Most flows are unstable...

1. Intro-definitions
2. Rayleigh-Taylor
3. Rayleigh Plateau (destabilization through surface tension)
4. Rayleigh-Benard (convection)
5. Benard-Marangoni
6. Taylor Couette-Centrifugal instability
7. Kelvin-Helmholtz
8. Inflection point theorem Rayleigh
9. Orr sommerfeld, transient growth
10. Spatial growth
11. Spatio-temporal growth

Which of these waves are unstable?

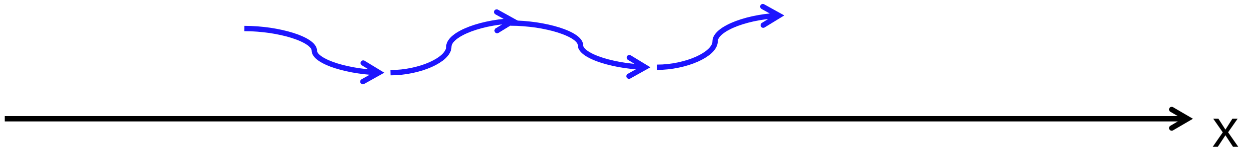
$Im(k)<0$?

$Im(k)>0$?

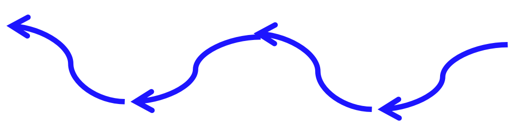
Recall : $\exp(i(kx-\omega t))$

The stability of a spatial wave can be only determined if one knows in which direction it propagates!

k^+ waves propagate towards positive x

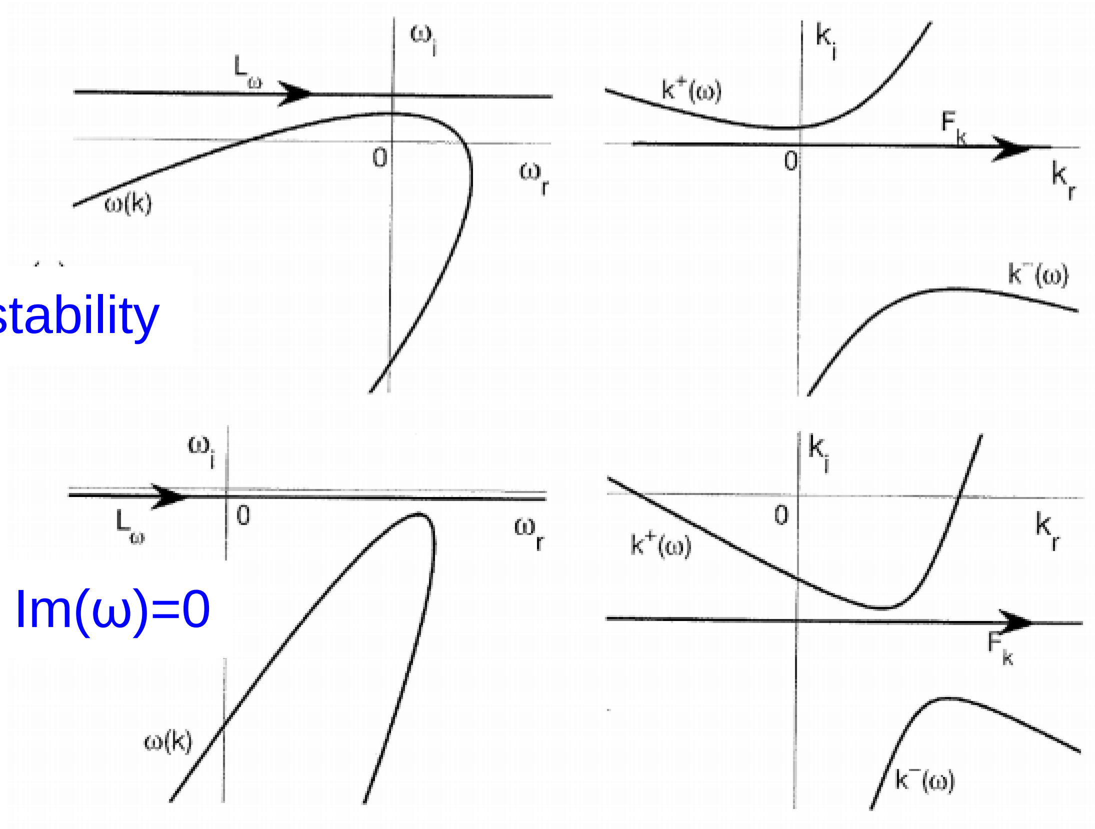


k^- waves propagate towards negative x



However, determining this direction of propagation is particularly difficult, except in the case of a temporally stable flow.

The addition of a positive imaginary offset to the frequency makes the temporal problem stable!
This separates the spatial waves into k+ and k- waves.



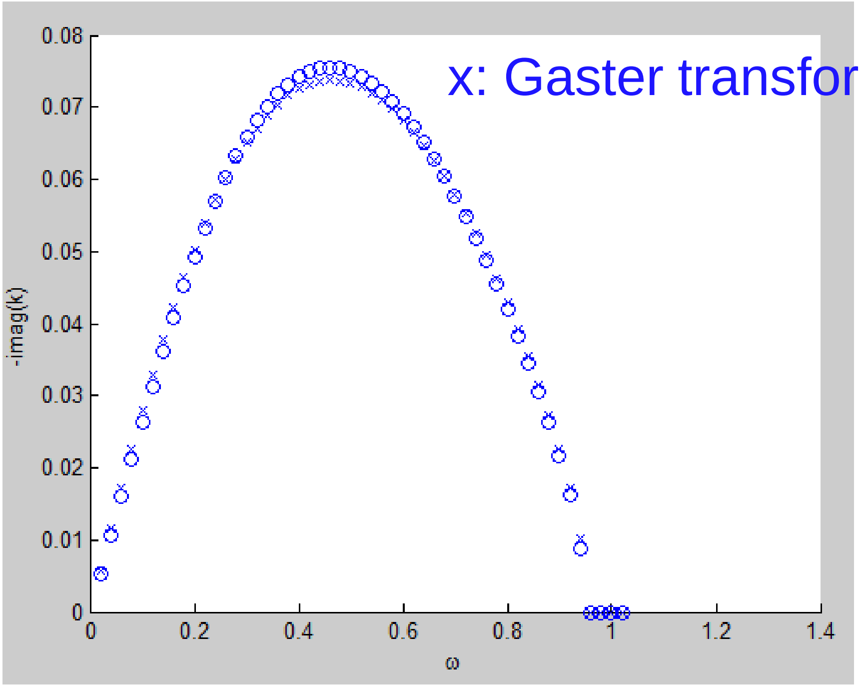
offset spatial stability

spatial stability: Im(ω)=0

The branches are then followed by continuity

Validity of Gaster transformation?

R=0.4

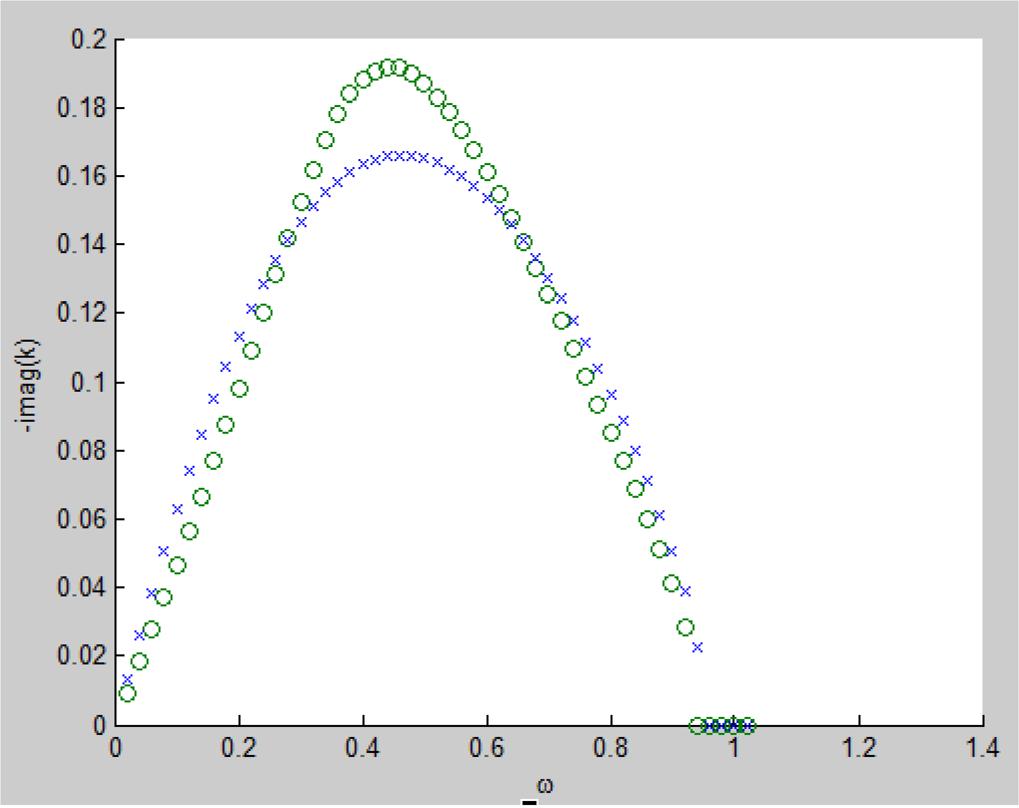


$$R \ll 1$$

$$-k_i(\omega, R) \sim R\omega_{1,i}(\omega)$$

Validity of Gaster transformation?

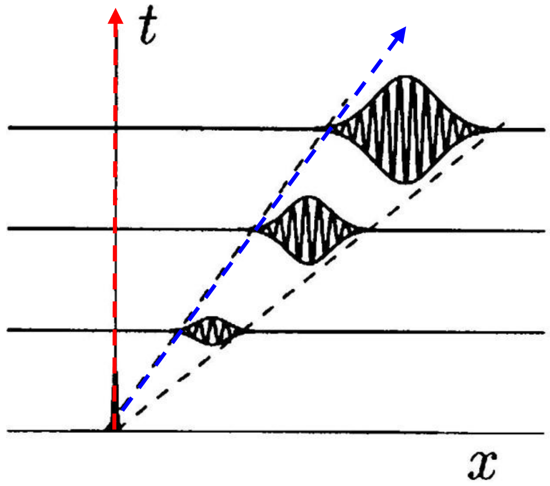
R=0.9 x: Gaster transformation



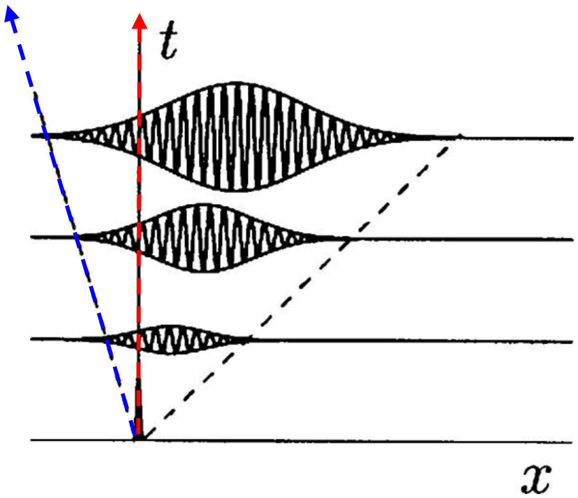
$$-k_i(\omega, R) \sim R\omega_{1,i}(\omega)$$

Spatio-temporal instability theory

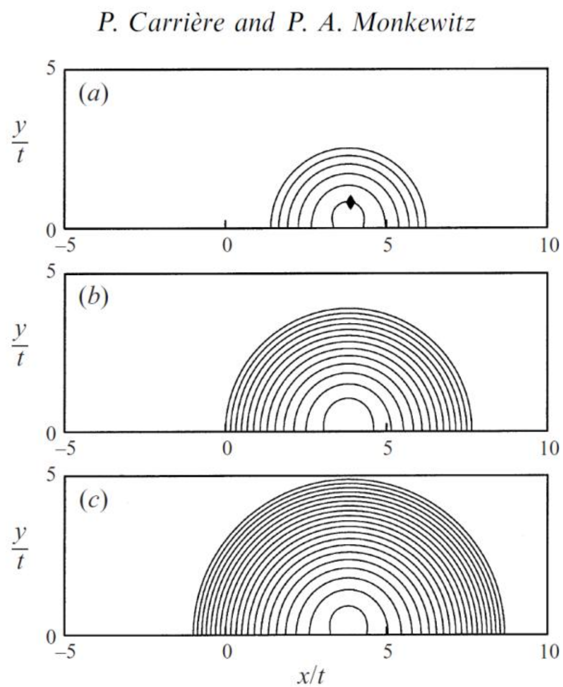
☞ Convective instability



F Absolute instability



A/C analysis in Rayleigh-Benard Poiseuille Carrière and Monkewitz (1999)



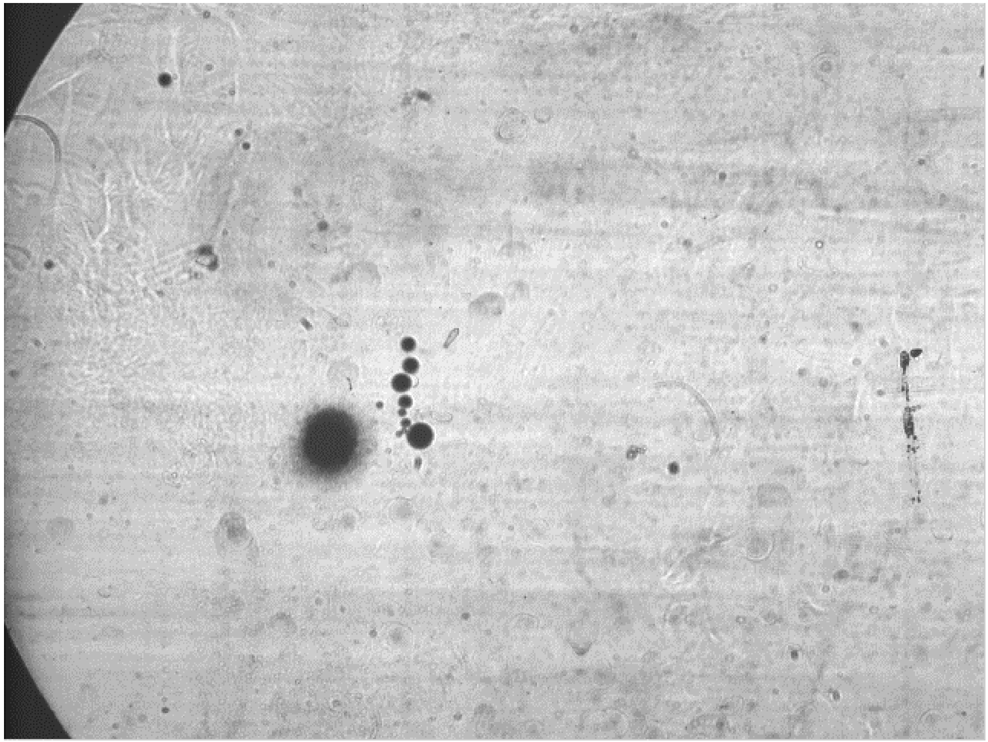
Ra=1760

Ra=1828

Ra=1890

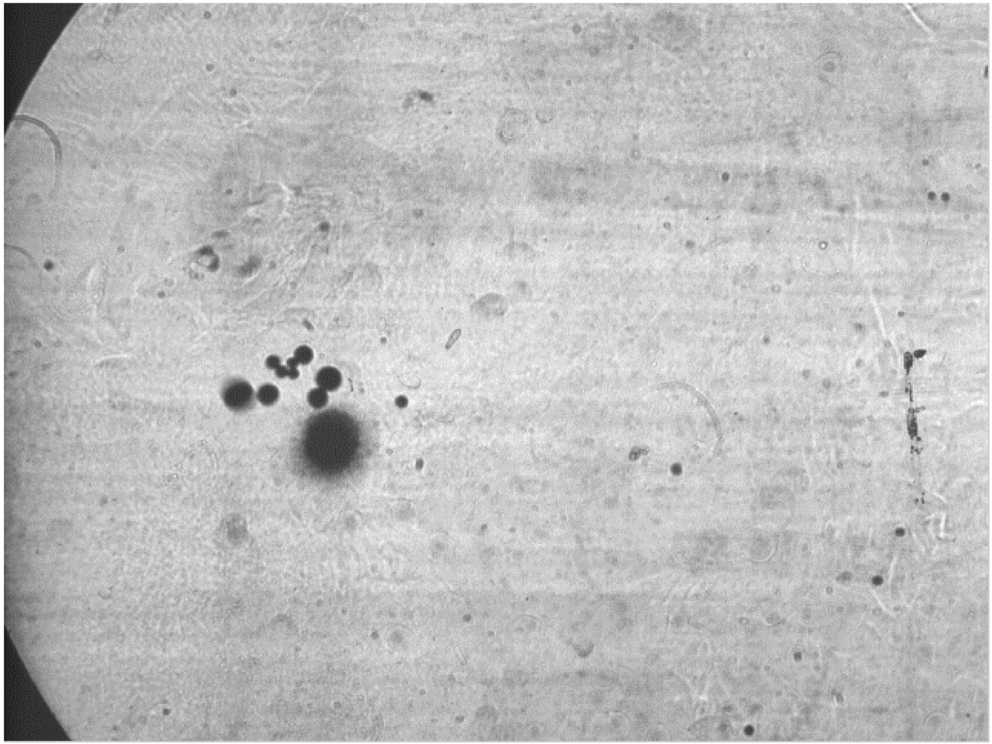
Re=0.63

Impulse in Rayleigh-Benard Poiseuille Grandjean and Monkewitz (2009)



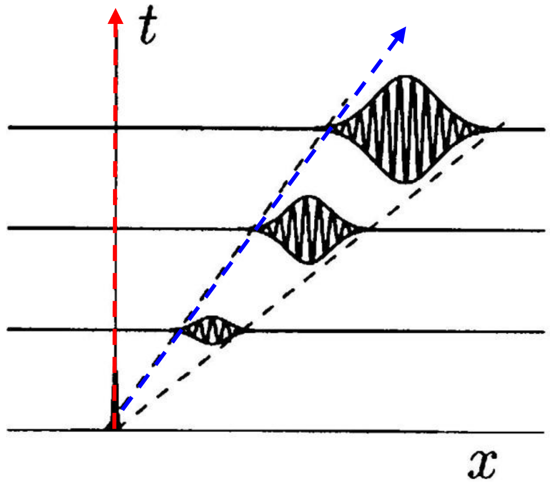
Ra>1707, Re=0.03

Impulse in Rayleigh-Benard Poiseuille
Grandjean and Monkewitz (2009)

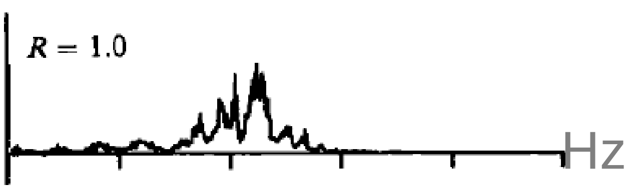


Ra>1707, Re=0.003

☞ Convective instability

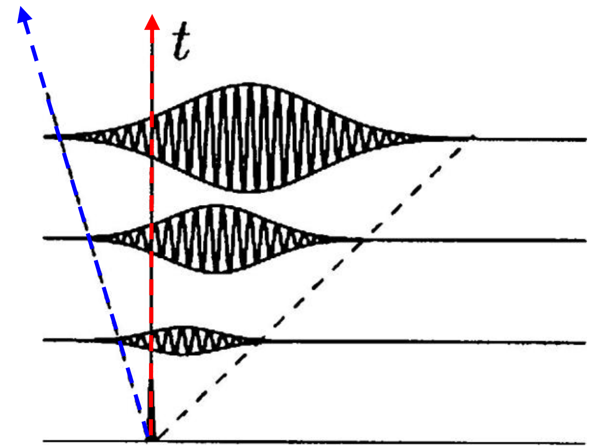


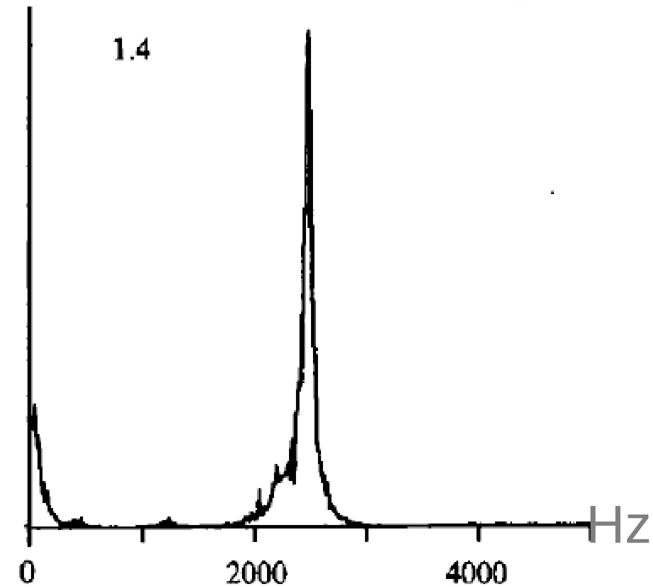
Spectrum



☞ amplifier

F Absolute instability



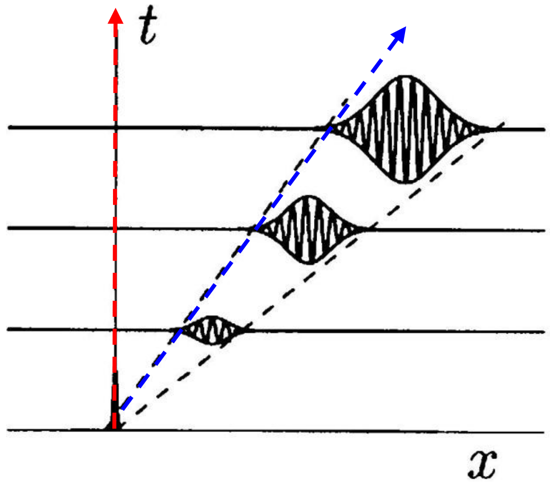


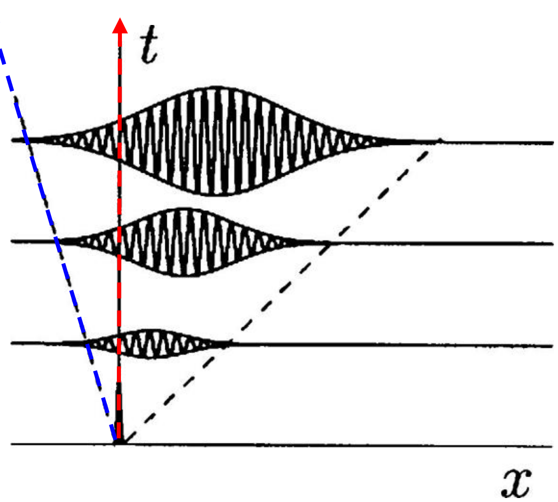
☞ oscillator

Mixing layer experiments by Strikowsky and Niccum (1991)

Spatio-temporal instability theory

☞Convective instability   F Absolute instability





We need to generalize the concept of group velocity since ω (and why not k) is complex

For neutral waves, the group velocity is $d\omega/dk$
Here this quantity is the derivative of a complex function with respect to a complex variable. Cauchy-Rieman conditions apply.

