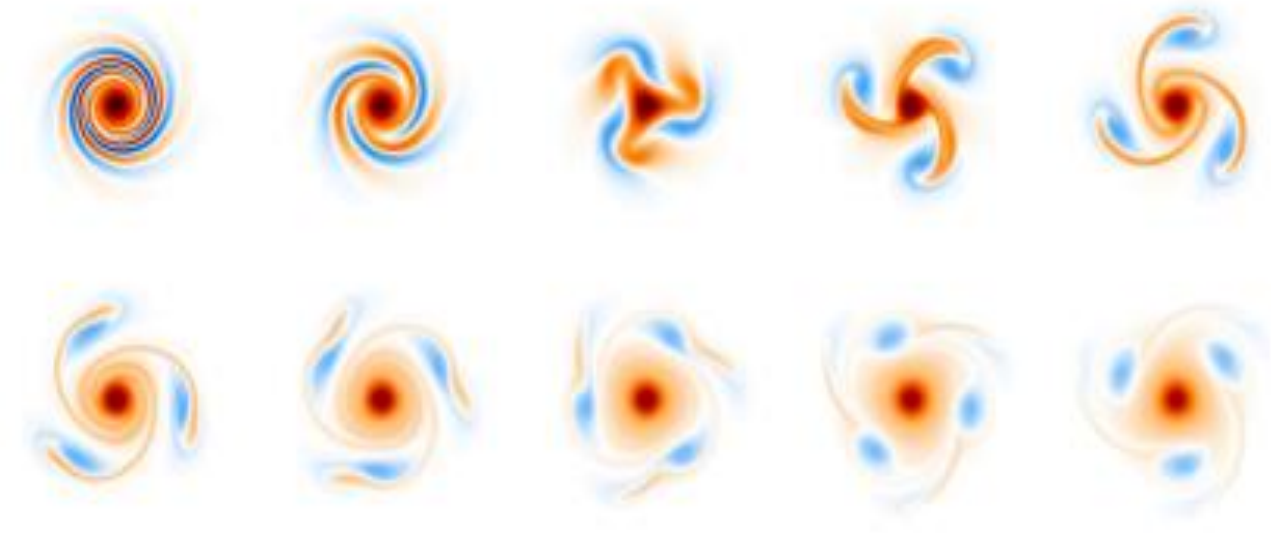
TOWARDS AN ELLIPTICAL STATE

Reconstructed flow

