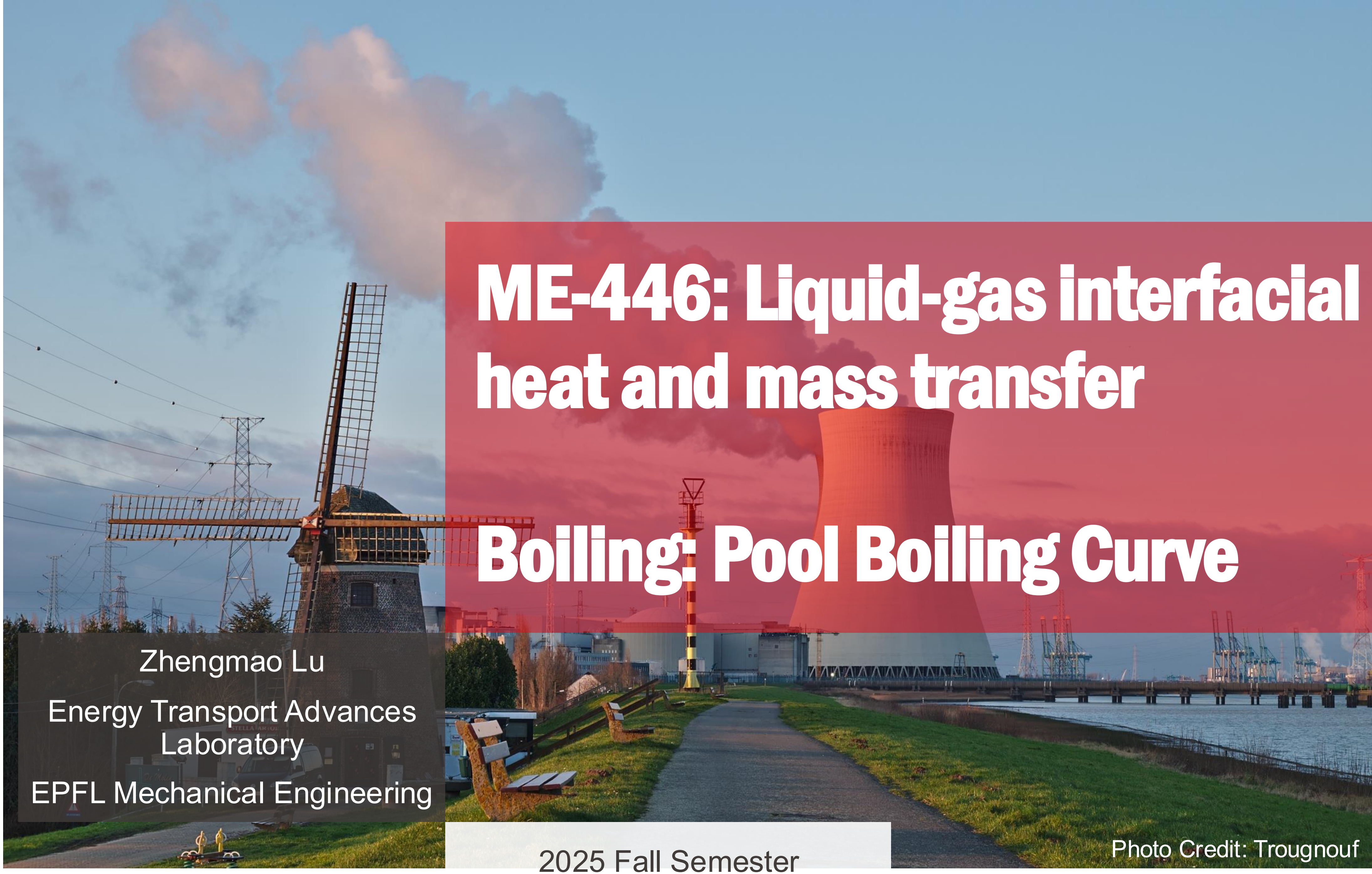
ETA-Lab
Energy Transport Advances



ME-446: Liquid-gas interfacial heat and mass transfer

Boiling: Pool Boiling Curve

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2025 Fall Semester

Photo Credit: Trougnouf

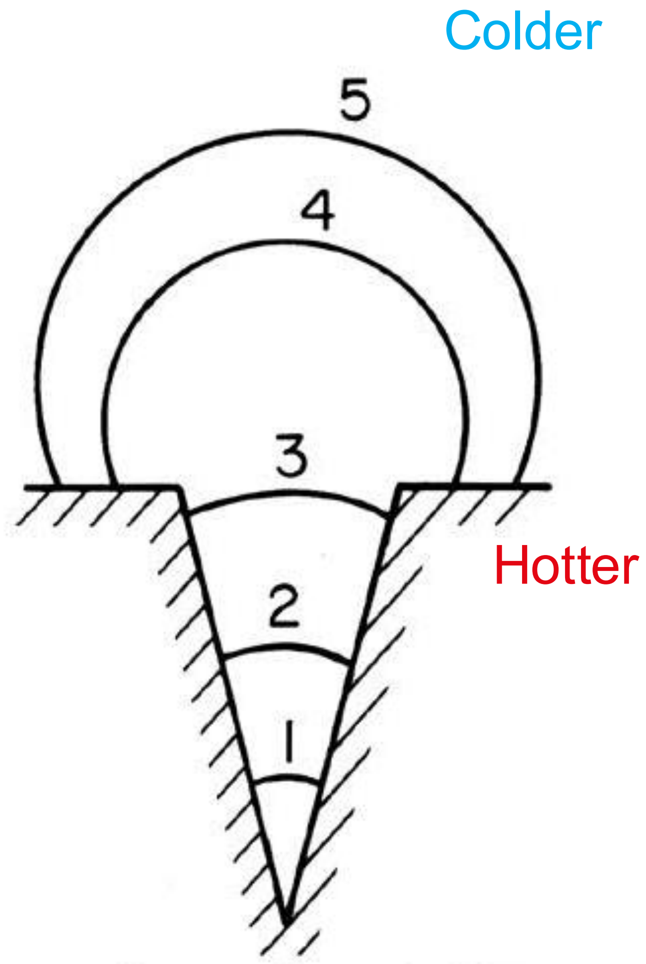
Logistics

- December 4th and 11th: journal presentation
- December 18th: final review and lab tour

Last Time

- Understand the mechanism for heterogeneous nucleation in practical systems (entrapped gas/vapor theory)
- Understand Hsu's criteria for nucleation site activation
- Analyze the timescales in the bubble cycle to evaluate bubble departure frequency

Criteria for Nucleation Site Activation



- Whether bubble can grow out of the cavity overcoming capillary pressure?
- Whether bubble can keep growing as it gets closer to the bulk fluid which is colder than the heated wall

