



ME-446: Liquid-gas interfacial heat and mass transfer

Boiling: Critical Heat Flux

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Energy Transport Advances
Laboratory

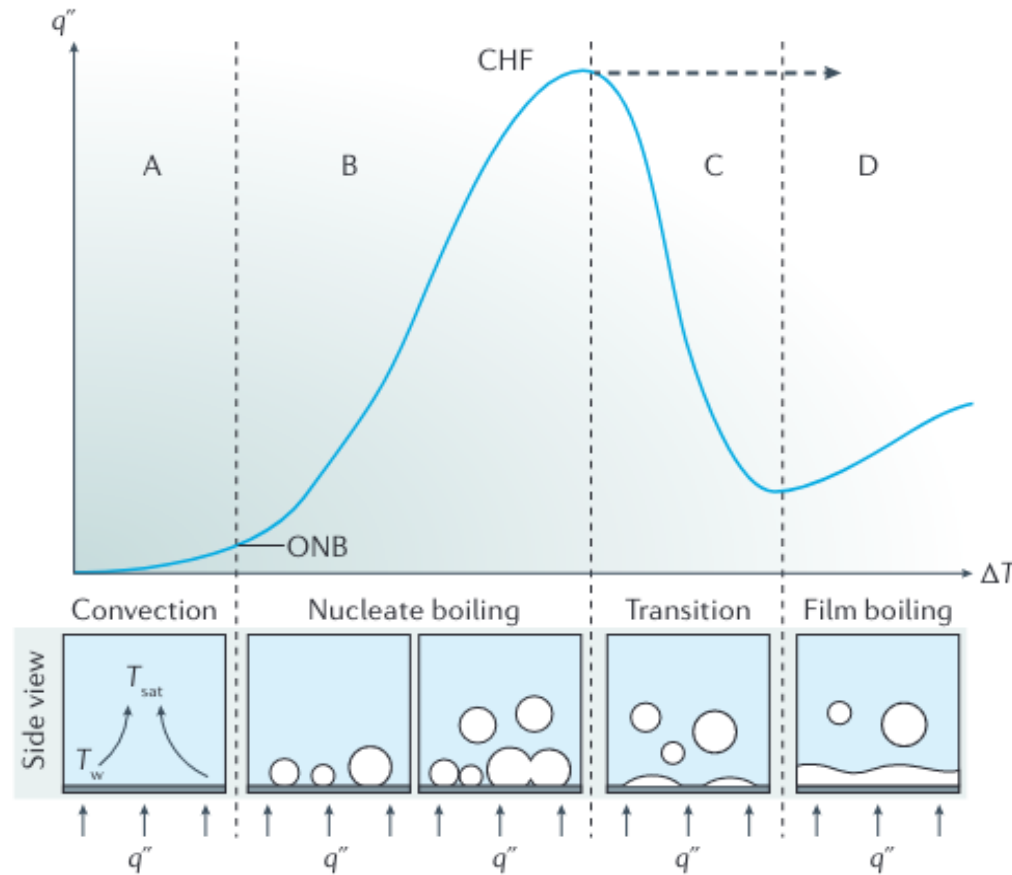
EPFL Mechanical Engineering

2025 Fall Semester

Photo Credit: Trougnouf

- Pool boiling curve
- Rohsenow's microconvection model for nucleate boiling
- Helmholtz and Taylor instabilities

Pool Boiling Curve



A. At very low superheats, heat transfer is mostly due to natural convection

B. After superheat is large enough to form vapor bubbles, nucleate boiling dominates, promoting bubble-motion-induced convection

C-D. After vapor generation becomes too much, passing the critical heat flux (CHF), insulating vapor film will start to form, decreasing the HTC

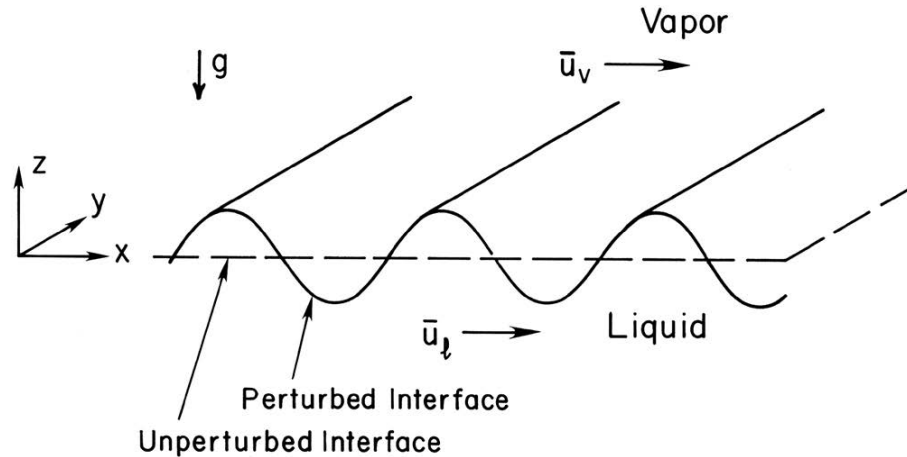
Convective transport facilitated by bubbles

$$\text{Nu}_b = \frac{hL_b}{k_l} \propto \text{Re}_b^{1-r} \text{Pr}_l^{1-s}$$

$$\text{Re}_b = \frac{\rho_v U L_b}{\mu_L} \quad U = \frac{q''}{\rho_v h_{lv}} = \frac{h \Delta T}{\rho_v h_{lv}}$$

$$\frac{q''}{\mu_l h_{lv}} \left[\frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2} = \left(\frac{1}{C_{sf}} \right)^{1/r} \text{Pr}_l^{-s/r} \left[\frac{c_{pl} \Delta T}{h_{lv}} \right]^{1/r}$$

Helmholtz Instability

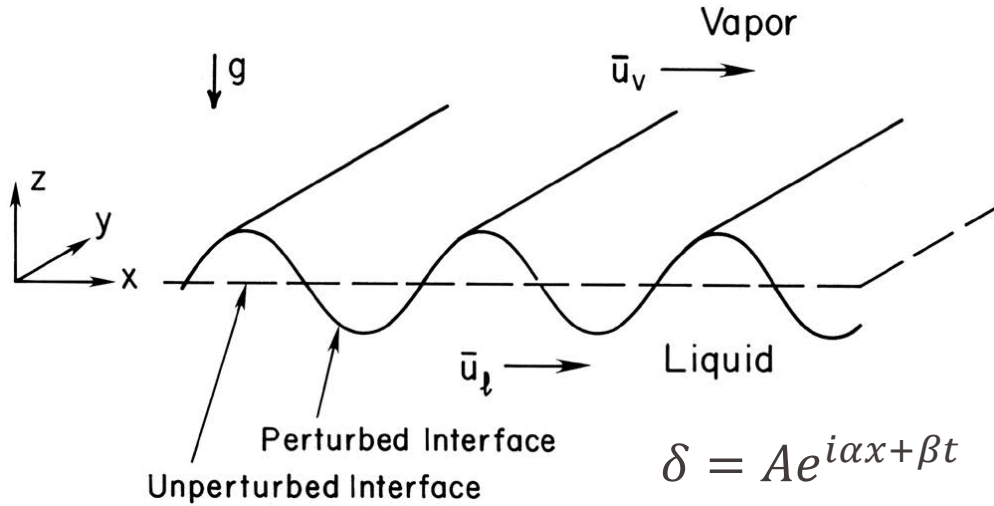


Perturbation $\delta(x, t = 0) = Ae^{i\alpha x}$
 Response: $\delta = Ae^{i\alpha x + \beta t}$

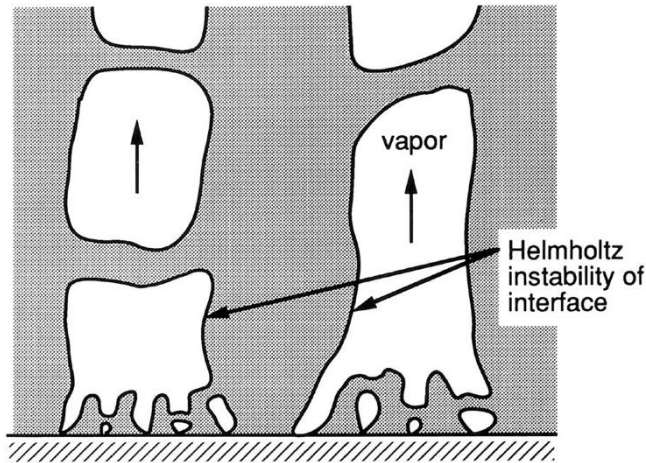
$$\beta = \pm \frac{\sqrt{\alpha^2 \rho_v \rho_l (\bar{u}_v - \bar{u}_l)^2 - (\sigma \alpha^3 + \Delta \rho g \alpha)}}{\rho_v + \rho_l} - i\alpha \frac{\rho_l \bar{u}_l + \rho_v \bar{u}_v}{\rho_v + \rho_l}$$

- Instability condition: $|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{(\sigma \alpha + \frac{\Delta \rho g}{\alpha})(\rho_l + \rho_v)}{\rho_l \rho_v}}$
- When $|\bar{u}_v - \bar{u}_l| > \left[\frac{2(\rho_l + \rho_v)}{\rho_l} \right]^{\frac{1}{2}} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{\frac{1}{4}}$, there will be some wave number α growing exponentially
- $\bar{u}_v - \bar{u}_l$ promotes instability while gravity and surface tension suppressing instability, we can adjust the value of g based on the orientation of the system.

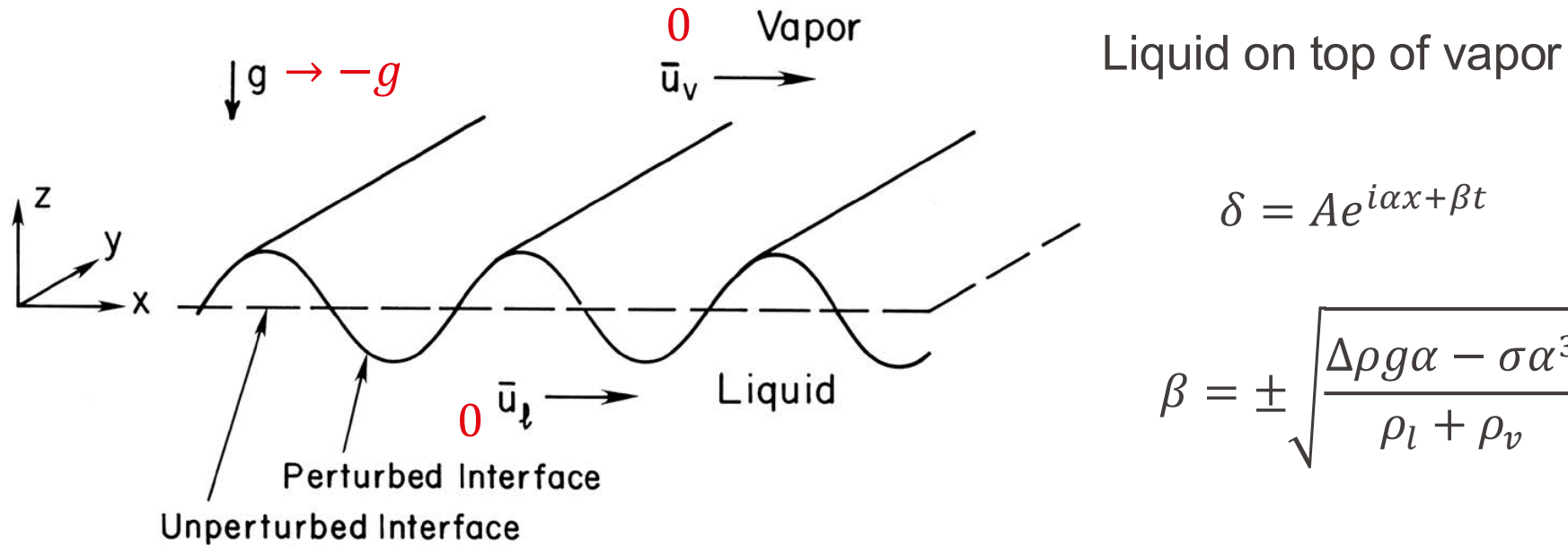
Vapor Column Instability



Setting $g = 0$ for in Helmholtz instability for vertical surfaces



$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\sigma \alpha (\rho_l + \rho_v)}{\rho_l \rho_v}} = \sqrt{\frac{2\pi\sigma (\rho_l + \rho_v)}{\rho_l \rho_v \lambda_H}}$$



$$\delta = Ae^{i\alpha x + \beta t}$$

$$\beta = \pm \sqrt{\frac{\Delta\rho g\alpha - \sigma\alpha^3}{\rho_l + \rho_v}}$$

