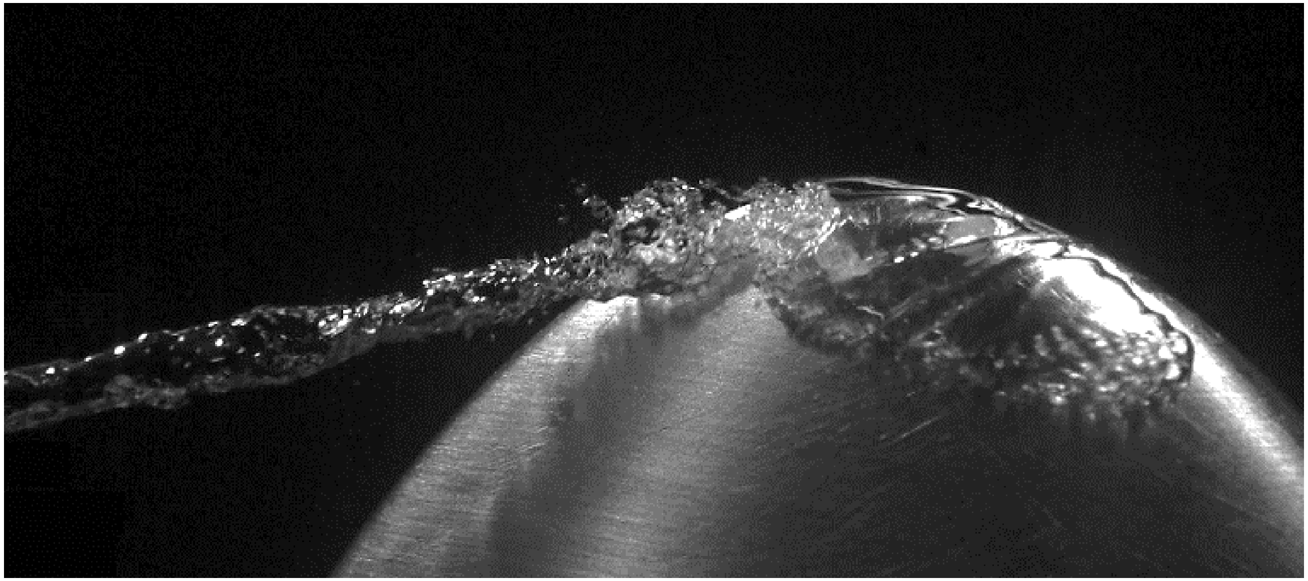
ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
SGM - 6th & 8th Semester, Fall 2025

CAVITATION AND INTERFACE PHENOMENA

Chapter 5: Vortex Cavitation
5.2: Effect of Gas Content on Tip Vortex Cavitation



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Velocity and pressure fields in a vortex

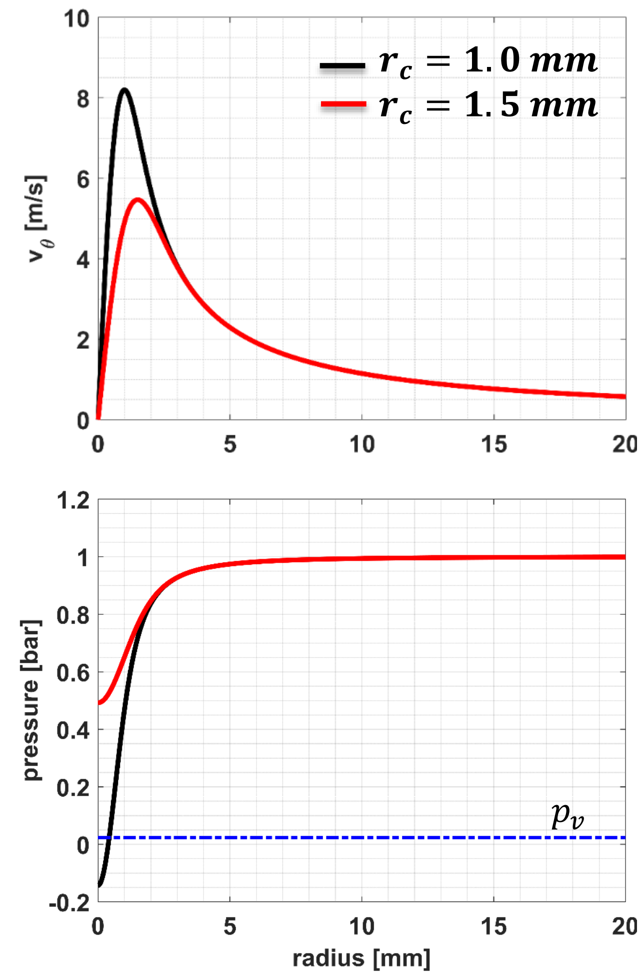
Lamb-Oseen vortex model:

$$v_\theta(r) = \frac{\Gamma}{2\pi r}\left(1 - e^{-1.256(r/r_c)^2}\right)$$

- r_c is the ***viscous core radius*** (location of $u_{\theta_{max}}$).
- Γ is the vortex strength or circulation.

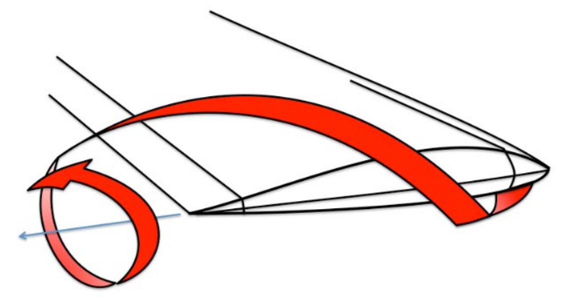
Let's try an example. Consider a vortex with $\Gamma = 0.07\,\frac{m^2}{s}$

- ➢ What is the effect of the viscous core?
  - ▪ Velocity profile for $r_c = 1$ and $1.5\ mm$
  - ▪ Significant influence on $v_{\theta_{max}}$ and p_{min}
- ➢ Pressure profile $\left(\frac{\partial p}{\partial r} = \frac{\rho v_\theta^2(r)}{r}\right)$

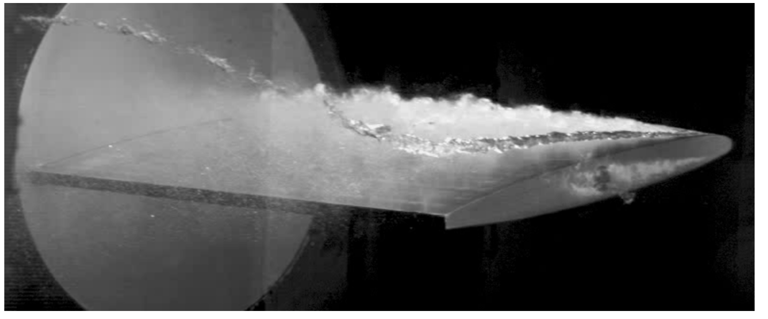


Tip vortex flow and cavitation

- **Finite-span lifting bodies** generate tip vortices
- **Low static pressures** develop in the vortex core
  - ➢ Tip vortices are very prone to cavitation
  - ➢ Usually the **first** type of cavitation that incepts



- Occurrence of Tip Vortex Cavitation (TVC) may lead to **severe erosion, noise emission, and structural vibrations**
- Very sensitive to gas content



Visualization of TVC on NACA-0009 hydrofoil

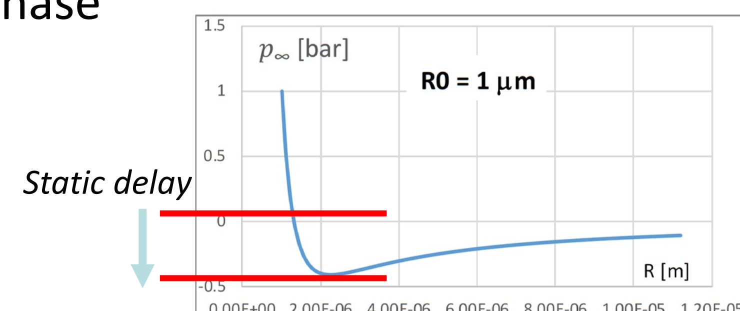
What is gas content?