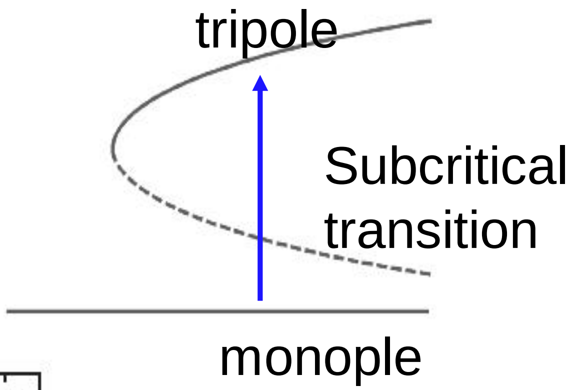


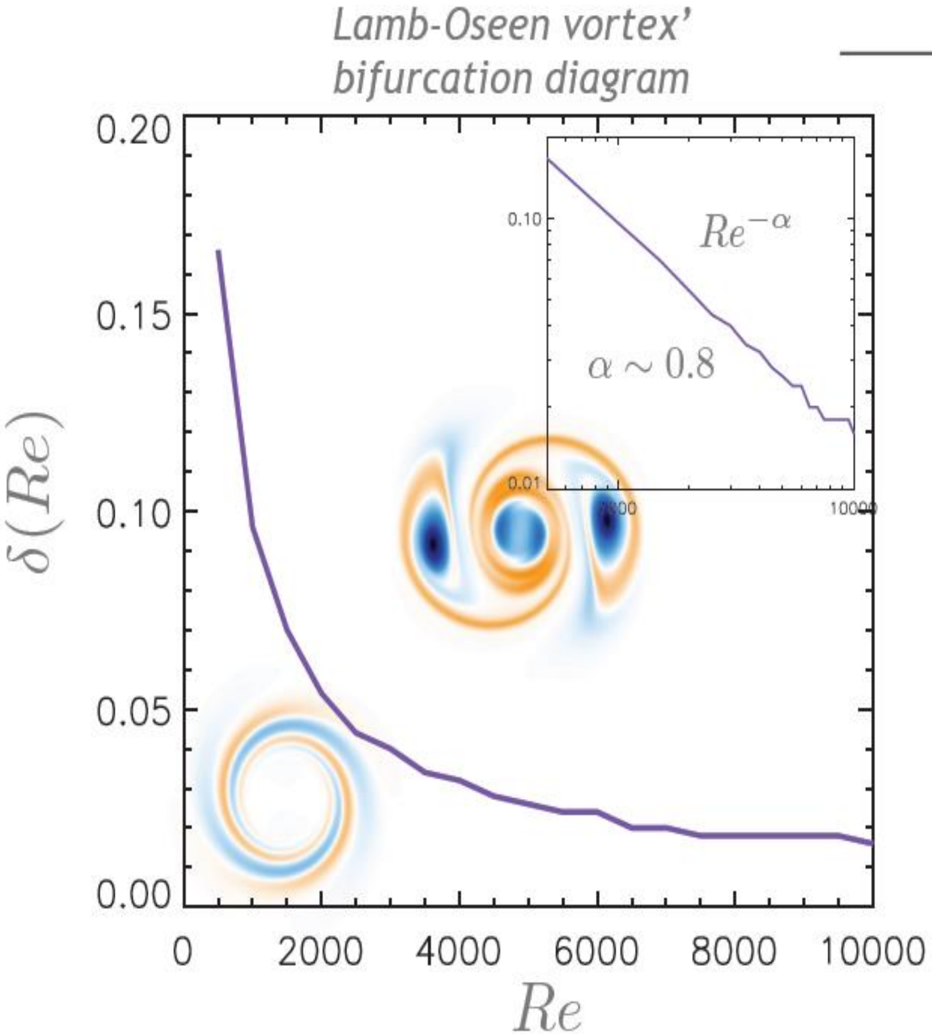
Re = 1000

Tripolar vortices?

