ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 9 - Solution

Problem 1: Capillary wicking

Part A. The surface is modeled as a thin porous layer of thickness h through which liquid spreads radially from a supply tube. Assuming creeping flow and negligible gravity/tube losses relative to the porous layer, the 1-D radial form of Darcy's law applies (paper Eq.1):

$$\mu u_r(r) = -K\frac{dp}{dr} \tag{1}$$

with permeability K, viscosity μ and Darcy velocity u_r. By continuity in an axisymmetric thin layer, the instantaneous volumetric flow rate is

$$q(t) = \int u_r dA = 2\pi r h u_r(r) \tag{2}$$

giving us an expression for the radial velocity,

$$u_r(r) = \frac{q(t)}{2\pi r h} \tag{3}$$

Substitute into Darcy's law and integrate from the tube contact radius r_{in} to the wetting front $r_{out}(t)$:

$$\mu\frac{q(t)}{2\pi r h}\int_{r_{in}}^{r_{out}(t)} \frac{1}{r}dr = -K\int_{P_{in}}^{P_{out}} dp = -K(P_{out} - P_{in}) \tag{4}$$

Assuming that the hydrostatic pressure in the tube is negligible compared to the capillary pressure, we can set $P_{in} = P_{atm}$ and we know that at the wetting front we have the Laplace capillary pressure $P_{out} = P_{atm} - \Delta P_{cap}$. thus, integrating both sides of Eq.4, we obtain the instantaneous volumetric flow rate:

$$q(t) = \frac{2\pi K h \Delta P_{cap}}{\mu \ln\left(\frac{r_{out(t)}}{r_{in}}\right)} \tag{5}$$

Part B The experiment uses a small tube held just above the test surface. When the liquid meniscus at the tip of the tube reaches contact, the surface wicks liquid from the tube. The meniscus height in the tube is tracked over time, giving the volume drawn into the surface versus time. With pixel-tolength calibration of the image taken of the meniscus over time during the wicking process, we have:

$$V_{out}(t) = V_{tube}(t_1) - V_{tube}(t) = A_t[h(t_1) - h(t)] \tag{6}$$

with, $h(t)$ being the position of the meniscus over time, and $h(t_1)$ the position of the meniscus after $\sim 25ms$ (discarding the fluctuation generated by the formation of the meniscus between the tube and the surface).

The model uses volume instead of flow rate using the wicks porosity ε:

$$q(t) = \frac{dV_{out}(t)}{dt}, \qquad V_{out}(t) = \pi r_{out}^2 h\varepsilon, \qquad V_{in} = \pi r_{in}^2 h\varepsilon \tag{7}$$

Using this, and setting $\omega = Kh\Delta P_{cap}$, a fluid-specific hydrophilicity metric, Eq.5 becomes:

$$\frac{dV_{out}(t)}{dt} = \frac{4\pi\omega}{\mu}\frac{1}{\ln(V_{out}(t)/V_{in})} \tag{8}$$

Following the paper suggestion, integrating between two arbitrary times, t_1 and t_2 gives the following formula (Eq. 4 in the paper):

$$t_2 - t_1 = \frac{\mu}{4\pi\omega}\left[V_{out}(t_2)\ln\left(\frac{V_{out}(t_2)}{V_{in}}\right) - V_{out}(t_2) - \left(V_{out}(t_1)\ln\left(\frac{V_{out}(t_1)}{V_{in}}\right) - V_{out}(t_1)\right)\right] \tag{9}$$

After the meniscus has stabilized and setting $V_1 = V_{out}(t_1)$, we have for each measured sample $(t_i, V_i = V_{out}(t_i))$:

$$F(V_i) - F(V_1) = \frac{4\pi\omega}{\mu}(t_i - t_1) \qquad \text{with, } F(V_i) = V_i \ln\left(\frac{V_i}{V_{in}}\right) - V_i \tag{10}$$

Which is a simple linear relation between $t_i - t_1$ and $F(V_i) - F(V_1)$ and allows us to retrieve the value of ω.

Problem 2: Flow boiling heat transfer coefficient

Following the Kandlikar correlation for saturated flow boiling explained in Carey book (p. 585-586), the heat transfer coefficient is plotted on Fig.1.

The correlation from Gungor and Winterton (Carey, p.584) is plotted on the same figure. We can observe a difference of approximately +30% in the heat transfer coefficient for the Kandlikar correlation compared to Gungor and Winterton.

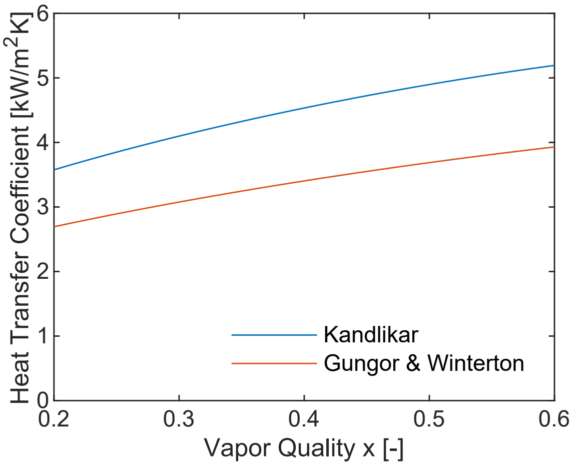

Figure 1: Saturated boiling heat transfer coefficient comparison for Kandlikar and Gungor and Winterton correlations.

Problem 3: Onset of nucleate boiling

For a constant heat flux condition and assuming the single-phase heat transfer coefficient $h_l e$ is constant along the tube, using the definition of h_{le} and energy conservation, we end up with the following equation for wall superheat as a function of the position z along the tube (see Carey p.561):

$$T_w(z) - T_{sat} = \frac{q''}{h_{le}} \left[1 + 4 \left(\frac{h_{le}}{Gc_{pl}} \right) \left(\frac{z}{d_h} \right) \right] - (T_{sat} - T_{l,in}) \tag{11}$$

When the onset of nucleate boiling (ONB) occurs, we know using Eq. 12.14 in Carey that the wall superheat has to be:

$$\Delta T_{w,ONB} = \sqrt{\frac{8\sigma q'' T_{sat}}{k_l h_{lv} \rho_v}} \tag{12}$$

Solving Eq.11 for z we can get the relation of the position of the ONB from the inlet of the tube z_{ONB}:

$$z_{ONB} = \frac{Gc_{pl} d_h}{4 h_{le}} \left[\frac{h_{le}}{q''} (\Delta T_{w,ONB} + \Delta T_{sub}) - 1 \right] \tag{13}$$

with ΔT_{sub} being the inlet subcooling $T_{sat} - T_{l,in}$.

The saturation properties of nitrogen are gathered using *CoolProp*, the single-phase liquid heat transfer coeficient is calculated using the Dittus-Boetler correlation. Using the *Matlab* code provided with this solution, we end up with $z_{ONB} = 0.6m$.