Spatio-temporal spectral analysis

Inverse Fourier Transform

$$u(x,t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{u}\,(k,\omega)\, e^{i(kx-\omega t)}\, dk\, d\omega$$

$$\hat{u}(k,\omega) = (2\pi)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u\,(x,t)\, e^{-i(kx-\omega t)}\, dx\, dt$$

Direct Fourier Transform

Spatio-temporal spectral analysis

Inverse Fourier Transform

$$u(x,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{u}\,(k,\omega)\, e^{i(kx-\omega t)}\, dk\, d\omega$$

Use dispersion relation ω(k)!

Spectral analysis

Fourier transform:

$$u(x,t) = \frac{1}{2}\int_0^{+\infty} \hat{u}(k)\, \mathrm{e}^{\mathrm{i}(kx-\omega(k)t)}\mathrm{d}k + c.c.$$

Carrier/enveloppe

Carrier/enveloppe : $$u(x,t) = \frac{1}{2} A(x,t)\, \mathrm{e}^{\mathrm{i}(k_0 x - \omega_0 t)} + c.c.$$

Spectral analysis

Fourier transform:

$$u(x,t) = \frac{1}{2}\int_0^{+\infty} \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(kx-\omega(k)t)}\mathrm{d}k + c.c.$$

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Spectral analysis

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Carrier/enveloppe :

$$u(x,t) = \frac{1}{2} A(x,t)\, \mathrm{e}^{\mathrm{i}(k_0 x - \omega_0 t)} + c.c.$$

Enveloppe :

$$A(x,t) = \int_0^{\infty} \hat{u}(k)\, \mathrm{e}^{\mathrm{i}(k-k_0)x - \mathrm{i}(\omega-\omega_0)t} \mathrm{d}k.$$

Spectral analysis at time=0

Fourier transform:

$$u(x,0) = \frac{1}{2}\int_0^{+\infty} \hat{u}(k)\, e^{i(kx)}\, dk + c.c.$$

$\hat{u}(k)$ is given by Fourier transform at time t=0

Enveloppe :

$$A(x,0) = \int_0^{\infty} \hat{u}(k)\, e^{i(k-k_0)x}\, dk + c.c.$$

Spectral analysis

Gaussian spectrum:

$$\hat{u}(k) = u_0 \, \mathrm{e}^{-\sigma^2 (k-k_0)^2}$$

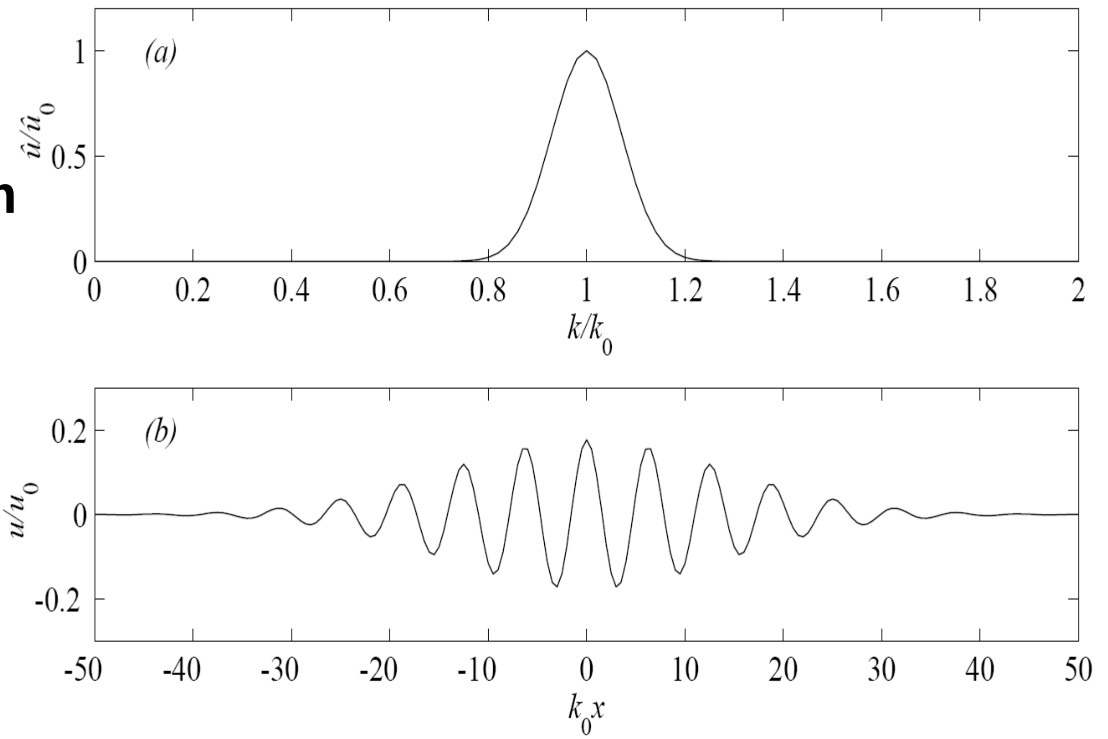
Initial enveloppe :

$$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma} \, \mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

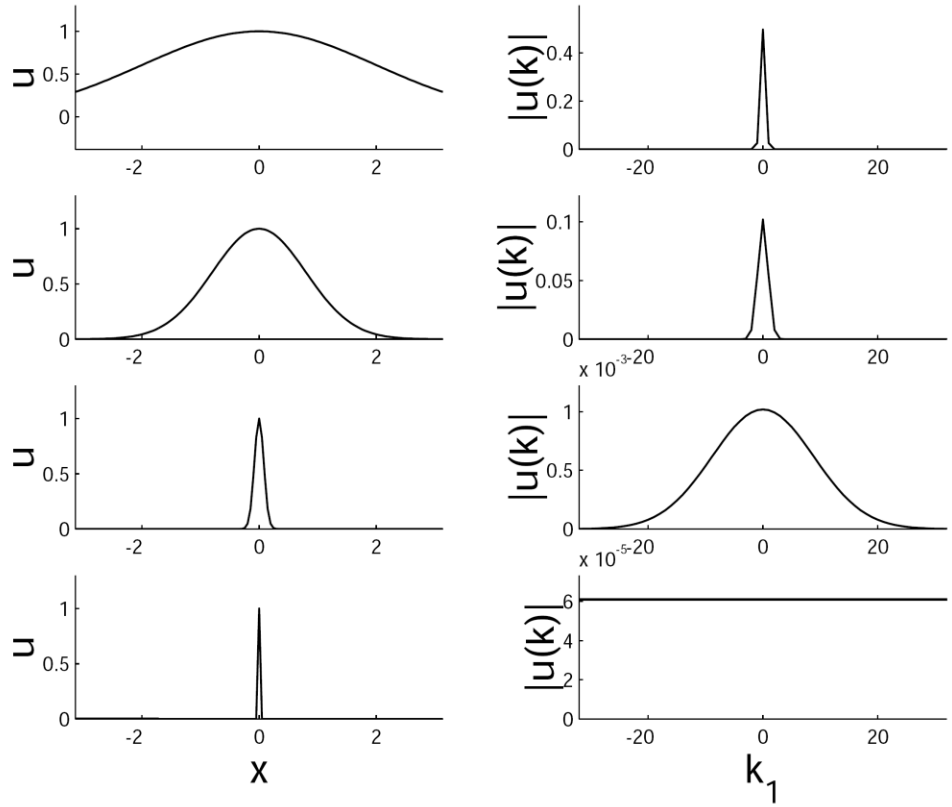
Gaussian spectrum

spectrum

wave



Gaussian wavepackets



$u_0(x)$	$\widehat{u}_0(k_1)$
$\exp\left(-\frac{x^2}{2\,\sigma^2}\right)$	$\frac{\sigma}{\sqrt{2\pi}}\;\exp\left(-\frac{\sigma^2\,k^2}{2}\right)$
$\delta(x)$	$\frac{1}{2\pi}$

Spectral analysis

Initial enveloppe :

$$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma}\,\mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

Gaussian spectrum:

$$\hat{u}(k) = u_0\,\mathrm{e}^{-\sigma^2(k-k_0)^2}$$

Spectral analysis

Initial enveloppe :

$$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma}\,\mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

Gaussian spectrum:

$$\hat{u}(k) = u_0\,\mathrm{e}^{-\sigma^2(k-k_0)^2}$$

Evolution of enveloppe :

$$A(x,t) = \int_0^\infty \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

Spectral analysis

Initial enveloppe :

$$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma}\,\mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

Gaussian spectrum:

$$\hat{u}(k) = u_0\,\mathrm{e}^{-\sigma^2(k-k_0)^2}$$

Evolution of enveloppe :

$$A(x,t) = \int_0^\infty \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

Definition group velocity

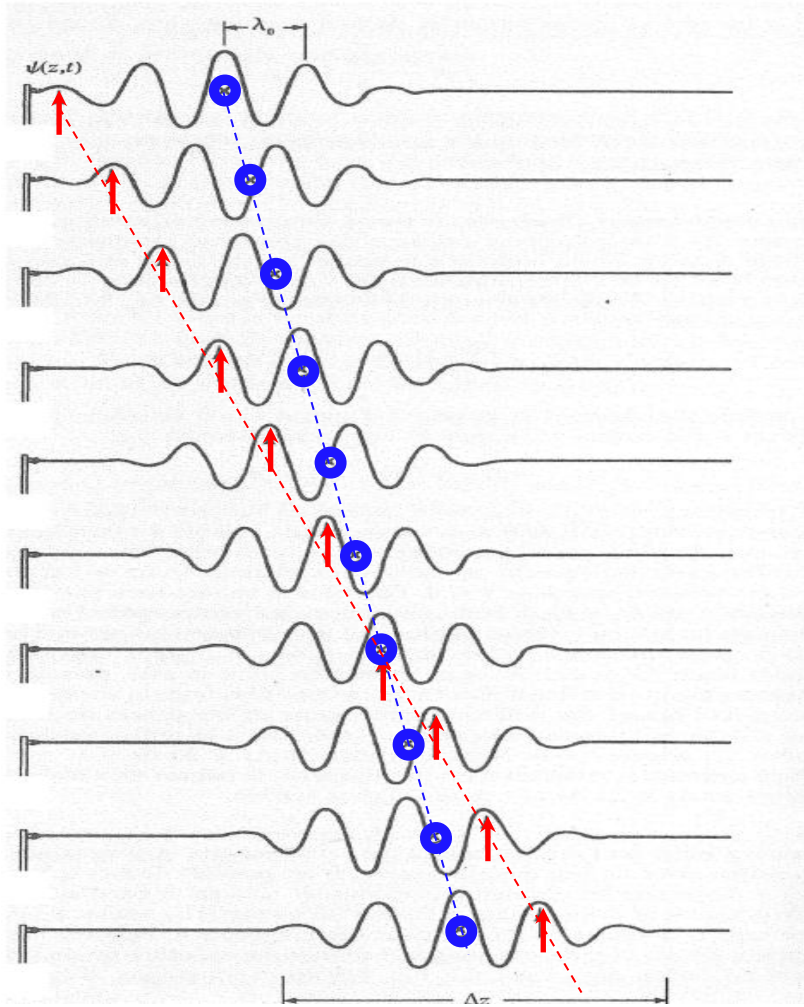
$$\omega - \omega_0 = c_g(k-k_0), \qquad c_g = \frac{\partial\omega}{\partial k}(k_0)$$

Spectral analysis

Definition of group velocity $\omega - \omega_0 = c_g(k - k_0), \qquad c_g = \frac{\partial \omega}{\partial k}(k_0)$

$$A(x,t) = \frac{u_0\sqrt{\pi}}{2\sigma}\,\mathrm{e}^{-\frac{(x-c_g t)^2}{4\sigma^2}}$$

Group velocity



Wavepacket

Spectral analysis

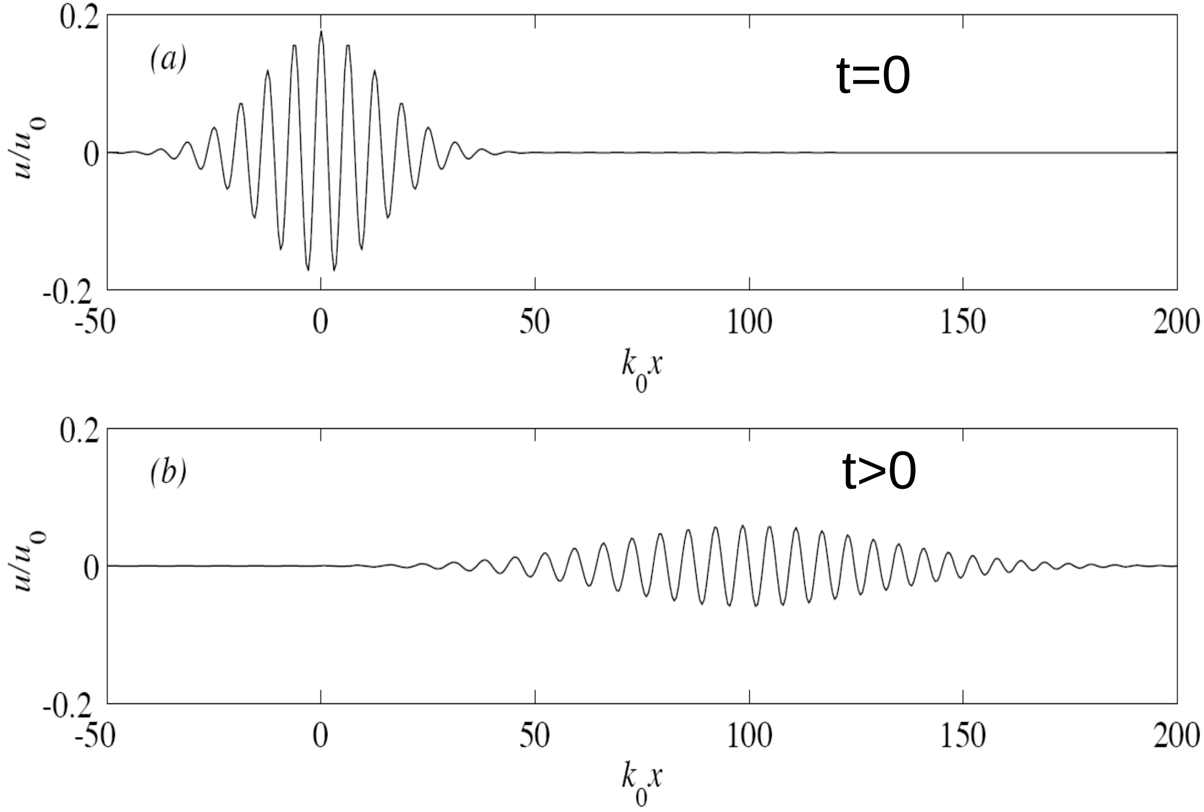
Higher order development

$$\omega - \omega_0 = c_g(k - k_0) + \frac{\omega_0''}{2}(k - k_0)^2$$

$$c_g = \frac{\partial \omega}{\partial k}(k_0), \qquad \omega_0'' = \frac{\partial^2 \omega}{\partial k^2}(k_0)$$

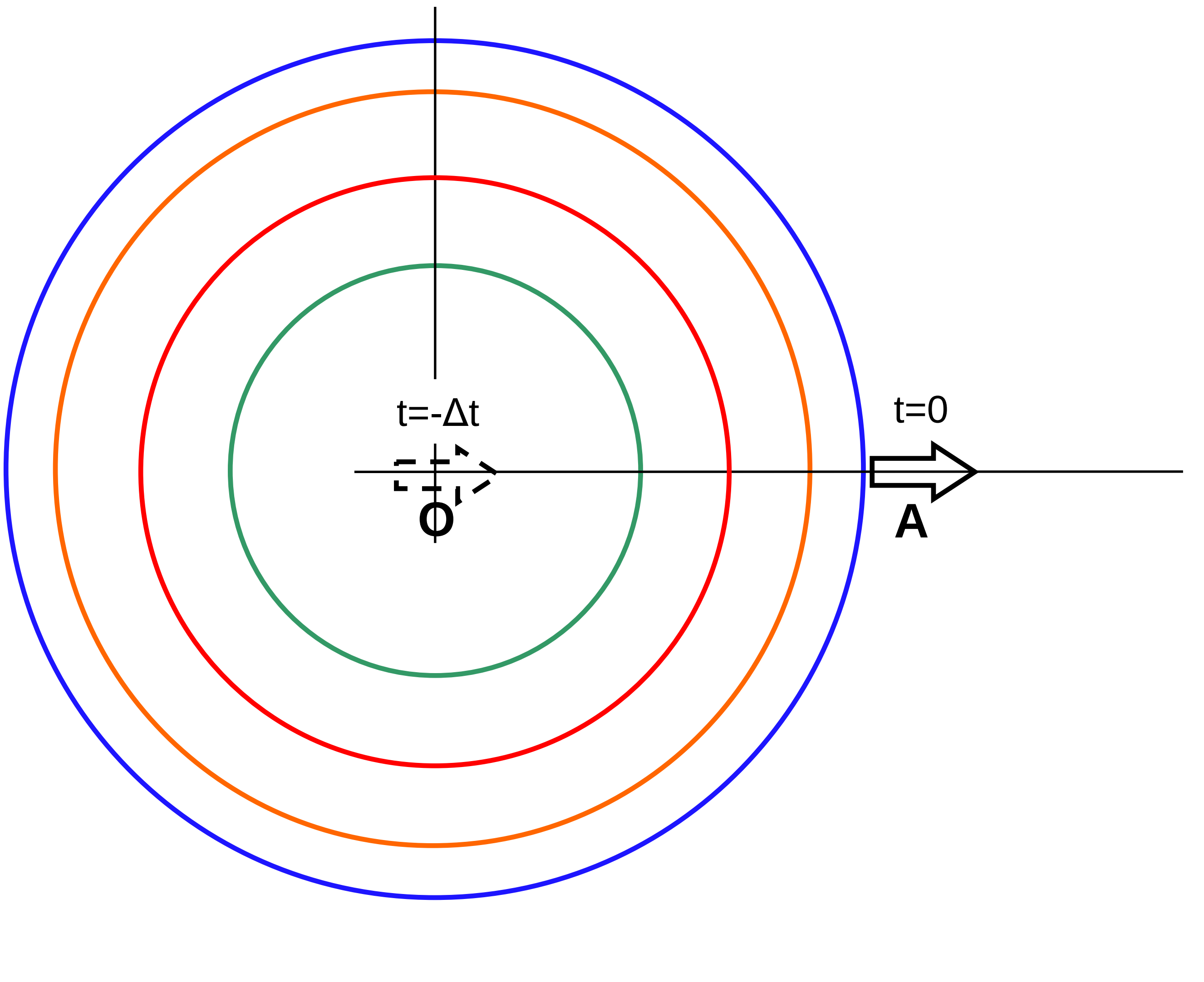
$$A(x,t) = \frac{u_0}{2}\sqrt{\frac{\pi}{\sigma^2 + \frac{1}{2}\mathrm{i}\omega_0'' t}}\,\exp\left(-\frac{(x - c_g t)^2}{4(\sigma^2 + \frac{1}{2}\mathrm{i}\omega_0'' t)}\right)$$

Wave packet dispersion

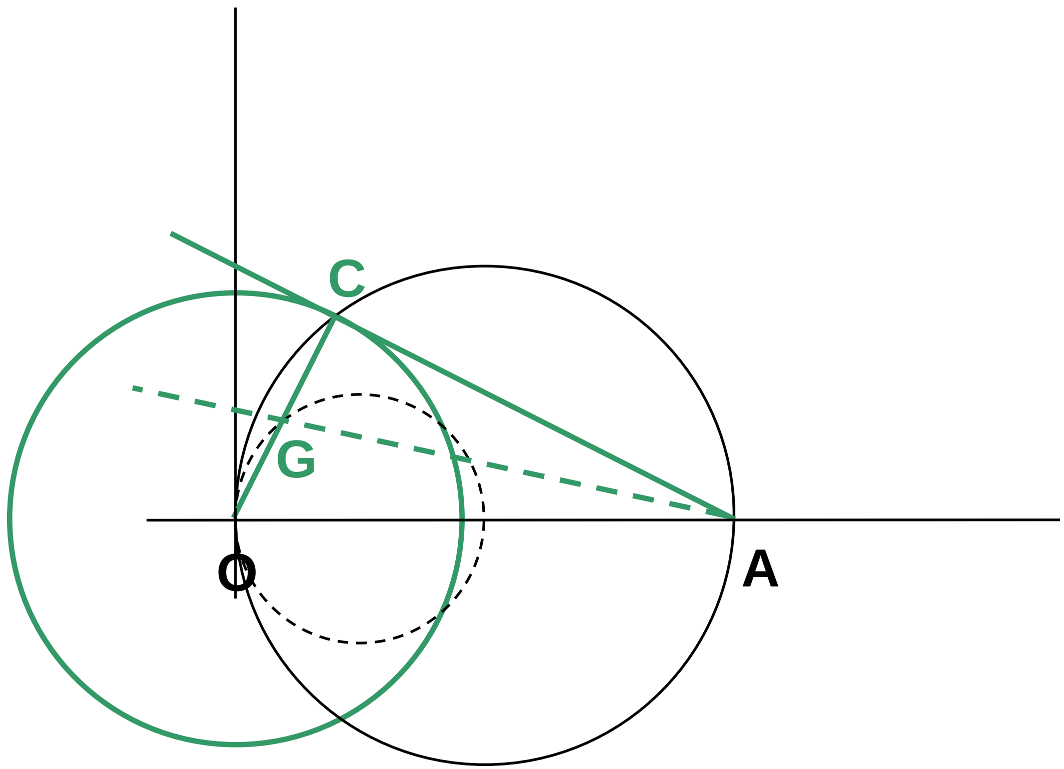


Onde correspondant à l'enveloppe ( ) *pour* $\sigma^{-1}k_0 = 0{,}1$ *et* $\omega_0'' = 4c_g/k_0$: *(a), instant initial* $t = 0$; *(b),* $c_g t = 100/k_0$.