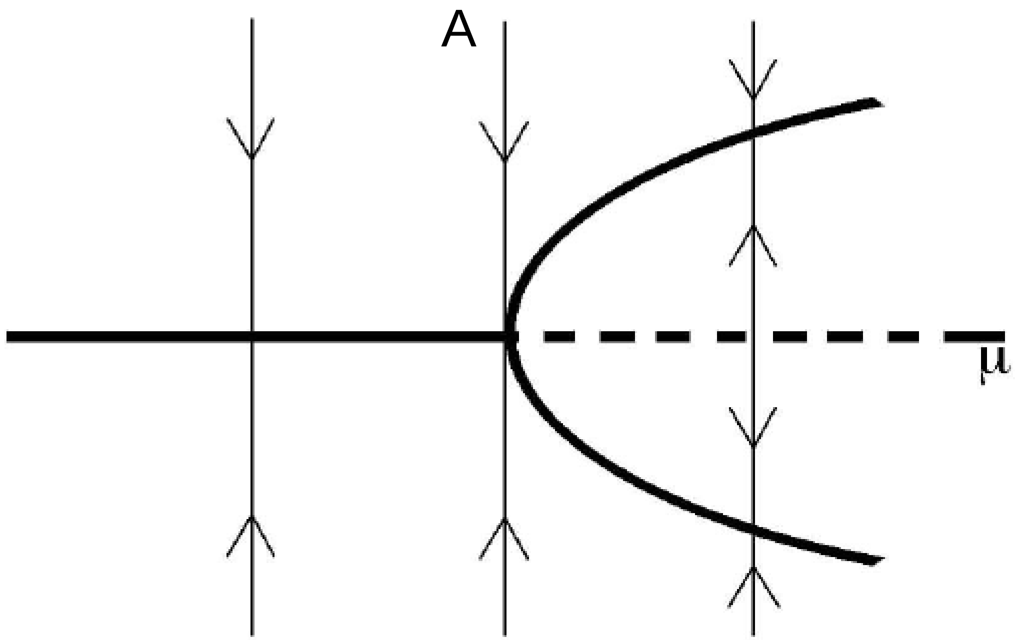


BASIN OF ATTRACTION'S SHRINKAGE



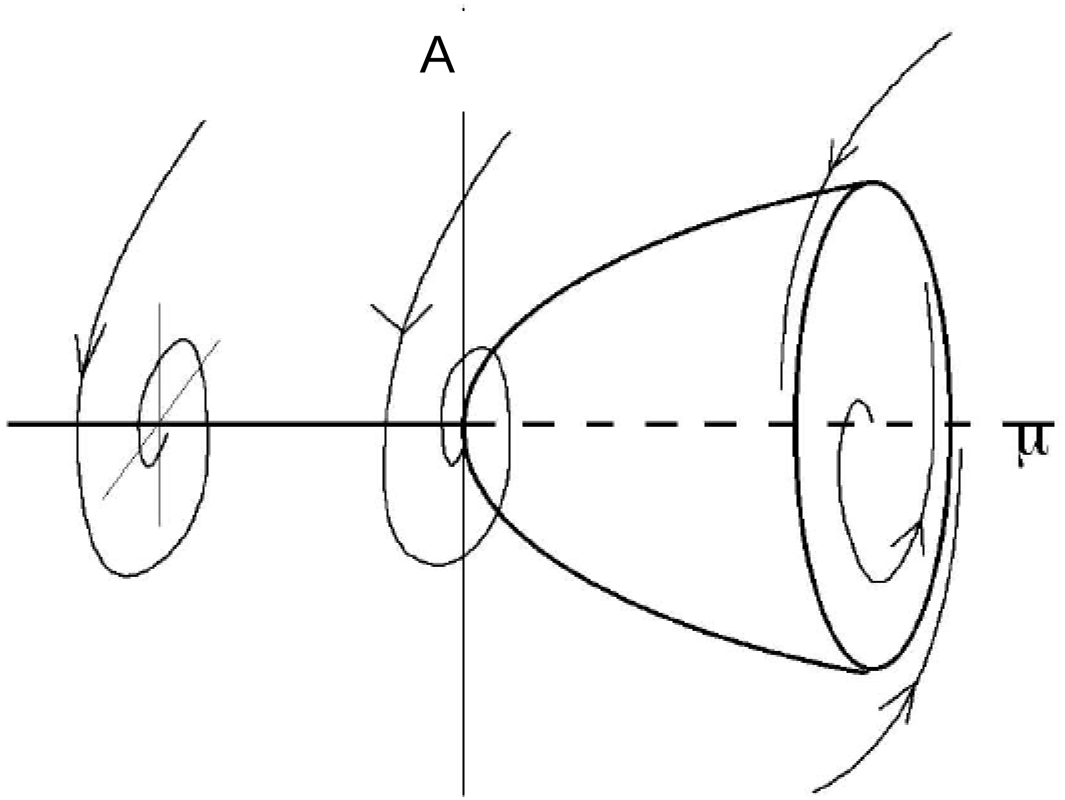


What about nonlinearities?



