ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 10 - Solution

Problem 1: Thermal conduction in a droplet

A) Following what has been done in Homework 4, Problem 2, the domain is shown in Figure 1a, where $E1$ is the liquid/substrate interface, $E2$ is the axisymmetric axis, and $E3$ is the liquid/gas interface.

Boundary conditions:

- $E1$: Substrate constant temperature T_s.
- $E2$: Symmetry axis, no flux normal to the interface.
- $E3$: Convection boundary condition, h, T_a

B) The code for solving the heat conduction problem has been uploaded along with this solution. Figure 1b show the heat flow rate across the droplet, q_d as a function of ΔT_{cond}. The heat rate can be calculated at either edge $E1$ or $E3$. Additionally, Eq. 9.28 from the Carey book is plotted for comparison, highlighting that it remains an approximation.

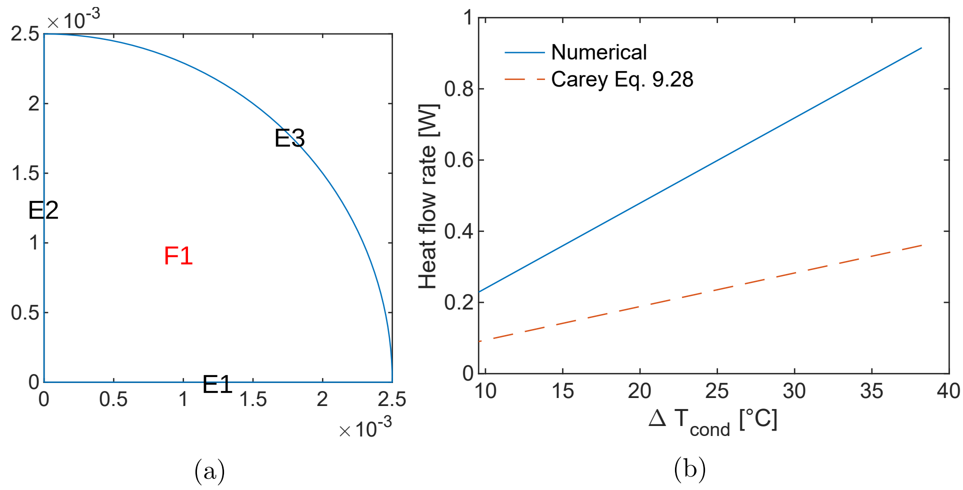

Figure 1: (a) Simulation domain with edges, $E1$ (liquid/substrate interface), $E2$ (axisymmetric axis), $E3$ (liquid/gas interface). (b) Numerical result of the heat flow rate q_d as a function of ΔT_{cond} and comparison with Eq. 9.28 from Carey's book.

Problem 2: Laminar film condensation (Carey P9.15)

A) The force balance is similar to the case without the hot surface, thus we have:

$$\dot{m}' = \frac{\rho_l(\rho_l - \rho_v)g\delta^3}{3\mu_l}$$

$$\frac{d\dot{m}'}{d\delta} = \frac{\rho_l(\rho_l - \rho_v)g\delta^2}{\mu_l}$$

Energy balance, including the conduction, radiation, and condensation:

$$\frac{k_l(T_{sat} - T_w)}{\delta}dx = \sigma_{SB}(T_h^4 - T_{sat}^4)dx + h_{lv}d\dot{m}'$$

The left-hand part is energy that is cooling the control volume while the radiation and condensation are heating it.

Dividing by dx and using the chain rule, we obtain the differential equation below where the only unknown is $\delta(x)$. This equation cannot be solved analytically.

$$\frac{d\delta}{dx} = \frac{1}{h_{lv}}\left[\frac{k_l(T_{sat} - T_w)}{\delta} - \sigma_{SB}(T_h^4 - T_{sat}^4)\right]\frac{\mu_l}{\rho_l(\rho_l - \rho_v)g\delta^2}$$

This first-order nonlinear differential equation can be solved numerically to get $\delta(x)$. The heat transfer coefficient along the x-direction is then:

$$h(x) = \frac{k_l}{\delta(x)}$$

B) If we assume an asymptotic variation of the liquid film far from the leading edge, at large x, we have $\frac{d\delta}{dx} = 0$. Thus, there is no more condensation. The film thickness depends only on the radiation and conduction:

$$\delta = \frac{k_l(T_{sat} - T_w)}{\sigma_{SB}(T_h^4 - T_{sat}^4)} = 3.2mm$$

Problem 3: Dropwise condensation correlation (Carey P9.1)

Rose at al. empirical correlation for steam dropwise correlation:

$$h_{dc} = T_v^{0.8}[5 + 0.3(T_{sat} - T_w)]$$

This equation is dimensional, T_v is the saturated vapor temperature in $°C$ and the heat transfer coefficient is determined in kW/m^2K. Note that the correlation is valid until 1 atm, we are looking at the correlation outside this range.