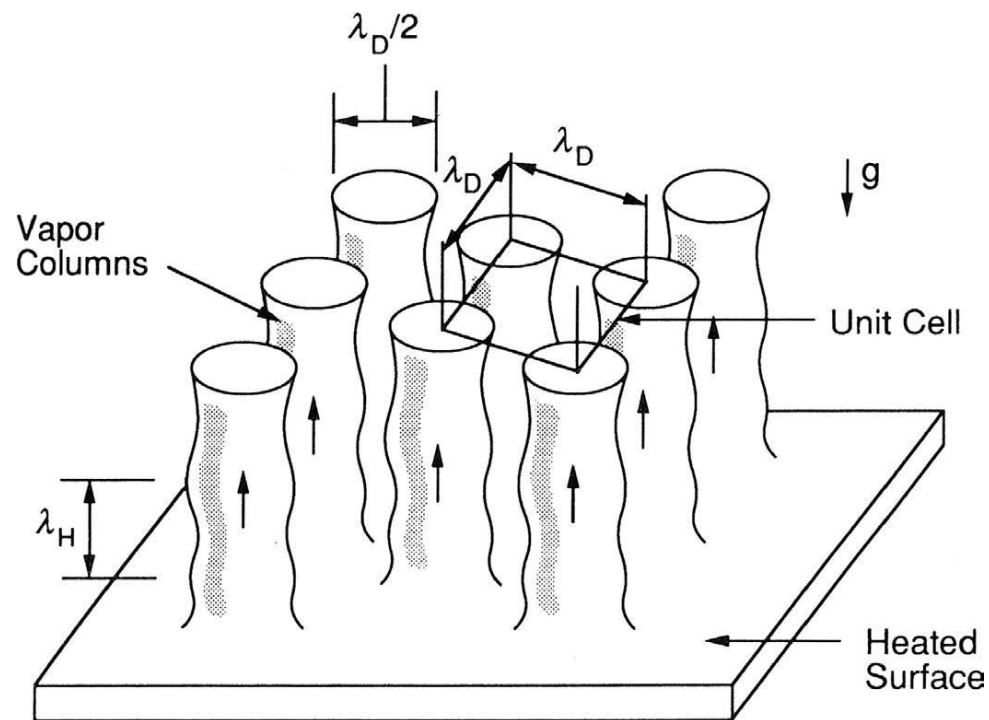
The fastest growing perturbation (α_{max}) in this case can be found by setting $\frac{d\beta}{d\alpha} = 0$

The corresponding most dangerous wavelength $\lambda_D = \frac{2\pi}{\alpha_{max}} = 2\pi \sqrt{\frac{3\sigma}{\Delta\rho g}}$

- Zuber' CHF model based on hydrodynamic instability
- Force balance model for CHF
- Statistical approach for CHF
- Wicking surfaces to enhance CHF

Zuber's Model Assumptions

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\sigma \alpha (\rho_l + \rho_v)}{\rho_l \rho_v}} = \sqrt{\frac{2\pi\sigma (\rho_l + \rho_v)}{\rho_l \rho_v \lambda_H}}$$



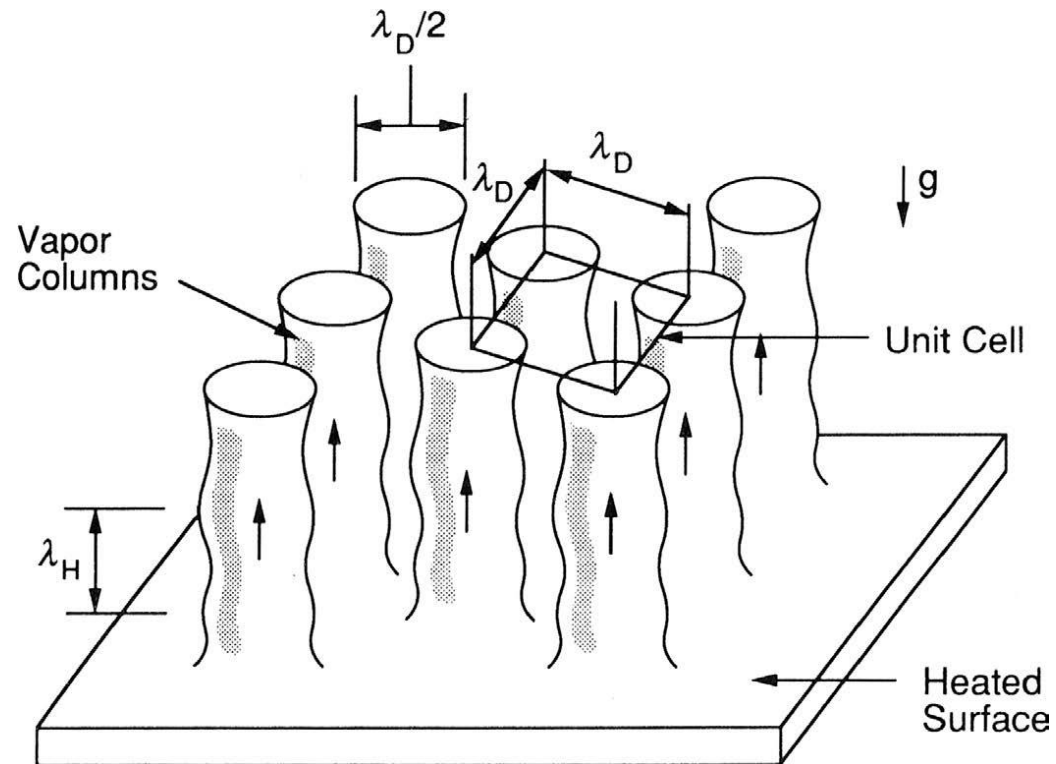
- Vapor columns are formed near CHF
- CHF is reached when interface of vapor columns becomes Helmholtz unstable (λ_H)
- The pitch of the vapor columns coincides with the most dangerous wavelength in Taylor instability

$$\lambda_D = 2\pi\sqrt{3\sigma/\Delta\rho g}$$

- The diameter of vapor column is $\lambda_D/2$
- Assuming $\lambda_H = \lambda_D$

Zuber's CHF Prediction

$$u_c = \sqrt{\frac{2\pi\sigma}{\rho_v\lambda_D}} \quad \lambda_H = \lambda_D = 2\pi \sqrt{\frac{3\sigma}{\Delta\rho g}}$$



Comments on Zuber's Model

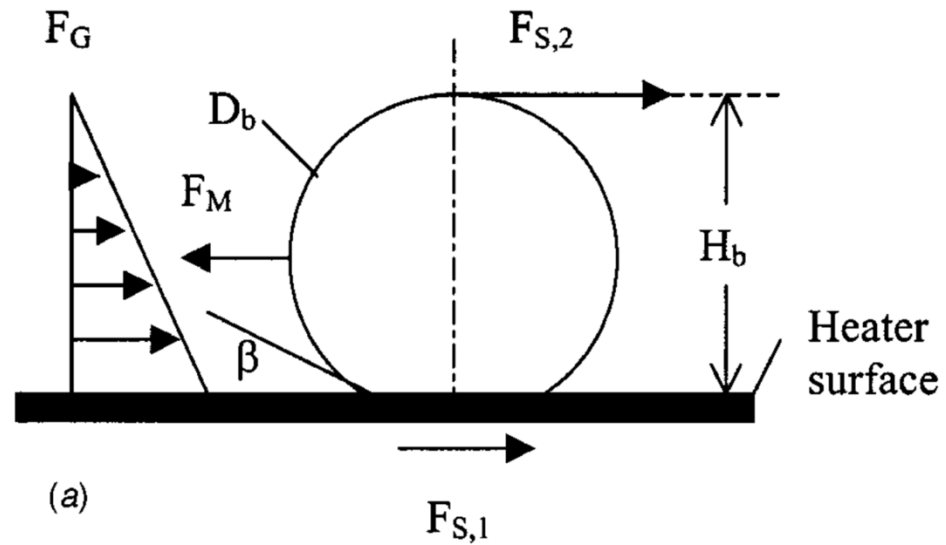
- No way to accommodate effects from geometry and surface wettability
- No clear justification for the choice of vapor column diameter as $\lambda_D/2$
- No visual observation of Helmholtz instability during boiling to date
- Still widely used as a reference model for all subsequent CHF models