Gas content

- Micro bubbles → Required for cavitation inception
  - Critical pressure and radius:

$$R_c = \sqrt{\frac{3 p_{g_0} R_0^3}{2S}} \qquad \text{and} \qquad p_c = p_v - \frac{4S}{3R_c}$$

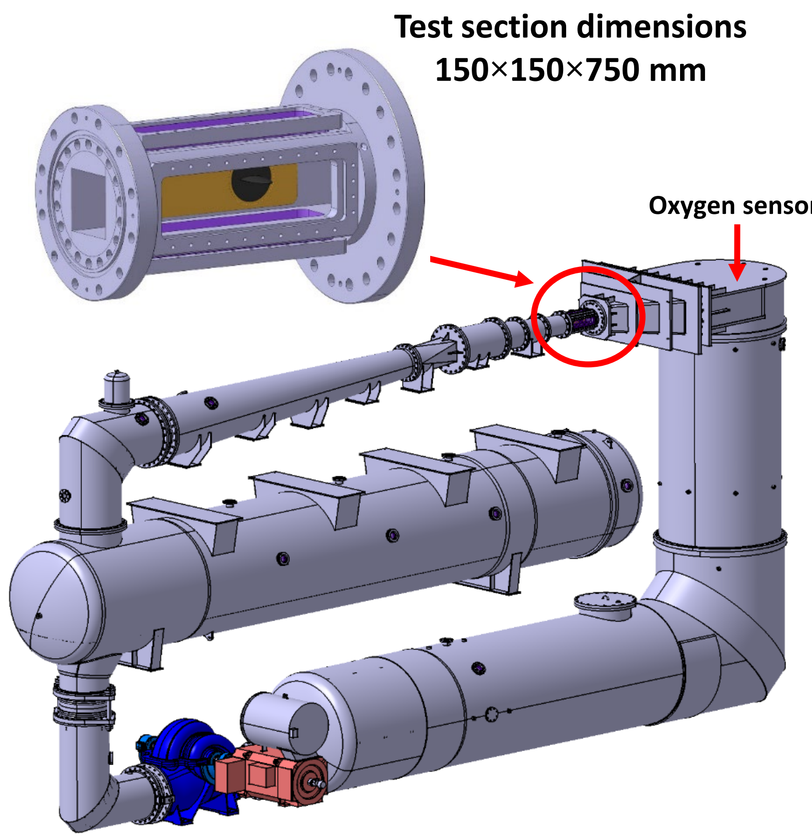
- Dissolved gases → molecular Oxygen, Nitrogen, etc.
  - Diffusion into the vaporous phase



Experimental Setup

High-speed cavitation tunnel

- Maximum velocity → **50 m/s**
- Maximum pressure → **16 bars**
- Velocity measurements
  - Stereo-PIV & LDV
- Various sensors
  - Oxygen saturation level
  - Lift and drag forces
- Flow visualization
  - Still photography
  - High-speed visualization