Bubble Nucleation Criteria

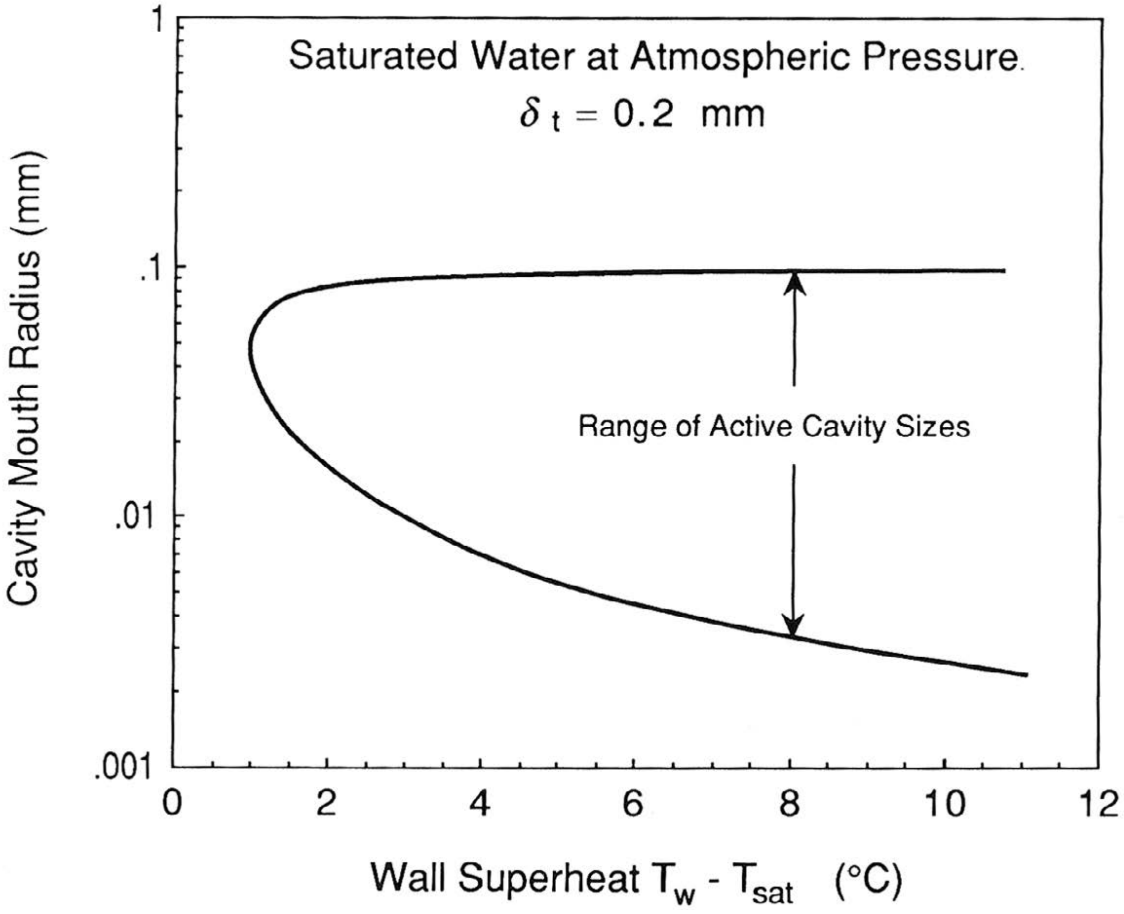
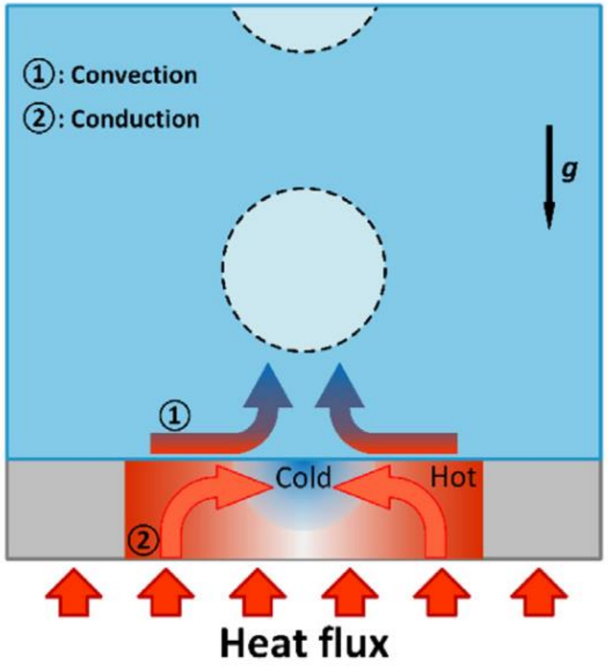


Figure 6.13 in Carey

- If the bubble is too small, the Laplace pressure will be too large for nucleation to occur
- If the bubble is too large, the top of the bubble may be surrounded by liquid of not-high-enough temperature

Timescale Analysis



- Rewetting and heat conduction are two competing mechanisms for temperature recovery

$$\frac{\tau_d}{\tau_w} \sim D^{1.5}$$

When *D* is relatively large, rewetting-induced convection dominates

$$f = \frac{1}{\tau_w} \sim D^{-0.5}$$

When *D* is relatively small, conduction dominates

$$f = \frac{1}{\tau_d} \sim D^{-2}$$

Bubble Departure Diameter

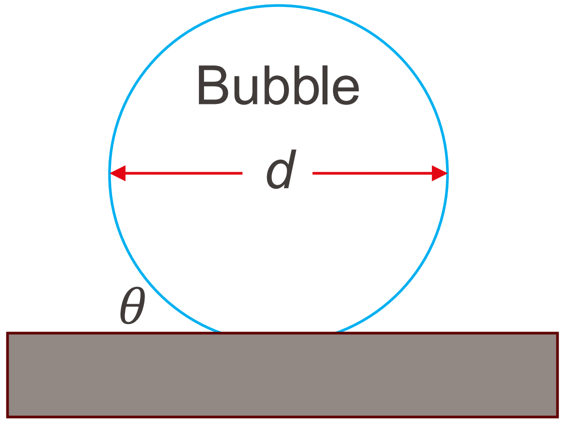
Buoyancy force $\sim(\rho_l - \rho_v)d^3 g$

Surface tension force $\sim\sigma_{lv} d$

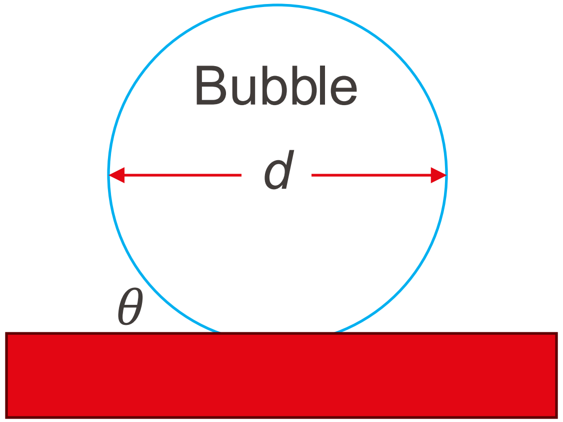
Bond number $$\mathrm{Bo}_d = \frac{(\rho_l - \rho_v)d^2}{\sigma_{lv}}$$