Lateral Force Balance Model

Kandlikar 2001

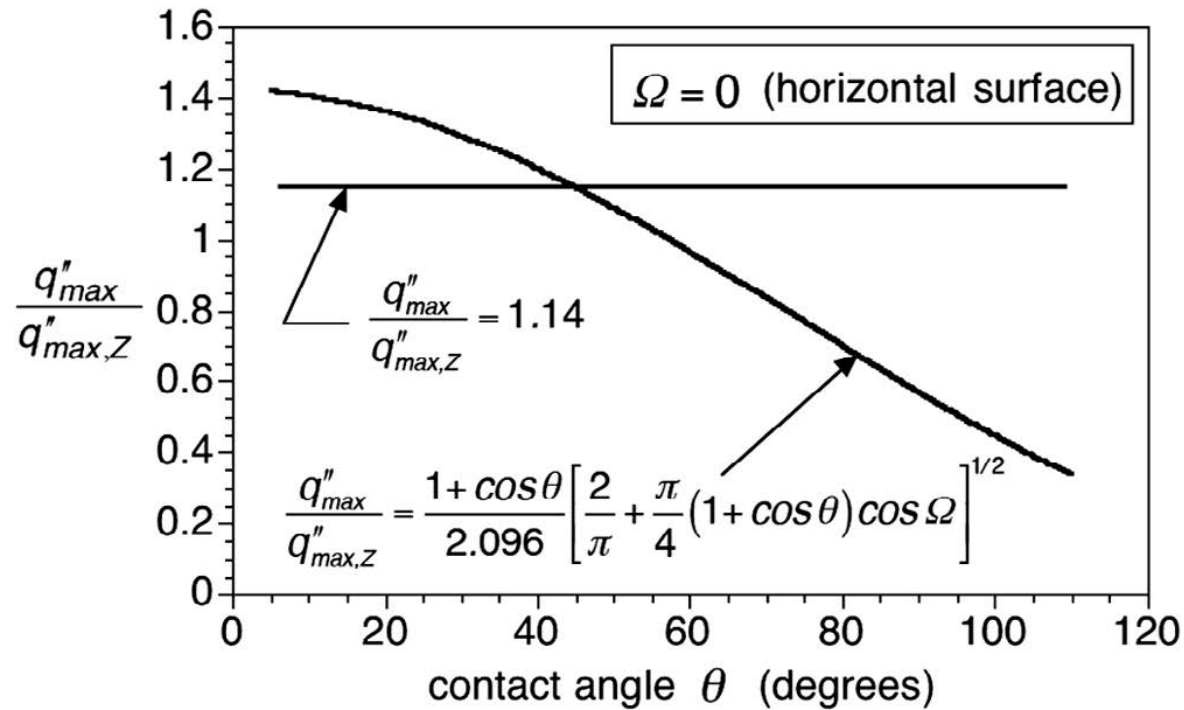
<https://doi.org/10.1115/1.1409265>

Considering the lateral direction
on half of a bubble



Contact Angle Dependence

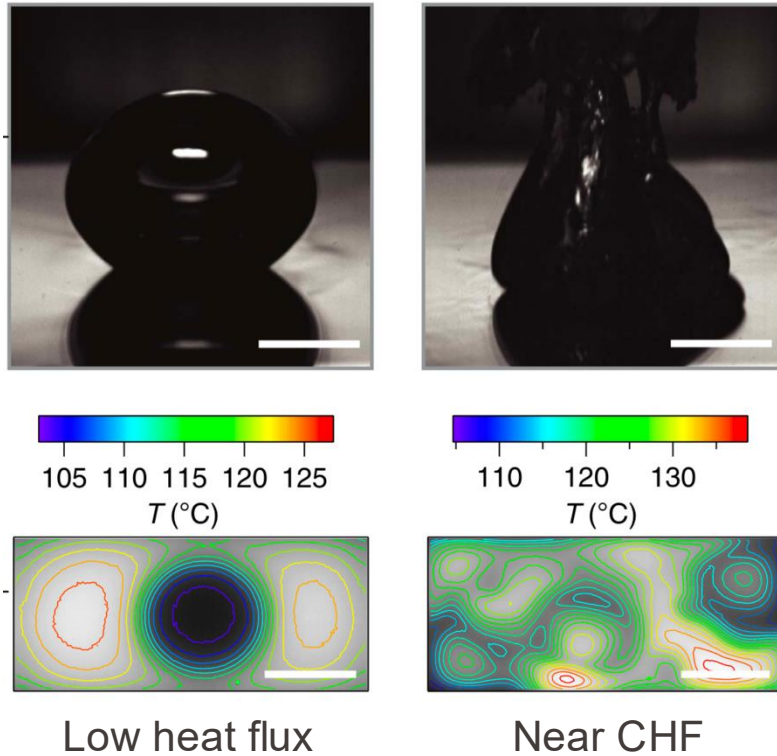
$$q''_K = \rho_v h_{fg} \left(\frac{1 + \cos \beta}{16} \right) \left[\frac{2}{\pi} + \frac{\pi}{4} (1 + \cos \beta) \right]^{\frac{1}{2}} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{1/4} \quad (\text{Horizontal surface})$$



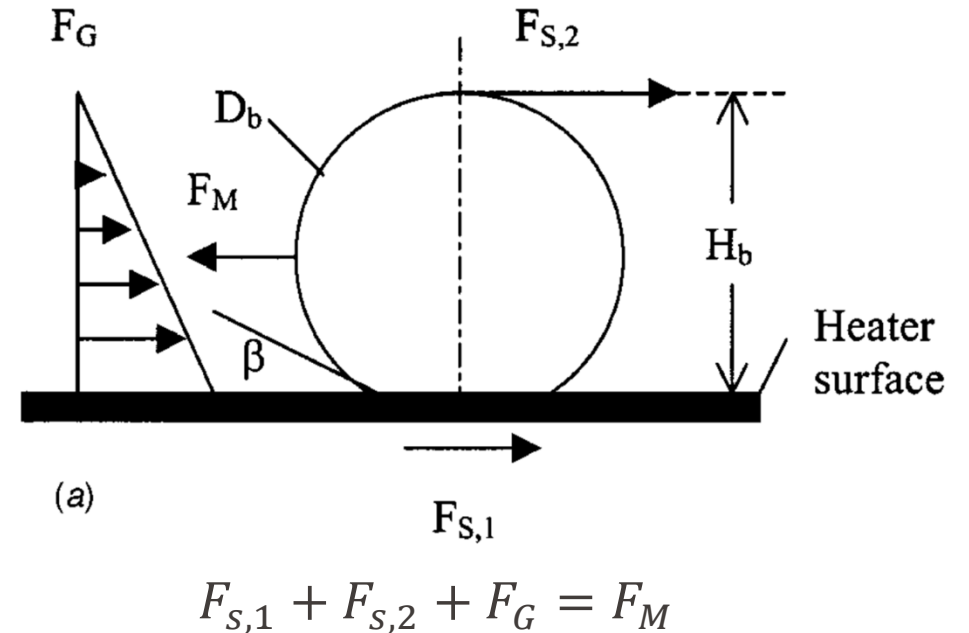
$$q''_{max,Z} = 0.149 \rho_v h_{lv} \left(\frac{\sigma \Delta \rho g}{\rho_v^2} \right)^{1/4}$$

FIGURE 7.19 in Carey

Dhillon *et al.*, *Nat Commun* 2015



Choice of geometric parameters (e.g., bubble shape and influence area) not quite justified at CHF



Liquid-vapor pressure difference not accounted for in force balance

Statistical Approach for Flat Surface Boiling

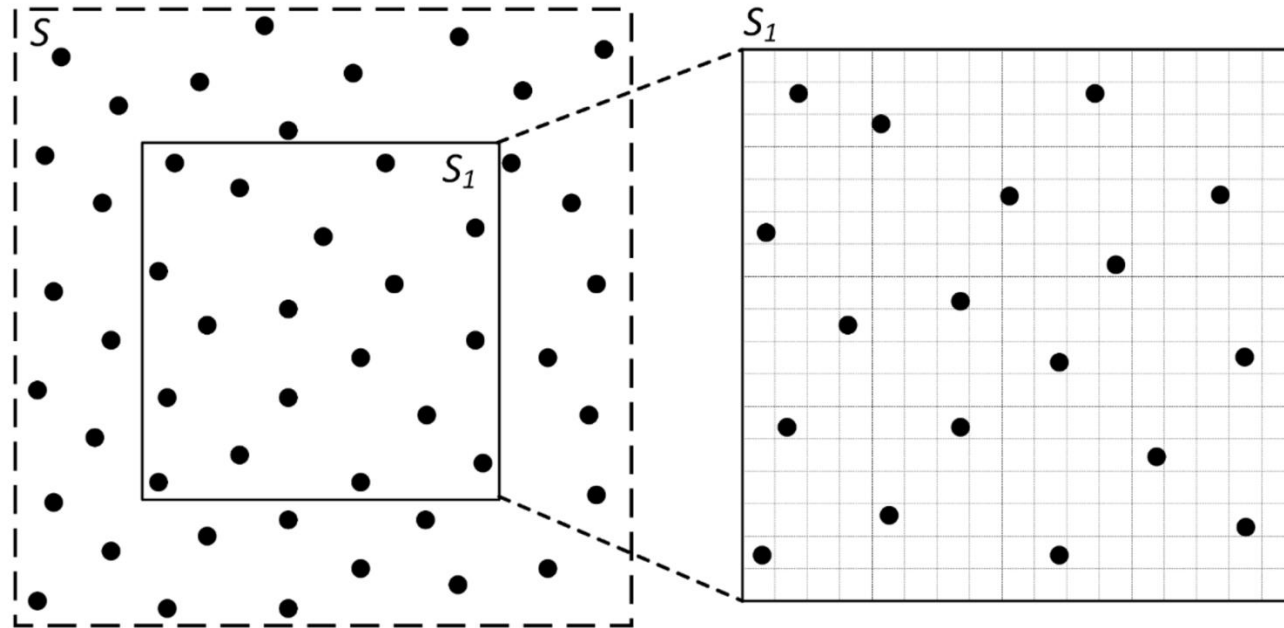


Working fluid: R134a

Heat flux:
from 1.5 W/cm² to 38 W/cm²

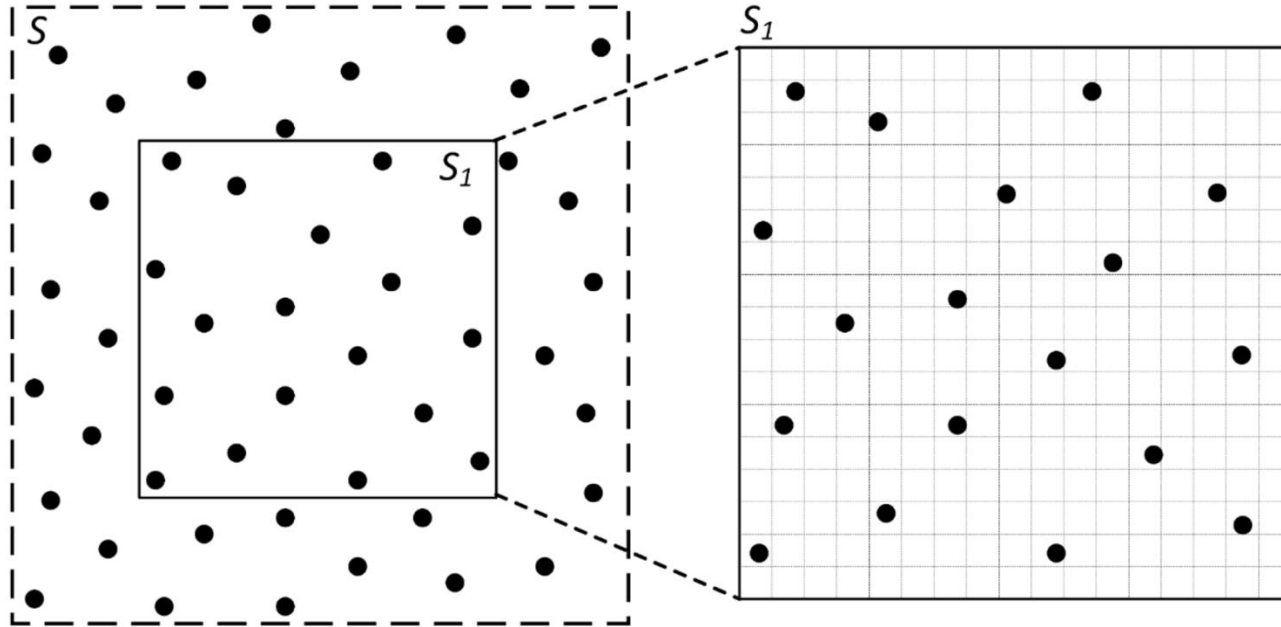
Yazdani, 2016
<https://doi.org/10.1063/1.4940042>