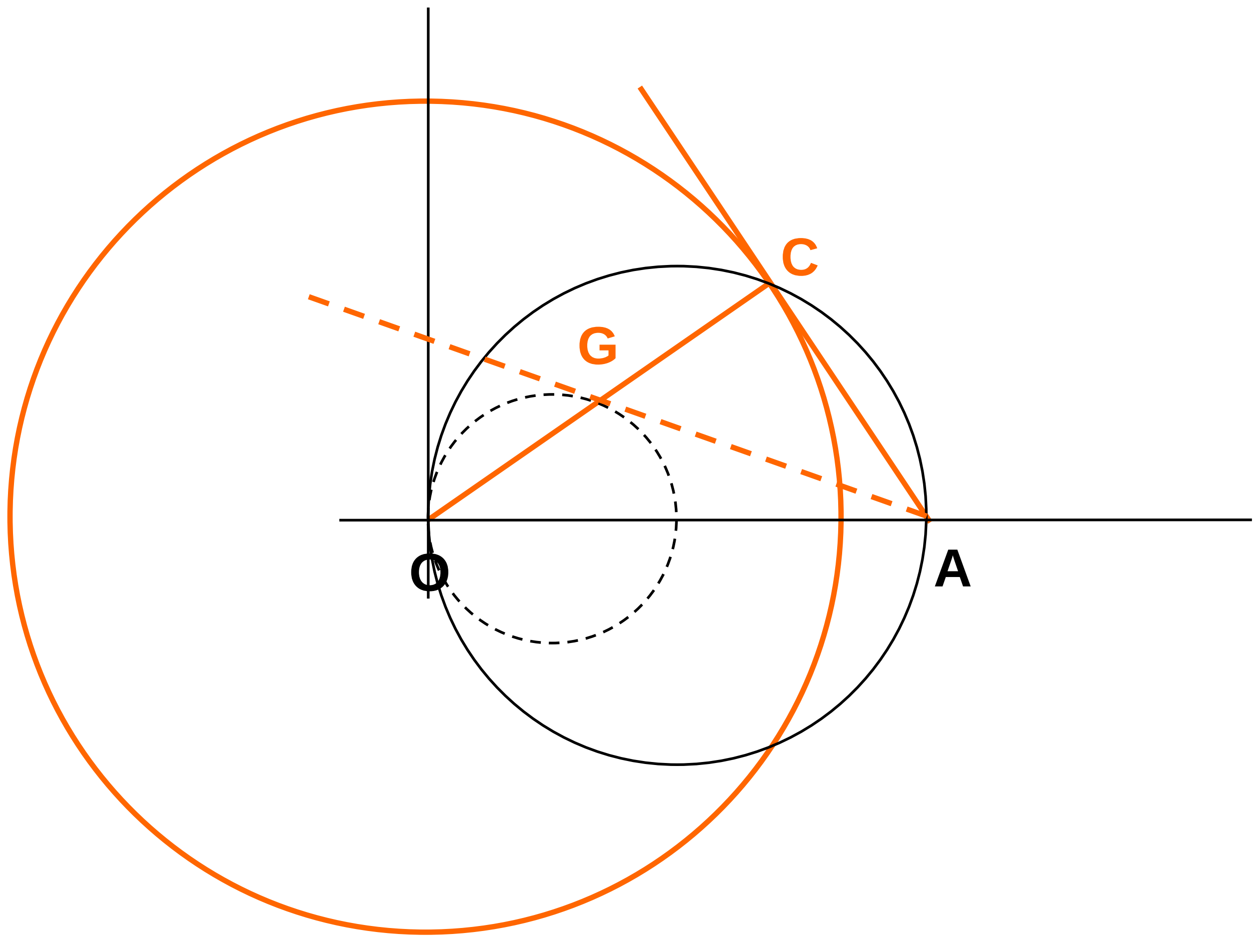
Kelvin's wake



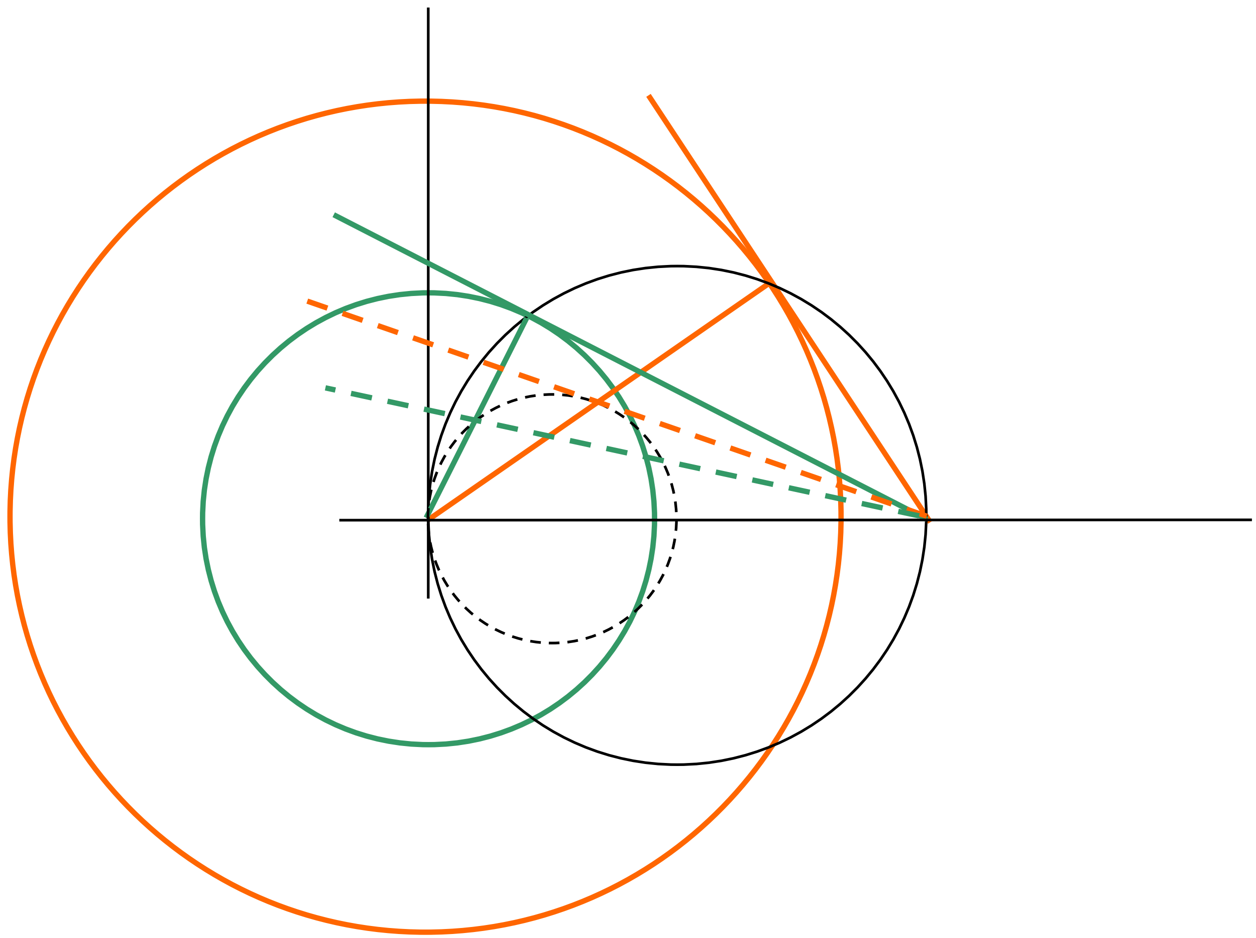
Gravity waves created by a ship



Gravity waves created by a ship



Gravity waves created by a ship



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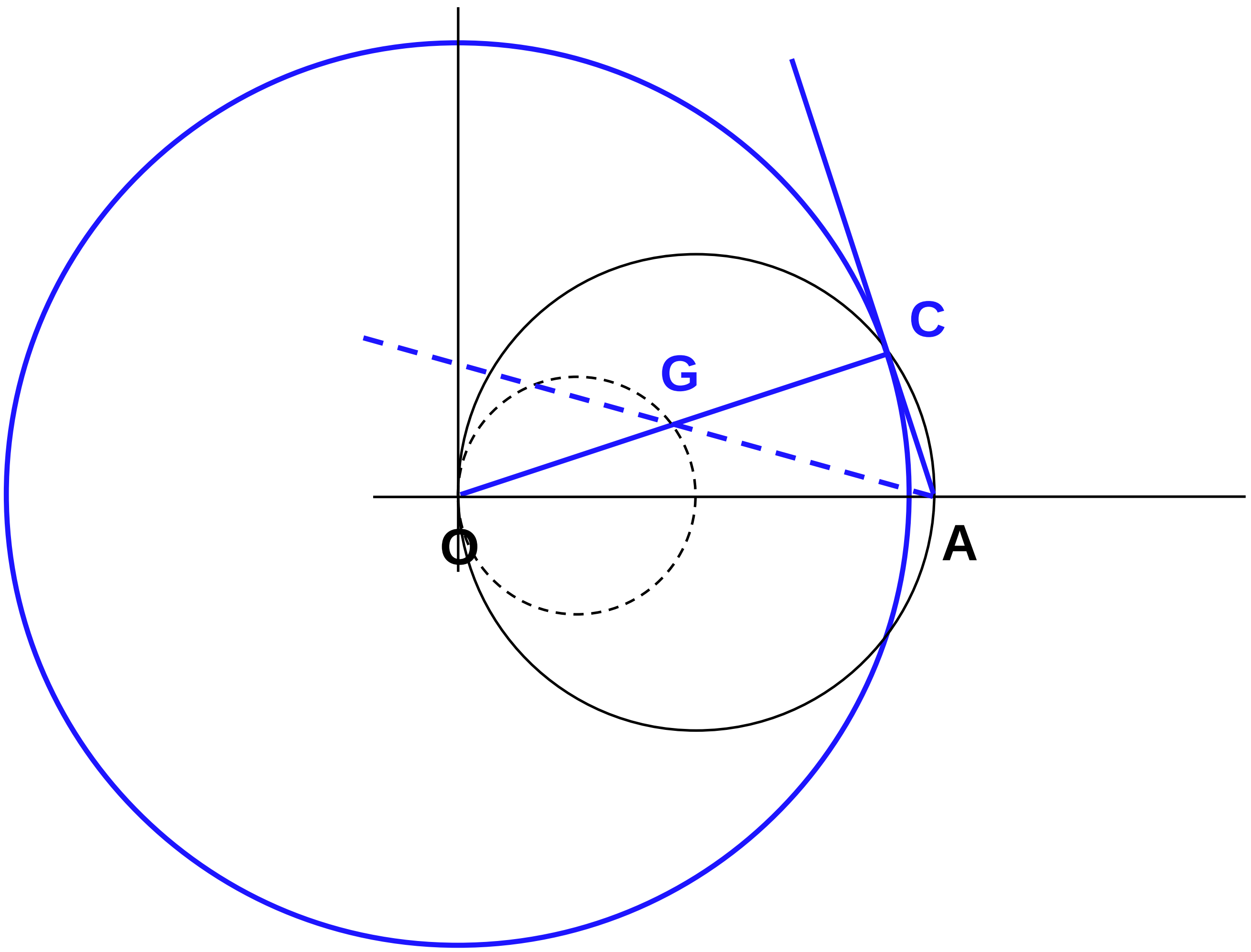

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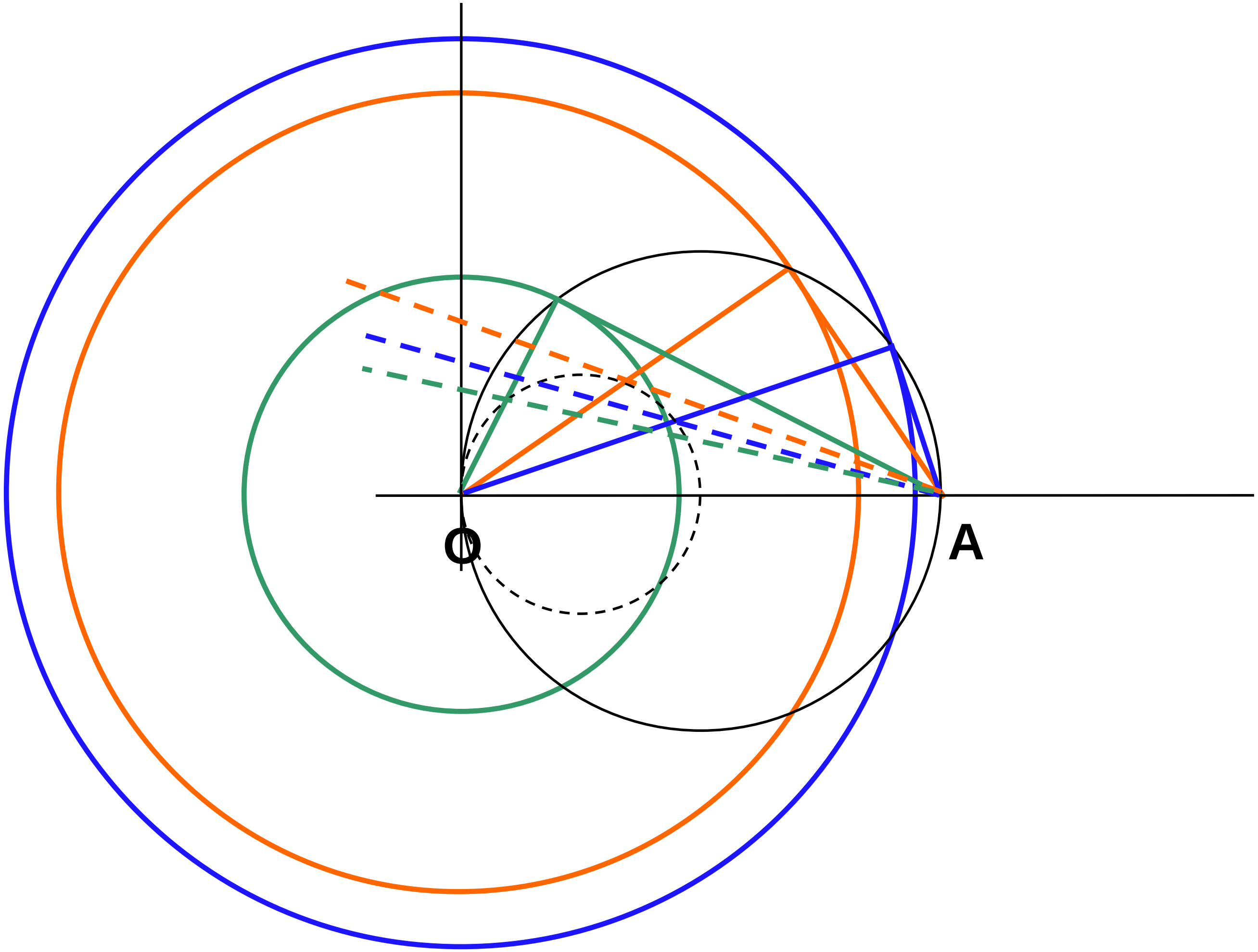
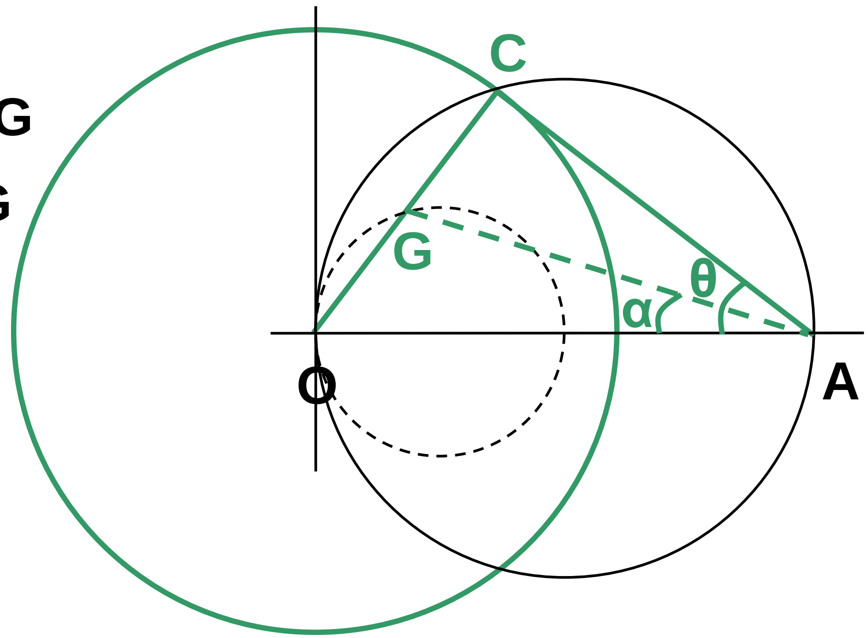