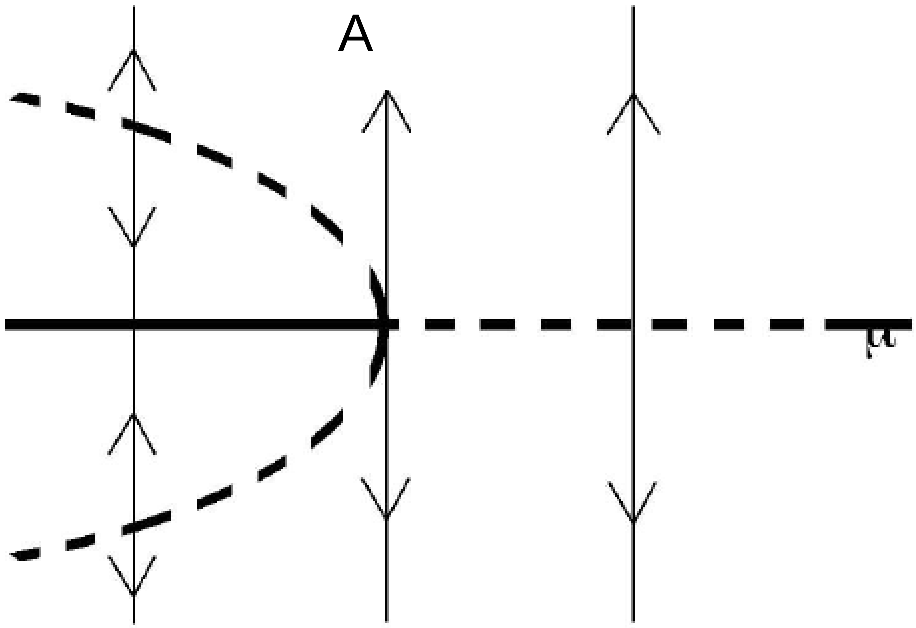
Supercritical fork bifurcation

What about nonlinearities?



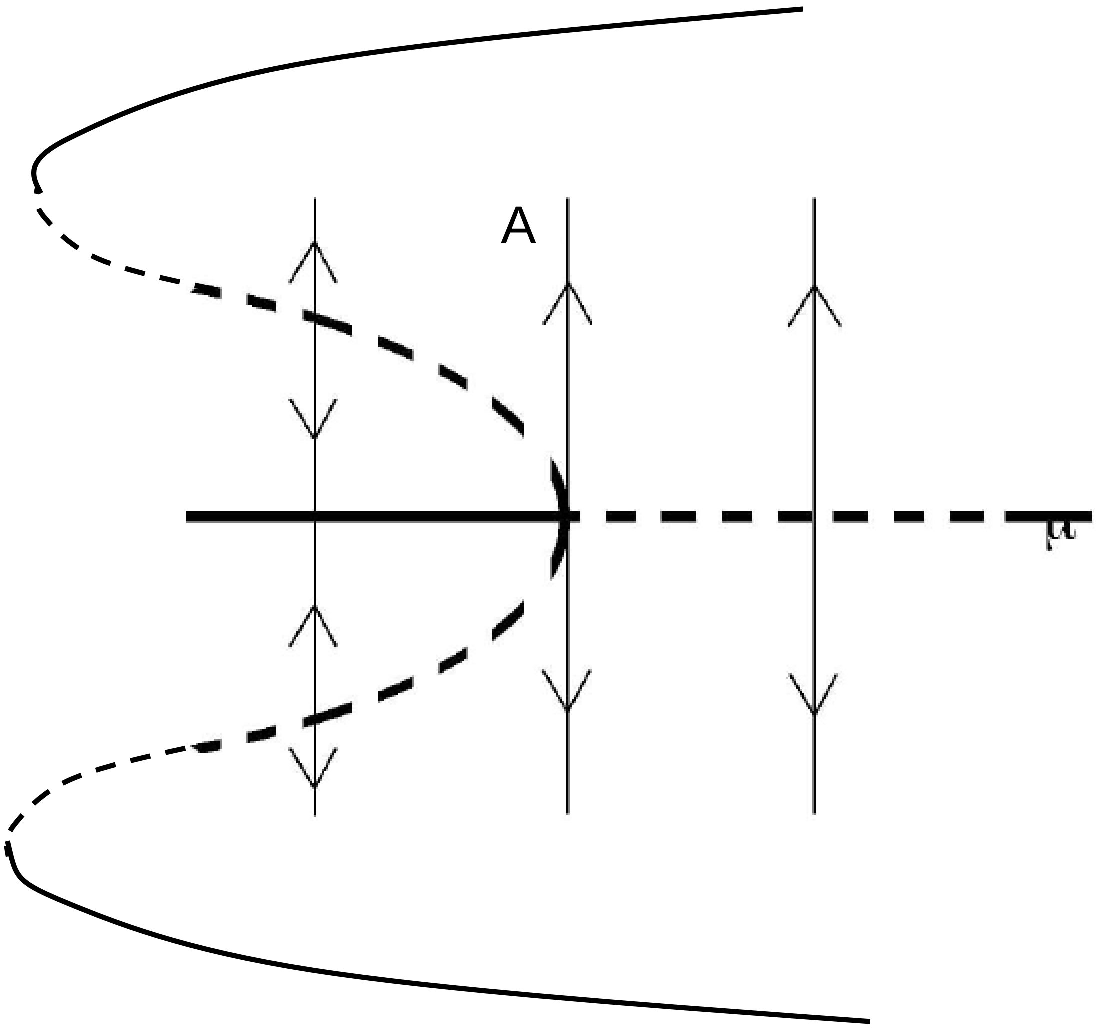
Hopf bifurcation

What about nonlinearities?



