ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 5 - Solution

Problem 1: Difference between Schrage equation and moment method for evaporation

A) When the vapor temperature is close to the one of the liquid surface, thus assuming $T_V \approx T_L = T_0$, the mass flux across the interface can be written as:

$$j = \frac{2\sigma}{2-\sigma}\sqrt{\frac{1}{2\pi R}}\left(\frac{P_L - P_V}{\sqrt{T_0}}\right)$$

Dividing both sides by $P_L/\sqrt{2\pi R T_L}$ we obtain a linear function between the two dimensionless quantities j^* and ΔP^*:

$$j^* = \frac{2\sigma}{2-\sigma}\left(\frac{P_L - P_V}{P_L}\right) = \frac{2\sigma}{2-\sigma}\Delta P^*$$

B) On Figure 1, one can see that when σ is small, the deviation of Schrage equation from the moment method is small. When σ is large, the deviation is also larger.

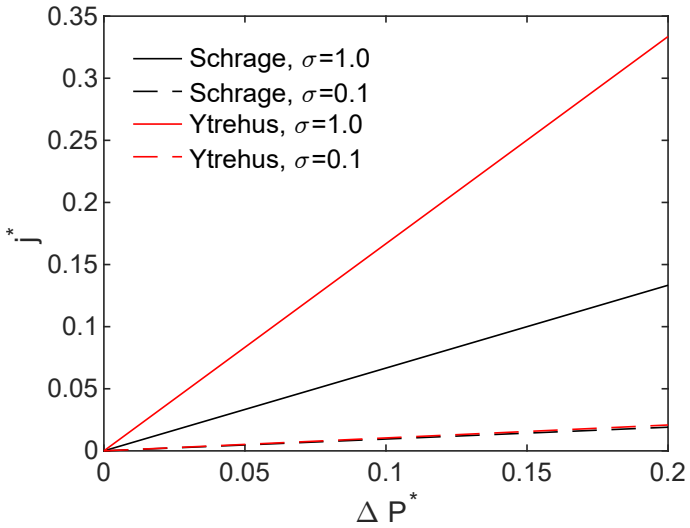

Figure 1: j^* as a function of ΔP^* for the Schrage equation and moment methods for $\sigma = 0.1$ and $\sigma = 1.0$

Problem 2: Solar thermal desalination

A) Energy balance

Incoming heat flux:

$$q_{in} = \alpha q_{sun}$$

Outgoing heat flux:

$$q_{out} = h_{loss}(T_s - T_\infty) + \eta_{evap}(\rho_{sat} - \rho_\infty)h_{fg}$$

The first part of the equation is the heat loss due to conduction, convection and radiation. The second part is due to evaporation.

The evaporation mass transfer coefficient η_{evap} is related to the Sherwood number Sh and the vapor density in the far field is related to the relative humidity:

$$Sh = \frac{\eta_{evap} L}{D_{va}}$$

$$\rho_\infty = \phi \rho_{sat}(T_\infty)$$

By energy balance we obtain:

$$\alpha q_{sun} = h_{loss}(T_s - T_\infty) + Sh \frac{D_{va}}{L}[\rho_{sat}(T) - \phi \rho_{sat}(T_\infty)]h_{fg}$$

B) The code has been uploaded separately. Note that we evaluated D_{va} at $(T + T_\infty)/2$ and plotted the absorber temperature as well.

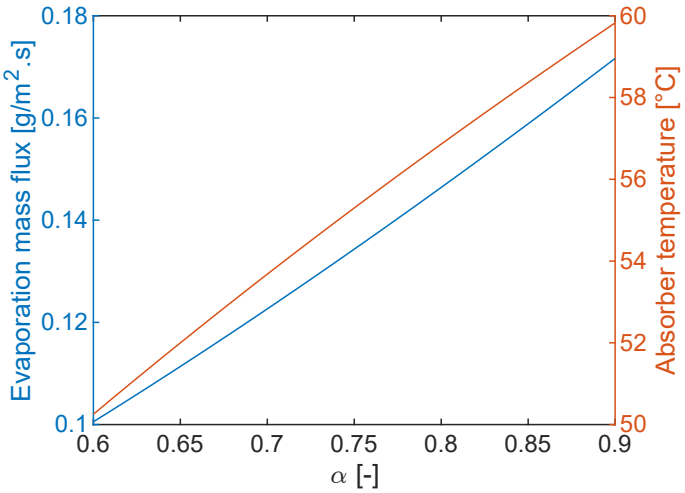

Figure 2: Evaporation mass flux and absorber temperature as a function of the solar absorptivity α.

Problem 3: Homogeneous nucleation in vapor-liquid systems

Starting with the first derivative

$$\frac{d\Delta G}{dr} = \frac{d\hat{N}_v}{dr}(\hat{g}_v - \hat{g}_l) + 4\pi r^2 \left(\frac{2\sigma_{lv}}{r} + P_l - P_v\right)$$

$$\frac{d^2\Delta G}{dr^2} = \frac{d^2\hat{N}_V}{dr^2}(\hat{g}_v - \hat{g}_l) + \frac{d\hat{N}_v}{dr}\hat{v}_v\frac{dP_v}{dr} + 8\pi r\left(\frac{2\sigma_{lv}}{r} + P_l - P_v\right) - 4\pi r^2\left(\frac{dP_v}{dr} + \frac{2\sigma_{lv}}{r^2}\right)$$

At equilibrium, we have ($r = r_e$), we have $\hat{g}_v = \hat{g}_l$, and assuming mechanical equilibrium is always sattisfied $P_v - P_l = 2\sigma/r$

$$\left.\frac{d^2\Delta G}{dr^2}\right|_{r=r_e} = \cancel{\frac{d^2\hat{N}_V}{dr^2}(\hat{g}_v - \hat{g}_l)} + \frac{1}{P_v}\frac{d(P_vV_v)}{dr}\left.\frac{dP_v}{dr}\right|_{r=r_e} + \cancel{8\pi r\left(\frac{2\sigma_{lv}}{r} + P_l - P_v\right)} - \cancel{4\pi r^2\left(\frac{dP_v}{dr} + \frac{2\sigma_{lv}}{r^2}\right)}$$

$$\left.\frac{d^2\Delta G}{dr^2}\right|_{r=r_e} = \left.\frac{1}{P_v}\left(\frac{dP_v}{dr}\right)^2 V_v\right|_{r=r_e} + \left.\frac{dV_v}{dr}\frac{dP_v}{dr}\right|_{r=r_e}$$

From Laplace equation, we have

$$\frac{dP_v}{dr} = -\frac{2\sigma_{lv}}{r^2}$$

Thus we obtain

$$\left.\frac{d^2\Delta G}{dr^2}\right|_{r=r_e} = \frac{1}{P_l + \frac{2\sigma_{lv}}{r_e}}\frac{4\sigma_{lv}^2}{r_e^4}\frac{4\pi r_e^3}{3} - 8\pi\sigma_{lv}$$

Rearranging the terms

$$\left.\frac{d^2\Delta G}{dr^2}\right|_{r=r_e} = -\frac{8\pi\sigma_{lv}}{3}\left(2 + \frac{1}{1 + \frac{2\sigma_{lv}}{r_eP_l}}\right) < 0$$

From this last expression we can see that $\frac{d^2\Delta G}{dr^2}$ is always negative when r is close to r_e. In other words, small deviation from the equilibrium radius will always result in a decrease of Gibbs free energy, the system is unstable. Fluctuation will either lead to collapse or further growth.