Figure 2 shows the variation of the heat transfer coefficient between 1 atm and 9460 kPa. Potential fluid properties influencing the prediction include the surface tension σ, liquid viscosity μ_l, and liquid thermal conductivity k_l. The first two are more sensitive to the temperature of the saturated fluid. Variations of these fluid properties as a function of the saturation pressure are plotted in the same graph.

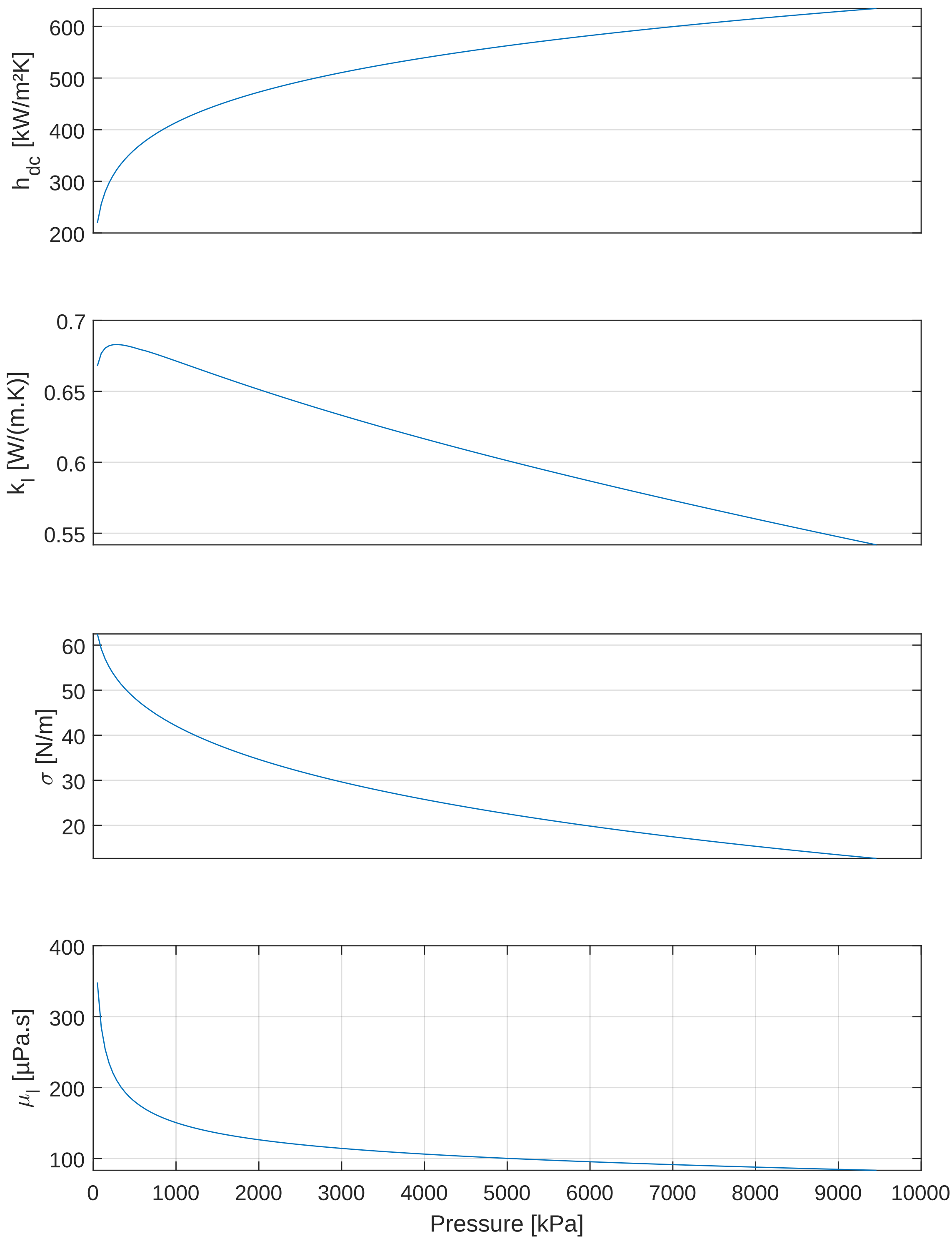

Figure 2: Rose et al. correlation for h_{dc} as a function of the saturation pressure (top) and fluid properties variation in the same range,