Most correlations are written in terms of $\mathrm{Bo_d}$

Fritz expression $\mathrm{Bo_d} = 0.0208\theta$

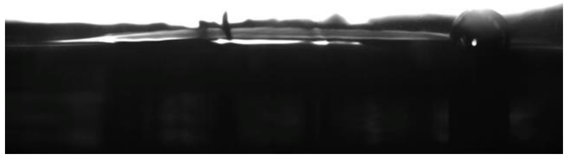


Comments on Fritz's Expression



Fritz expression $\mathbf{Bo_d = 0.0208\theta}$

Simple balance between surface tension force and buoyance force. The effect of the contact angle is taken into account in an empirical manner



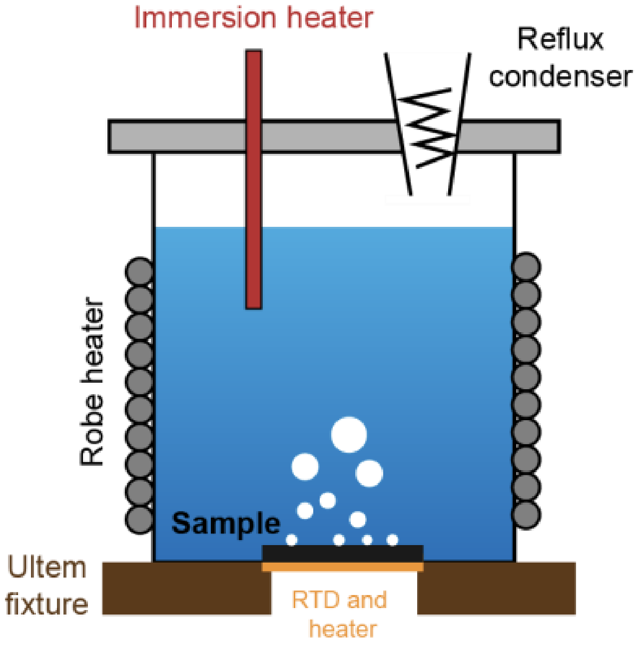
At different heat fluxes, the bubble may have different growth rate, corresponding to a different momentum force and nonspherical shape

Intended Learning Objectives Today

- Pool boiling (concept, heat transfer regimes,…)
- Rohsenow's microconvection model for nucleate boiling
- Zuber's CHF model based on Helmholtz and Taylor instabilities

Pool Boiling

: boiling a liquid on a heated surface that is submerged in the liquid, without any forced flow; the liquid motion is driven only by natural convection and bubble activity



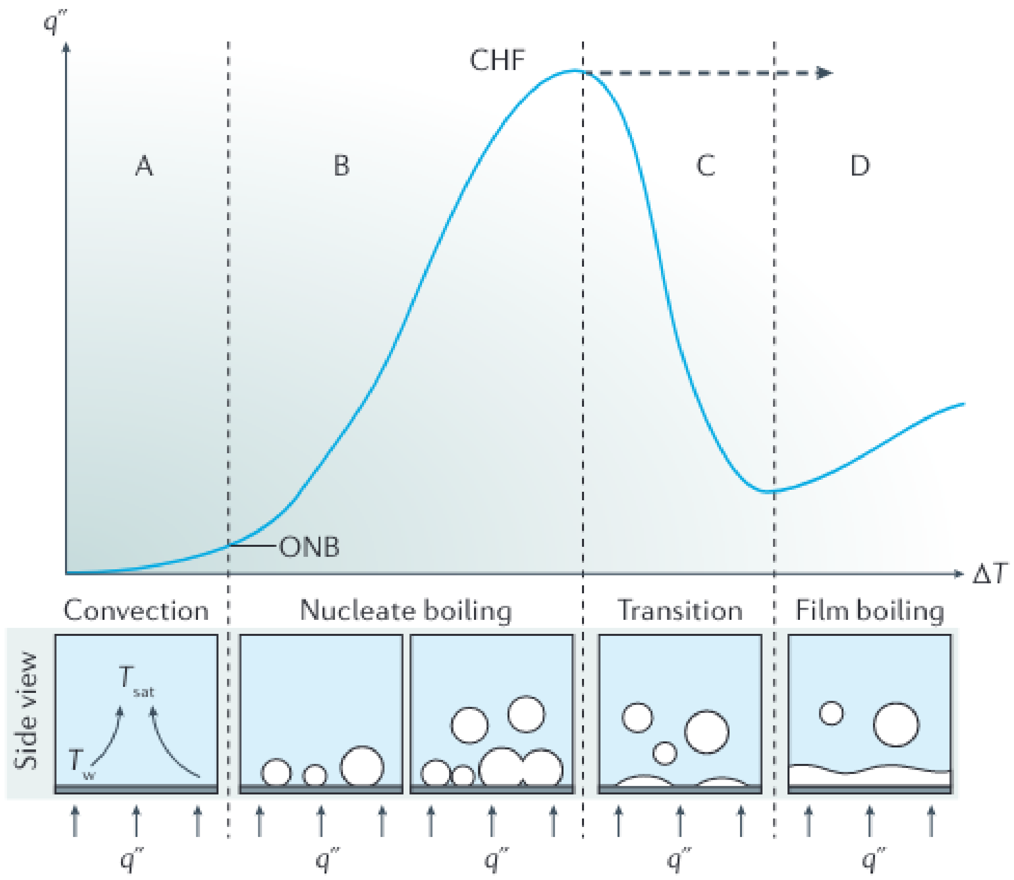
Pool boiling surface characteristic length $L >> L_b$

Capillary length $L_b = \sqrt{\dfrac{\sigma}{g(\rho_l - \rho_v)}}$ by setting Bo = 1

We are interested in the relationship between the average heat flux at the boiling surface q" and the surface super heat $\Delta T = T_w - T_{sat}(P_l)$

We focus on saturated pool boiling ($T_\infty = T_{sat}(P_l)$) unless stated otherwise

Boiling Curve



A. At very low superheats, heat transfer is mostly due to natural convection

B. After superheat is large enough to form vapor bubbles, nucleate boiling dominates, promoting bubble-motion-induced convection

C-D. After vapor generation becomes too much, passing the critical heat flux (CHF), insulating vapor film will start to form, decreasing the HTC