Population Distribution of Nucleation Sites

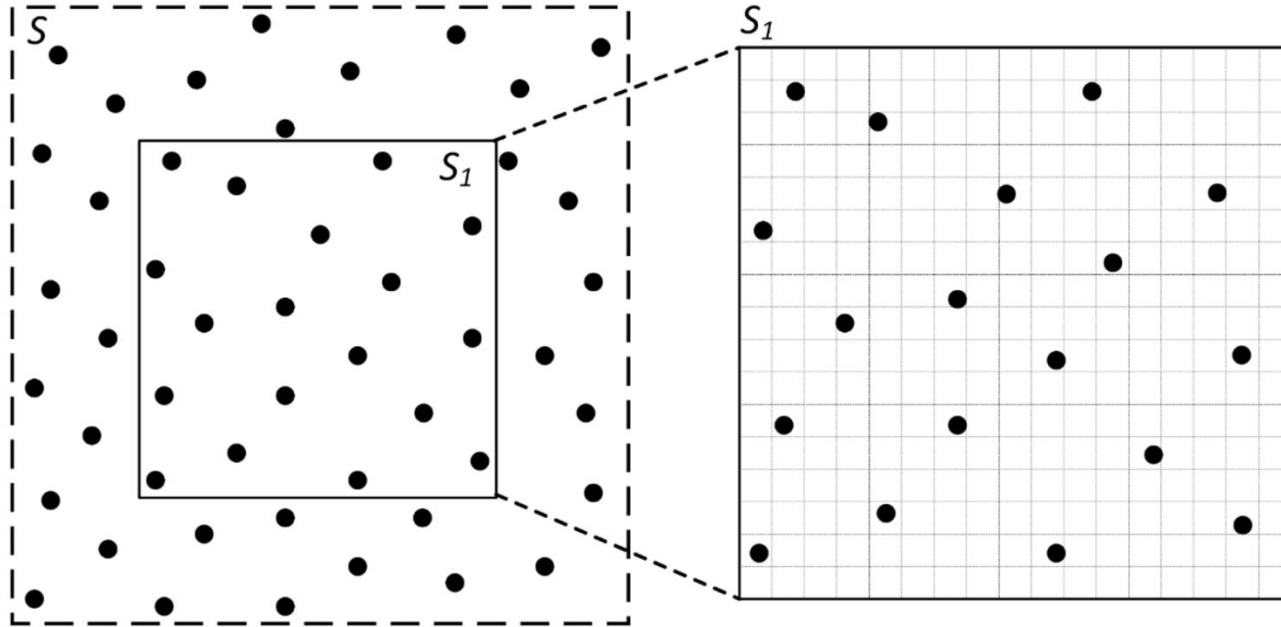


<https://doi.org/10.1016/j.ijheatmasstransfer.2021.121904>

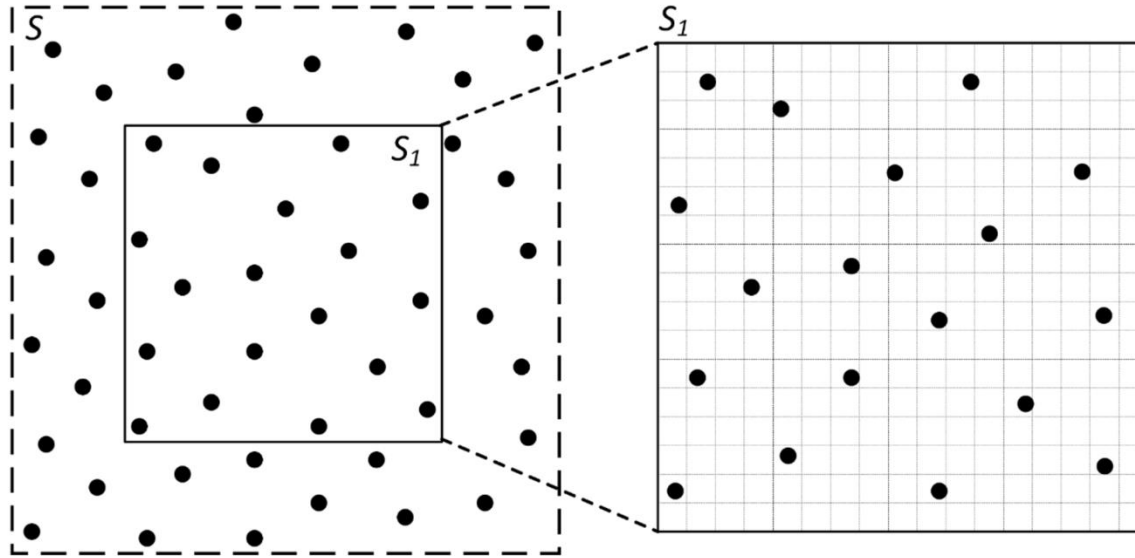
- Consider a large surface S (large enough to ignore edge effects)
- Assume that probability of each point on the surface becoming an active nucleation sites is equal



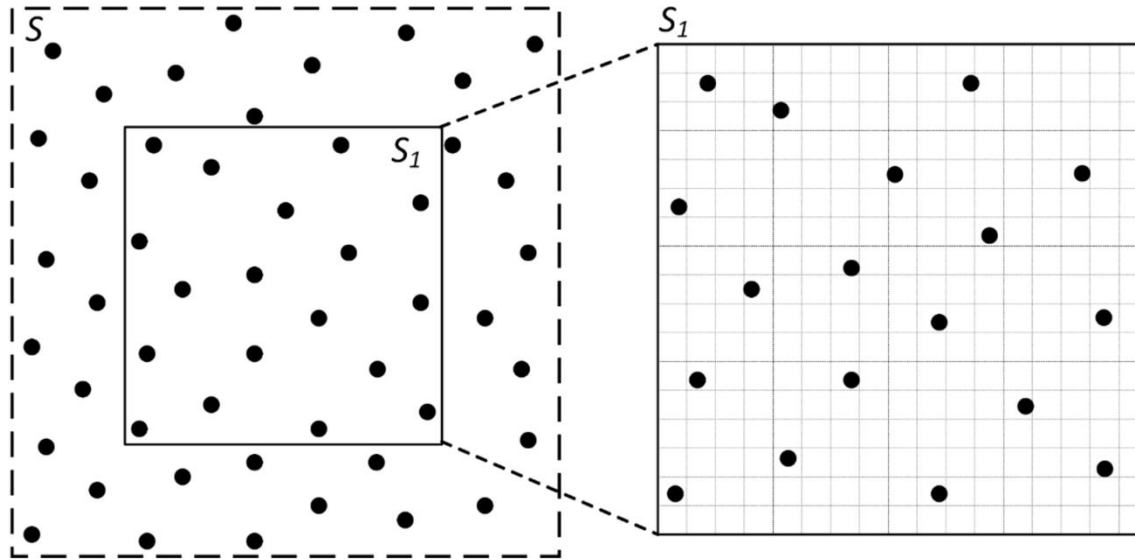
- The actual number of active nucleation sites in S_1 , N , can be considered as a random variable with an expectation value N_0



- Divide S_1 into M segments, each with the same area A/M



- Probability of finding N squares that contain one nucleation site is a binomial distribution

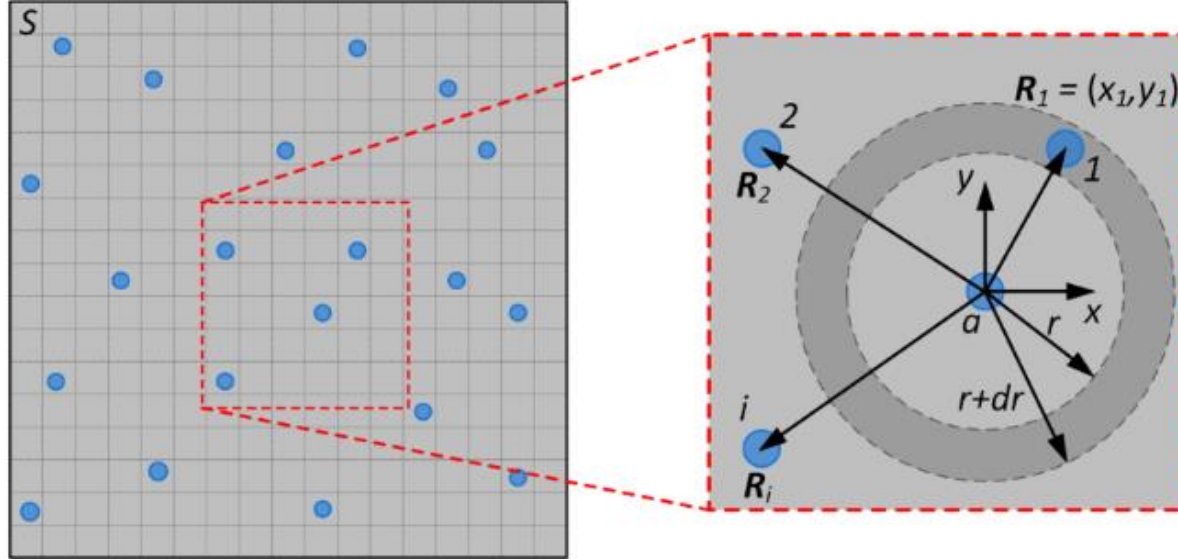


$$P(N, N_0, M) = \frac{1}{N!} \cdot \frac{M!}{(M - N)! M^N} \cdot N_0^N \left(1 - \frac{N_0}{M}\right)^{M-N}$$

$$\text{Po}(N, N_0) = \frac{N_0^N}{N!} e^{-N_0}$$

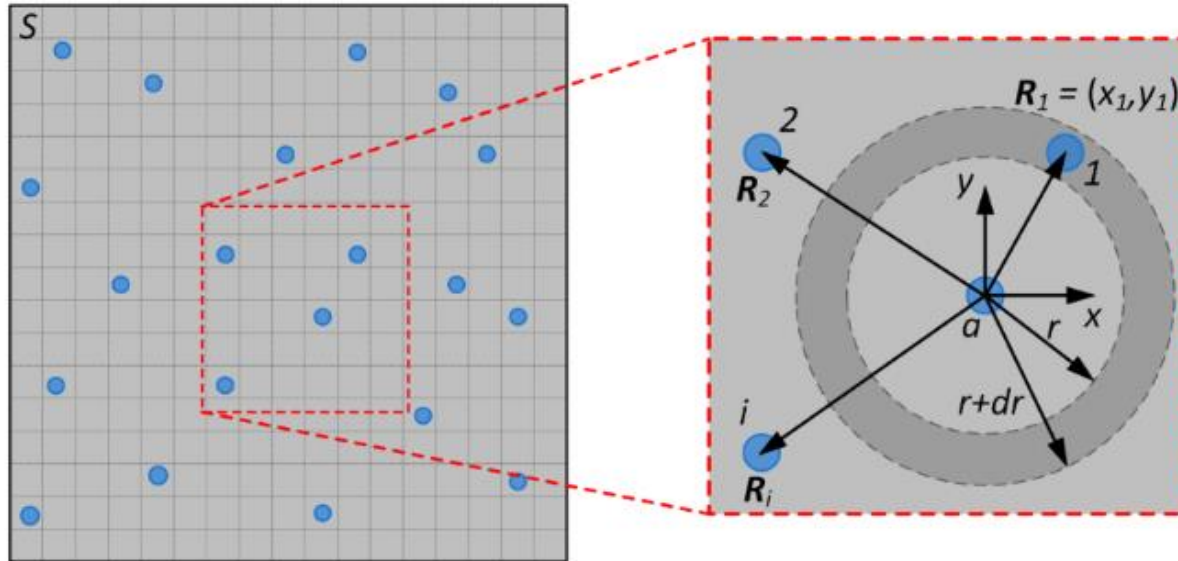
- Isolated bubbles dissipate heat better than merged bubbles