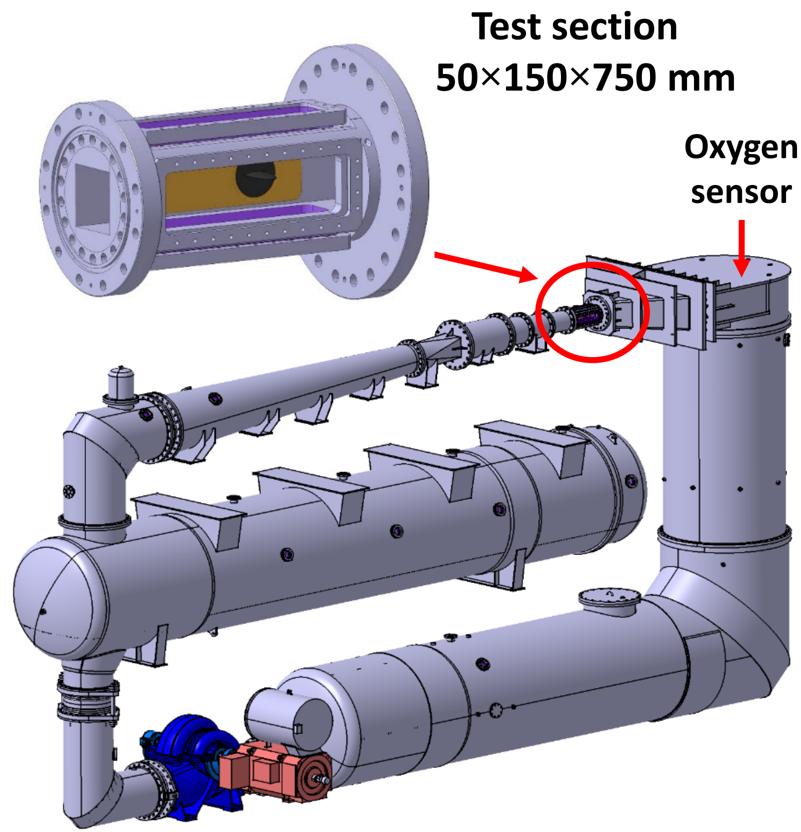


Experimental Setup

High-speed cavitation tunnel

- Maximum velocity → **50 m/s**
- Maximum pressure → **16 bars**

- **Control of gas content level:**



Super-cavitation
Degassing

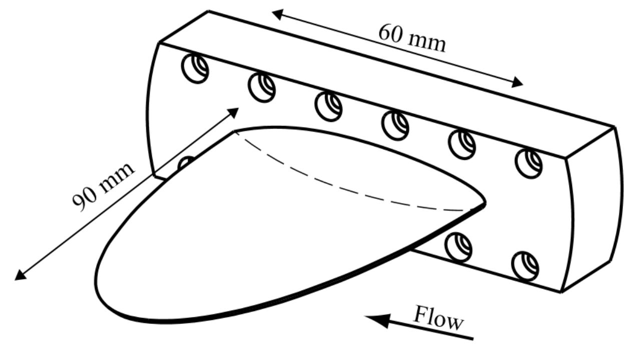
Free-surface flow
Engassing

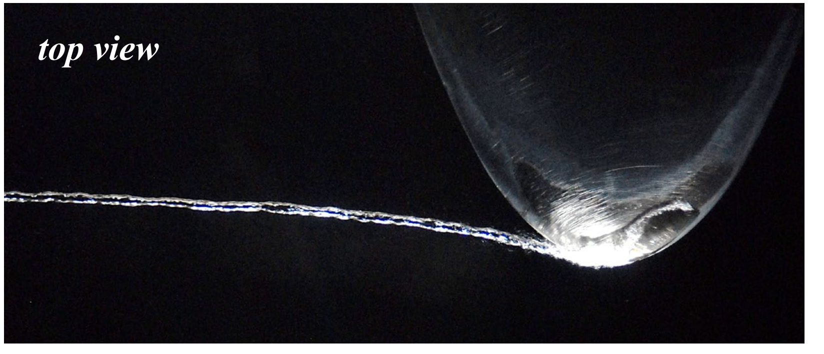
Experimental Setup

– **Cross-section:**
  - NACA 16-020
  - Symmetrical

– **Planform:**
  - Elliptical

– **Main feature:**
  - Well-defined tip vortex



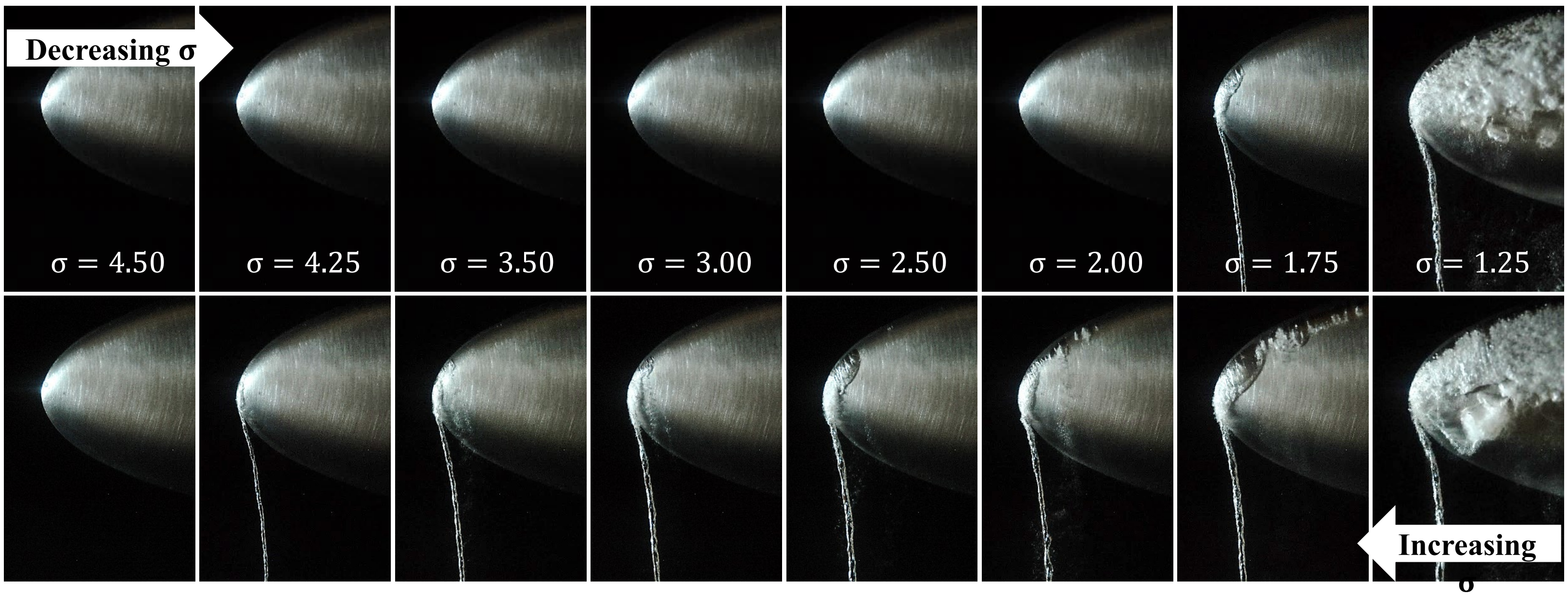


TVC Hysteresis

Hysteresis in TVC inception and desinence

Large inception-desinence thresholds observed for specific flow conditions

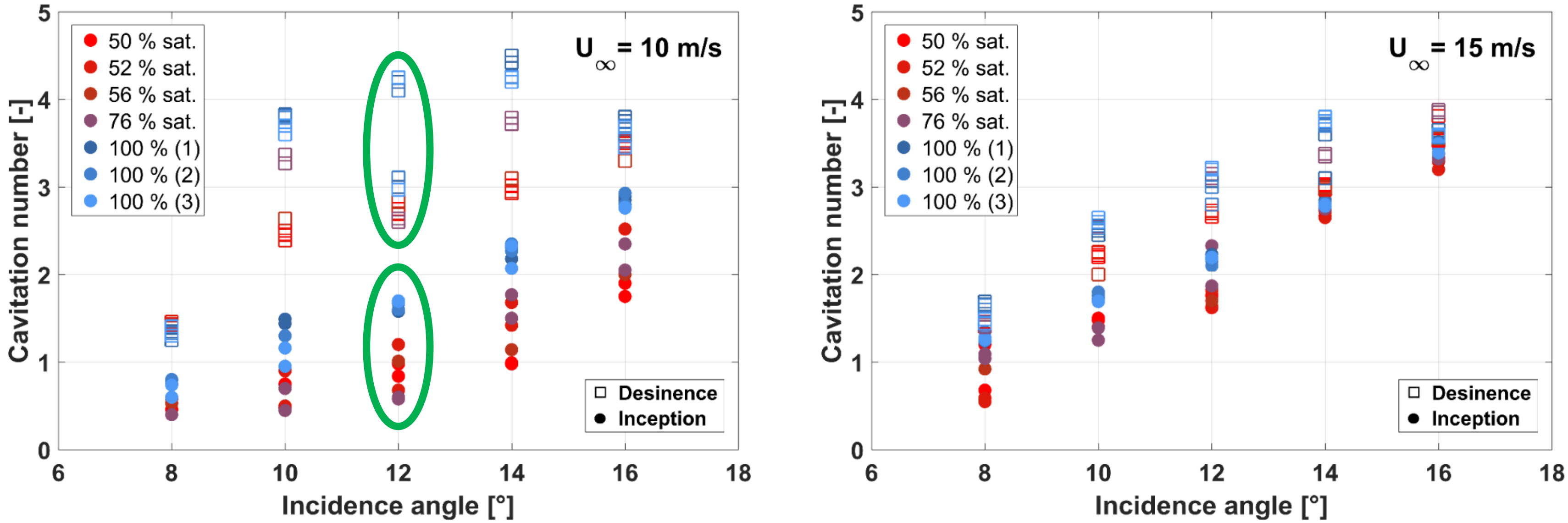
$U_\infty = 10\ \mathrm{m/s}, \alpha = 12°$



Ali Amini, Martino Reclari, Takeshi Sano, Masamichi Iino, Matthieu Dreyer, and Mohamed Farhat. "On the physical mechanism of tip vortex cavitation hysteresis." *Experiments in Fluids* 60, no. 7 (2019): 118.

TVC inception/desinence tests

- σ_i depends on gas content → more dissolved O_2 → higher concentrations of large nuclei
- σ_d depends on gas content → p_d is not necessarily equal to p_v and is usually higher
- The extent of the hysteresis depends on the bulk flow parameters.

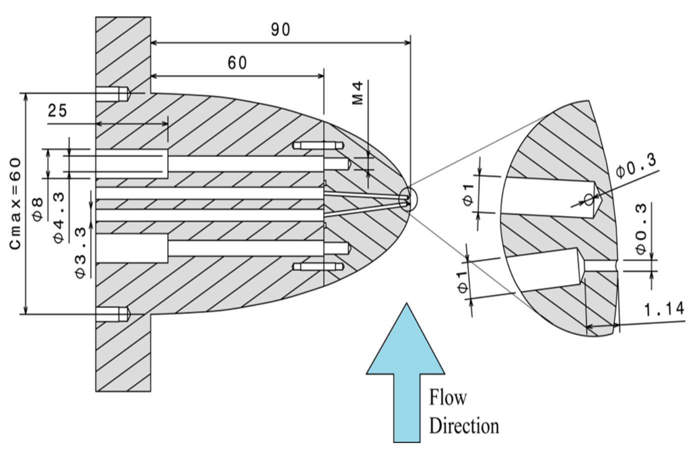


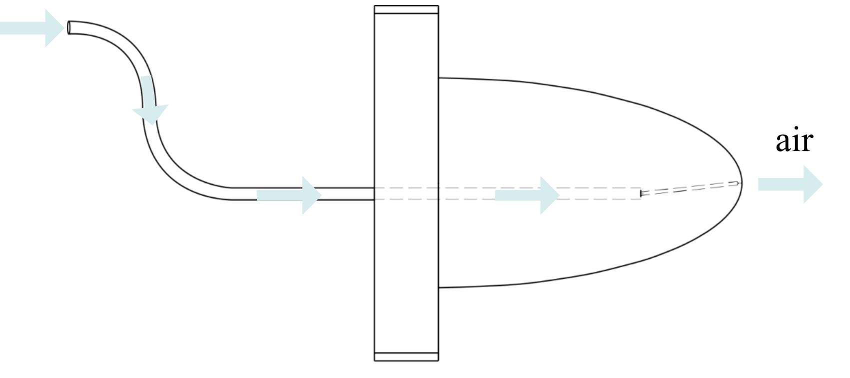
TVC hysteresis

- **The hysteresis depends on**
  - **The dissolved gas content** →
    - **Mass diffusion into TVC**
    - **Gaseous cavitation**
  - **Freestream velocity**
  - **Angle of attack** →
    - **Boundary layer characteristics**
    - **Wake flow patterns**