Rohsenow's Microconvection Model

Convective transport facilitated by bubbles

$$\mathrm{Nu}_b = \frac{hL_b}{k_l} \propto \mathrm{Re}_b^{1-r}\mathrm{Pr}_l^{1-s}$$

$$\mathrm{Re}_b = \frac{\rho_v U L_b}{\mu_L} \qquad U = \frac{q''}{\rho_v h_{lv}} = \frac{h\Delta T}{\rho_v h_{lv}}$$

$$\frac{q''}{\mu_l h_{lv}}\left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{1/2} = \left(\frac{1}{C_{sf}}\right)^{1/r} \mathrm{Pr}_l^{-s/r}\left[\frac{c_{pl}\Delta T}{h_{lv}}\right]^{1/r}$$

Rohsenow's Microconvection Model

$$\frac{q''}{\mu_l h_{lv}}\left[\frac{\sigma}{g(\rho_l-\rho_v)}\right]^{1/2}=\left(\frac{1}{C_{sf}}\right)^{1/r}\mathrm{Pr}_l^{-s/r}\left[\frac{c_{pl}\Delta T}{h_{lv}}\right]^{1/r}$$

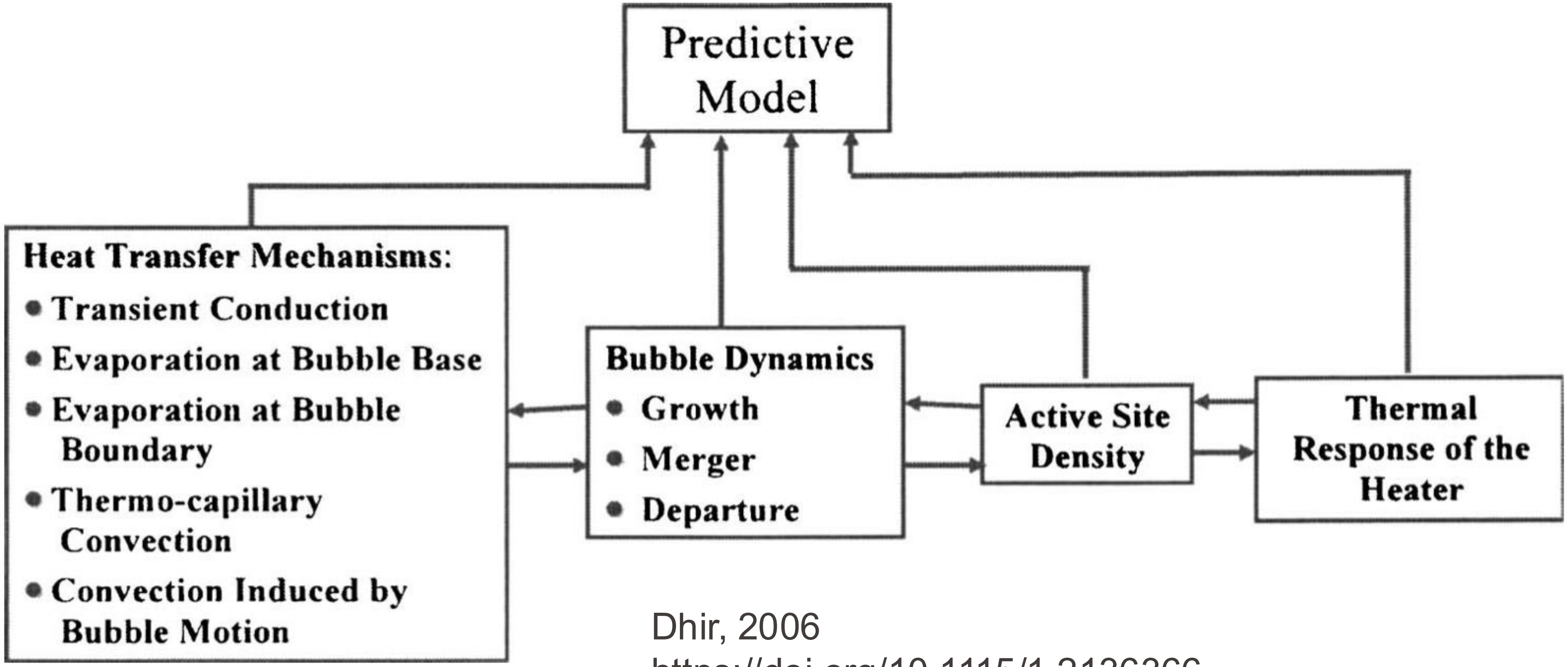
See Chapter 7.7 in Carey for recommended values of r, s, C_{sf}

Most commonly used correlation for nucleate boiling heat transfer

For hydrophobic surfaces, C_{sf} is smaller.