Nearest Neighbor (NN) Distance Between Nucleation Sites



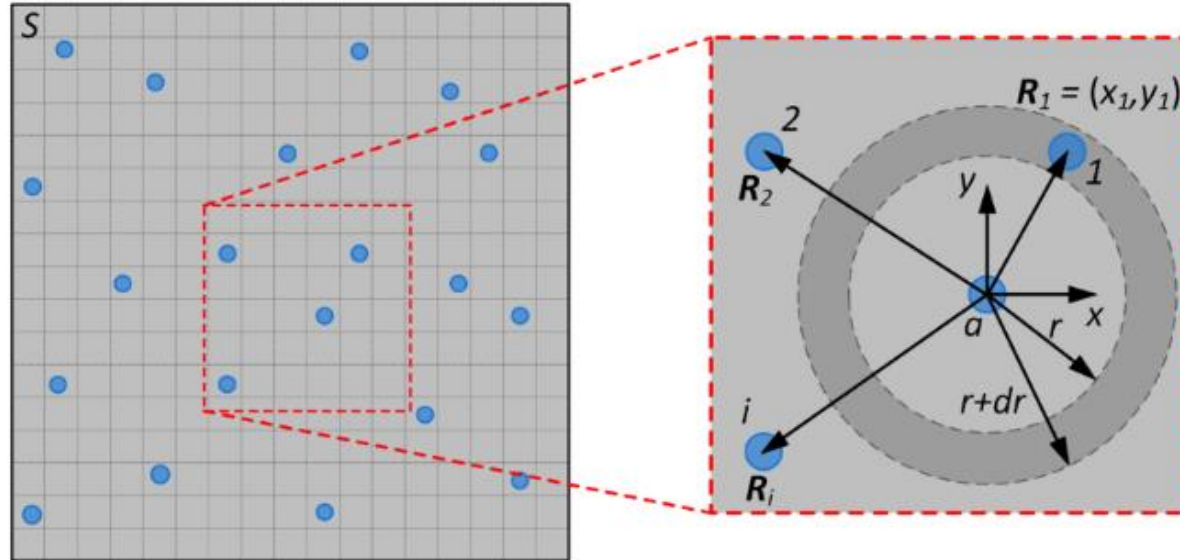
- We would like to know the probability distribution function for distance between two nearest activated nucleation sites

Nearest Neighbor (NN) Distance Between Nucleation Sites



- Assuming the surface is sufficiently large (no edge effects)

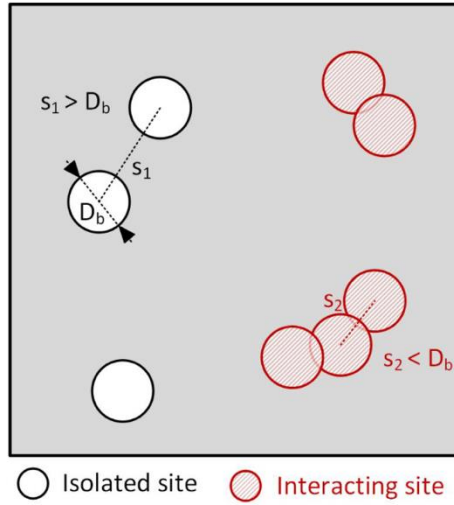
Nearest Neighbor (NN) Distance Between Nucleation Sites



$$\Pr(r \leq s) = 1 - \left(1 - \frac{\pi S^2}{A}\right)^{N-1}$$

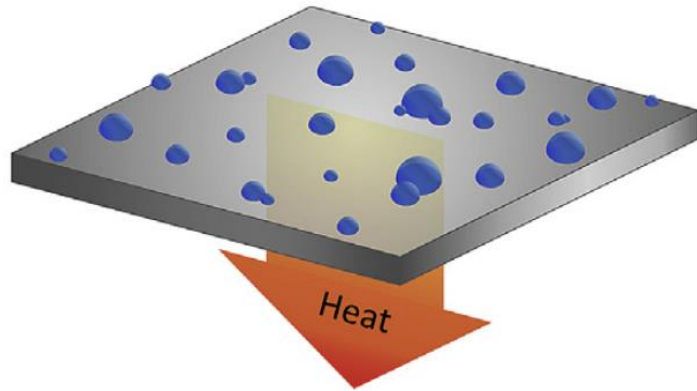
The probability distribution function for NN distance $f(s)$

Number of Isolated Bubbles



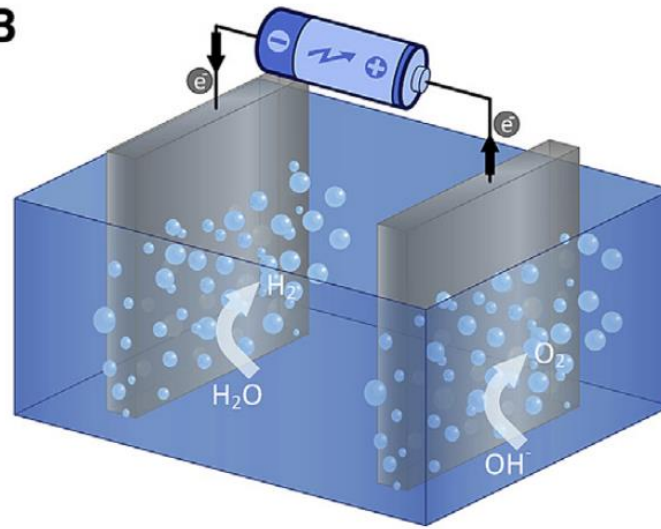


A



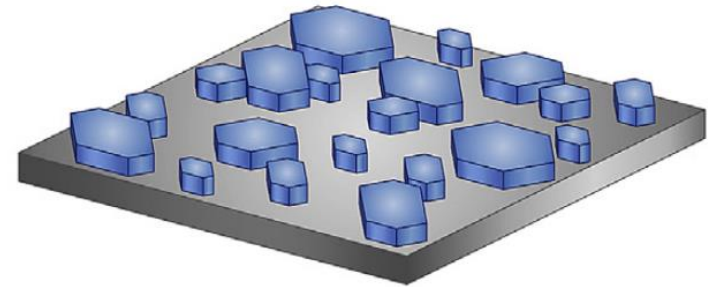
Condensation

B



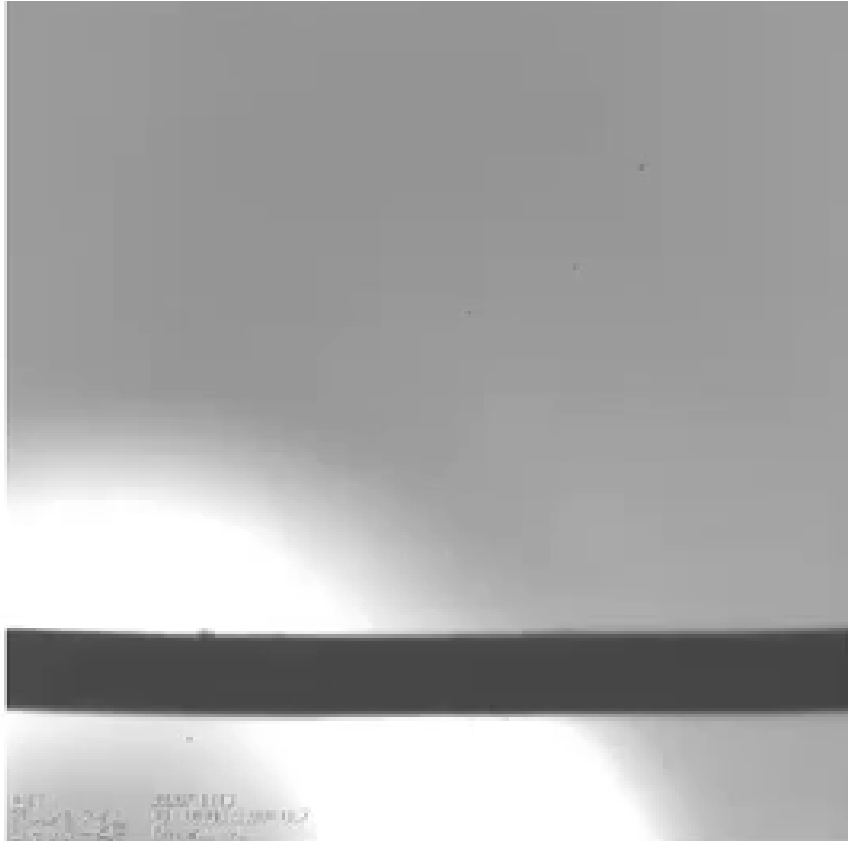
Water splitting

C



Material growth

Gas Evolving Reaction (GER)



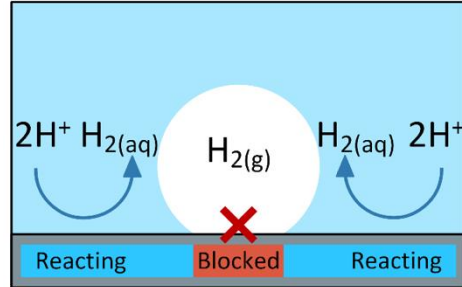
Industrial electrochemical processes that involves bubbles

Water splitting

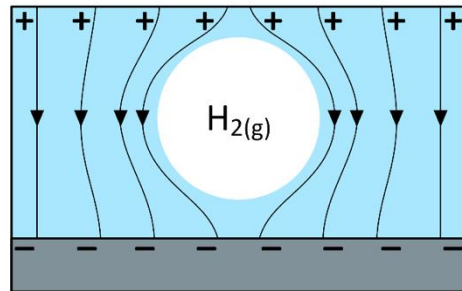
Aluminum production

Sodium chlorate production

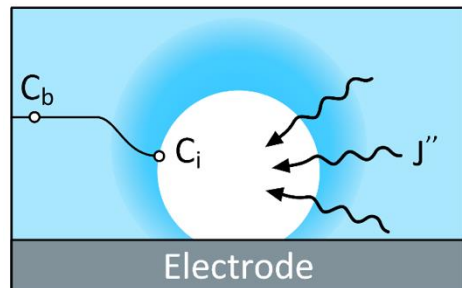
Chlorine and sodium hydroxide production