Gravity waves created by a ship

sin(α)/OG=cos(θ)/AG

sin(θ-α)AG=GC=OG

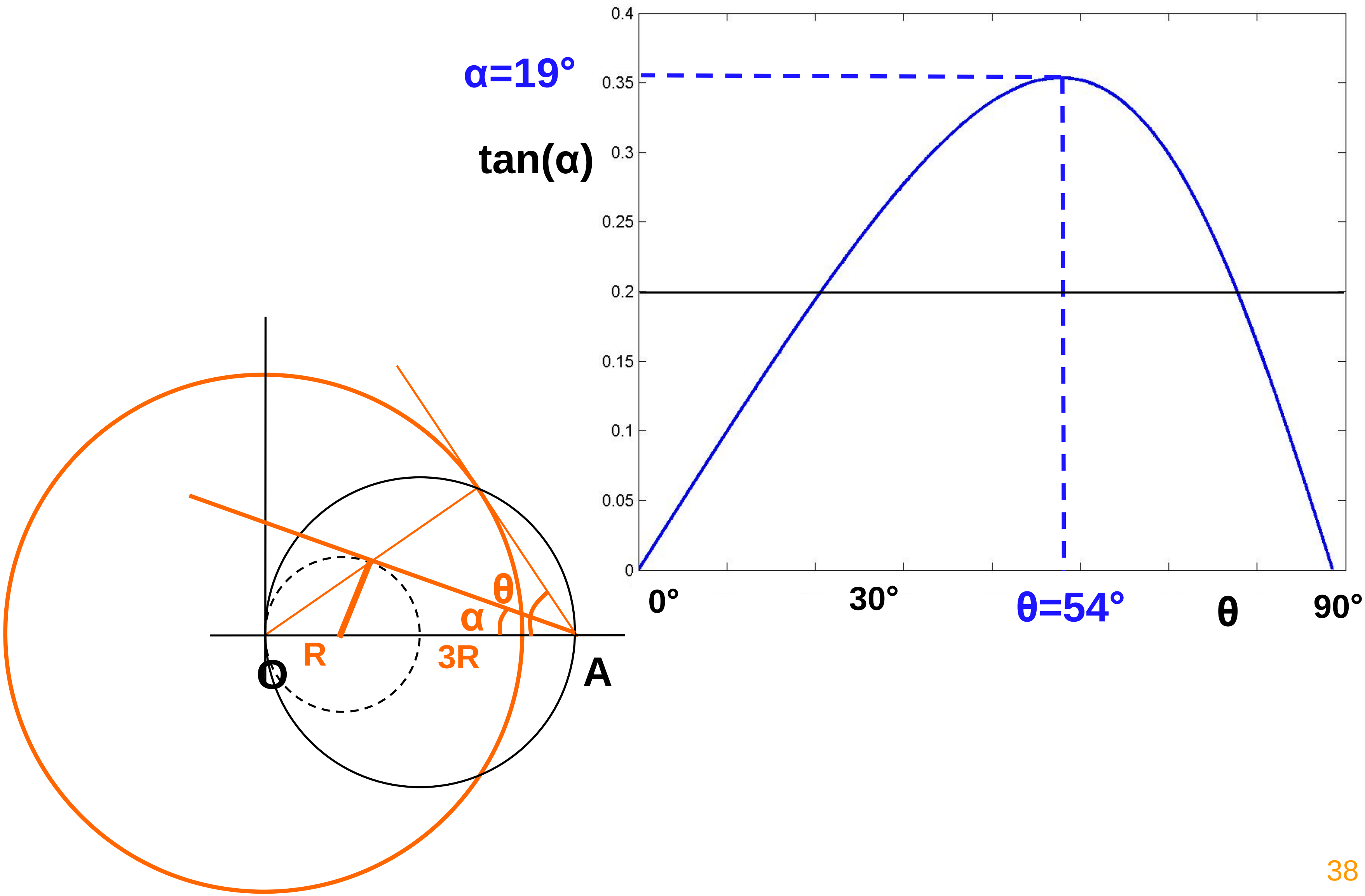


⇨ **sin(α)=cos(θ)sin(θ-α)**

⇨ **sin(α)=cos(θ)(sin(θ)cos(α)+cos(θ)sin(α))**

⇨ **tan(α)=cos(θ)sin(θ)/(1+cos^2(θ))**

Gravity waves created by a ship



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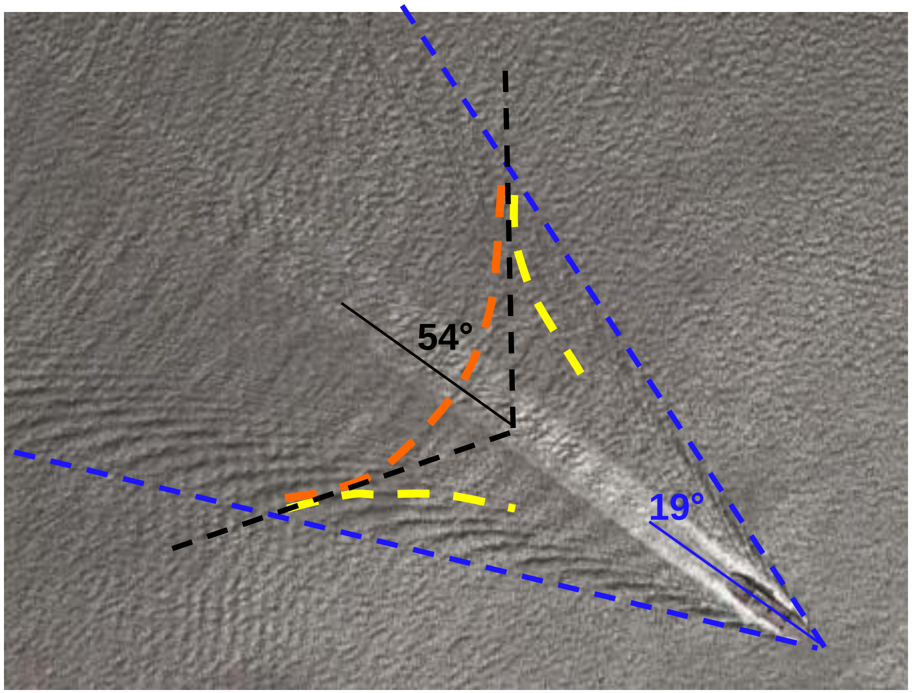
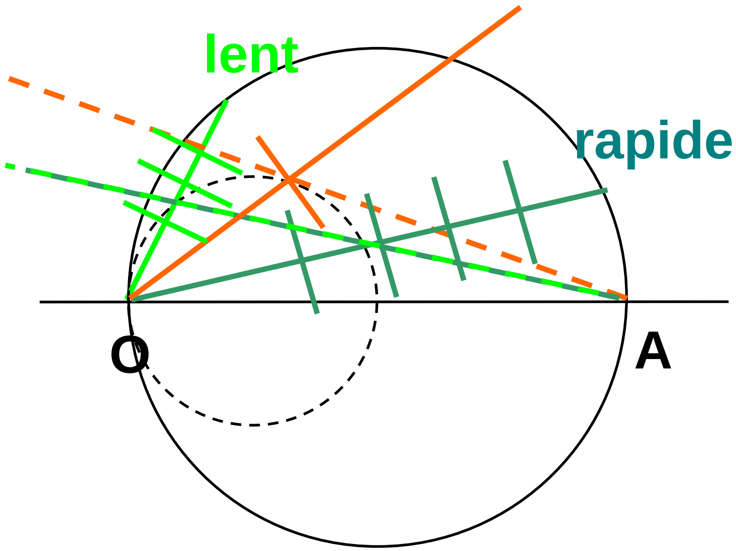
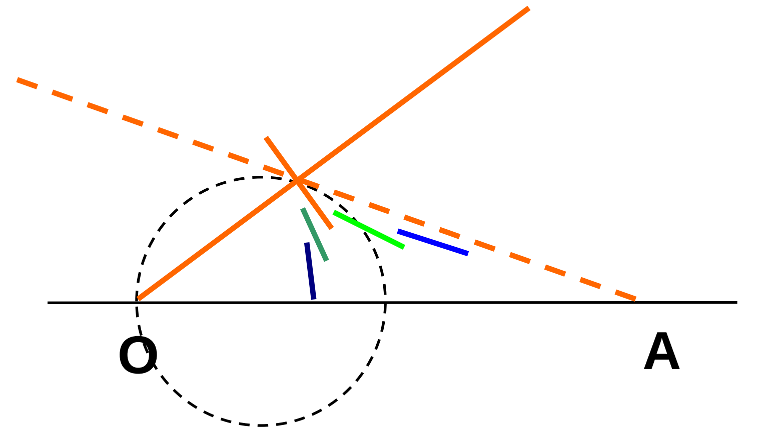
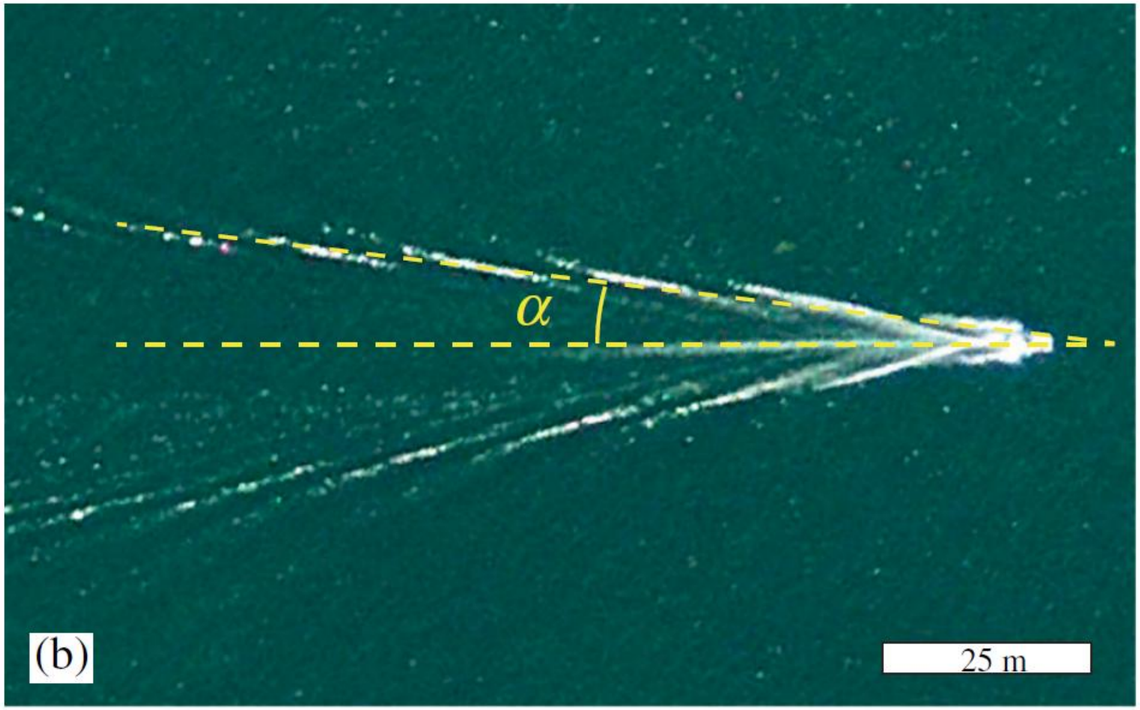


Gravity waves created by a ship



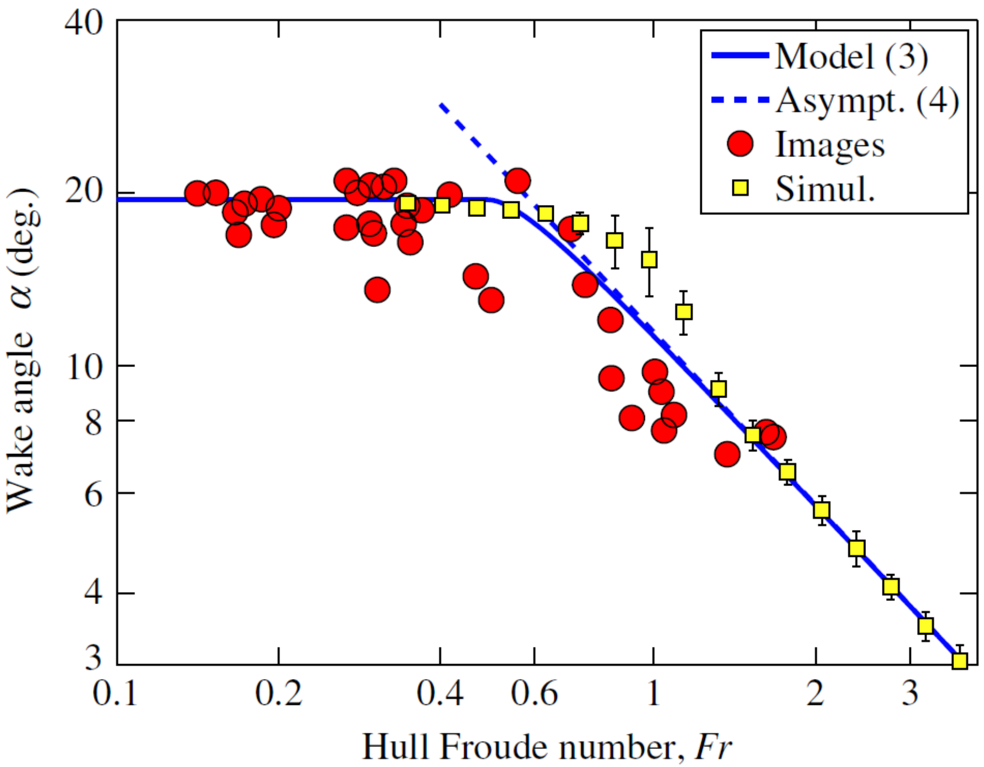


But observations show



Moisy and Rabaud 2013

But observations show

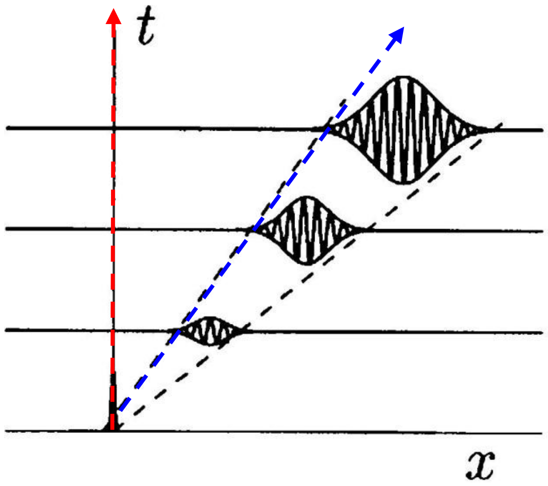


Moisy and Rabaud 2013

Generalization: Spatio-temporal instability theory

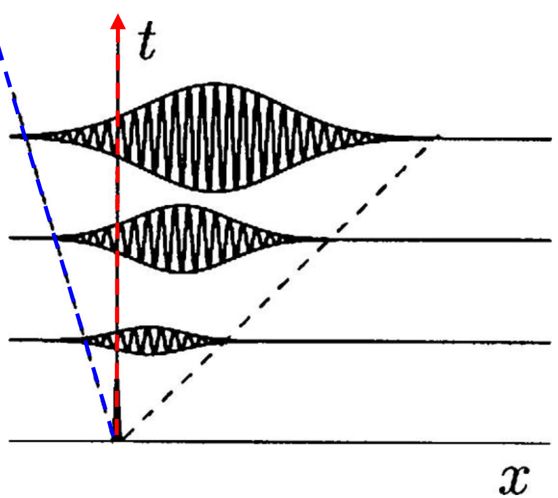
First find the zero group velocity wave: $d\omega/dk=0 \Rightarrow (k_0,\omega_0)$
and consider the sign of $\mathrm{Im}(\omega_0)$

☞ Convective instability



$\mathrm{Im}(\omega_0)<0$

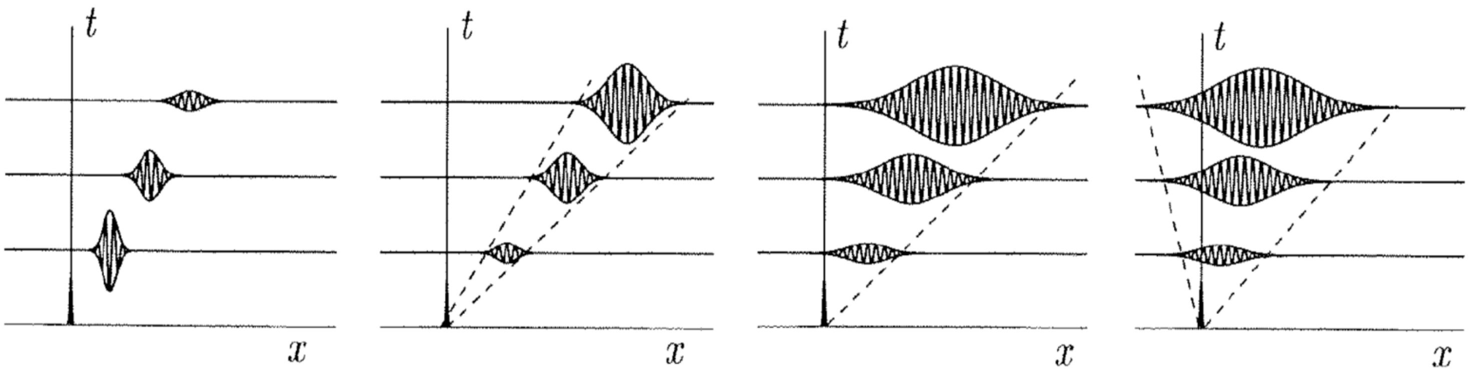
F Absolute instability



$\mathrm{Im}(\omega_0)>0$

LINEAR IMPULSE RESPONSE: ABSOLUTE/CONVECTIVE INSTABILITY

Green's function or impulse response



Stable Unstable Unstable Unstable

Briggs (1964) Bers (1983)

Huerre and Monkewitz (1985)

LINEAR IMPULSE RESPONSE: ABSOLUTE/CONVECTIVE INSTABILITY

Linearly stable flow

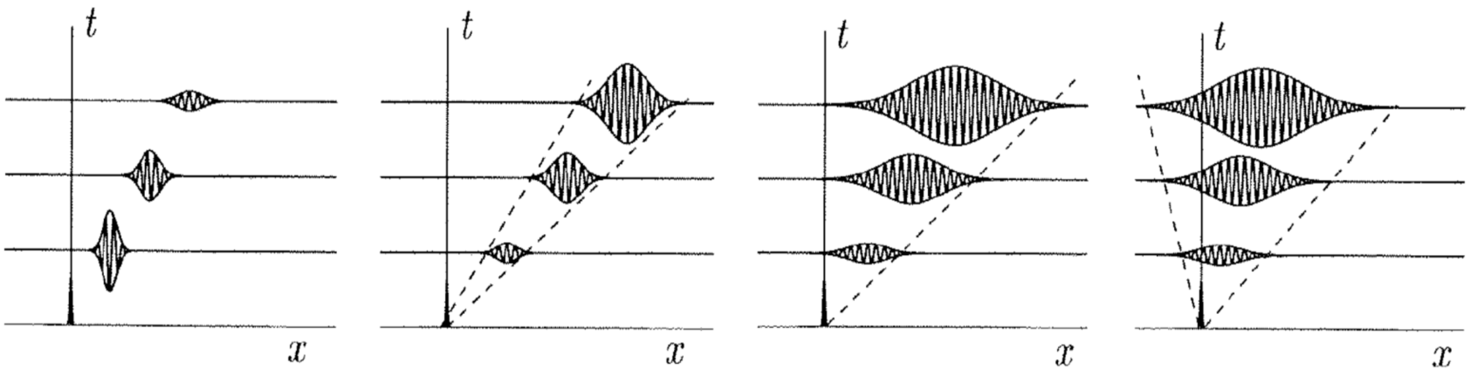
$$\lim_{t \to \infty} G(x,t) = 0$$ along all rays x/t = const.

Linearly unstable flow

$$\lim_{t \to \infty} G(x,t) = \infty$$ along at least one ray x/t = const.

LINEAR IMPULSE RESPONSE: ABSOLUTE/CONVECTIVE INSTABILITY

Green's function or impulse response



Stable | convectively Unstable | Unstable | absolutely Unstable

Briggs (1964) Bers (1983)

Huerre and Monkewitz (1985)

LINEAR IMPULSE RESPONSE: ABSOLUTE/CONVECTIVE INSTABILITY

Convectively unstable flow

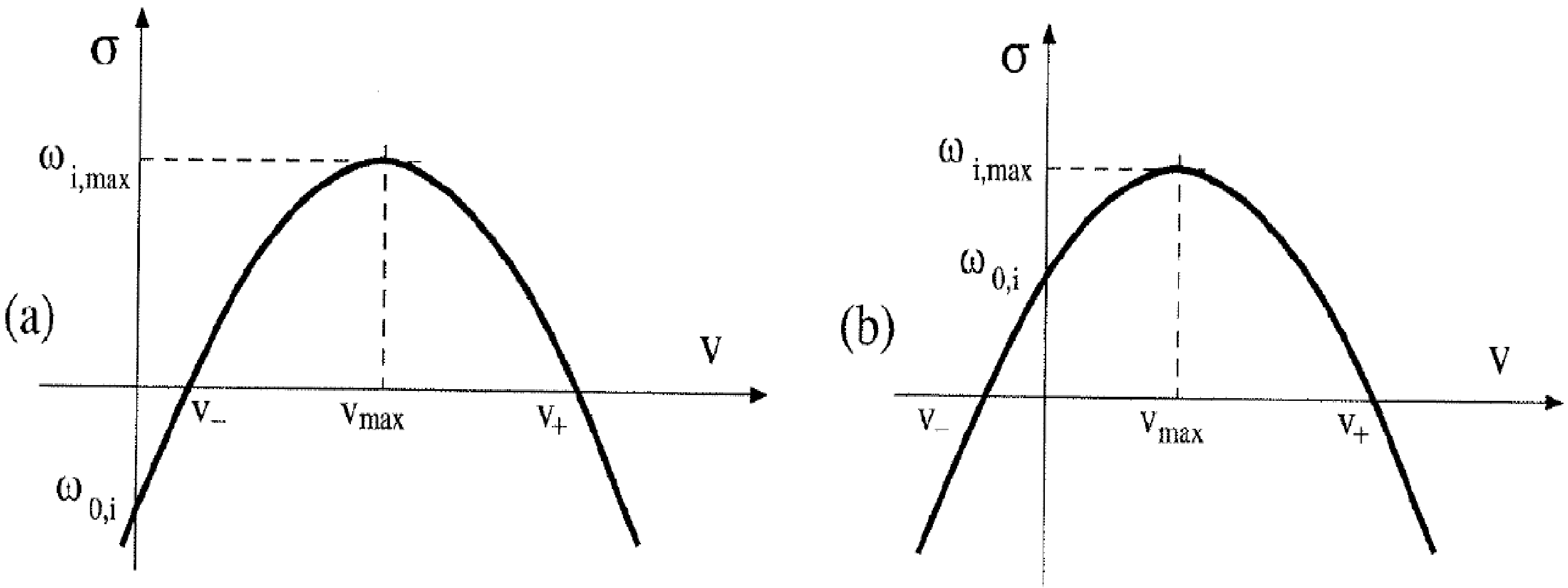
$$\lim_{t\to\infty} G(x,t) = 0$$ along the ray x/t = 0

Absolutely unstable flow

$$\lim_{t\to\infty} G(x,t) = \infty$$ along the ray x/t = 0

ANALYSIS IN COMPLEX FOURIER SPACE: AU/CU CRITERION

Temporal growth rate « at velocity v »



Convective instability   Absolute instability

ANALYSIS IN COMPLEX FOURIER SPACE: AU/CU CRITERION

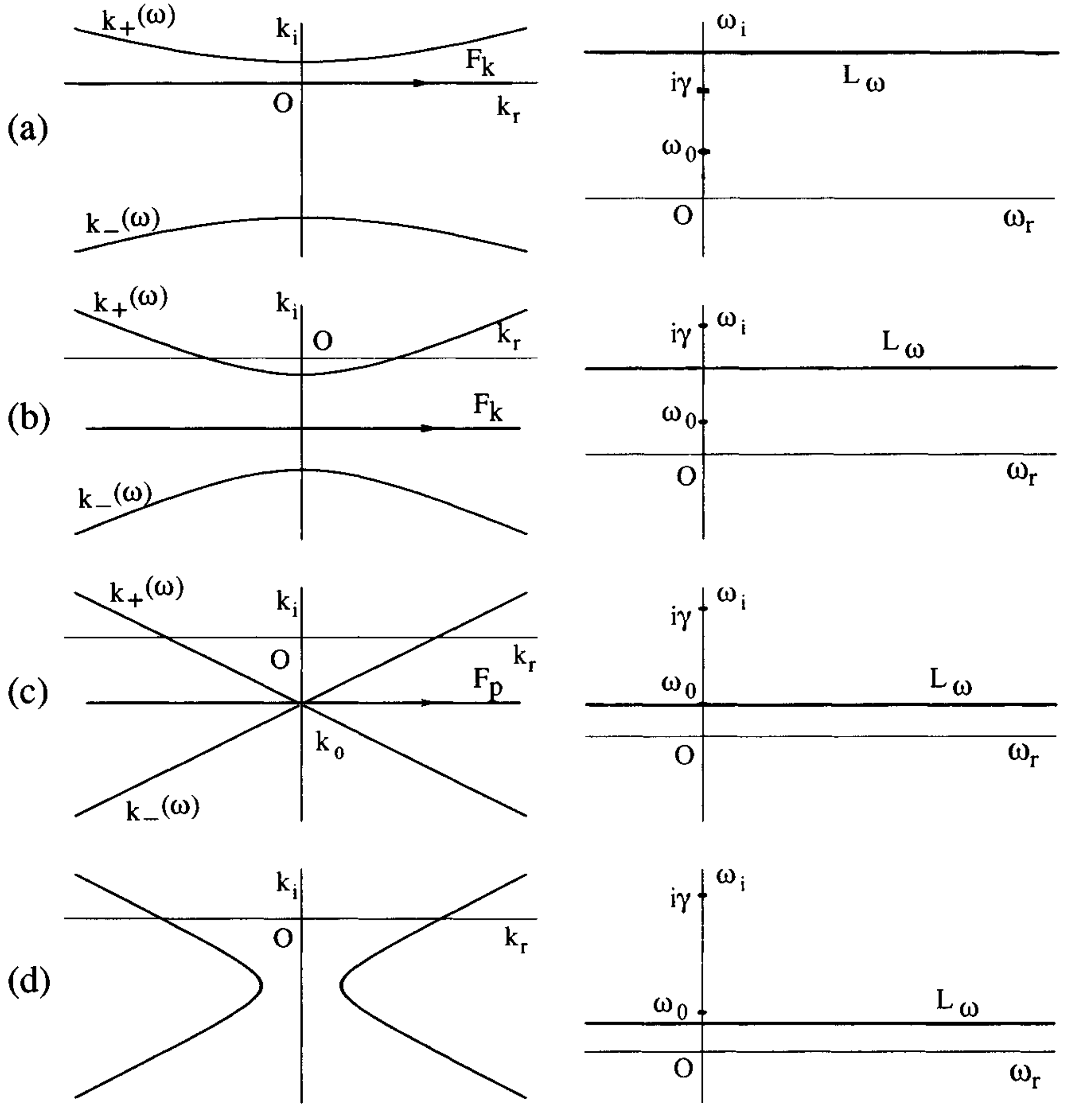
Important notions

Absolute wavenumber k_0 and frequency $\omega_0 = \omega(k_0)$
observed along ray v = 0, i.e. for a stationary observer,
defined by

$$\frac{\partial \omega}{\partial k}(k_0) = 0$$

Absolute growth rate is

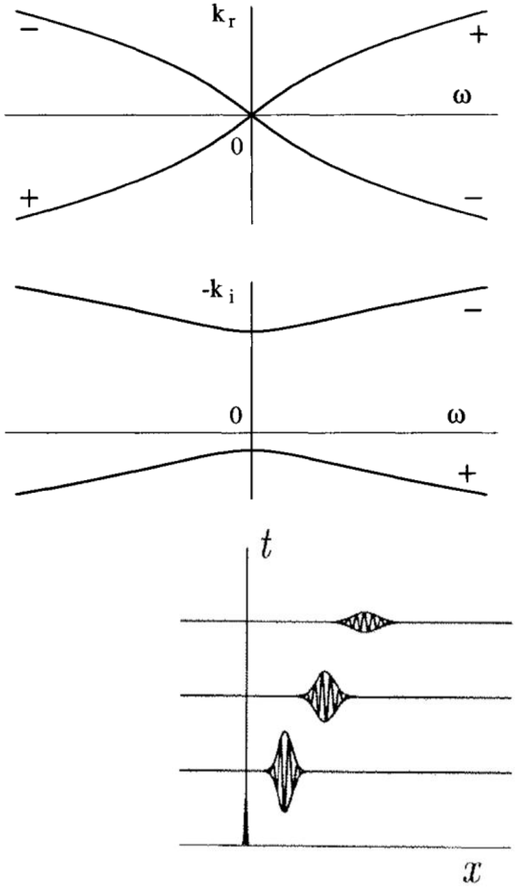
$$\sigma(0) = \omega_{0,i}$$



(a)
$k_+(\omega)$
k_i
F_k
O
k_r
$k_-(\omega)$
ω_i
$i\gamma$
L_ω
ω_0
O
ω_r
(b)
$k_+(\omega)$
k_i
O
k_r
F_k
$k_-(\omega)$
$i\gamma$
ω_i
L_ω
ω_0
O
ω_r
(c)
$k_+(\omega)$
k_i
O
k_r
F_p
k_0
$k_-(\omega)$
ω_i
$i\gamma$
ω_0
L_ω
O
ω_r
(d)
k_i
O
k_r
ω_i
$i\gamma$
ω_0
L_ω
O
ω_r

Spatial analysis

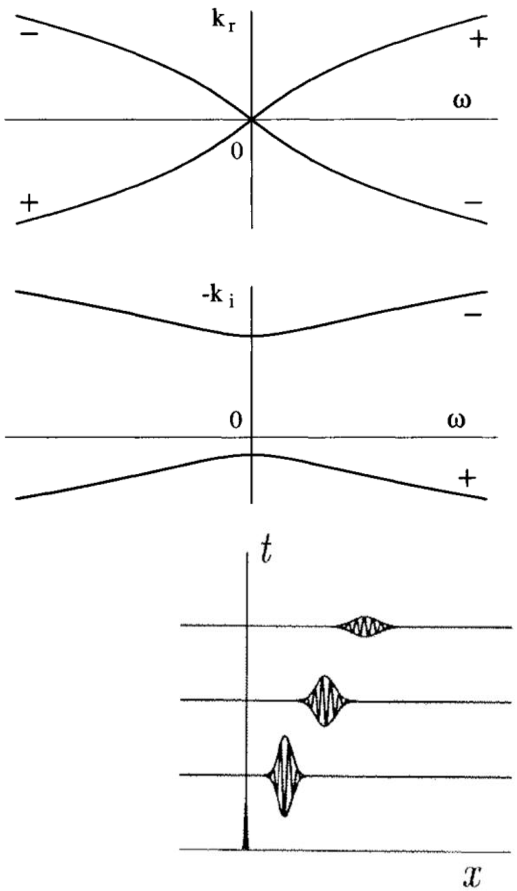
ω is real, k complex, k+ and k- waves, plotting kr(ω) and -ki(ω) :