Trapping gas/vapor is easier ⇒ More active bubble nucleation sites

What's Needed for Mechanistic Understanding



Dhir, 2006
https://doi.org/10.1115/1.2136366

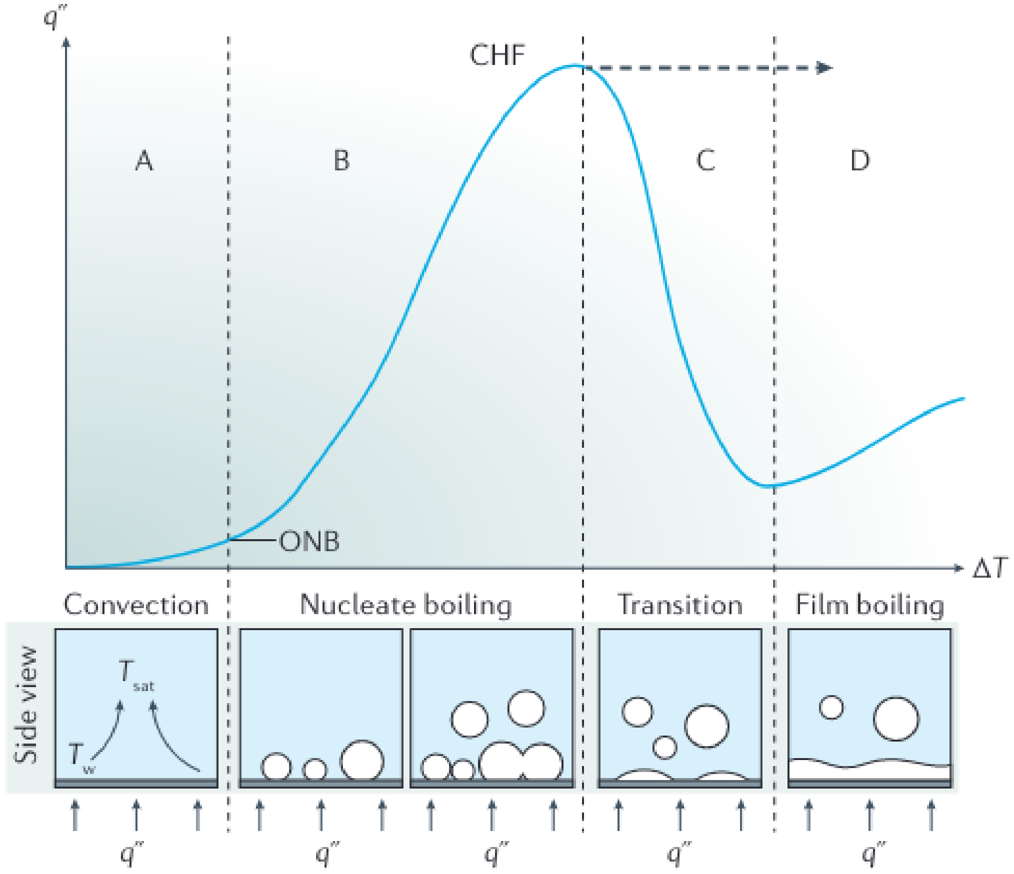
Numerical Simulation

Working fluid: R134a

Heat flux:
from 1.5 W/cm^2 to 38 W/cm^2

Yazdani, 2016
https://doi.org/10.1063/1.4940042

Critical Heat flux



U.S. Department of Energy
Test of Nuclear Rods

Helmholtz Instability of Vapor Columns

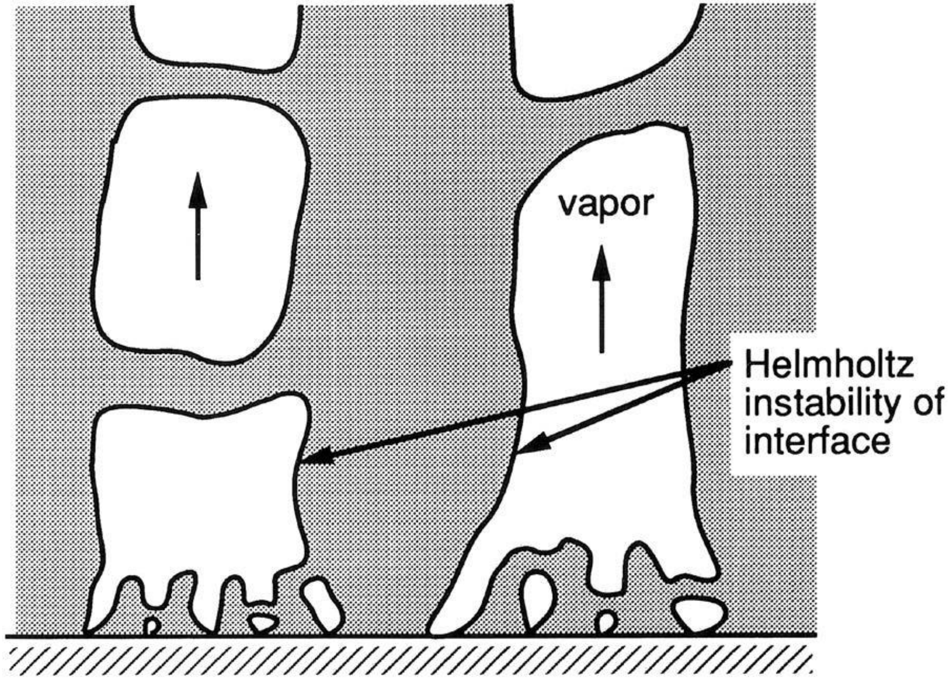


Figure 7.16 Carey

Video credit: Dr. Rameez Iqbal

Vapor columns form at high heat fluxes

Helmholtz Instability

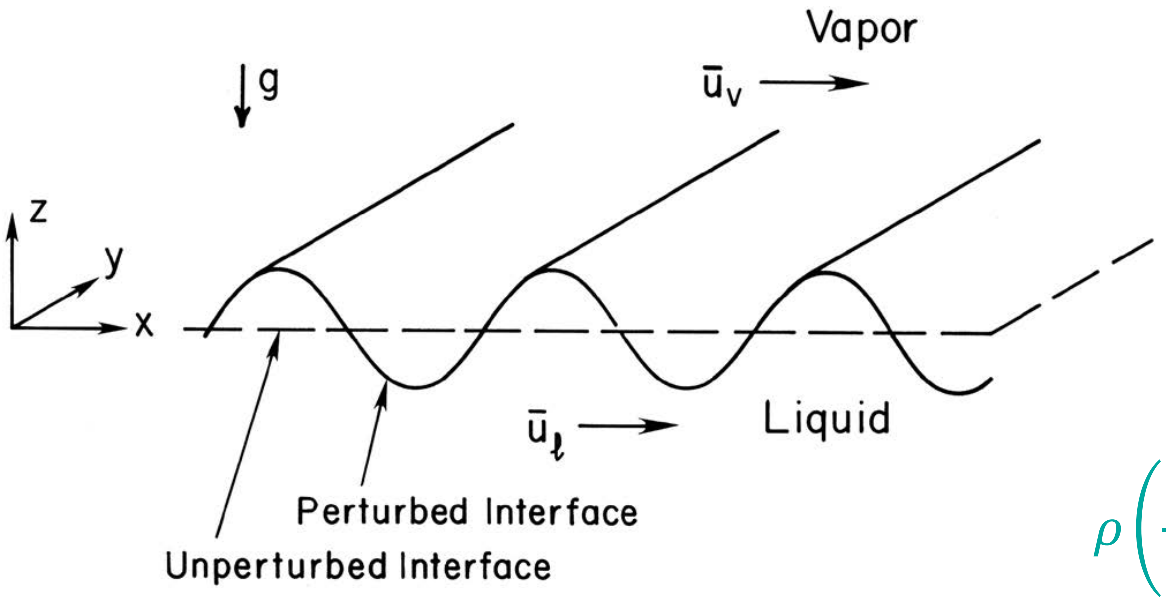


Figure 4.4 in Carey

Continuity

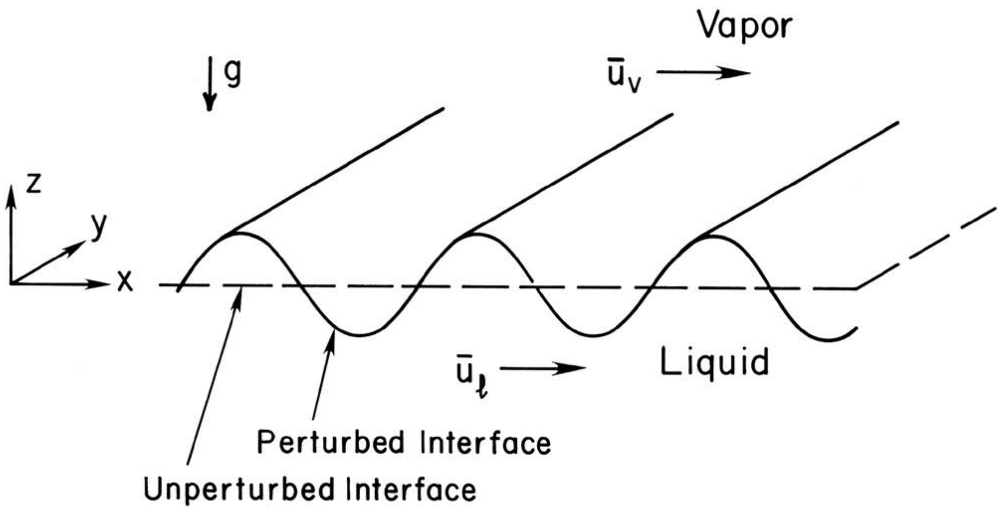
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Momentum balance (neglecting viscosity)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} - \rho g$$