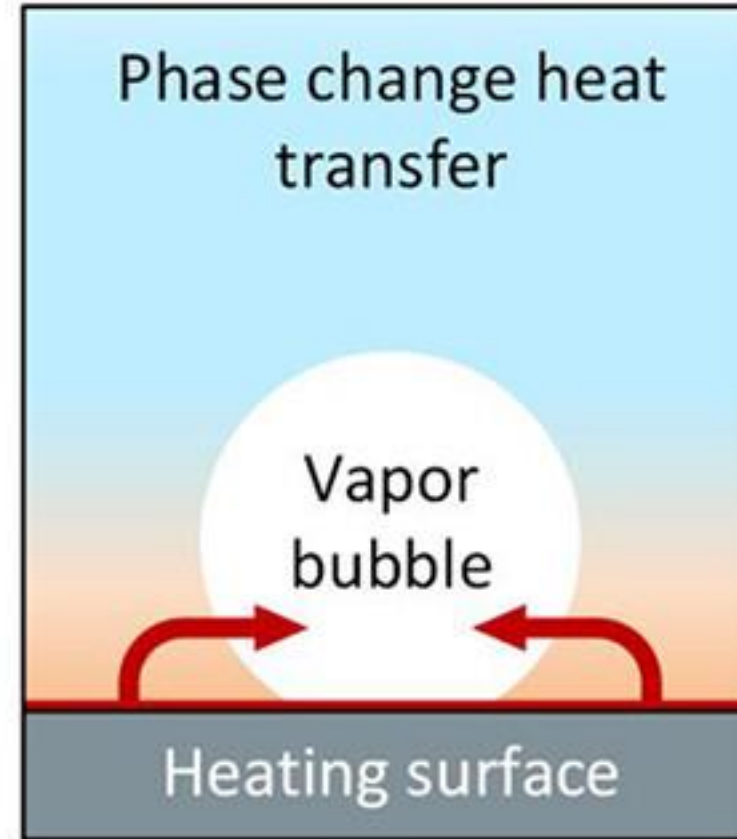
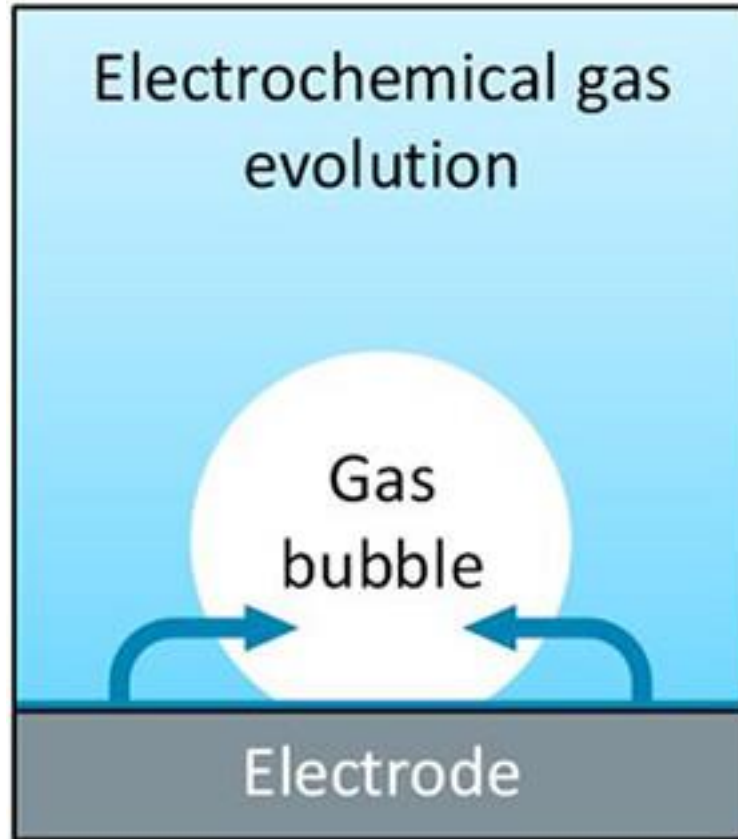
Blocking reaction area

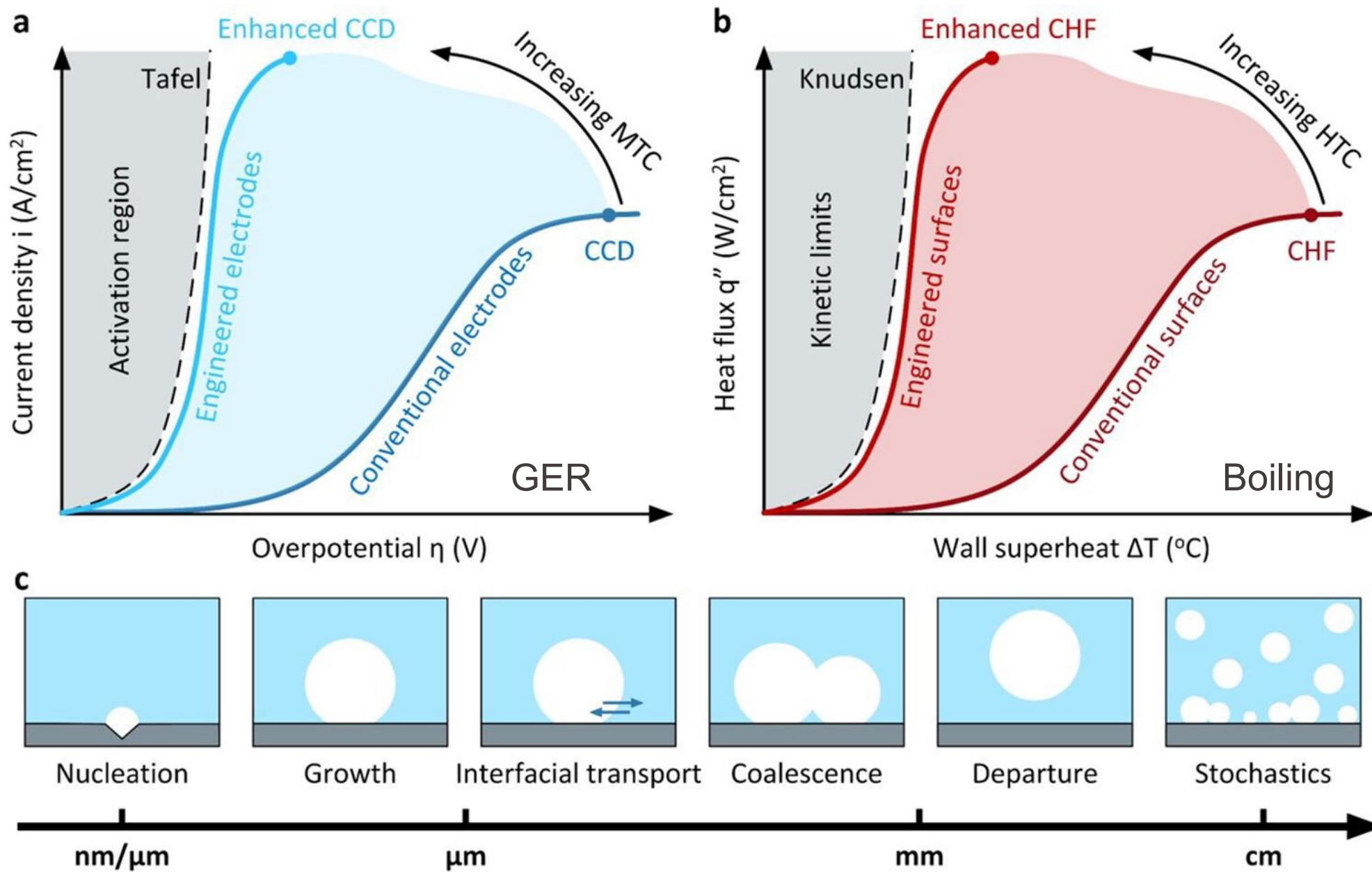


Additional resistance for ion transport

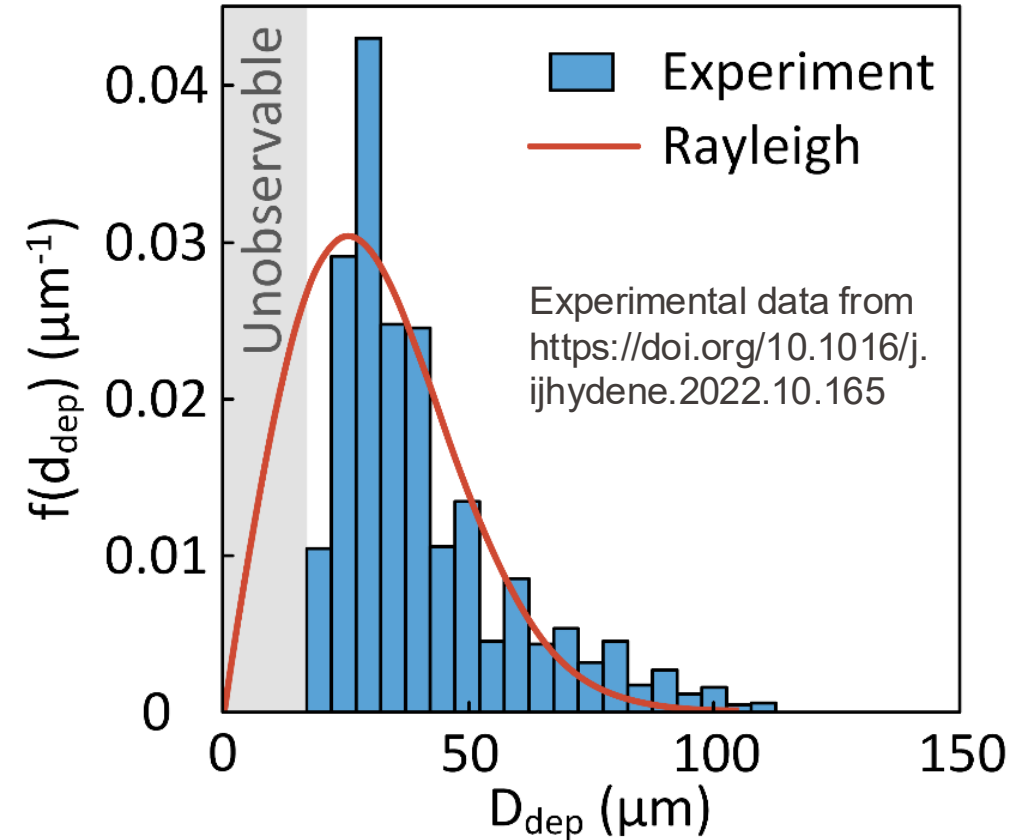
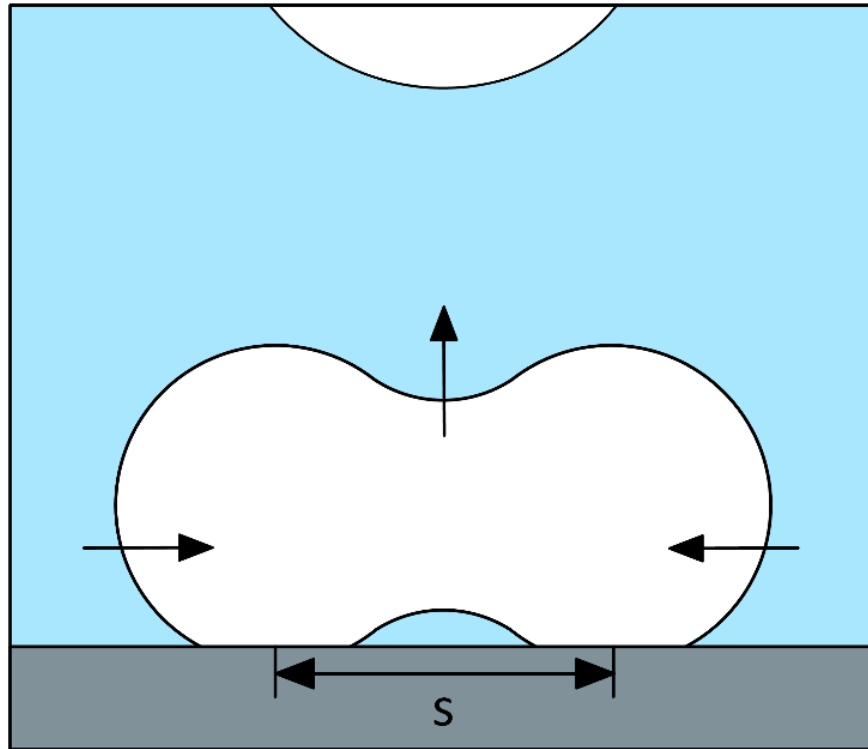