Transient air injection

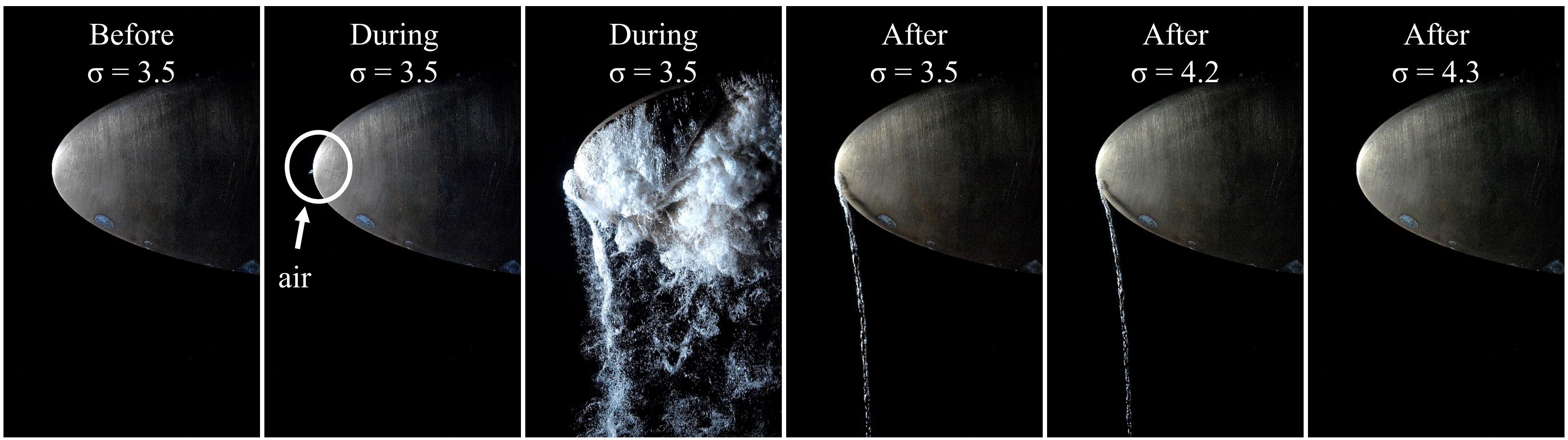
- A specific hydrofoil equipped with a 0.3-mm ejection channel at the tip
- Flow conditions: $U_\infty = 10\ m/s\,, \alpha = 12°$
  - ➢ Initial saturation level ~ **98 %**
  - ➢ At injection: $\sigma_{initial} = 3.5$ **(No cavitation)**





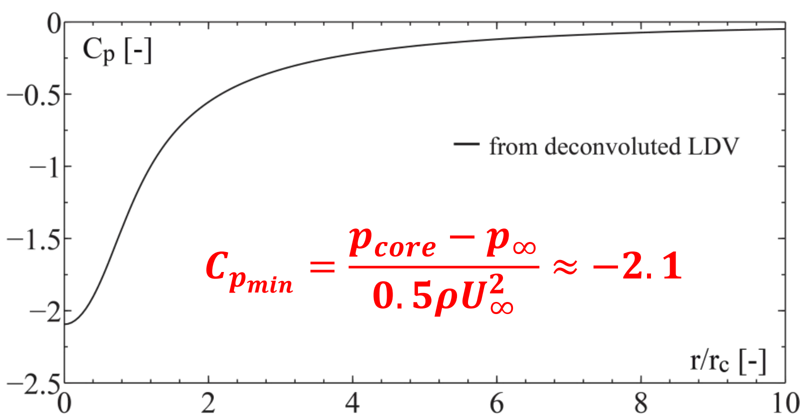
Transient air injection

- **Flow conditions:** $U_\infty = 10\ m/s, \alpha = 12°, \sigma_{initial} = 3.5$



Transient air injection

- **What is the pressure in the vortex core?**
  - ➢ LDV measurements have revealed that for $U_\infty = 10\ m/s$ and $\alpha = 12°$:



Calculation of Cp_{min} by Dr. M. Dreyer in his PhD thesis (EPFL)

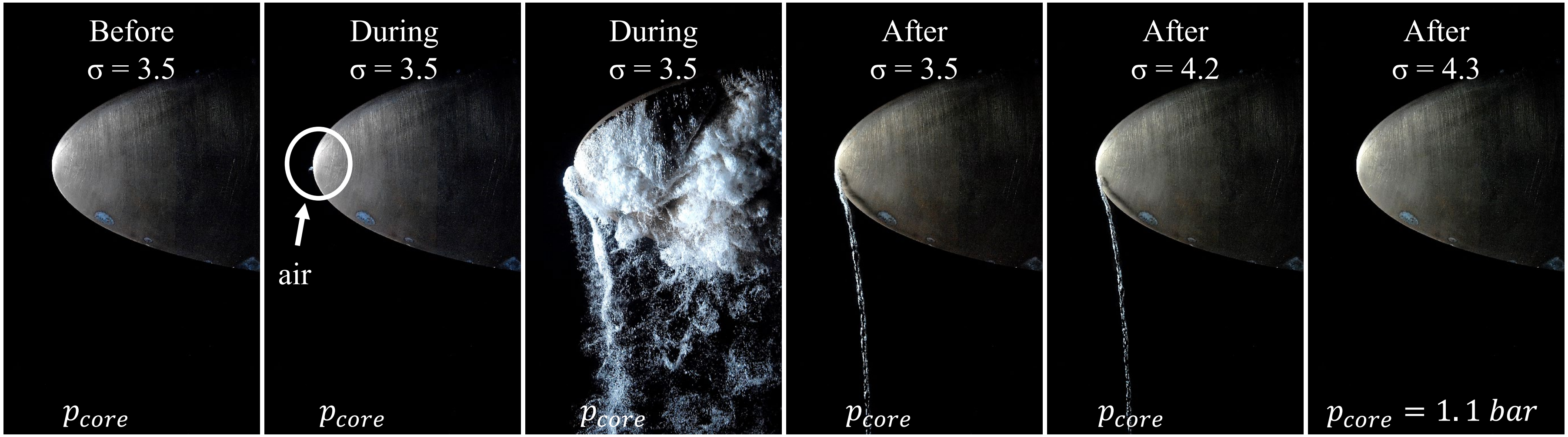
Cp_{min} is equal to -2.1

From the definitions of cavitation number and pressure coefficient ($p_v \approx 0$):

$$p_{core} = \frac{1}{2}\rho U_\infty^2(\sigma + C_{p_{core}})$$

Transient air injection

- **Flow conditions:** $\boldsymbol{U_\infty = 10\ m/s\,, \alpha = 12°, \sigma_{initial} = 3.5}$
- Pressure coefficient at the core is measured: $\boldsymbol{C_{p_{min}} \approx -2.1}$

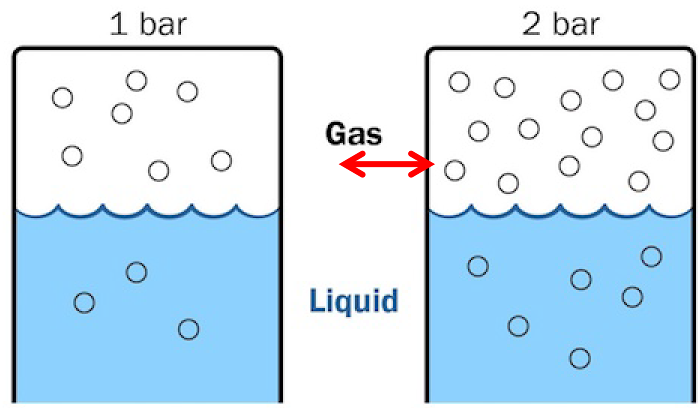


TVC disappears at ≈1 bar, which is the initial saturation pressure (p_{sat_i}) of water

Transient air injection

Henry's law: solubility of a gaseous species in a liquid is proportional to its partial pressure

$$c = \mathcal{H} p_g$$



c: concentration of the i species in the liquid phase $[\frac{mol}{m^3}]$

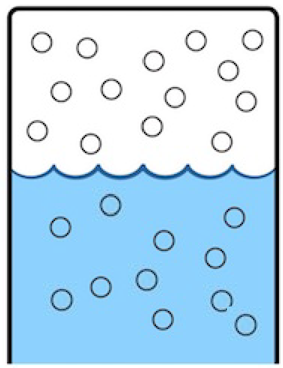
$\mathcal{H}$: Henry's law constant $[\frac{mol}{m^3.atm}]$ (varies for different substances and with temperature)

p_g: partial pressure of the i species in the gas phase $[atm]$

Transient air injection

What happens for a cavitating tip vortex?