Helmholtz Instability



Consider after a perturbation $\delta(x, t=0) = Ae^{i\alpha x}$

$$u: \bar{u} \to \bar{u} + u', \qquad w: 0 \to w', \qquad P: \bar{P} \to \bar{P} + P'$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0$$

$$\rho\left(\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x}\right) = -\frac{\partial P'}{\partial x}$$

$$\rho\left(\frac{\partial w'}{\partial t} + \bar{u}\frac{\partial w'}{\partial x}\right) = -\frac{\partial P'}{\partial z}$$

$$\Rightarrow \frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0$$

Postulate the form of the response function:

$$\delta = Ae^{i\alpha x + \beta t} \qquad w' = \hat{w}(z)e^{i\alpha x + \beta t} \qquad P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

We are interested in β

Helmholtz Instability

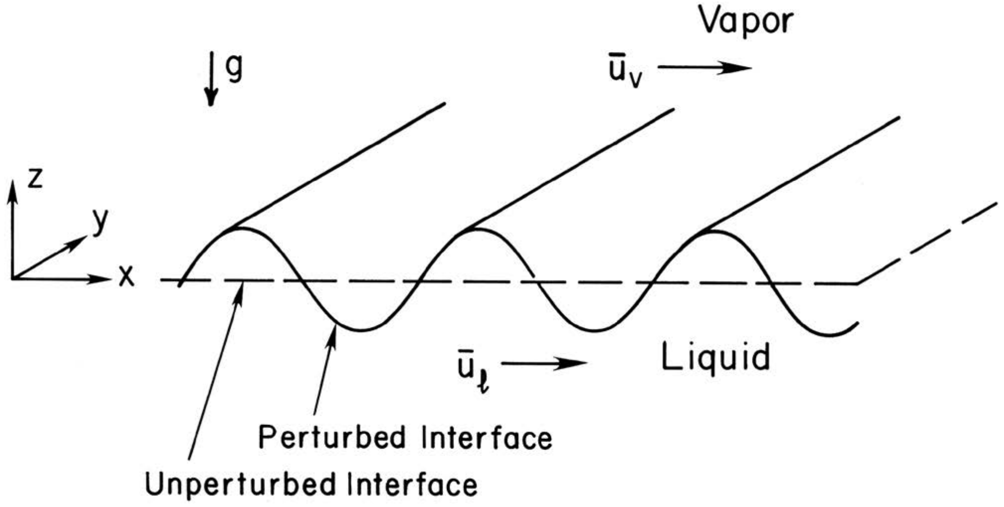
$$\frac{\partial^2 P'}{\partial x^2} + \frac{\partial^2 P'}{\partial z^2} = 0 \qquad P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

$$\frac{d^2\hat{P}}{dz^2} = \alpha^2 \hat{P}$$

$\hat{P} \to 0$ far from interface

$$\hat{P}_v = a_v e^{-\alpha z} \qquad \hat{P}_l = a_l e^{\alpha z}$$

Typo in 4.29



$$\hat{w}(z) = -\frac{1}{\rho(\beta + i\alpha\bar{u})}\frac{d\hat{P}}{dz}$$

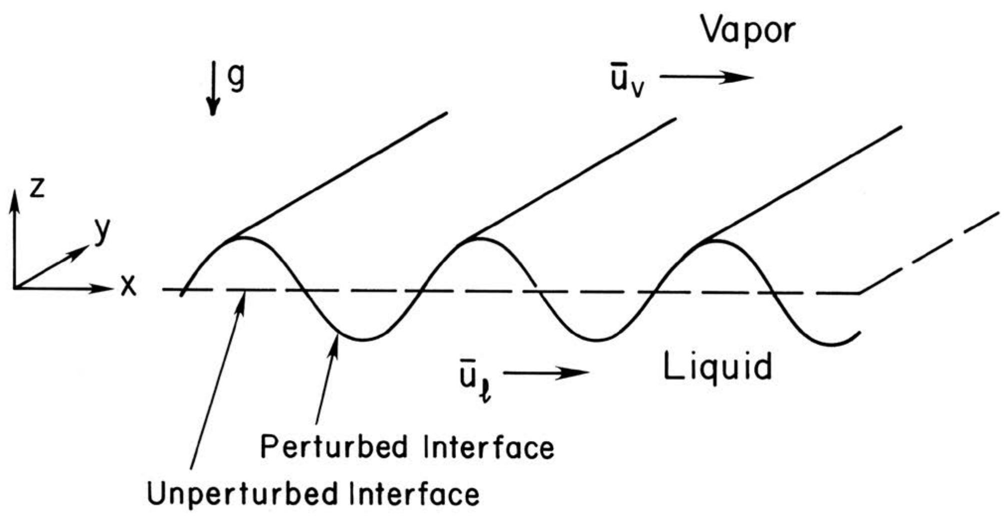
$$\hat{w}_v(z) = \frac{a_v \alpha}{\rho_v(\beta + i\alpha\bar{u})}e^{-\alpha z}$$

$$w' = \hat{w}(z)e^{i\alpha x + \beta t} \qquad \rho\left(\frac{\partial w'}{\partial t} + \bar{u}\frac{\partial w'}{\partial x}\right) = -\frac{\partial P'}{\partial z}$$

$$\hat{w}_l(z) = -\frac{a_l \alpha}{\rho_l(\beta + i\alpha\bar{u})}e^{\alpha z}$$

Helmholtz Instability

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \qquad w' = \widehat{w}(z)e^{i\alpha x+\beta t}$$



$$\delta = Ae^{i\alpha x+\beta t}$$

$$\widehat{w}_v(z) = \frac{a_v \alpha}{\rho_v(\beta + i\alpha\bar{u})}e^{-\alpha z}$$

$$\widehat{w}_l(z) = -\frac{a_l \alpha}{\rho_l(\beta + i\alpha\bar{u})}e^{\alpha z}$$