Additional diffusion resistance
Bubble-induced convection

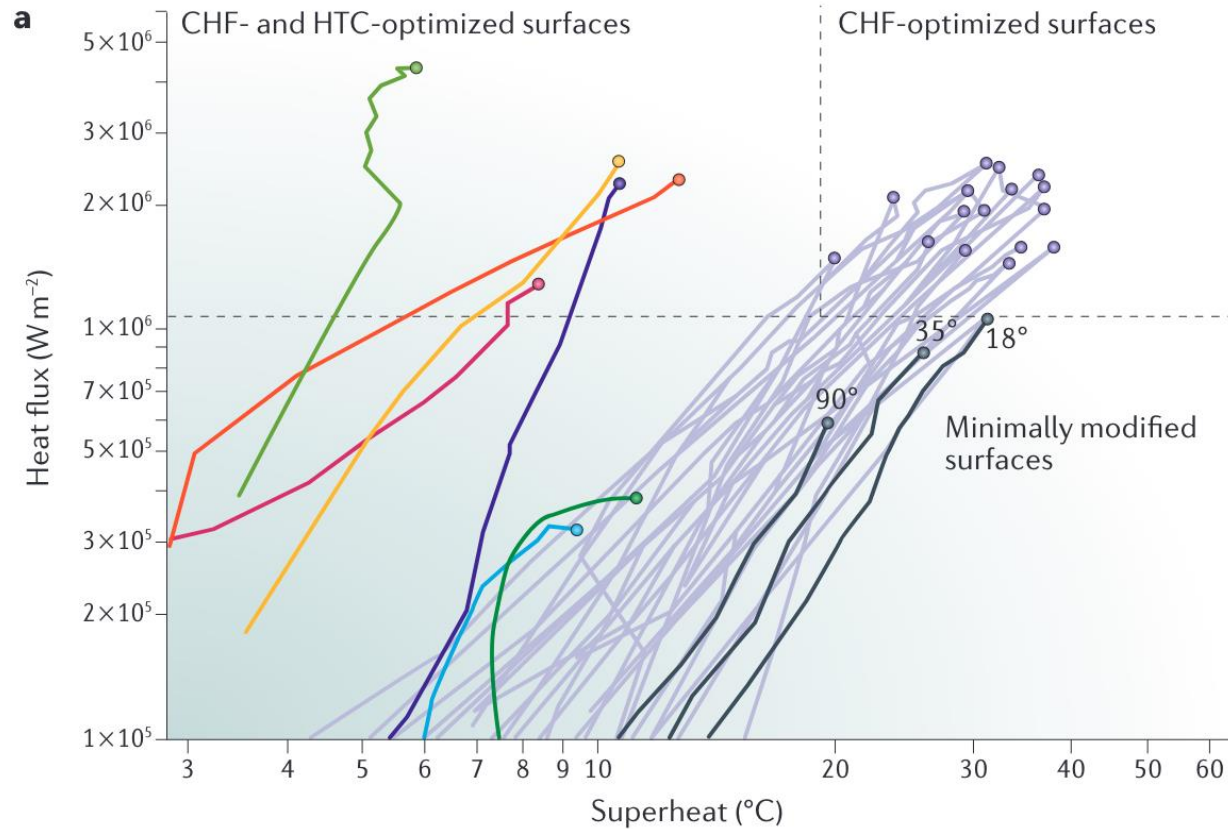




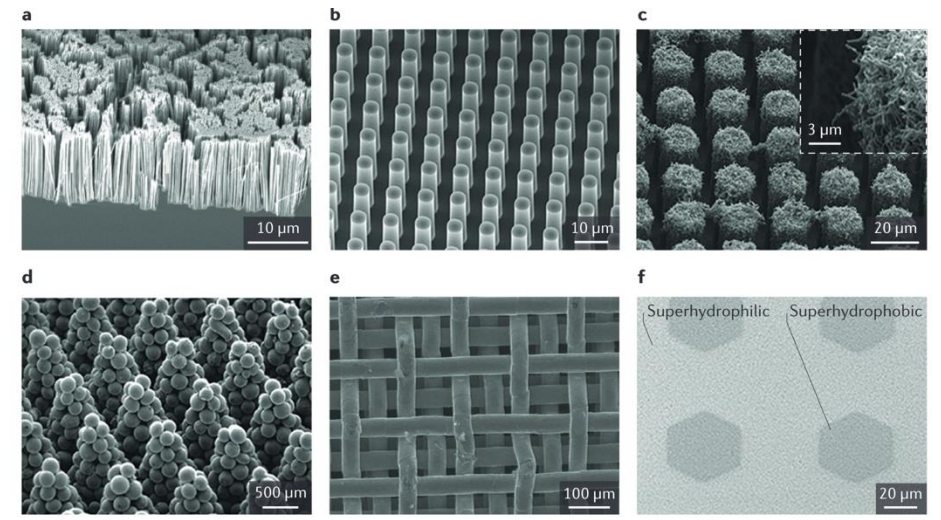
Coalescence-Driven Bubble Departure



- When two bubbles coalesce into one, surface area becomes smaller. Excess surface energy needs to be released.
- This may cause bubble to depart immediately after contacting its nearest neighbor

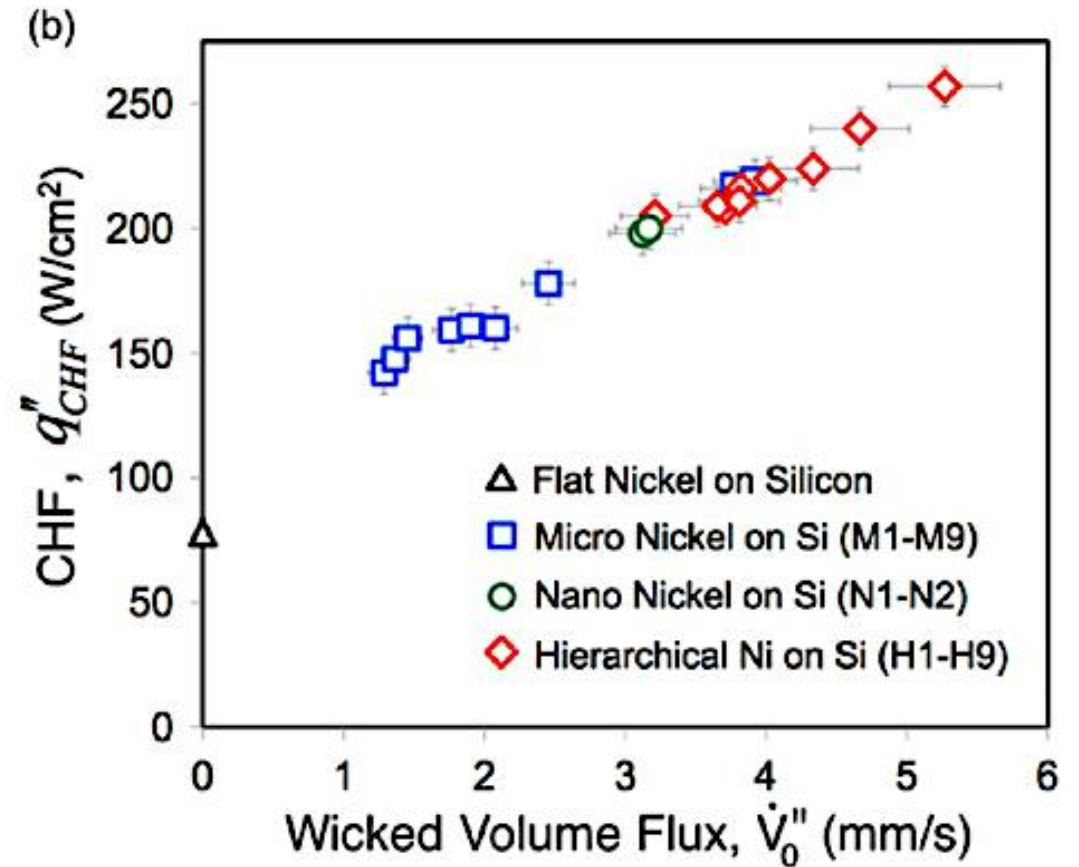
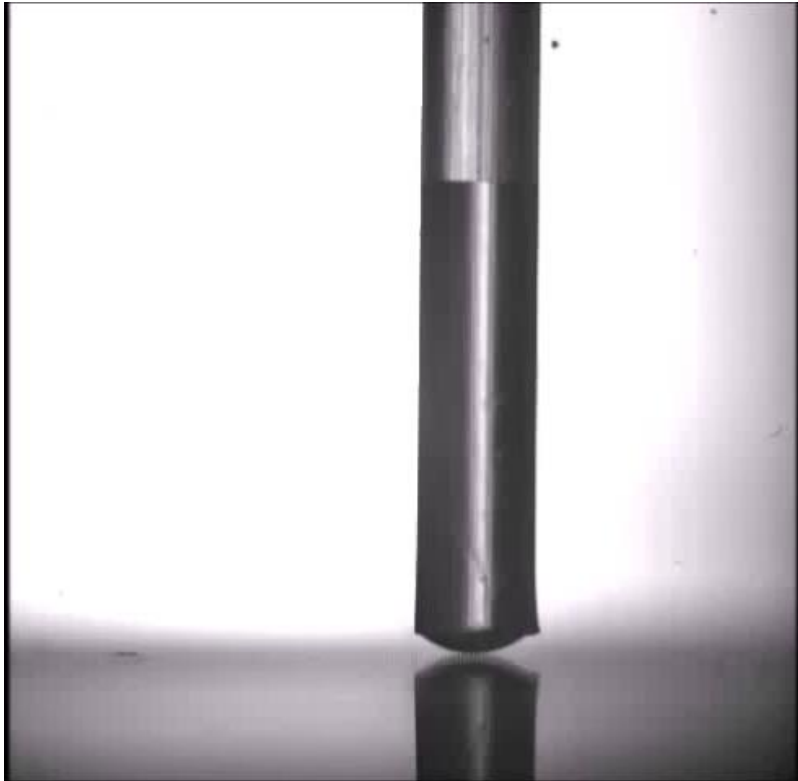
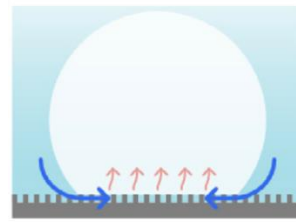