$p_{sat_i} = 1\ bar$



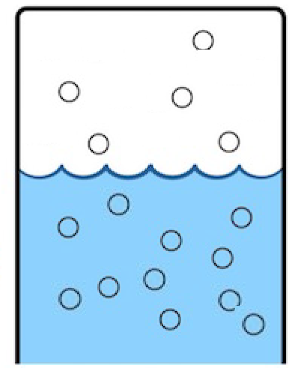
Equilibrium at rest

Saturated mixture

Formation of sub-atmospheric cavity within the tip vortex

$$\boldsymbol{c_i = \mathcal{H} p_i}$$

$p < p_{sat_i}$



Depressurized due to rotational motion

Diffusion of dissolved gases into the cavity

Super-saturated mixture

Transient air injection

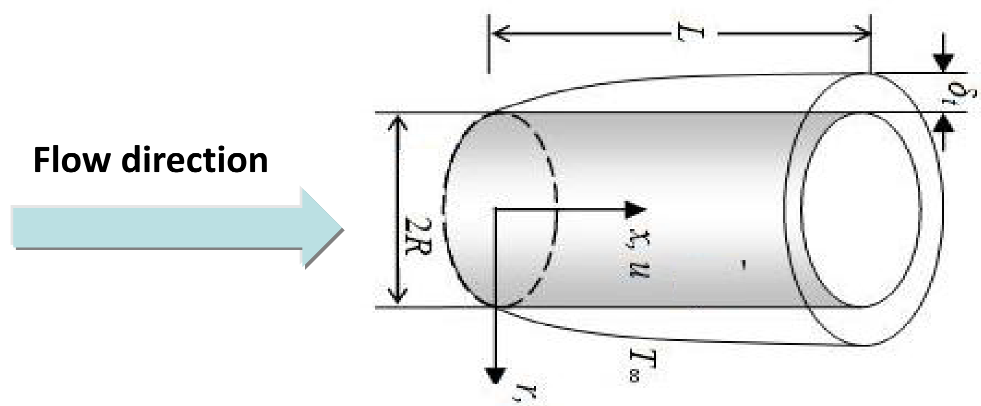
- **Once a cavity forms within the tip vortex:**
  - Water flowing around it is exposed to a gaseous medium with a pressure (p_{core}) that deviates from the initial saturation condition (p_{sat_i})
  - **While** there is **no change** in the amount of dissolved gases.
- **Therefore in our case:**
  - For $\boldsymbol{\sigma > 2.1 \rightarrow p_{min} > p_v}$
    - No more vaporization occurs.
  - For $\boldsymbol{\sigma < 4.2 \rightarrow p_{min} < p_{atm} = p_{sat_i}}$
    - Water flowing along the cavity is **supersaturated**. As a result, the dissolved air diffuses into the cavity and forms a gaseous TVC.

Modelling the outgassing process

- A concentration boundary layer develops along the *cylindrical* cavity:

$$\frac{\partial c}{\partial t} + v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r}\frac{\partial c}{\partial \theta} + v_z \frac{\partial c}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(D r \frac{\partial c}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D \frac{\partial c}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(D \frac{\partial c}{\partial z}\right)$$

- *c*: concentration of the species
- *D*: mass diffusion coefficient

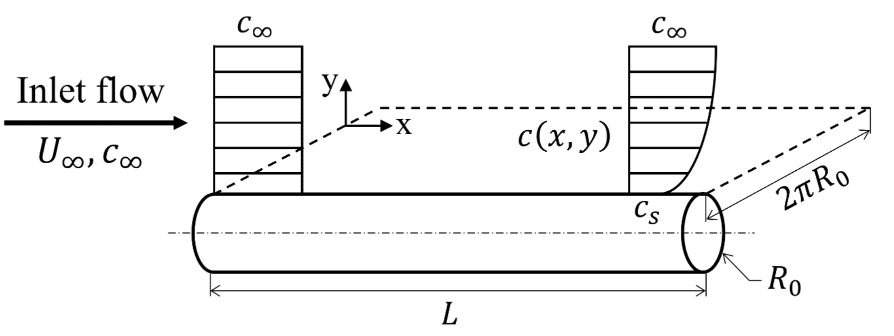


Development of a concentration boundary layer similar to a thermal boundary layer

Modelling the outgassing process

- **Outgassing** → A convective diffusion that occurs along a hollow cylinder
- We employ the '**wrap-around**' technique → Cylindrical to planar coordinates
- Two-dimensional Cartesian species transfer equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right)$$



Modelling the outgassing process

❑ **Assumptions:**

- Constant velocity field: $\boldsymbol{u = U_\infty, v = 0}$
- Steady state
- Large Peclet numbers $\left(\boldsymbol{Pe = \frac{U_\infty L}{D}}\right)$
  - ➢ Diffusion in *x* is negligible compared to diffusion in *y*
- A concentration boundary layer develops along the TVC interface
  - ➢ The thickness (δ) scales with $\sqrt{\boldsymbol{Dx/U_\infty}}$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right)$$

⇩

$$\frac{\partial c}{\partial x} = \frac{D}{U_\infty}\frac{\partial^2 c}{\partial y^2}$$

$$\boldsymbol{BC's} \rightarrow \begin{cases} x = 0 & \rightarrow & c(0, y) = c_\infty \\ y = 0 & \rightarrow & c(x, 0) = c_s \\ y \rightarrow \infty & \rightarrow & c(x, \infty) = c_\infty \end{cases}$$

Modelling the outgassing process

- Similarity solution in the form of

$$\frac{\partial c}{\partial x} = \frac{D}{U_\infty}\frac{\partial^2 c}{\partial y^2}$$

$$f(\eta) = \frac{c(\eta) - c_s}{c_\infty - c_s}$$

where $\eta = y/\delta = y\sqrt{U_\infty/Dx}$ is the similarity variable and $f(\eta)$ is the normalized concentration of the dissolved gases.

$$f'' + 0.5f' = 0$$

$$\begin{cases} \eta = 0 \rightarrow f(0) = 0 \\ \eta \rightarrow \infty \rightarrow f(\infty) \rightarrow 1 \end{cases}$$

Modelling the outgassing process

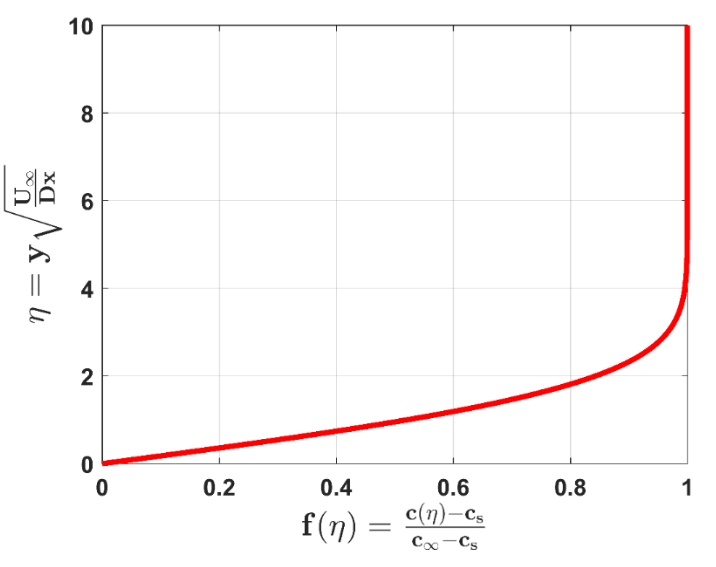
- This equation has an analytical solution:
- Local mass flux into TVC can be calculated by *Fick's second law*:

$$m_y''(x) = -D\left.\frac{\partial c}{\partial y}\right|_{y=0}$$

- Wrapping the solution around the TVC with radius R_0 and taking the integral along the whole length L, the total mass transfer rate in [mol/s] is found to be

$$\dot{M}_{tot} = 4R_0(c_\infty - c_s)\sqrt{\pi U_\infty D L}$$

$$f(\eta) = \mathrm{erf}(\eta/2)$$



Modelling the outgassing process

- Variation of air concentration with time and the characteristic time of the system (from 100% down to 36.8%):

$$\frac{dc}{dt} = -\frac{\dot{M}_{tot}}{V_{tot}} \Longrightarrow c(t) = (c_\infty - c_s)e^{(-t/\tau)} + c_s$$

$$\tau = \frac{V_{tot}}{4R_0\sqrt{\pi U_\infty D L}}$$

- With typical values of:
  - $R_0 = 1\,mm,\ U_\infty = 10\ m/s\,,\ L = 1\,m$
  - $D = 7 \times 10^{-5}\ m^2/s$ (calculated based on a mixing length model from Parkin & Ravindra (1991)).
  - $\sigma = 2.5$ and $C_{p_{min}} = -2.1$

Number of volume change per second is 4.25 and:

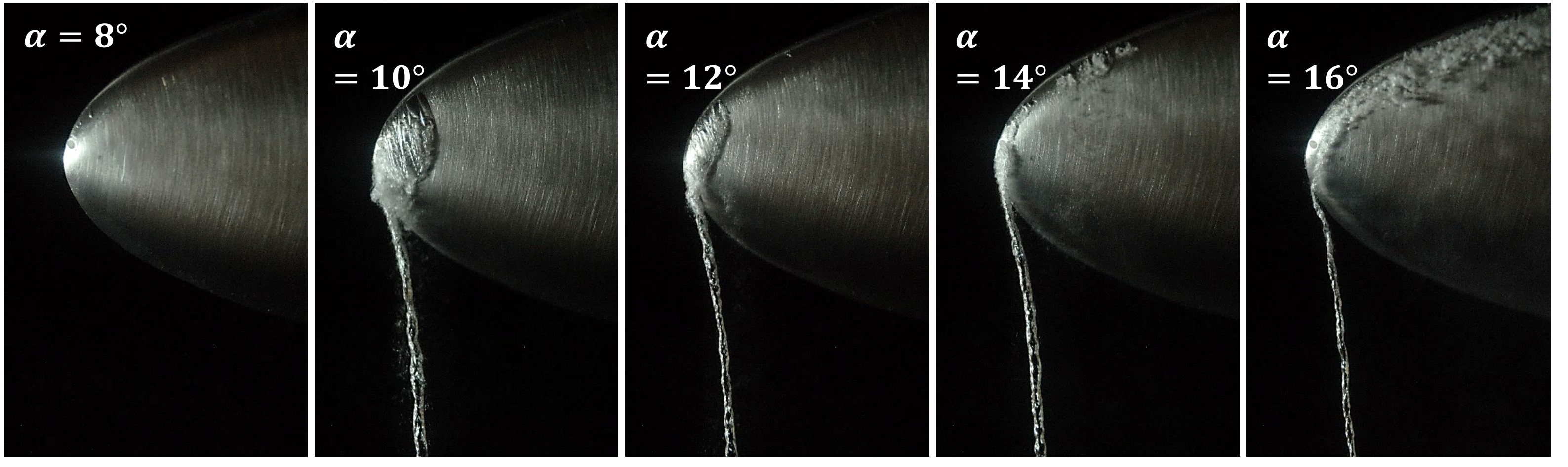
$$\frac{\dot{V}_{diff}}{\dot{V}_{conv}} = 0.43$$

→ $\tau = 6.8$ days !!!

Effect of Flow Structures on Outgassing Process

Effect of incidence angle on TVC Hysteresis

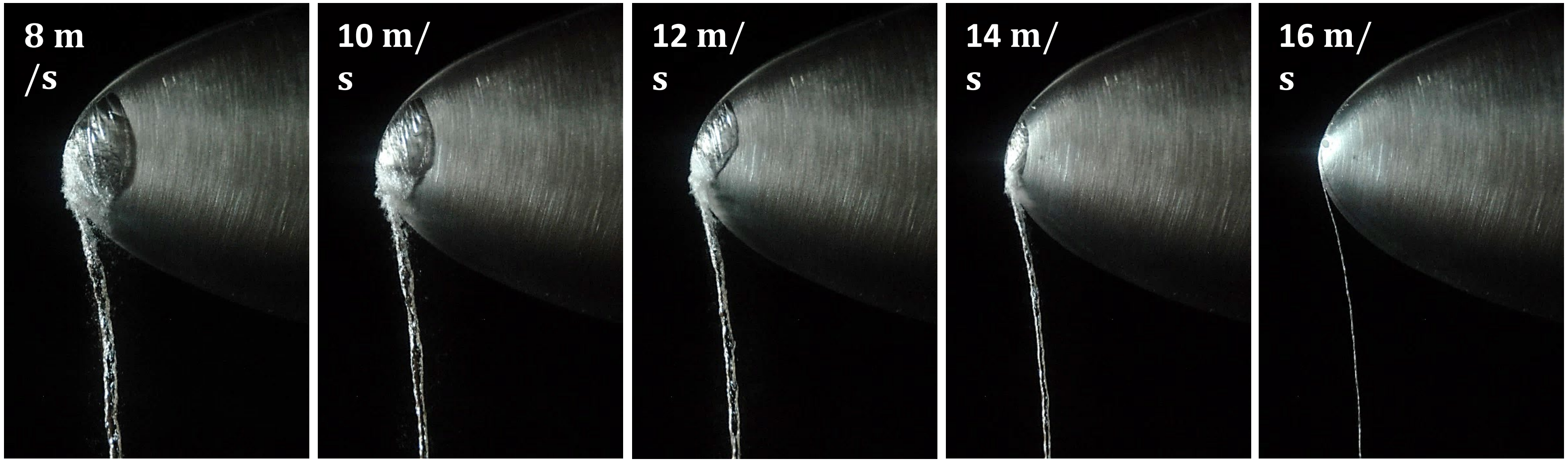
- **Flow conditions:** $U_\infty = 10\ \mathrm{m/s}\ ,\ \sigma = 2.5$



It is observed that the shape and aspect of the cavity at the tip of the hydrofoil changes drastically as the incidence angle is changed.

Effect of Reynolds number on TVC Hysteresis

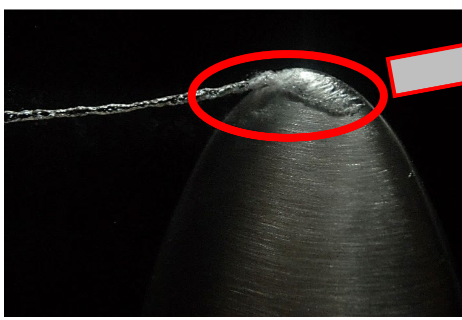
- **Flow conditions: $\alpha = 10°,\ \sigma = 2.4$**

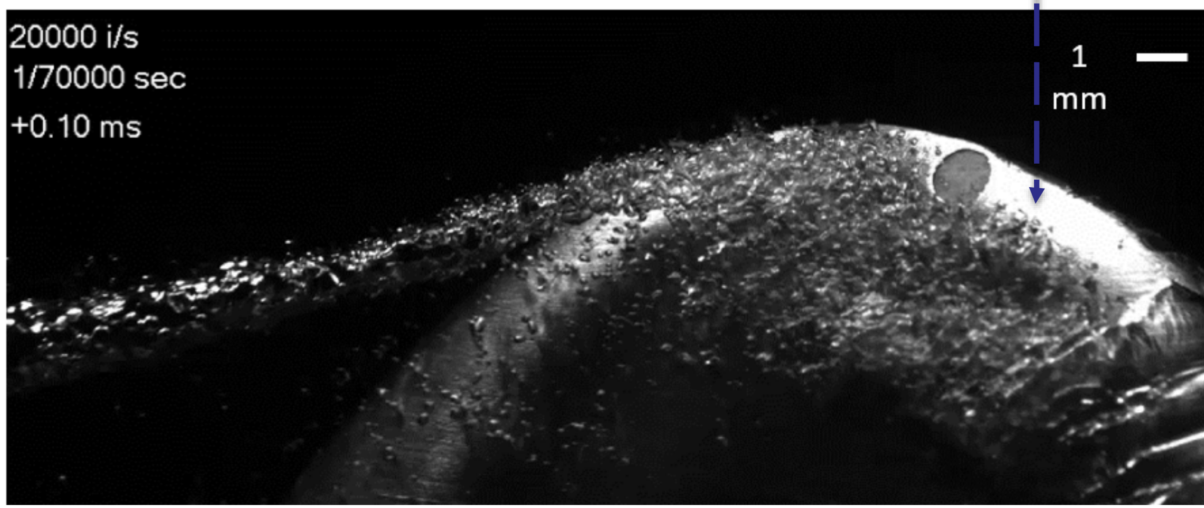


It is likely that the gas diffusion is enhanced when a laminar separation bubble forms at the leading edge near the tip