$u' \rightarrow 0$ far from interface

$$u' = \frac{i}{\alpha}\frac{d\widehat{w}}{dz}e^{i\alpha x+\beta t}$$

Interface vertical motion is due to the time evolution of the vibration and the traveling of the perturbation wave

$$w'_{z\rightarrow 0} = \frac{\partial \delta}{\partial t} + \bar{u}\frac{\partial \delta}{\partial x}$$

True for both two phases

$$a_v = \frac{\rho_v}{\alpha}(\beta A + i\alpha\bar{u}_v)^2 A$$

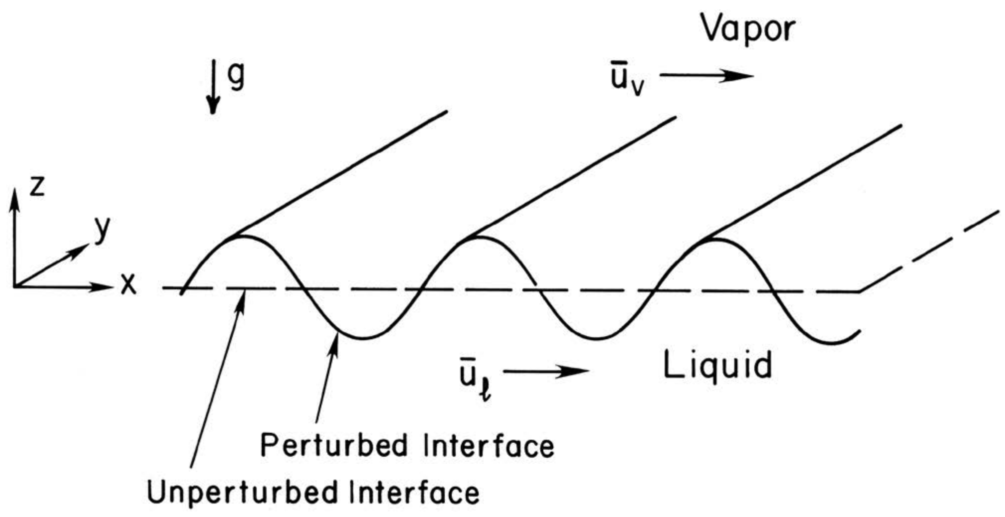
$$a_l = -\frac{\rho_l}{\alpha}(\beta A + i\alpha\bar{u}_l)^2 A$$

Helmholtz Instability

$$a_v = \frac{\rho_v}{\alpha}(\beta A + i\alpha\bar{u}_v)^2 A$$

$$a_l = -\frac{\rho_l}{\alpha}(\beta A + i\alpha\bar{u}_l)^2 A$$

$$\hat{P}_v = a_v e^{-\alpha z} \qquad \hat{P}_l = a_l e^{\alpha z}$$



$$P_{v,in} - P_{l,in} = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

r_2: interface radius of curvature in y-z plane

r_1: interface radius of curvature in x-z plane

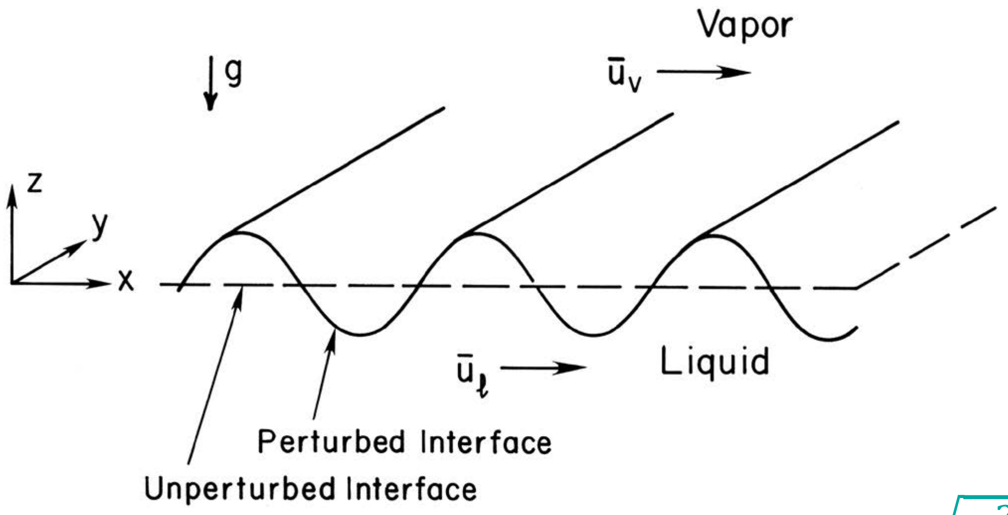
$$\frac{1}{r_1} \approx -\frac{\partial^2 \delta}{\partial x^2}$$

$$P_{v,in} = P_0 - \rho_v g\delta + \hat{P}_v e^{i\alpha x + \beta t}$$

$$P_{l,in} = P_0 - \rho_l g\delta + \hat{P}_l e^{i\alpha x + \beta t}$$

$$a_v - a_l = -(\Delta\rho g + \sigma\alpha^2)A$$

Helmholtz Instability



Perturbation $\delta(x, t = 0) = Ae^{i\alpha x}$

$$\delta = Ae^{i\alpha x + \beta t}$$

$$w' = \hat{w}(z)e^{i\alpha x + \beta t}$$

$$P' = \hat{P}(z)e^{i\alpha x + \beta t}$$

$$\beta = \pm\frac{\sqrt{\alpha^2\rho_v\rho_l(\bar{u}_v - \bar{u}_l)^2 - (\sigma\alpha^3 + \Delta\rho g\alpha)}}{\rho_v + \rho_l} - i\alpha\frac{\rho_l\bar{u}_l + \rho_v\bar{u}_v}{\rho_v + \rho_l}$$

The perturbation will cause a growing response if and only if β has a positive real part

Instability condition: $|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma\alpha + \frac{\Delta\rho g}{\alpha}\right)(\rho_l + \rho_v)}{\rho_l\rho_v}}$

RHS reaches minimum $\left[\frac{2(\rho_l + \rho_v)}{\rho_l}\right]^{\frac{1}{2}}\left(\frac{\sigma\Delta\rho g}{\rho_v^2}\right)^{\frac{1}{4}}$ when $\alpha = \alpha_c = \left(\frac{\Delta\rho g}{\sigma}\right)^{\frac{1}{2}}$

Helmholtz Instability

- When $|\bar{u}_v - \bar{u}_l| > \left[\frac{2(\rho_l+\rho_v)}{\rho_l}\right]^{\frac{1}{2}} \left(\frac{\sigma\Delta\rho g}{\rho_v^2}\right)^{\frac{1}{4}}$, there will be some range of disturbances that are unstable (grow exponentially)

- $\bar{u}_v - \bar{u}_l$ promotes instability while gravity and surface tension suppressing instability, we can adjust the value of g based on the orientation of the system.

