ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 8 - Solution

Problem 1: Maxima of number of isolated bubbles

Part A)

Number of isolated bubbles N_{iso}:

$$N_{iso} = \sum_{N=0}^{\infty} N \cdot P_{iso} \cdot P_0(N, N_0) = \sum_{N=1}^{\infty} \frac{N_0^N}{(N-1)!} e^{\left(-N_0 - \frac{\pi N D_b^2}{A}\right)} \tag{1}$$

Merging the power of N and taking out e^{-N_0} of the sum:

$$N_{iso} = e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^N}{(N-1)!} \tag{2}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} \tag{3}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} e^{-N_0 e^{-\frac{\pi D_b^2}{A}}} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \tag{4}$$

We recognize a Poisson distribution:

$$\sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} e^{-N_0 e^{-\frac{\pi D_b^2}{A}}} = 1 \tag{5}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \tag{6}$$

At the CHF point, we expect $\frac{\partial N_{iso}}{\partial T} = 0$.

$$\frac{\partial}{\partial T}\left[N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}}\right] = 0 \tag{7}$$

Taking out the constant term and applying the product rule, we have:

$$e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \frac{\partial N_0}{\partial T} + N_0 e^{N_0 e^{-\frac{\pi D_b^2}{A}}} (-e^{N_0}) \frac{\partial N_0}{\partial T} + N_0 e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} e^{-\frac{\pi D_b^2}{A}} \frac{\partial N_0}{\partial T} = 0 \tag{8}$$

Assuming $\frac{\partial N_0}{\partial T} \neq 0$ and simplifying, we end up with:

$$1 - N_0 + N_0 e^{-\frac{\pi D_b^2}{A}} = 0 \tag{9}$$

For $D << A$ we can use the Taylor expansion of $e^{-\frac{\pi D_b^2}{A}}$ at 0:

$$e^{-\frac{\pi D_b^2}{A}} \approx 1 - \frac{\pi D_b^2}{A} \tag{10}$$

Then we obtain:

$$1 - N_0 + N_o \left(1 - \frac{\pi D_b^2}{A}\right) = 0 \tag{11}$$

$$\frac{N_0}{A} \pi D_b^2 = 1 \tag{12}$$

Part B)

Substituing Equation (12) into Equation (6), we have

$$N_{iso} = N_0 e^{-\frac{1}{N_0}} e^{-N_0} e^{N_0 e^{-\frac{1}{N_0}}}$$

Using the same Taylor expansion as before, we have

$$N_{iso} = N_0 e^{-N_0} e^{N_0(1-\frac{1}{N_0})} e^{-\frac{1}{N_0}}$$

Simplifying and assuming $e^{-\frac{1}{N_0}} \approx 1$ if N_0 is large, we obtain

$$N_{iso} = \frac{N_0}{e}$$

Problem 2: Kandlikar Model

CHF prediction using the Kandlikar model for contact angles β between 5° and 50° are shown in Figure 1. The CHF decreases when the contact angle increases.

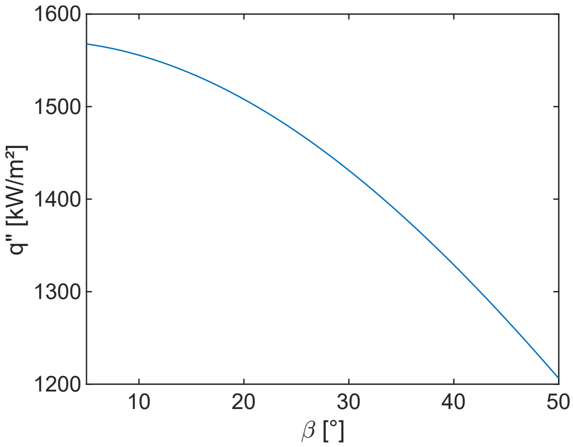


Figure 1: CHF Kandlikar Model for Water at 1 atm

Problem 3: Bubble coalescence

The sum of the two bubbles volume and surface area in the initial case are:

$$V_i = \frac{8}{3}\pi r^3$$

$$A_i = 8\pi r^2$$

Assuming the volume is conserved, we can write the radius of the final bubble as a function of r:

$$r_f = 2^{1/3} r$$

and,

$$A_f = 4\pi 2^{2/3} r^2$$

The surface energy change between the final and initial case is expressed as:

$$\Delta E = \gamma(A_f - A_i) = 4\pi r^2 \gamma(2^{2/3} - 2) < 0$$

Problem 4: Monte-Carlo simulation of spatial distribution of nucleation sites

A) Flat plate ($A = 1$ cm^2)

(a) MC simulation

Each realization places $N = 100$ sites randomly on the square plate of area

$$A = 1 \text{ cm}^2, \qquad n_0 = \frac{N}{A} = 100 \text{ cm}^{-2}$$

For each site i, the nearest–neighbor distance is

$$s_i = \min_{j \neq i} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

All the s_i from a single realization are gathered and plot as a normalized PDF on Fig. 2a.

(b) Expected PDF

For a spatially random (Poisson) distribution with density n_0, the nearest neighborgh distance PDF is expected to follow a Rayleigh distribution:

$$f(s) = 2\pi n_0 \, s \, e^{-\pi n_0 s^2}$$

This Rayleigh distribution is plotted on the same figure. The histogram does not follow perfectly the Rayleigh distribution because the nucleation site is kept quite low, increasing N or the number of realization will bring the histogram closer to the expected distribution.

B) Cylinder ($R = 0.5$ cm, $H = 1$ cm).

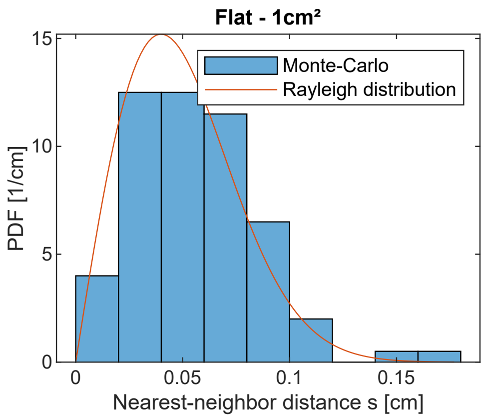
Unwraping the lateral surface to a rectangle of width $W = 2\pi R$ and height H (periodic in the circumferential direction). We obtain the following nearest-neighbor distance:

$$\Delta x_{\text{per}} = \min(|x_i - x_j|, \, W - |x_i - x_j|), \qquad \Delta y = y_i - y_j,$$

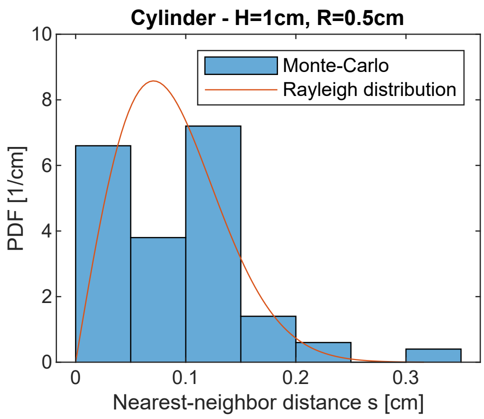
$$s_i = \min_{j \neq i} \sqrt{\Delta x_{\text{per}}^2 + \Delta y^2}$$

In this case, we have: $A = \pi$ cm$^2, n_0 = \dfrac{100}{\pi}$ cm^{-2}

The MC simulation results and corresponding Rayleigh distribution function are displayed on Fig. 2b.



(a)



(b)

Figure 2: Histograms of nearest–neighbor distances compared with the Rayleigh distribution. (a) Flat plate ($A = 1$ cm^2); (b) Cylinder ($R = 0.5$ cm, $H = 1$ cm) for $N = 100$ in a single realization.