- Sintered microchannels¹³¹
- Sintered wire mesh¹²⁵
- Microchannels¹³⁰
- Biconductive¹²⁹
- Superbiphilic¹²⁸
- Surfactant desorbed¹³⁵
- Surfactant adsorbed¹³⁵
- Hydrophilic micro- and nanostructured (structured, porous and nanofluid)^{34,35,37,39,40,42,43}
- 18° smooth²⁰
- 35° smooth²⁰
- 90° smooth²⁰



doi:10.1038/natrevmats.2016.92

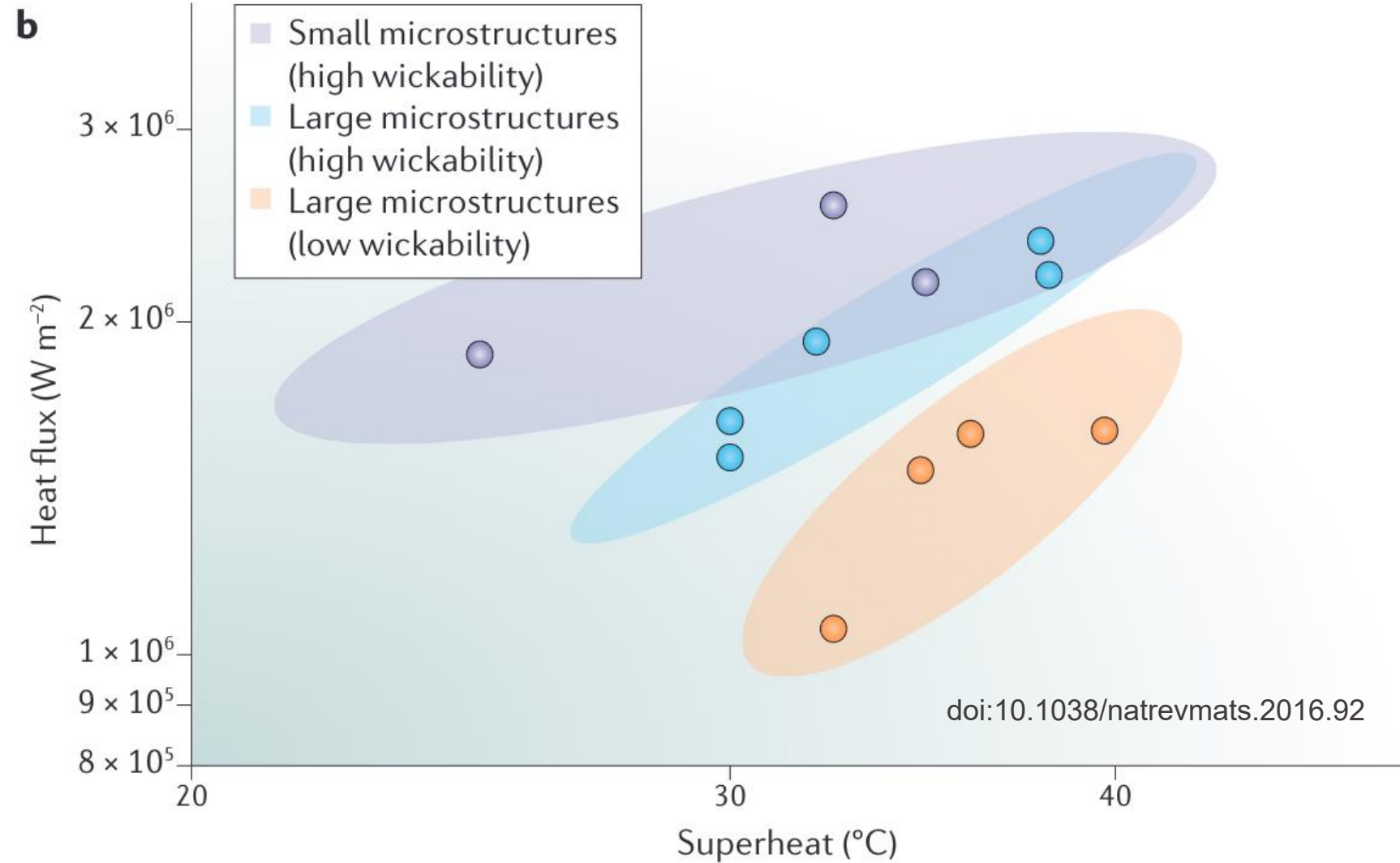
Wicking Helps Boiling



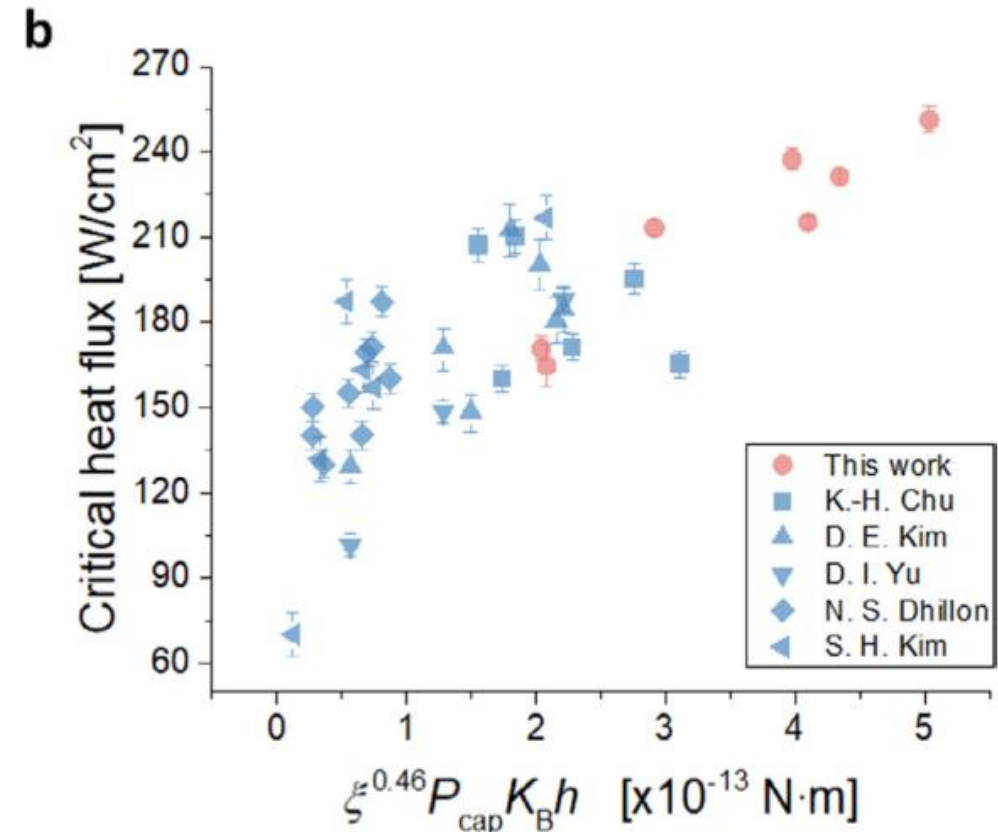
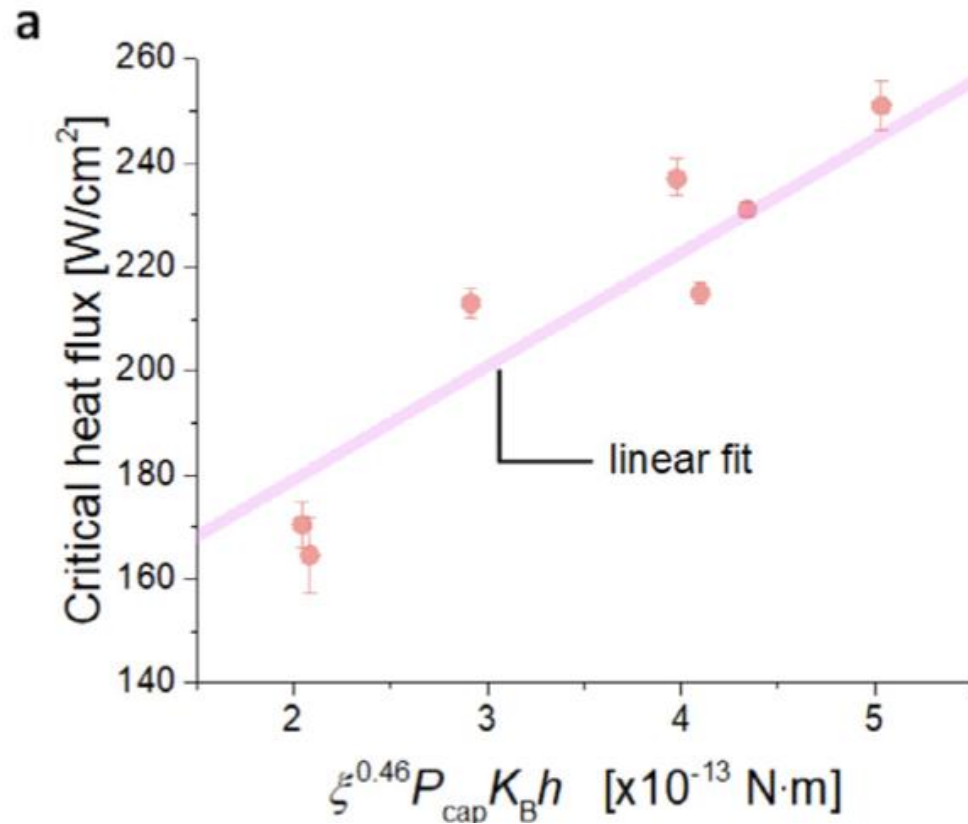
<https://doi.org/10.1021/acs.langmuir.7b01522>

<https://doi.org/10.1021/la5030923>

Wickability Helps Boiling



CHF on Wicking Surfaces

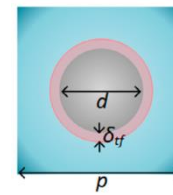


P_{cap} : capillary pressure

K_B : permeability

h : thickness of the wick

<https://doi.org/10.1016/j.ijheatmasstransfer.2021.122189>



ξ : "thin film" liquid area fraction

Summary on Boiling CHF

- On flat surfaces, CHF is thought to be caused by 1) hydrodynamic instabilities and/or 2) number of isolated bubbles reaching maximum
- With wicking surfaces, additional liquid supply (through capillarity) to the bubble can improve CHF