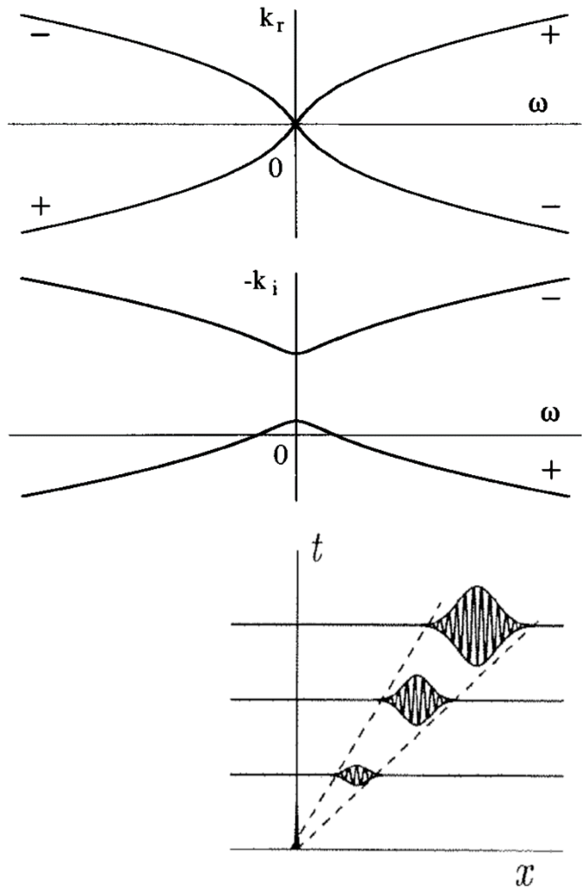
stable

Spatial analysis

ω is real, k complex, k+ and k- waves, plotting kr(ω) and -ki(ω) :



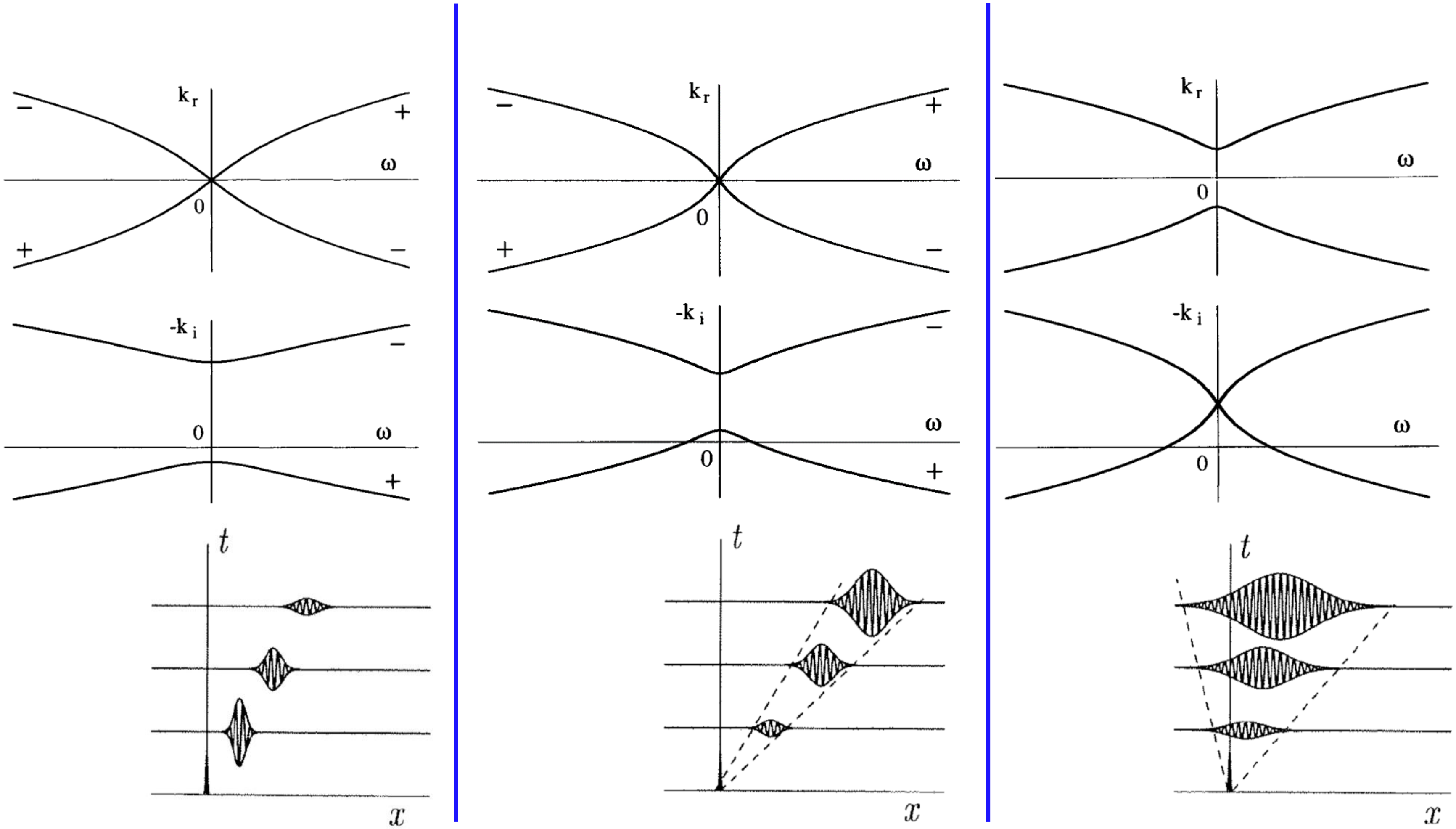
stable



convectively unstable

Spatial analysis

ω is real, k complex, k+ and k- waves, plotting kr(ω) and -ki(ω) :



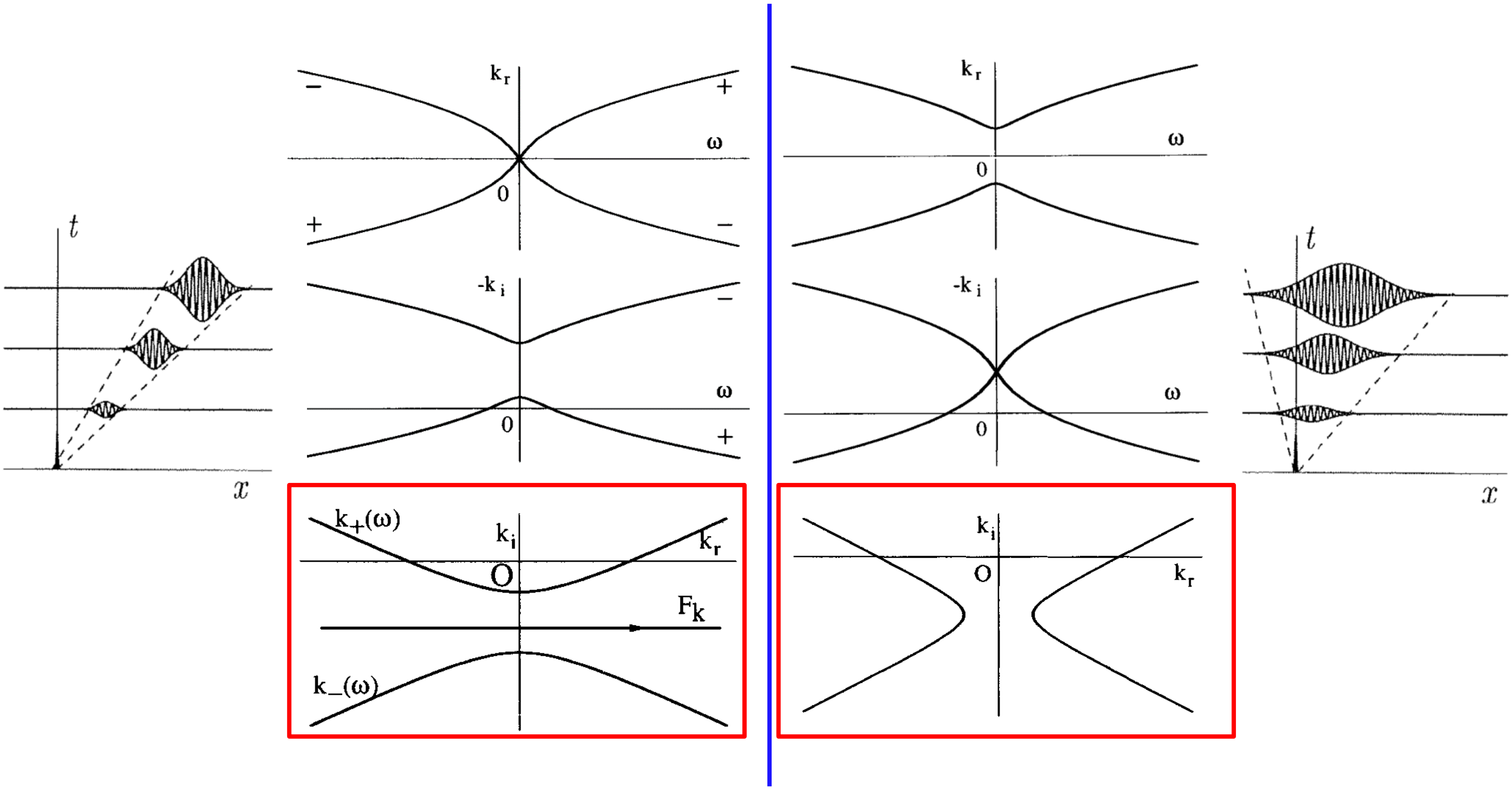
stable

convectively unstable

absolutely unstable

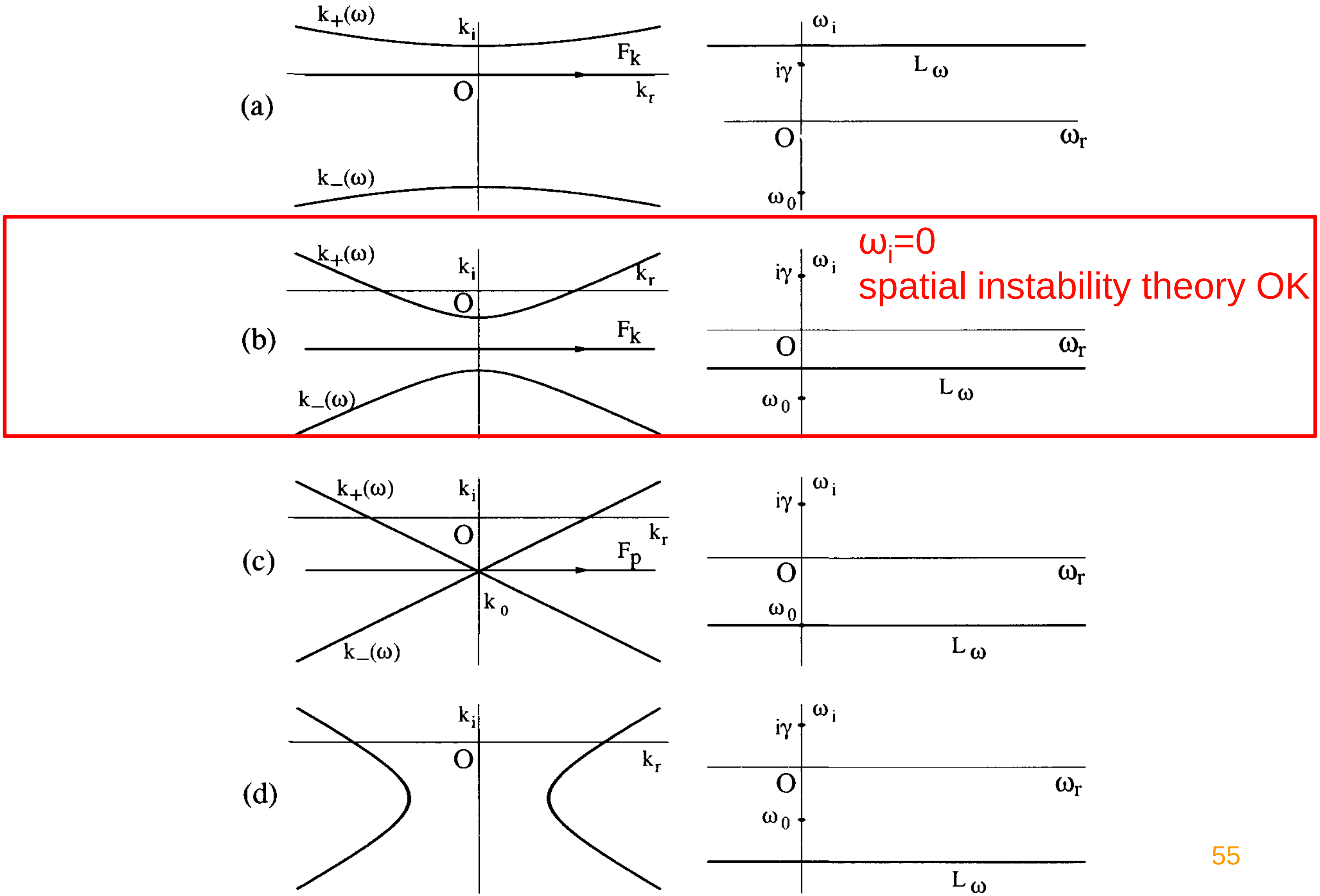
Spatial analysis

ω is real, k complex, k+ and k- waves, plotting kr(ω) and -ki(ω) and ki(kr)



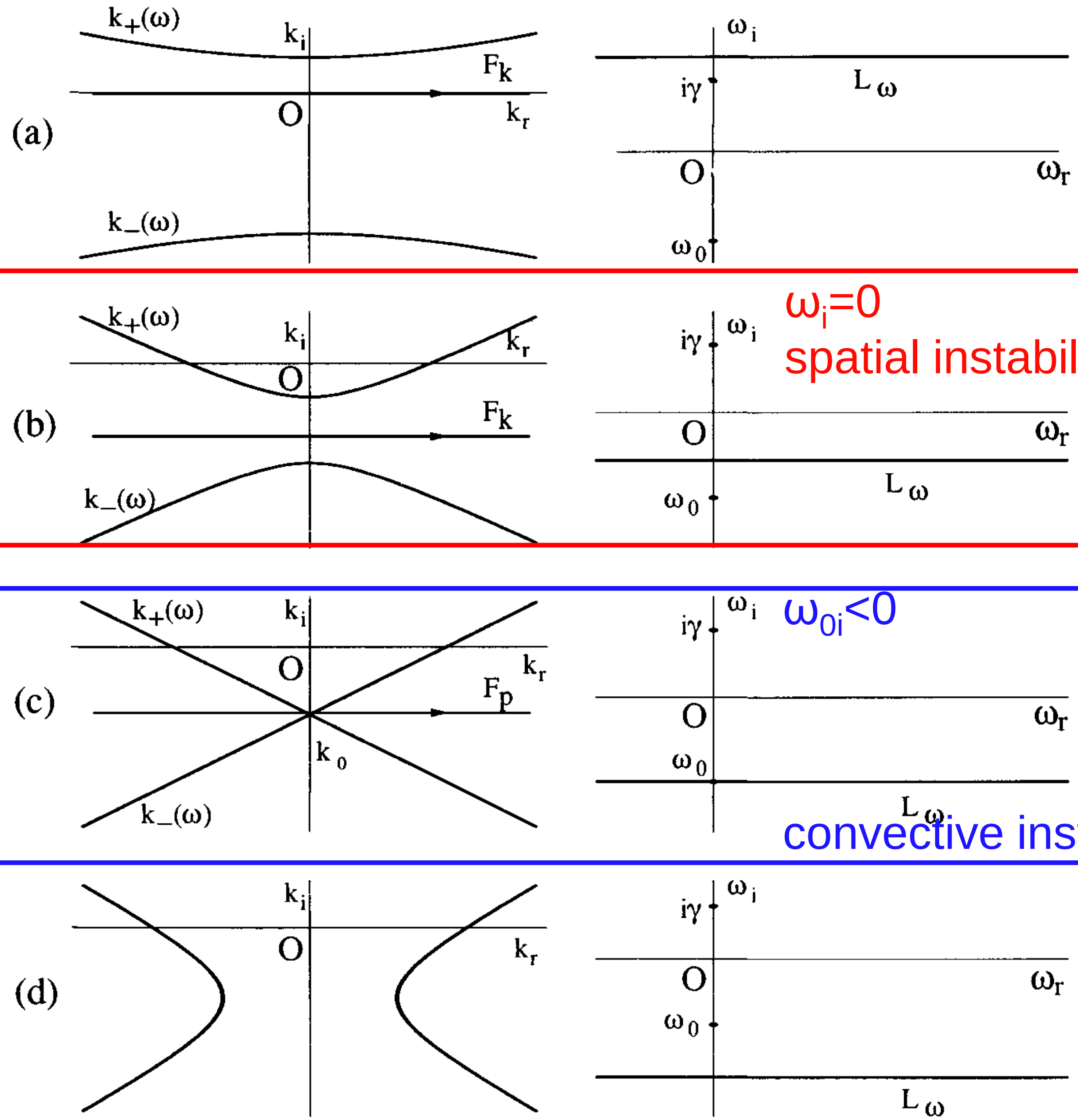
convectively unstable | absolutely unstable

Add ω_i offset and vary ω_r



Add ω_i offset and vary ω_r

Saddle point

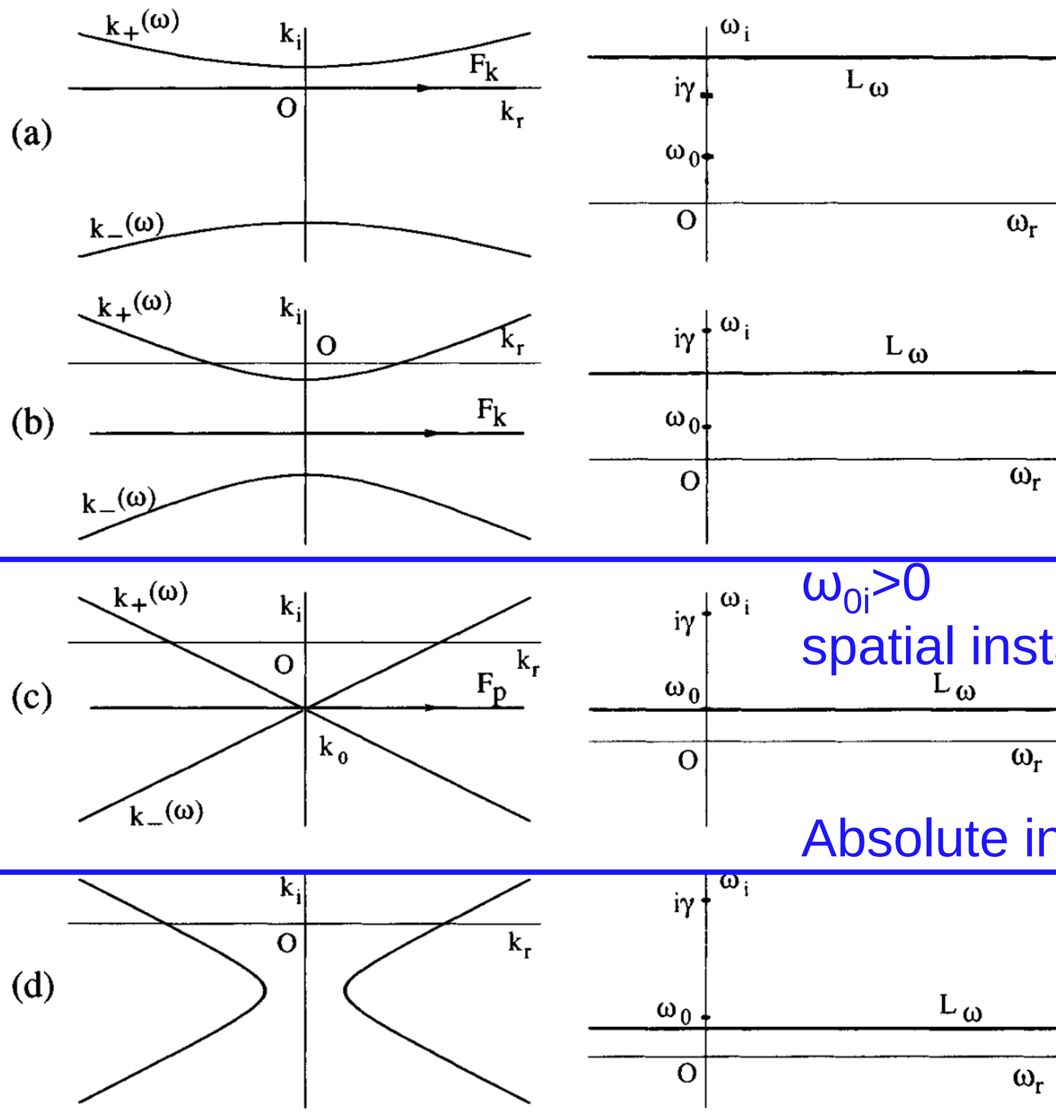


ω_i=0
spatial instability theory OK

ω_{0i}<0

convective instability

Add ω_i offset and vary ω_r
Saddle point condition



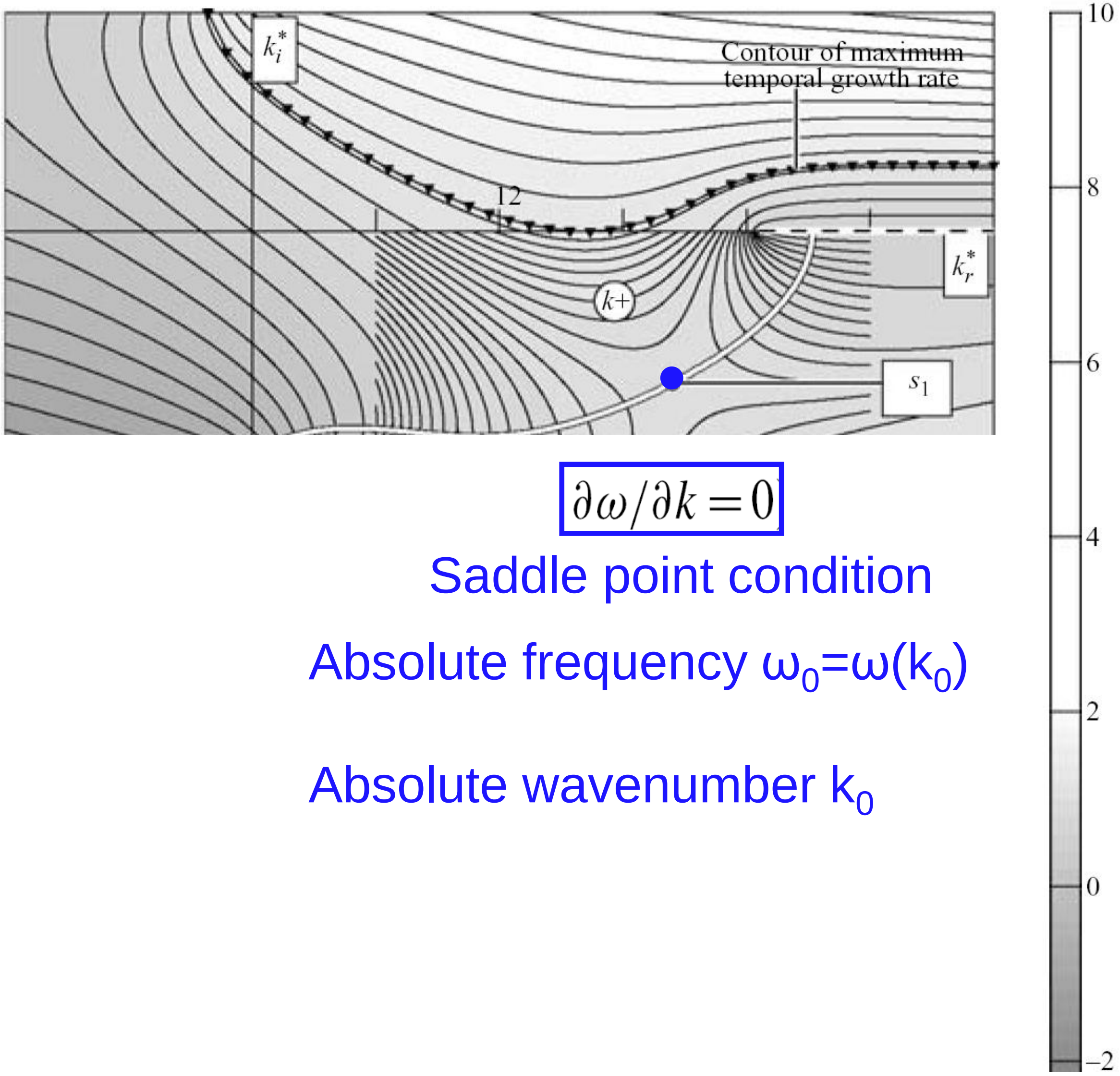
Saddle point

$\omega_{0i}>0$

spatial instability NOT OK

Absolute instability

Spatio-temporal analysis (vary both kr and ki)
Isocontours of ω_i in the kr-ki plane



$$\partial\omega/\partial k = 0$$

Saddle point condition

Absolute frequency $\omega_0=\omega(k_0)$

Absolute wavenumber k_0

ANALYSIS IN COMPLEX FOURIER SPACE: AU/CU CRITERION

Instability criteria

$\omega_{i,max} < 0$	linearly stable
$\omega_{i,max} > 0$	linearly unstable
$\omega_{0,i} < 0$	convectively unstable
$\omega_{0,i} > 0$	absolutely unstable