Laminar separation bubble

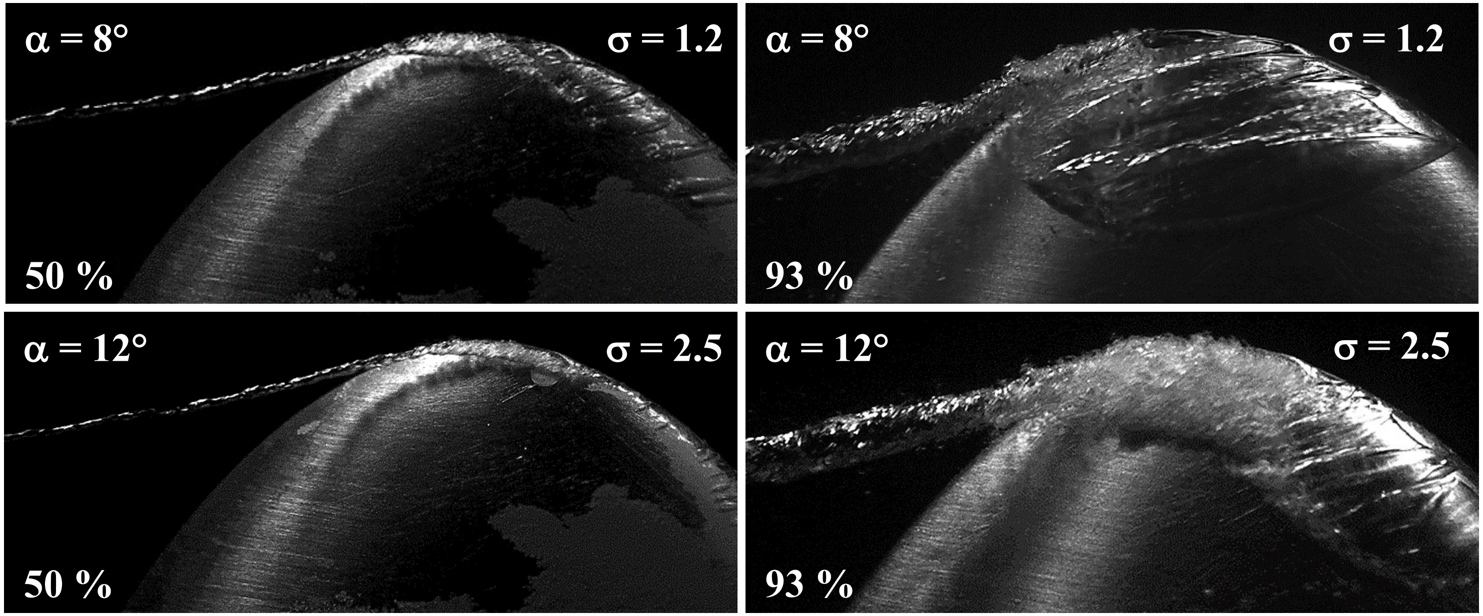
- **Flow conditions:** $\alpha = 12°, U_\infty = 5.3\ \mathrm{m/s},\ \sigma = 6.38$





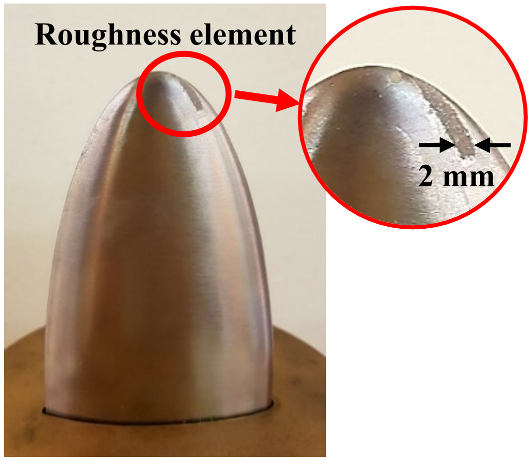
Effect of gas content on TVC

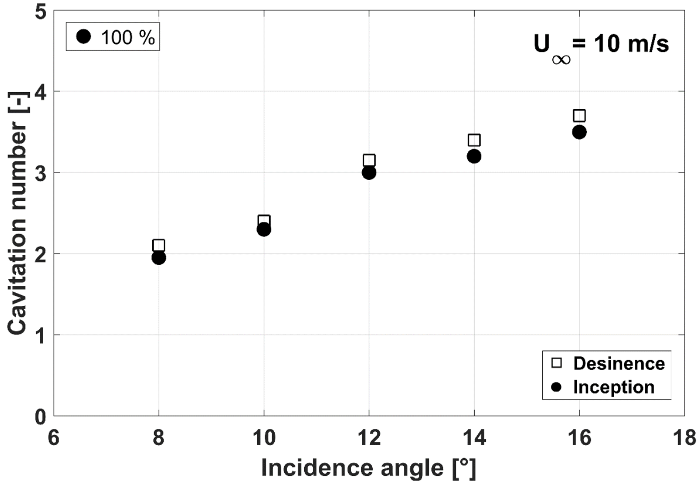
Gas diffusion occurs **spontaneously** for all the cavitating vortices, although it is the **nature of the BL separation** that defines its effectiveness in retarding the desinence.

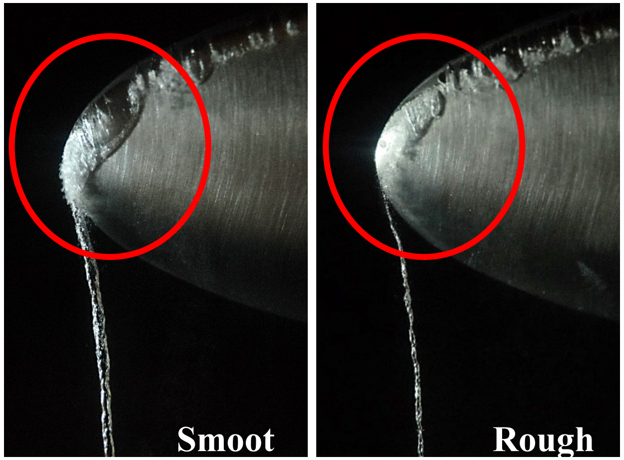


Let's Destroy the Shelter Zone!

- **Roughness** (125-μm-diameter sand) is added to only the **last 10 %** of the span.
- The hysteresis is totally suppressed due to turbulence.

