ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 7 - Solution

Problem 1: Rohsenow's correlation

A Matlab code is uploaded separately. CoolProp was used to obtain the thermophysical properties of saturated water. The boiling curve obtained with Rohsenow's correlation is shown below.

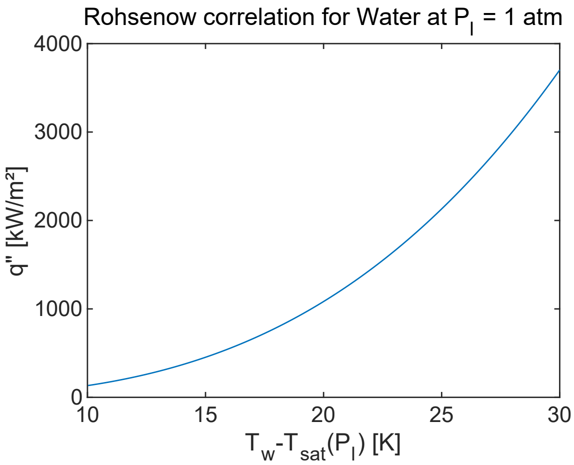

Figure 1: Boiling curve for water with $P_l = 1atm$.

Problem 2: Bubble departure

Consider the vertical force balance, the buoyancy force is

$$F_b = \Delta\rho V_B g$$

where, V_B is the volume of the bubble expressed as a function of r, the radius of the bubble:

$$V_B = \frac{\pi}{3} r^3 (2 - \cos\theta)(1 + \cos\theta)^2$$

The vertical component of the surface tension force is

$$F_{s,v} = \pi\sigma D_{cl} \sin\theta$$

where the contact line can be expressed as:

$$D_{cl} = 2r\sin\theta = 2\left(\frac{3V_B}{\pi(2-\cos\theta)(1+\cos\theta)^2}\right)^{\frac{1}{3}} \sin\theta$$

The force balance requires $F_s = F_B$

$$\Delta\rho V_B g = 2\pi\sigma \left(\frac{3V_B}{\pi(2-\cos\theta)(1+\cos\theta)^2}\right)^{\frac{1}{3}} \sin^2\theta$$

Therefore

$$V_B = 2\sqrt{6}\pi \left(\frac{\sigma}{\Delta\rho g}\right)^{\frac{3}{2}} (2-\cos\theta)^{-\frac{1}{2}} (1+\cos\theta)^{-1} \sin^3\theta$$

Problem 3: Transient conduction in a semi-infinite solid

A) The solution for the semi-infinite solid transient conduction for a sudden temperature T_f at the surface is (Eq. 5.50 in Lienhart):

$$\theta = \operatorname{erf}\left(\frac{\zeta}{2}\right)$$

where θ is the nondimensional temperature,

$$\theta = \frac{T(x,t) - T_f}{T_i - T_f}$$

and ζ is the second similarity variable used to solve the heat equation

$$\zeta = \frac{x}{\sqrt{\alpha t}}$$

The timescale τ of the thermal response at L_c is:

$$\tau = \frac{L_c^2}{4\alpha}$$

B) When the solid surface is suddenly exposed to a fluid of temperature T_f and a convective heat transfer coefficient h, the solution of the transient conduction is (Eq. 5.53 in Lienhart):

$$\theta = \operatorname{erf}\left(\frac{\zeta}{2}\right) + \exp(\beta\zeta + \beta^2)\left[\operatorname{erfc}\left(\frac{\zeta}{2} + \beta\right)\right]$$

where a new similarity variable β is needed to solve the heat equation

$$\beta = \frac{h\sqrt{\alpha t}}{k}$$

in this case we have two timescale

$$\tau_1 = \frac{L_c^2}{4\alpha}$$

and

$$\tau_2 = \frac{k^2}{h^2\alpha}$$

In the case of a bubble growing at the surface ($L_c = 0$) the characteristic timescale of the rewetting induced convection is given by τ_2.

Problem 4: Hydrodynamic instabiltiy

Part A)

When $a > 0$, $\lim_{t\to\infty} |e^{\beta t}| = \infty$ and the amplitude will grow in time.

When $a < 0$, $\lim_{t\to\infty} |e^{\beta t}| = 0$, and the perturbation will vanish.

Part B)

Only the positive β will grow. Set the first derivative with regard to α to zero:

$$\frac{d}{d\alpha}\left[\frac{(\rho_l - \rho_v)g\alpha - \sigma\alpha^3}{\rho_l - \rho_v}\right]^{\frac{1}{2}} = 0$$

$$\frac{d}{d\alpha}[(\rho_l - \rho_v)g\alpha - \sigma\alpha^3] = 0$$

$$(\rho_l - \rho_v)g = 3\sigma\alpha^2$$

$$\alpha = \sqrt{\frac{(\rho_l - \rho_v)g}{3\sigma}}$$

It's easy to verify that this α corresponds to the maximum of the positive β. The perturbation wavelength with the fastest growth can then be written as

$$\lambda = \frac{2\pi}{\alpha} = 2\pi\sqrt{\frac{3\sigma}{(\rho_l - \rho_v)g}}$$