Hyperbolic tangent mixing layer

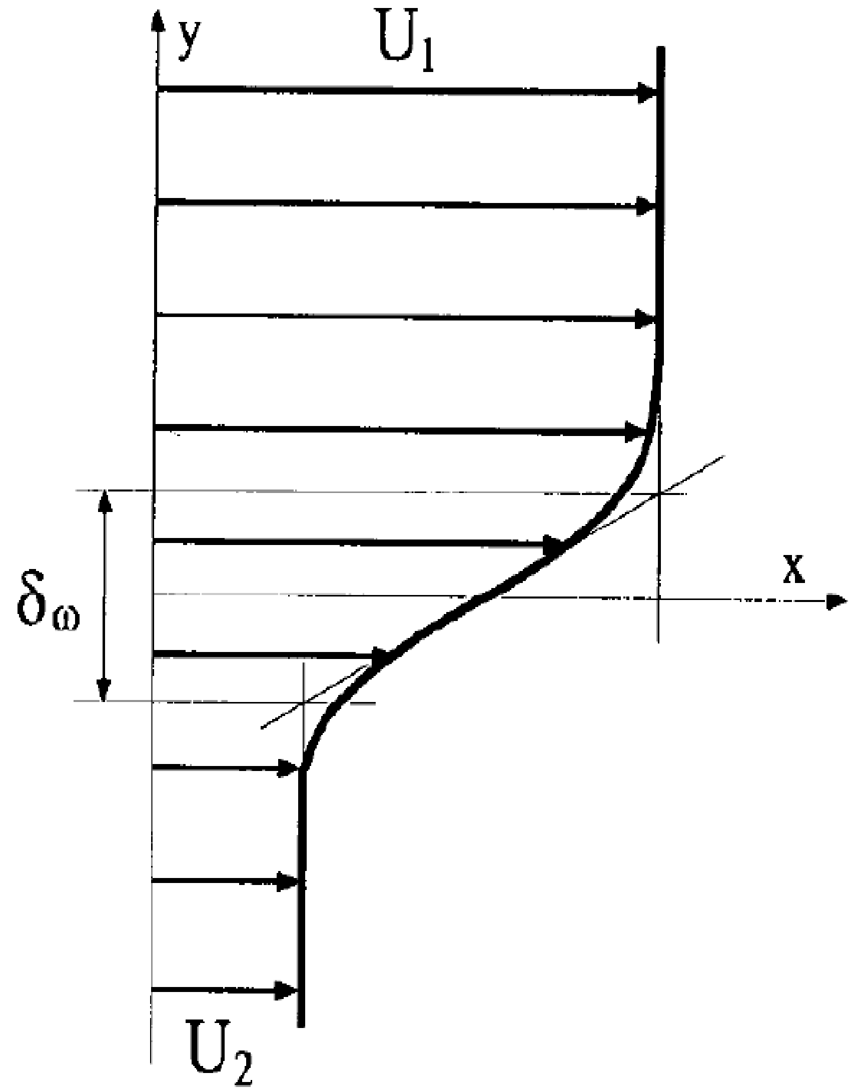
$$U(y) = \bar{U} + \frac{\Delta U}{2} \tanh\left(\frac{2y}{\delta_\omega}\right)$$

$$\delta_\omega(x) \equiv \frac{(U_1 - U_2)}{(dU/dy)_{\max}}$$

Velocity ratio

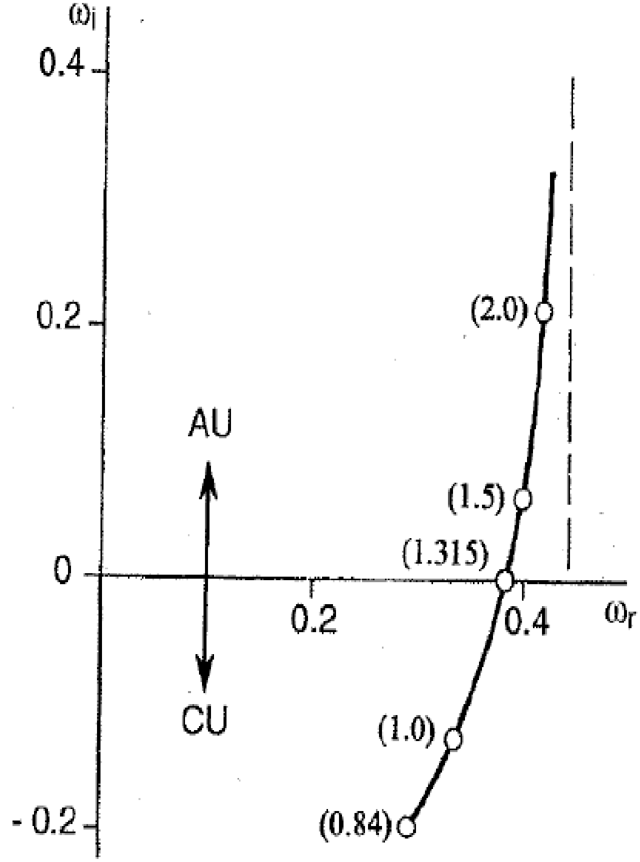
$$R \equiv \frac{U_1 - U_2}{U_1 + U_2} = \frac{\Delta U}{2\bar{U}}$$

$$U(y; R) = 1 + R \tanh y$$

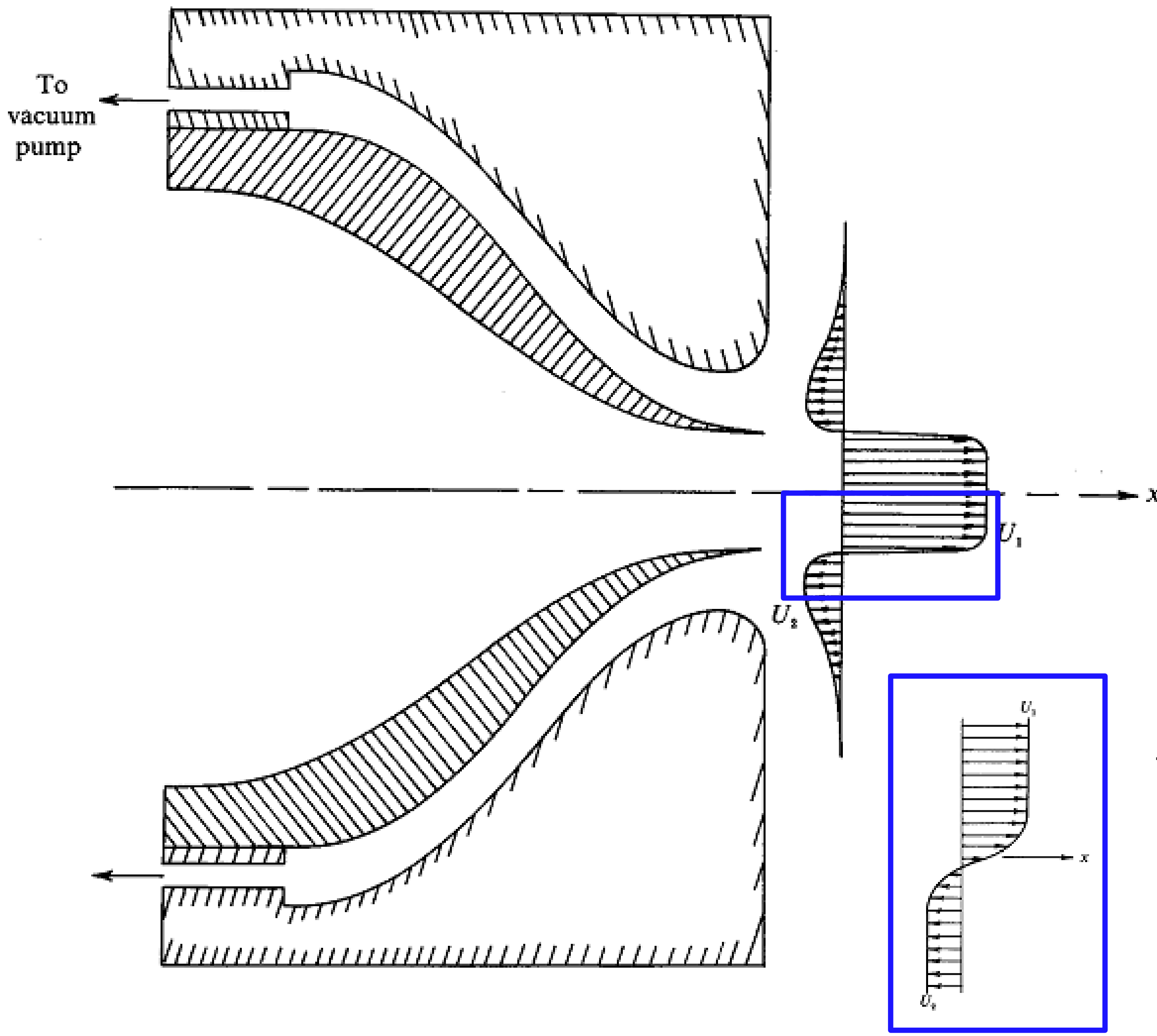


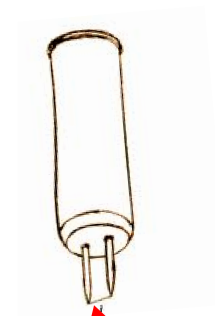
APPLICATION TO MIXING LAYERS

Locus of complex absolute frequency

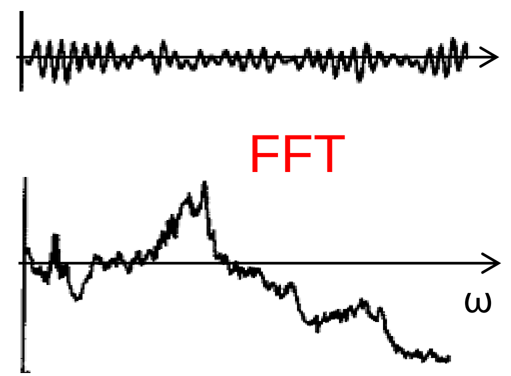


H.&Monkewitz (1985)