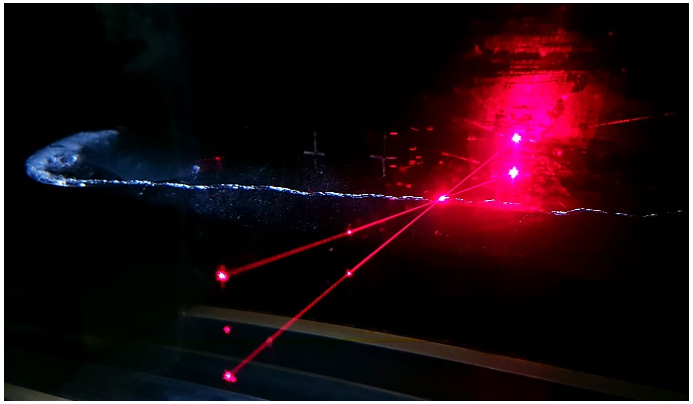


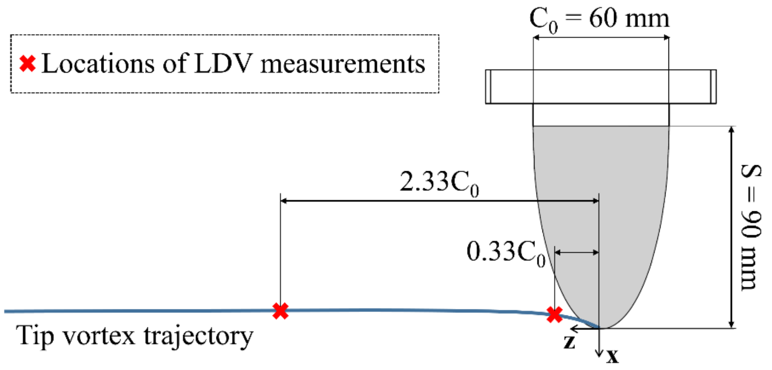




Velocity measurements in the wake flow

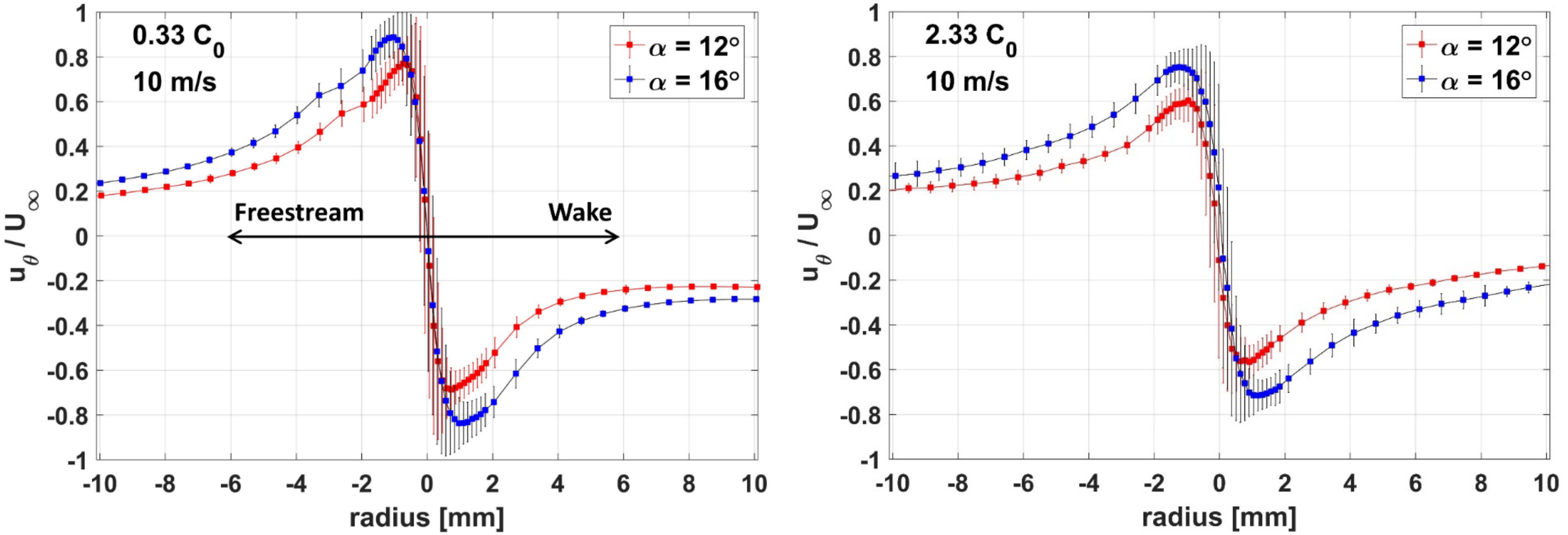
- LDV measurements of the tangential and axial velocity components





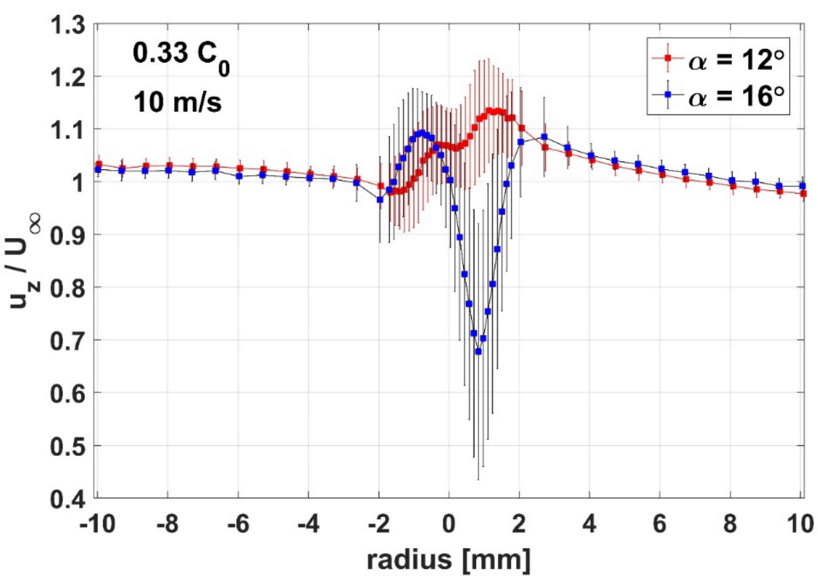
Velocity measurements in the wake flow

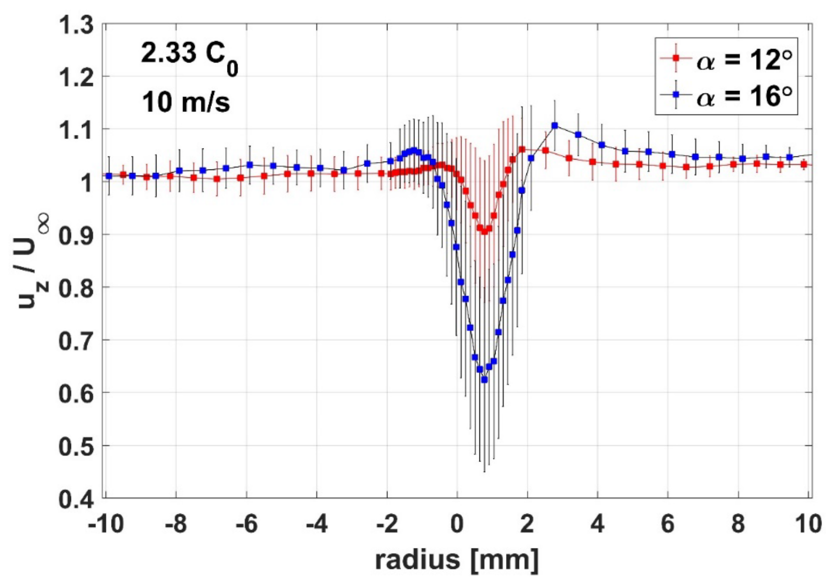
- **LDV measurements of the tangential velocity component:**
  - Evolution of tip vortex velocity profiles with the downstream distance from the tip
  - Similar patterns are observed for both angles of attack



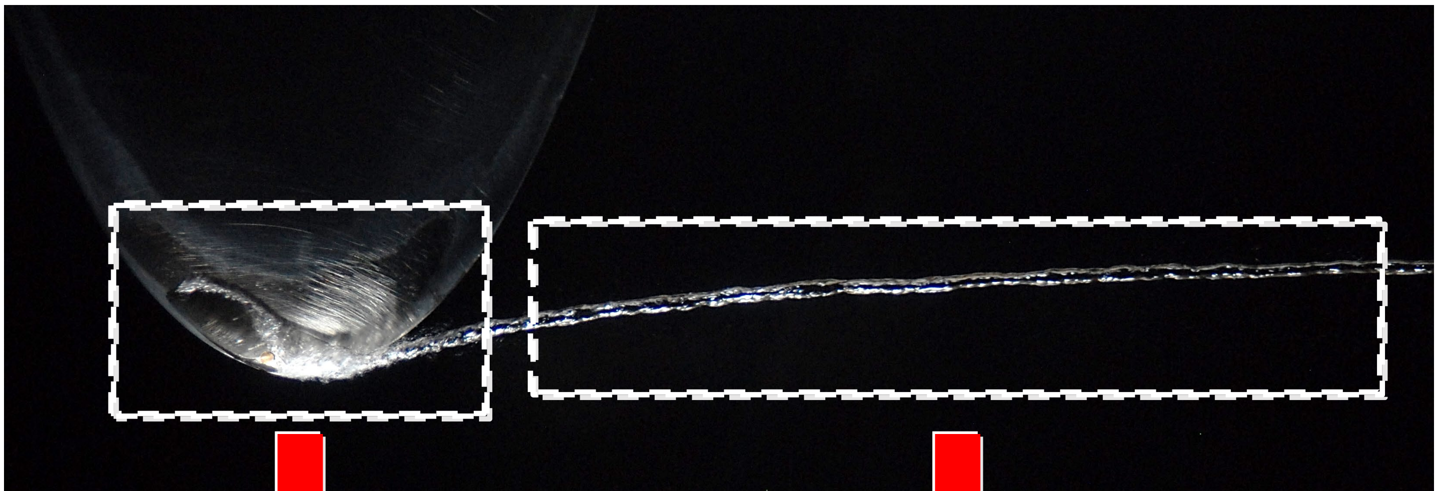
Velocity measurements in the wake flow

- **LDV measurements of the axial velocity component:**
  - Evolution of tip vortex velocity profiles with the downstream distance from the tip
  - A jet-like velocity profile for $\alpha = 12°$, while a wake-like one occurs for $\alpha = 16°$.





Conclusion



- **Recirculation area**
- **Roll-up process**
- **Shelter for TVC**

- **Air diffusion into the cavity**
- **Development of concentration BL**
- **Jet / wake-like behavior**