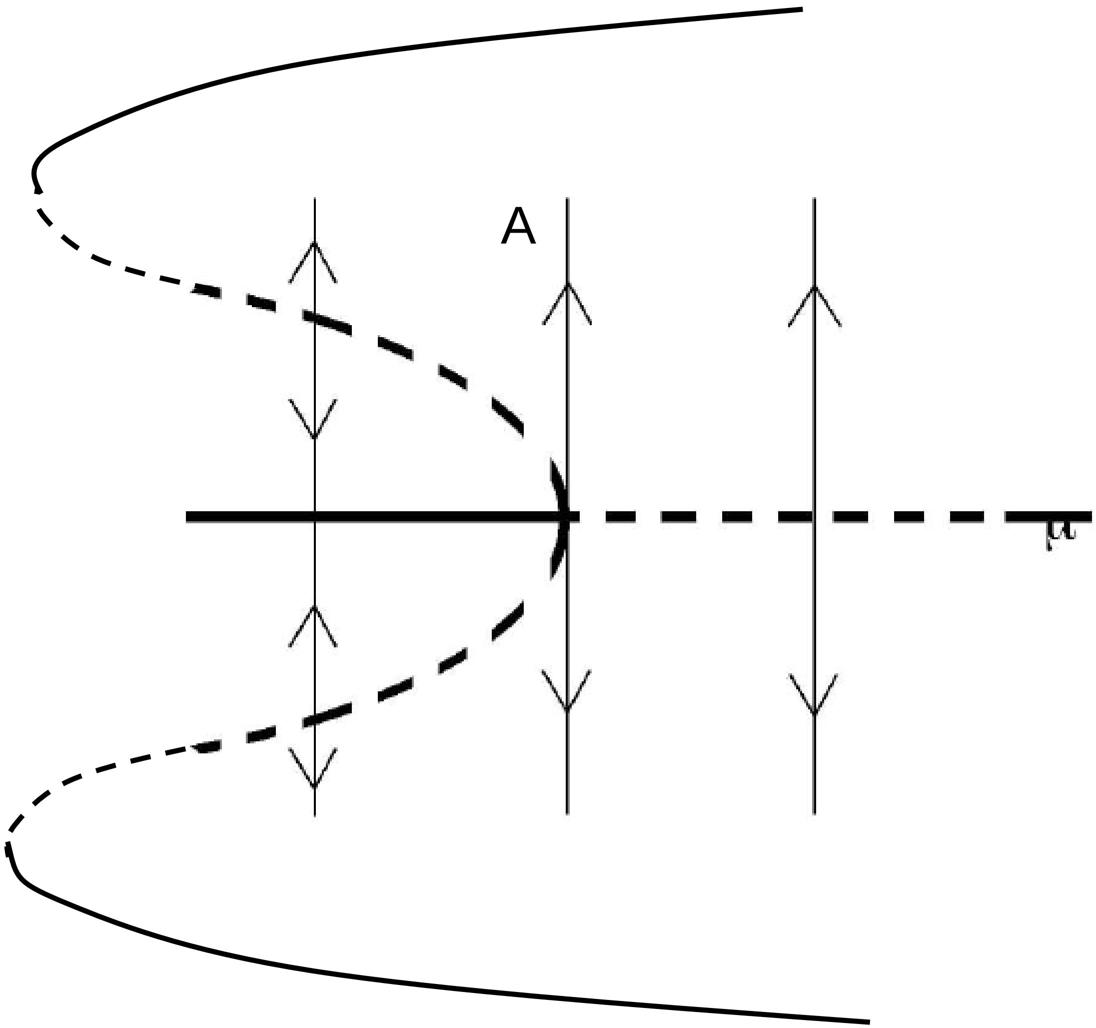
Subcritical fork bifurcation

What about nonlinearities?