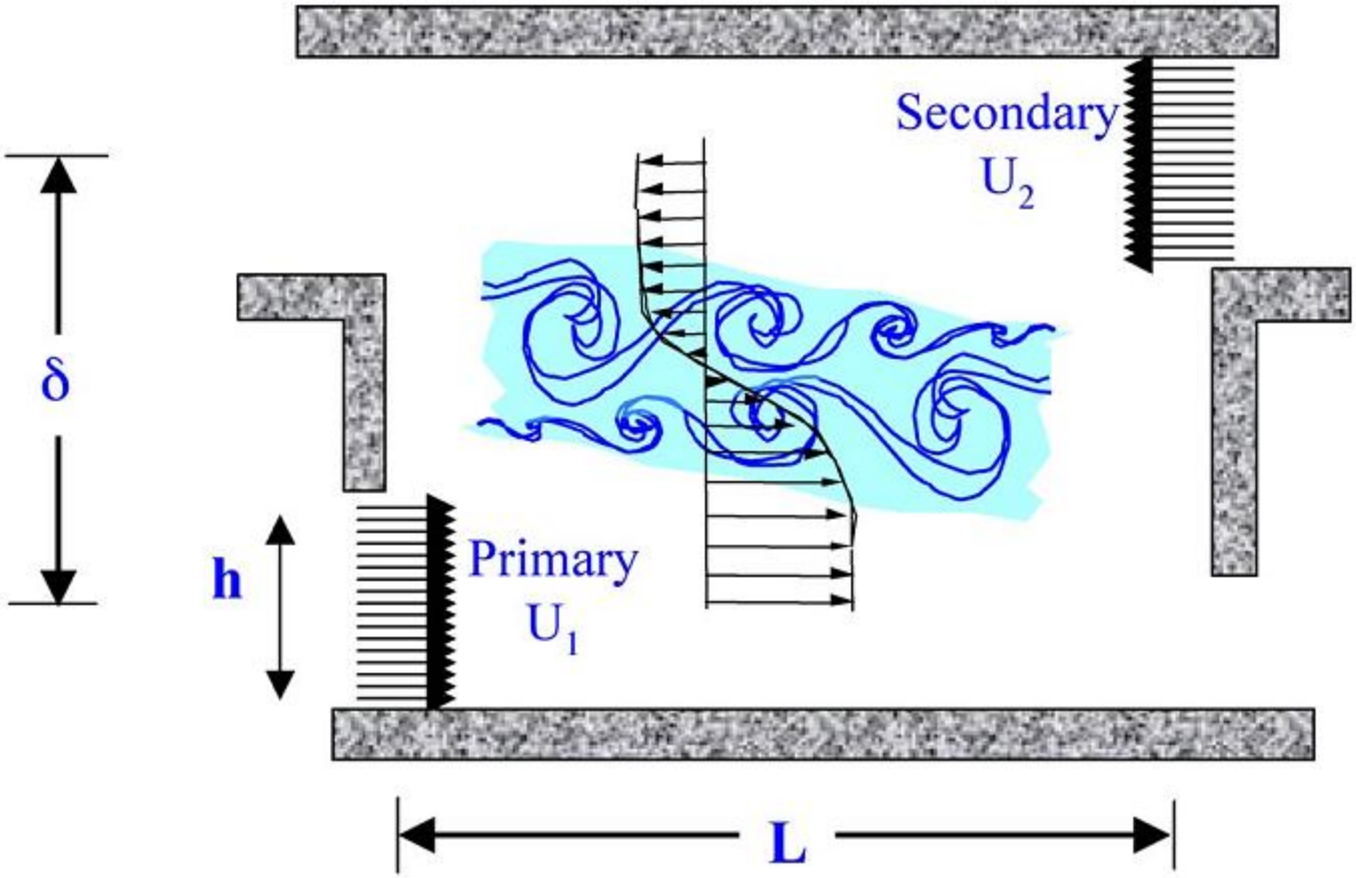




Hot wire measurement

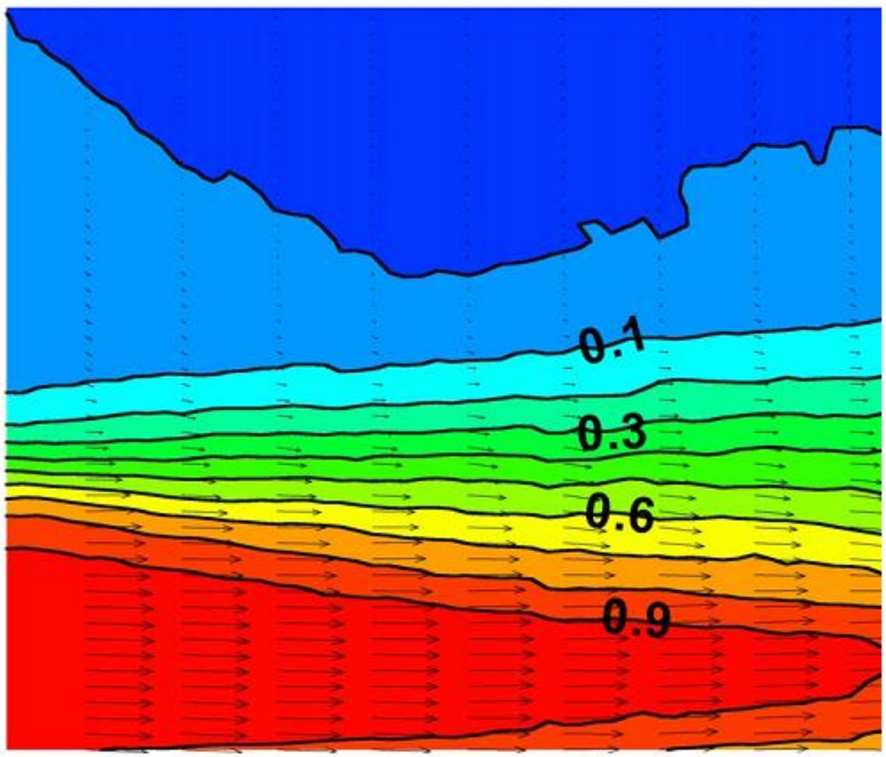


Influence of coutercurrent shear on turbulence level

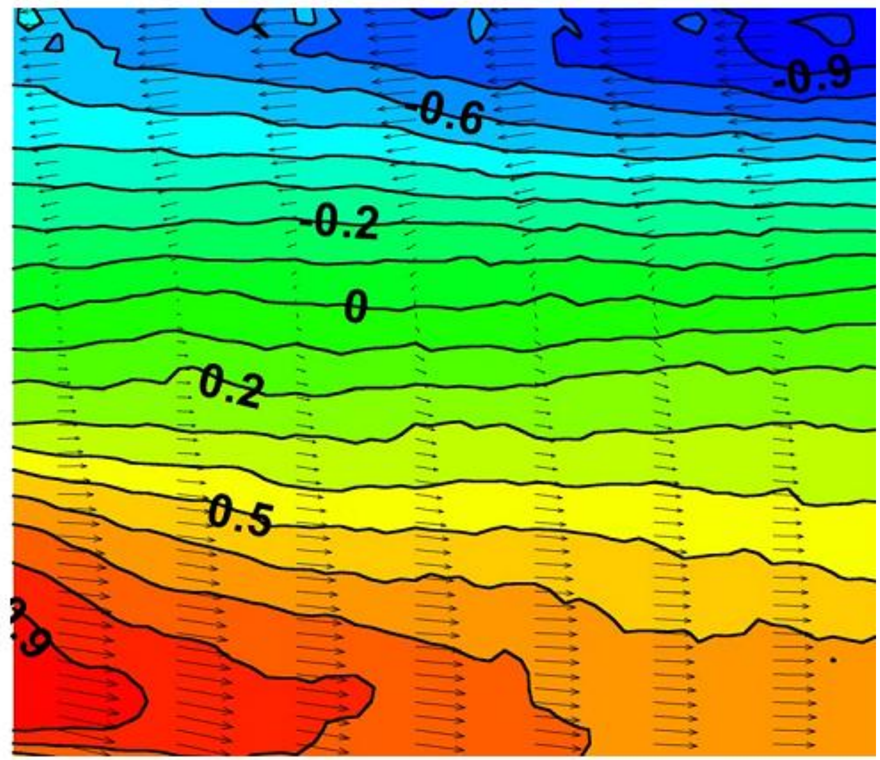


Influence of coutercurrent shear on turbulence level

Base flow



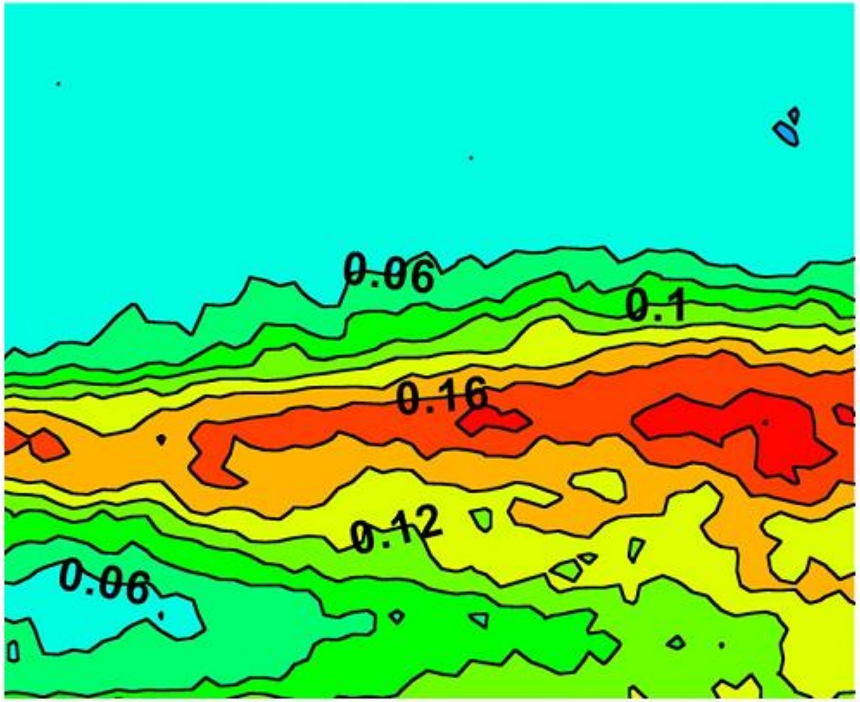
a) Single stream shear layer



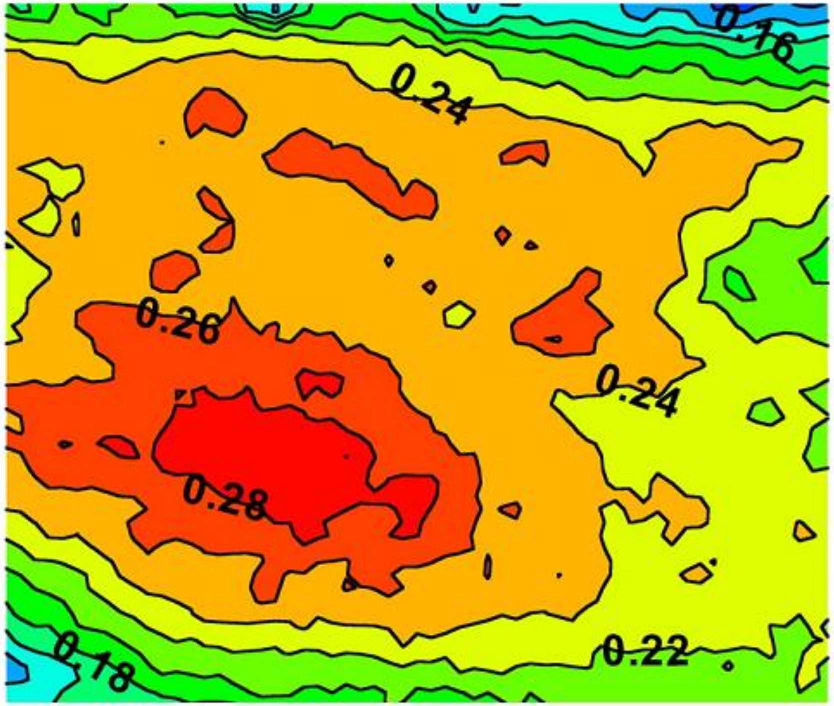
b) Countercurrent shear layer

Influence of coutercurrent shear on turbulence level

Turbulence intensity

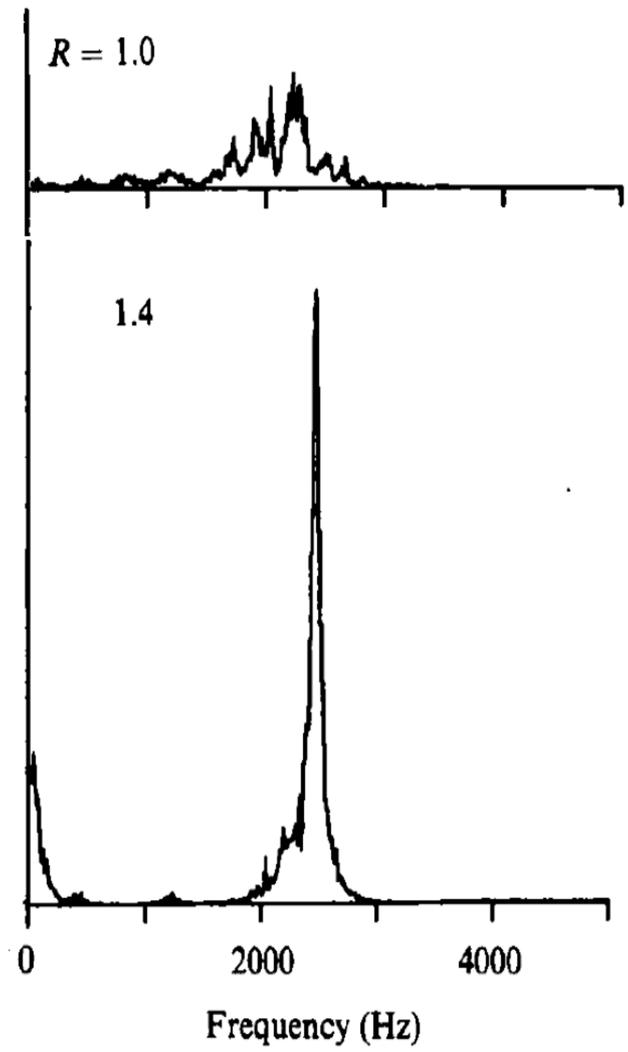


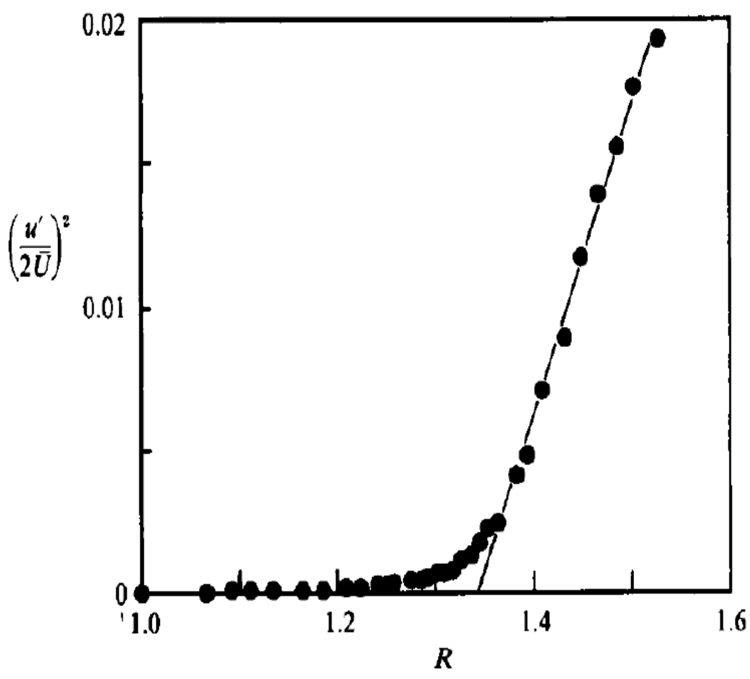
a) Single-stream shear layer



b) Countercurrent shear layer

THE MIXING LAYER: SHIFT TO OSCILLATOR !



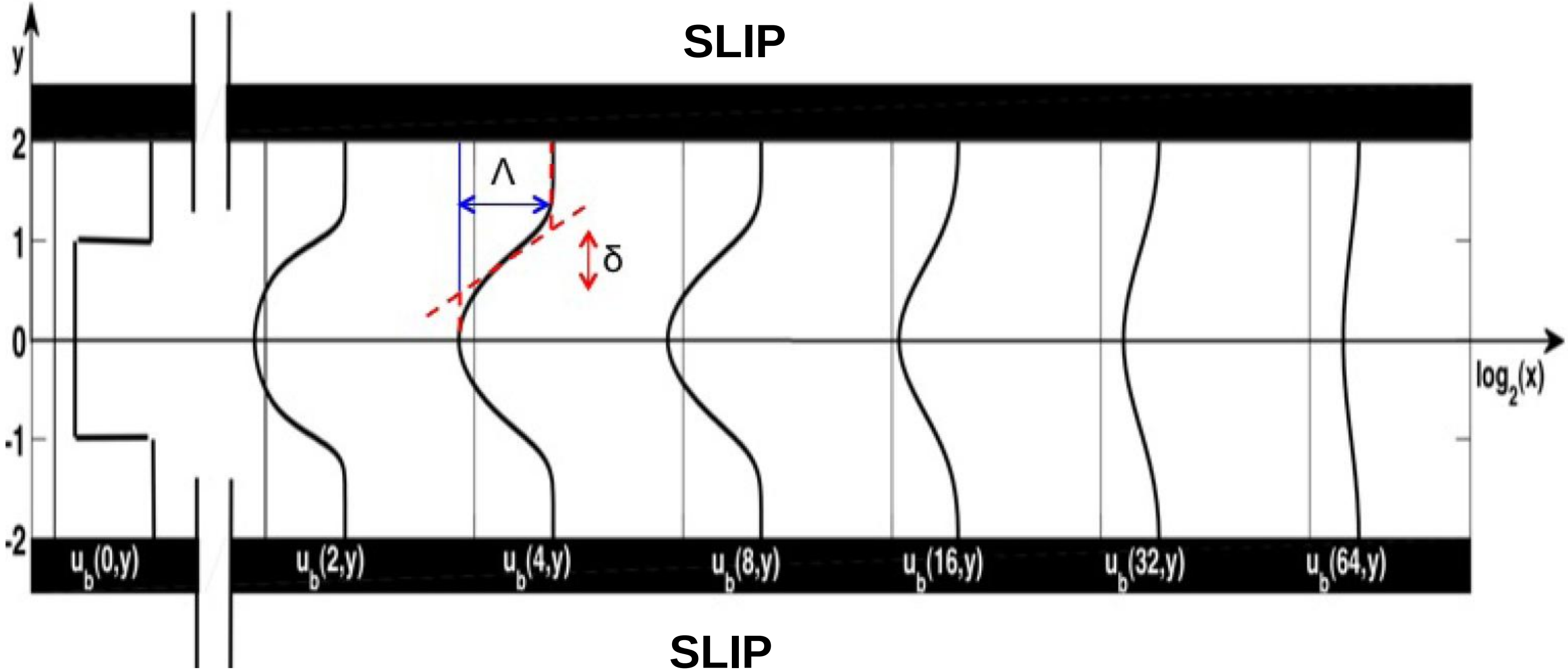


Strykowski & Niccum (1991)

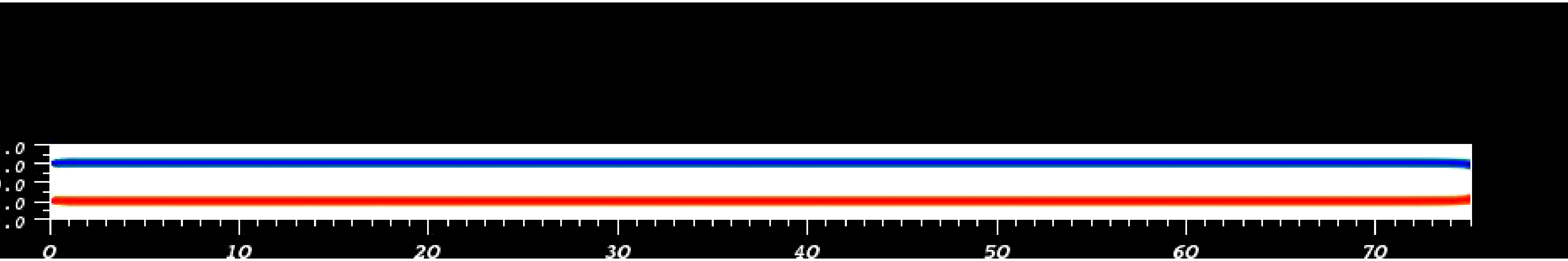
Direct Numerical Simulations with top-hat profile at inlet

Viscous diffusion → Non-parallel flow

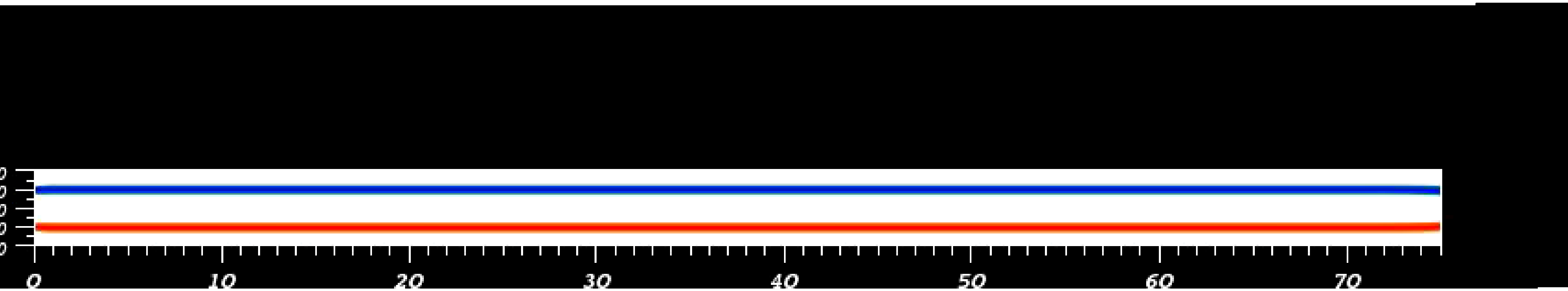
- $\Lambda_{loc} = (U_{max} - U_{min})/(U_{max} + U_{min})$
- $\delta = (U_{max} - U_{min})/(|dU/dy|_{max})$



Vorticity field: Re = 100, h = 1



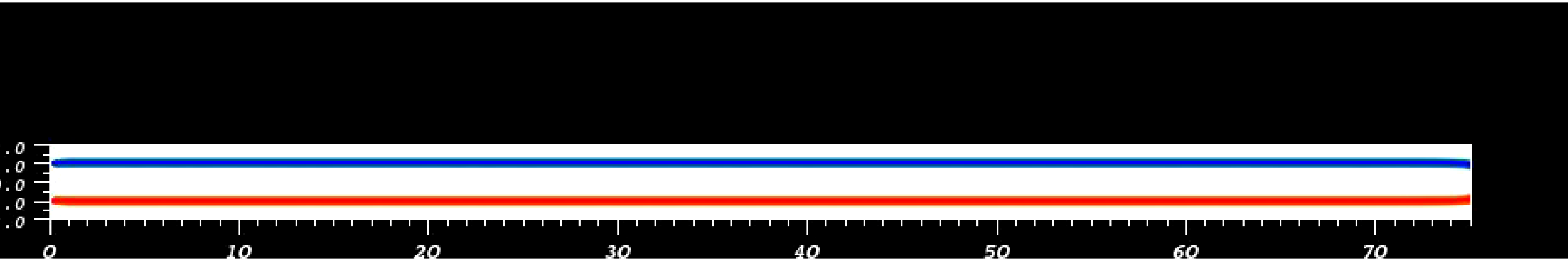
Λ = -0.739



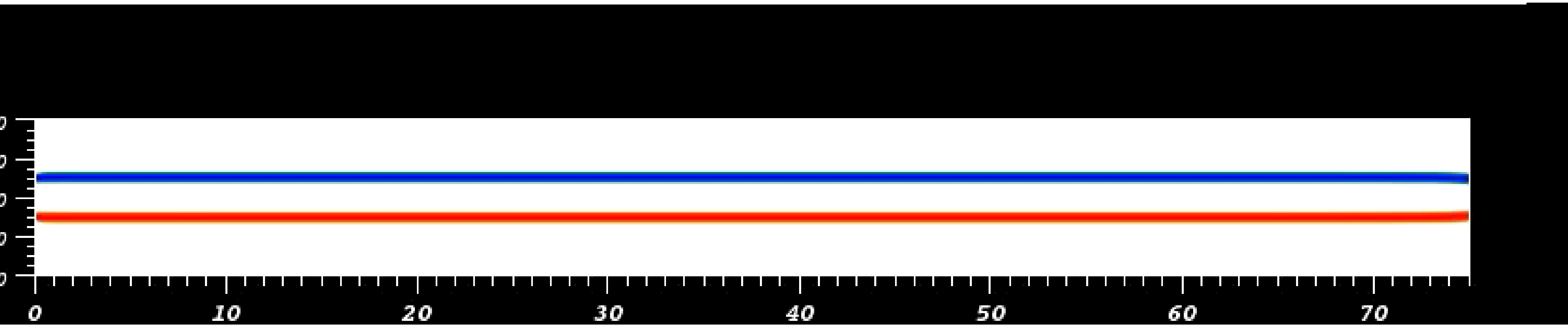
Λ = -0.667

An increase in Λ (more coflow) advects the perturbation

Vorticity field: Re = 100, Λ = -0.739



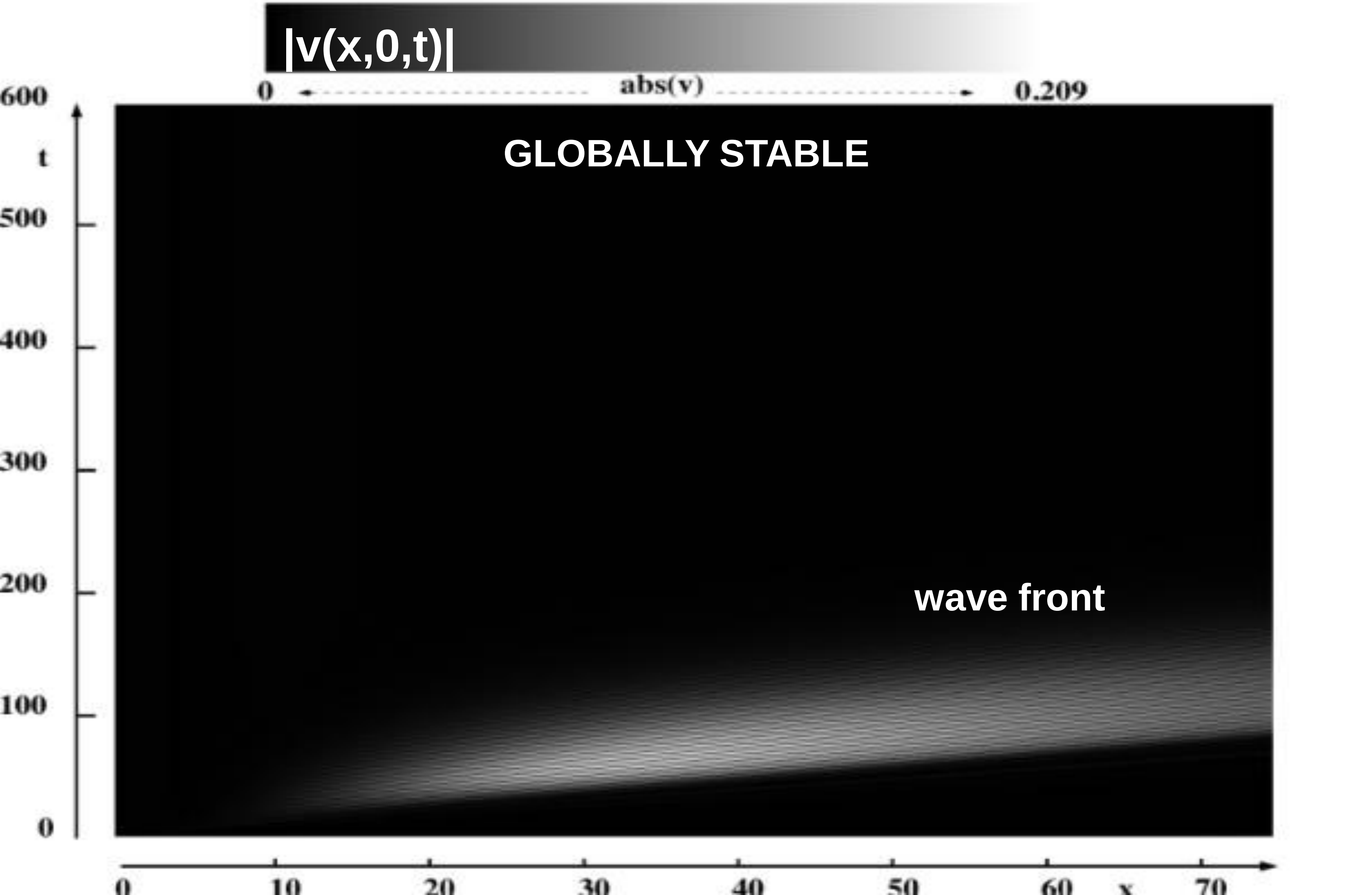
h=1



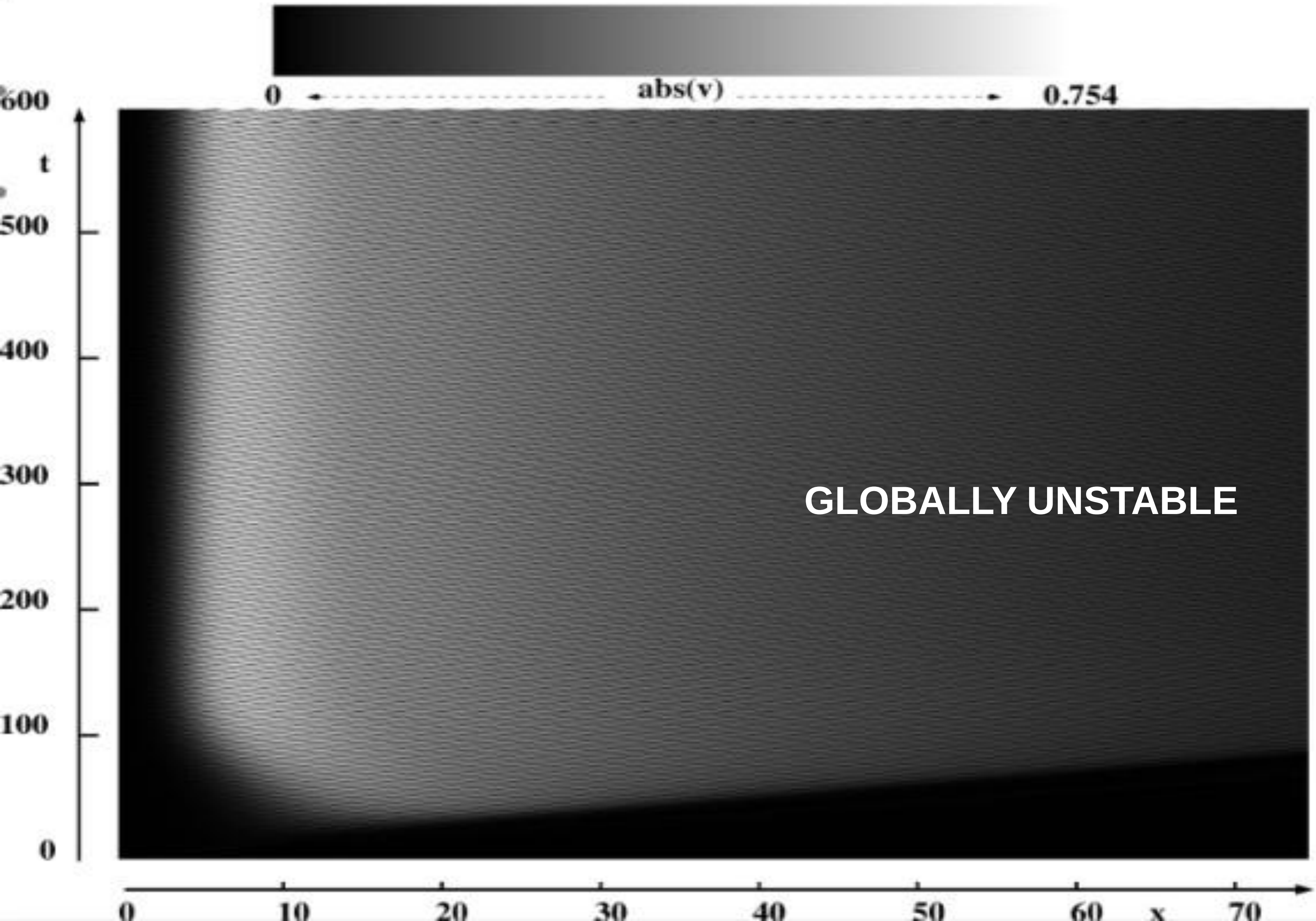
h=3

Destabilizing influence of confinement!