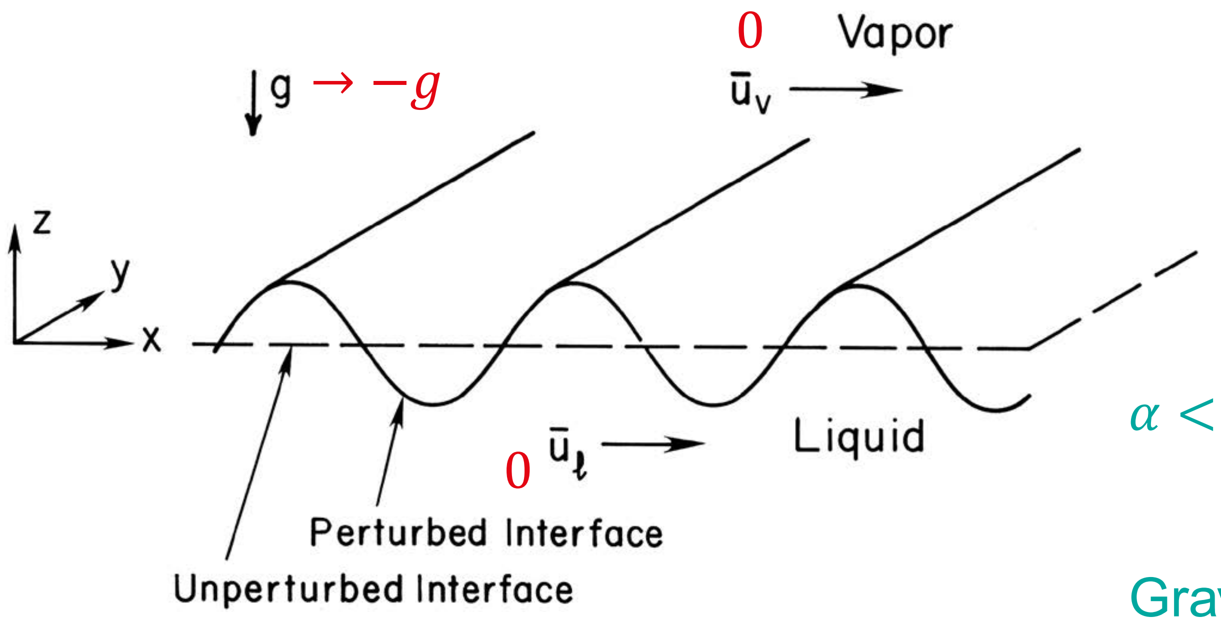
Helmholtz Instability

Helmholtz Instability


Facebook/ Rachel Gordon

Taylor Instability

$$\delta = Ae^{i\alpha x+\beta t}$$



$$\beta = \pm\sqrt{\frac{\Delta\rho g\alpha - \sigma\alpha^3}{\rho_l + \rho_v}}$$

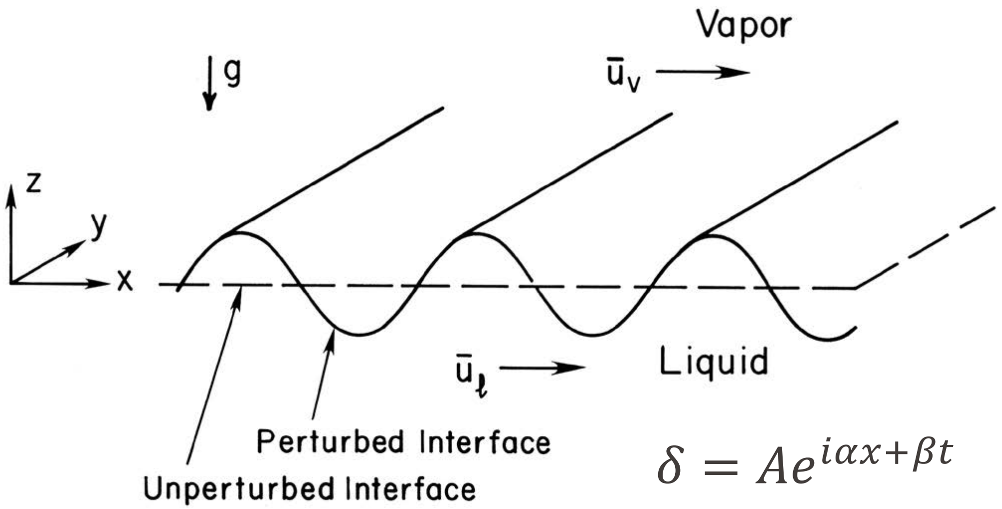
$\alpha < \alpha_c = \left(\frac{\Delta\rho g}{\sigma}\right)^{\frac{1}{2}}$ can grow exponentially

Gravity causes instability

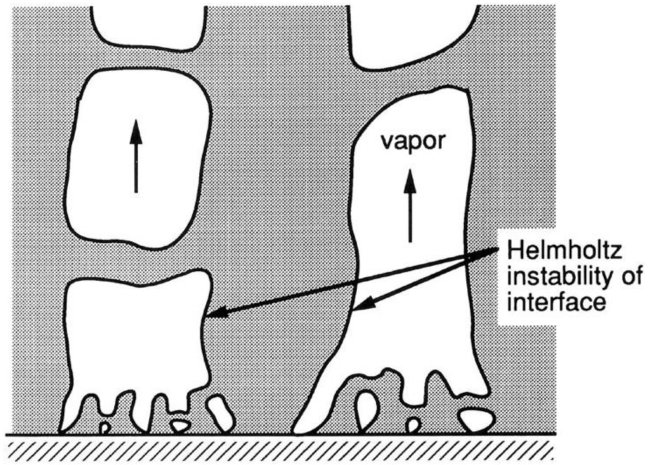
The fastest growing perturbation (α_{max}) in this case can be found by setting $\frac{d\beta}{d\alpha} = 0$

$$\alpha_{max} = \left(\frac{\Delta\rho g}{3\sigma}\right)^{\frac{1}{2}} \qquad \lambda_D = \frac{2\pi}{\alpha_{max}} = 2\pi\sqrt{\frac{3\sigma}{\Delta\rho g}}$$

How It's Related to Boiling



$$\delta = Ae^{i\alpha x+\beta t}$$



Helmholtz Instability

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\left(\sigma\alpha + \frac{\Delta\rho g}{\alpha}\right)(\rho_l + \rho_v)}{\rho_l\rho_v}}$$

Setting g = 0 for vertical interfaces

$$|\bar{u}_v - \bar{u}_l| > \sqrt{\frac{\sigma\alpha\,(\rho_l + \rho_v)}{\rho_l\rho_v}} = \sqrt{\frac{2\pi\sigma\,(\rho_l + \rho_v)}{\rho_l\rho_v\lambda_H}}$$