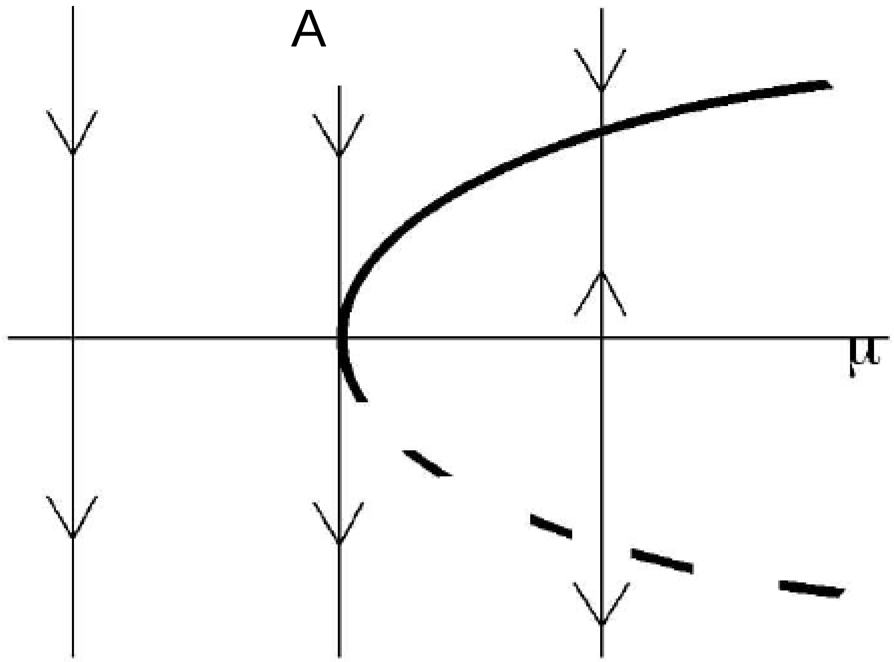
Subcritical bifurcation

What about nonlinearities?



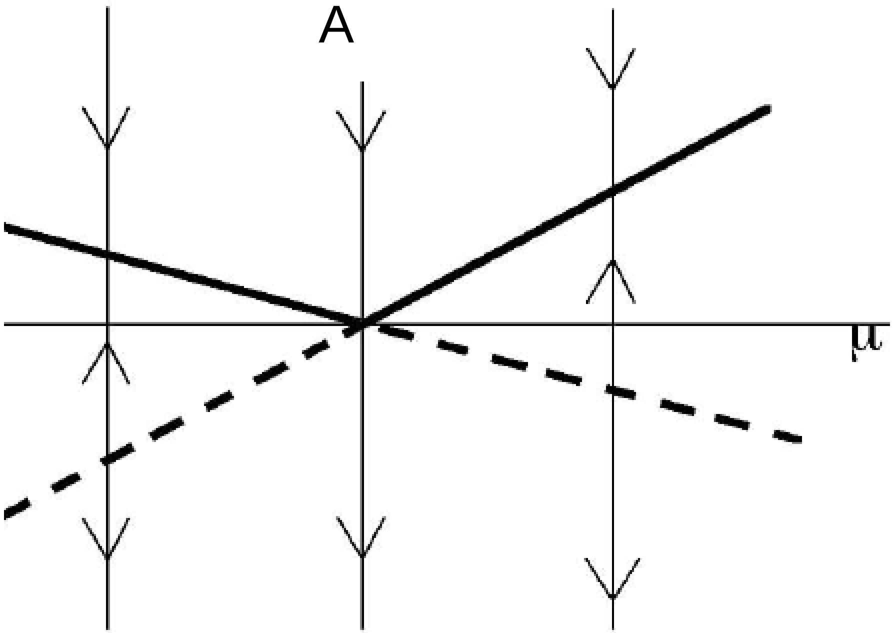
Hysteresis cycle!

What about nonlinearities?



Saddle Node bifurcation

What about nonlinearities?



Transcritical bifurcation