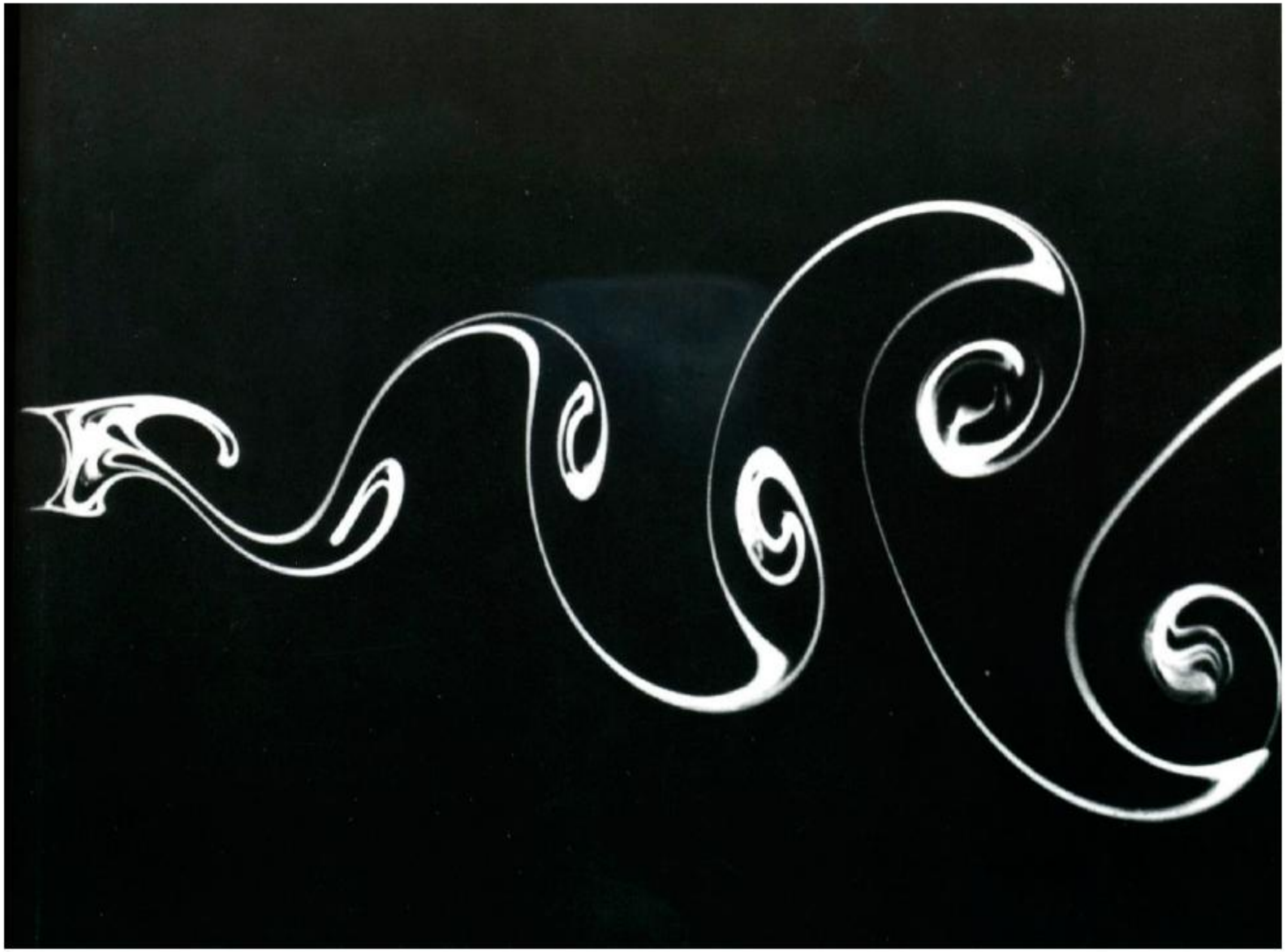
Spatio-temporal diagram, h=1 and Λ = -0.667



Spatio-temporal diagram, h=1 and Λ = -0.739



THE BLUFF BODY WAKE: A TYPICAL FLOW OSCILLATOR



Re = 140
Periodic
flow

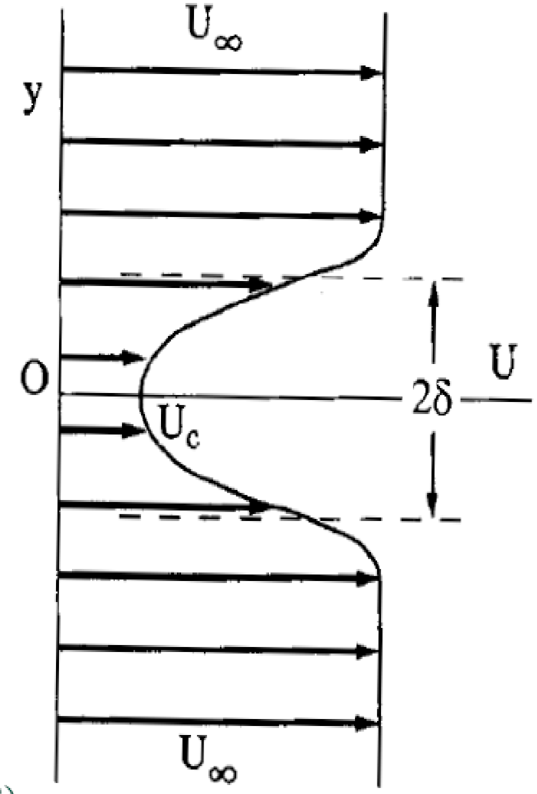
Taneda (1982)

ABSOLUTE/CONVECTIVE INSTABILITY IN PARALLEL WAKES

Family of wake profiles

$$U(y) = U_\infty + (U_c - U_\infty)\, U_1\left(\frac{y}{\delta}; N\right)$$

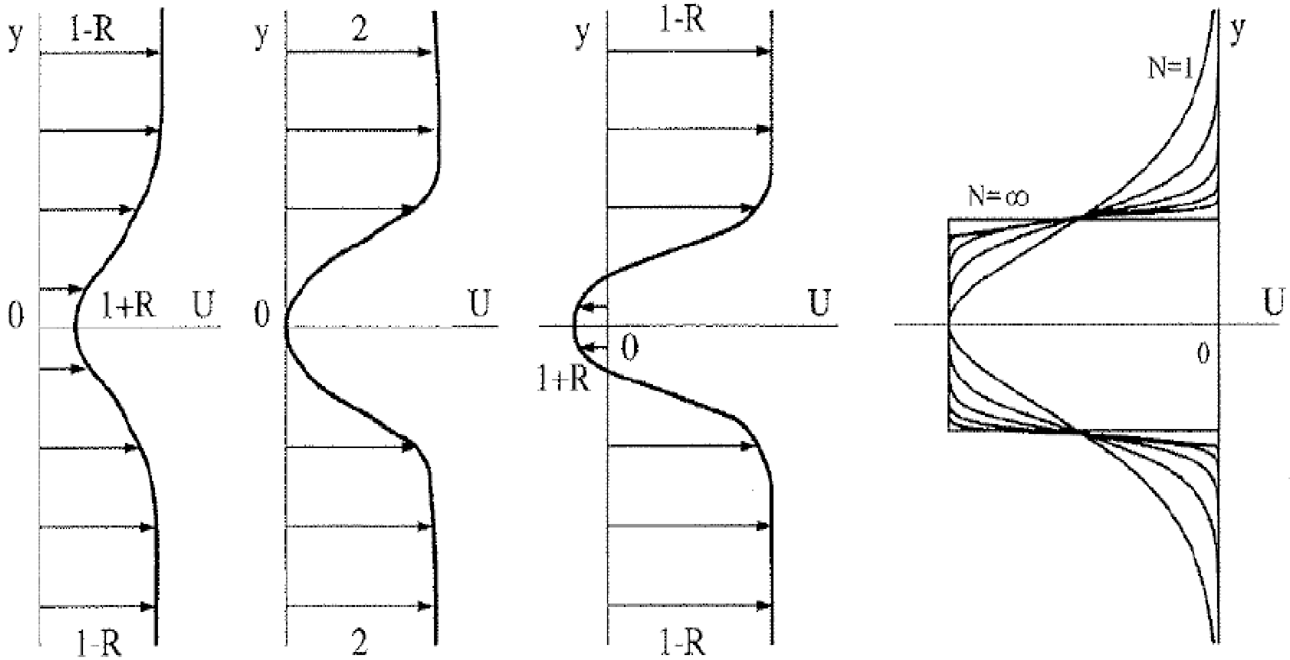
$$U_1(\xi; N) = [1 + \sinh^{2N}\{\xi \sinh^{-1}(1)\}]^{-1}$$



Monkewitz (1988)

ABSOLUTE/CONVECTIVE INSTABILITY IN PARALLEL WAKES

Family of wake profiles



(a) -1 < R < 0

(b) R = -1

(c) R < -1

(d)

Effect of velocity ratio R

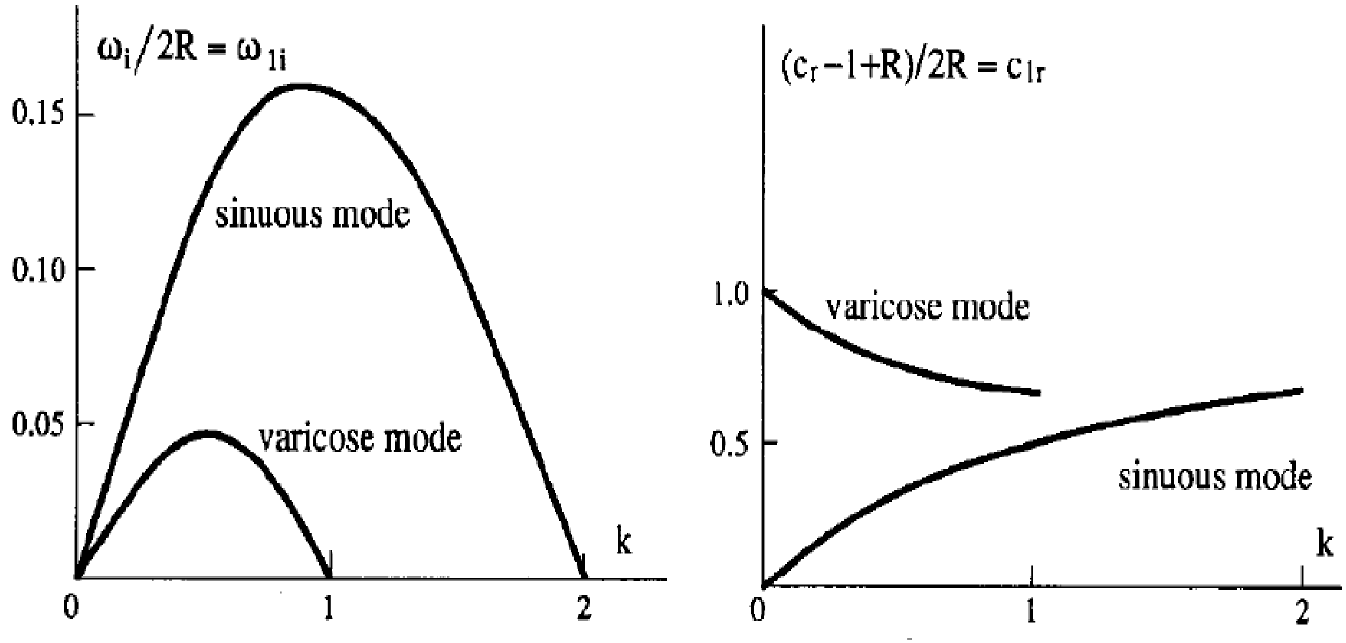
Effect of N

ABSOLUTE/CONVECTIVE INSTABILITY IN PARALLEL

2D PARALLEL FLOW CONCEPTS

$\mathrm{sech}^2\, y$ wake

Temporal approach

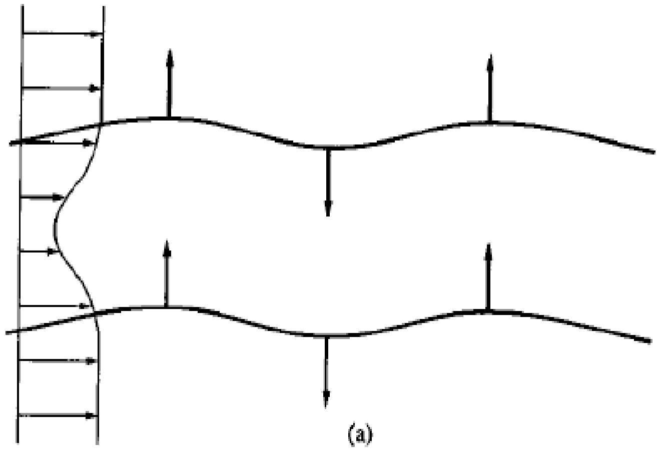


Betchov & Criminale (1966)

2D PARALLEL FLOW CONCEPTS

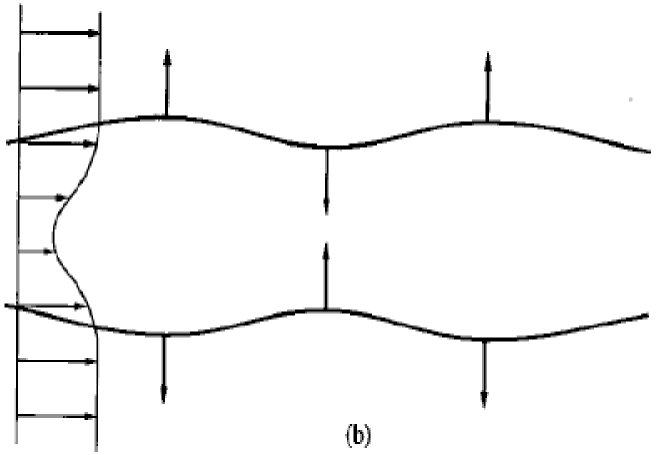
$\mathrm{sech}^2\, y$ wake

Sinuous and varicose modes



(a)

sinuous

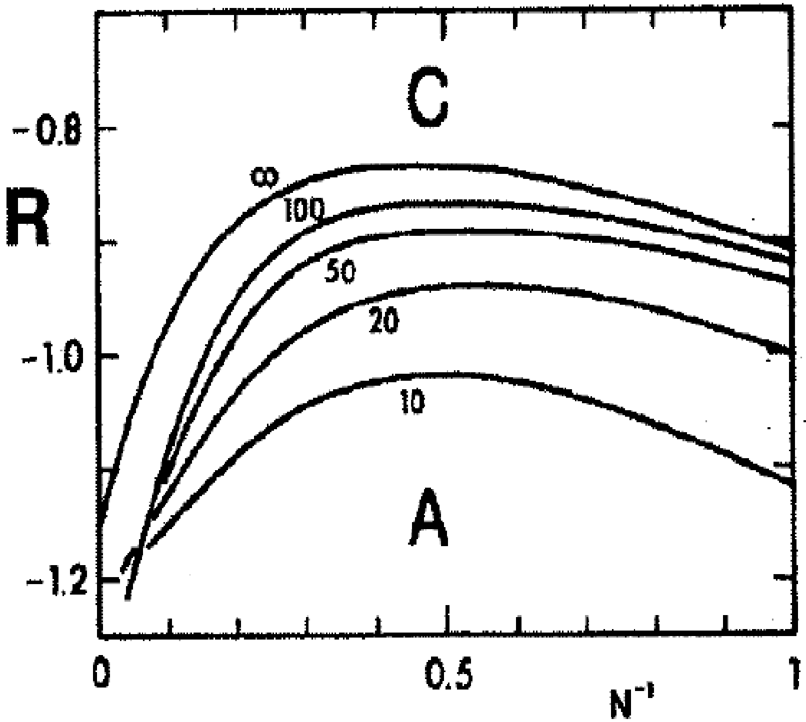


(b)

varicose

ABSOLUTE/CONVECTIVE INSTABILITY IN PARALLEL WAKES

Effect of steepness, velocity ratio and Reynolds number



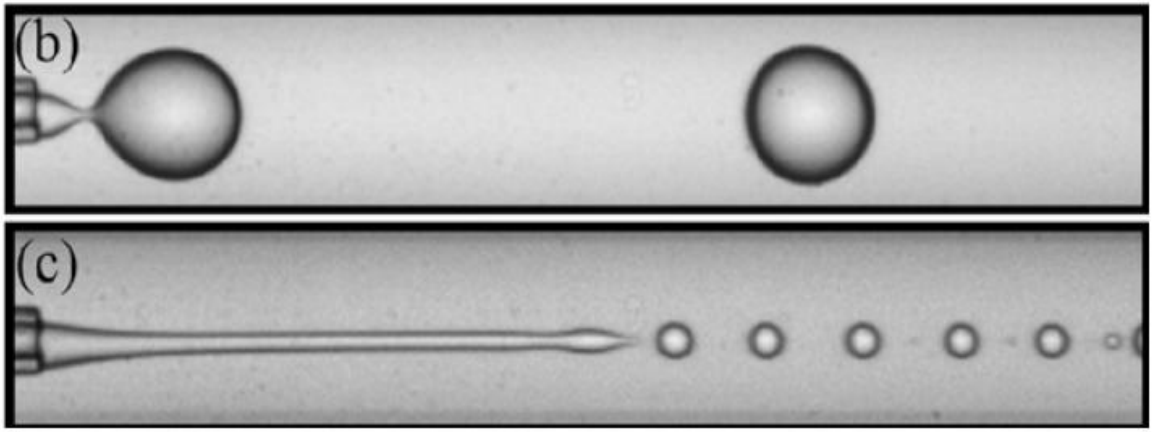
Monkewitz (1988)

LOCAL INSTABILITY BEHAVIOR OF CYLINDER WAKE

$5 < Re < 25$ **Convective instability**

$25 < Re < 48.5$ **Absolute instability**

Dripping/Jetting transition linked to absolute/convective transition?



Absolutely unstable

Convectively unstable

Guillot et al. (2008), Utada et al. (2008)

5. Dispersion relation

$$\omega = Uk \pm \sqrt{\frac{\gamma k^2}{\rho}\left(k^2 - \frac{1}{R_0^2}\right)\frac{I_0'(kR)}{I_0(kR)}}$$

- **Unstable if there exists one ω, Im(ω)>0** **at $k<1/R_0$**
- **Neutral if for all ω, Im(ω)=0** **at $k>1/R_0$**
- **Stable (or damped) if for all ω, Im(ω)<0:**

The flow considered is not damped, we have neglected dissipation by neglecting viscosity

Destabilisation d'un jet

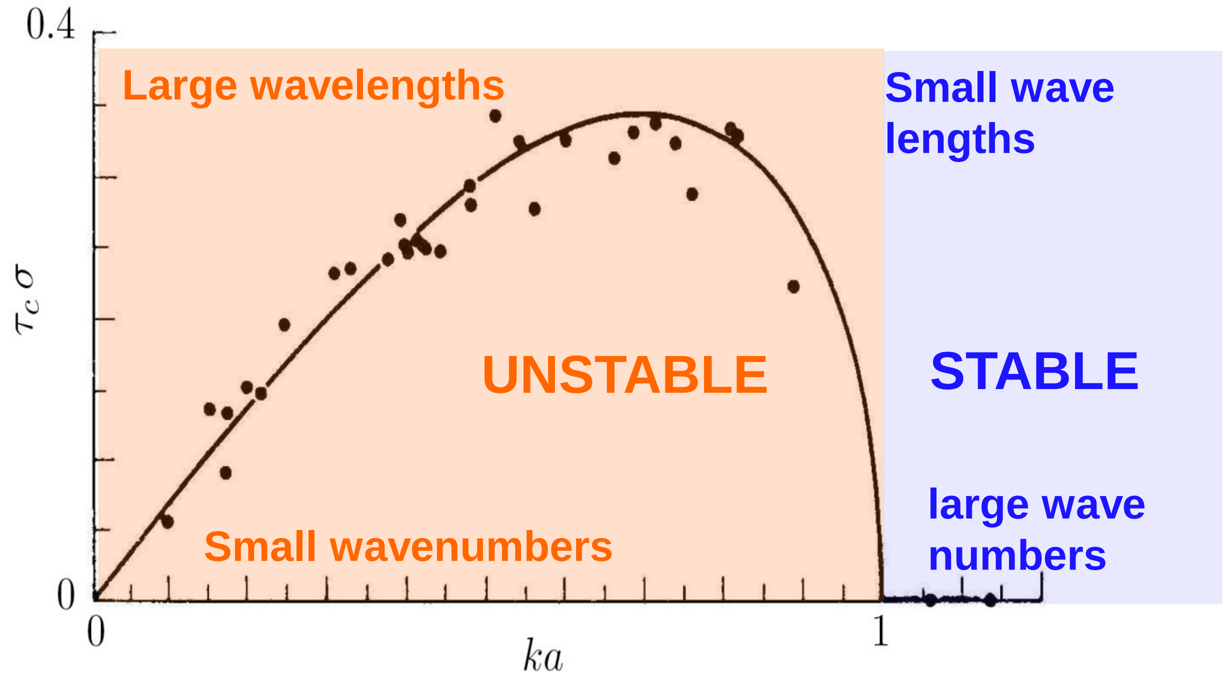
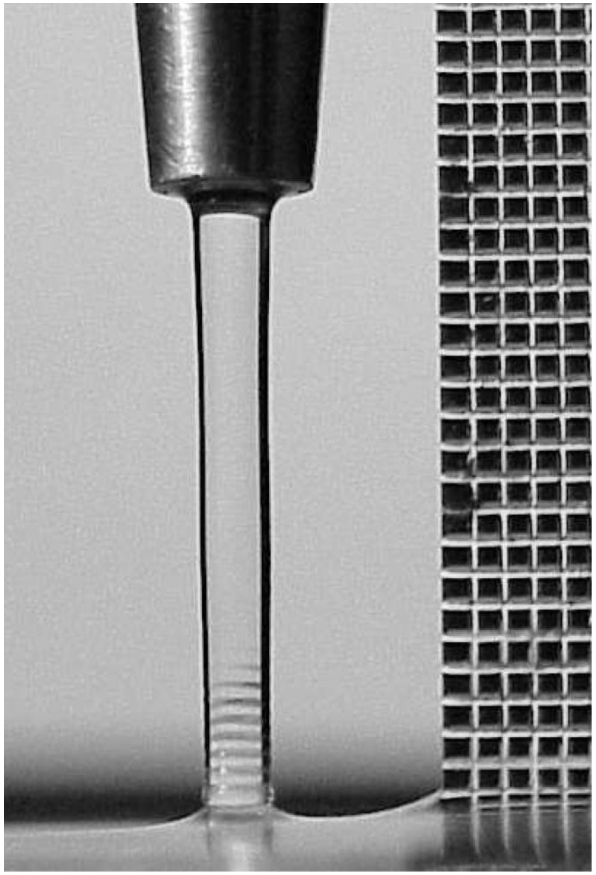


FIG. 2.10 – *Taux de croissance* $\tau_c\sigma$*, avec* $\tau_c = \sqrt{\rho a^3/\gamma}$*, de l'instabilité d'un filet fluide non visqueux, et points expérimentaux. D'après (Drazin & Reid 2004).*



Surface tension is destabilizing as a consequence of the radial curvature

Surface tension is stabilizing as a consequence of the axial curvature