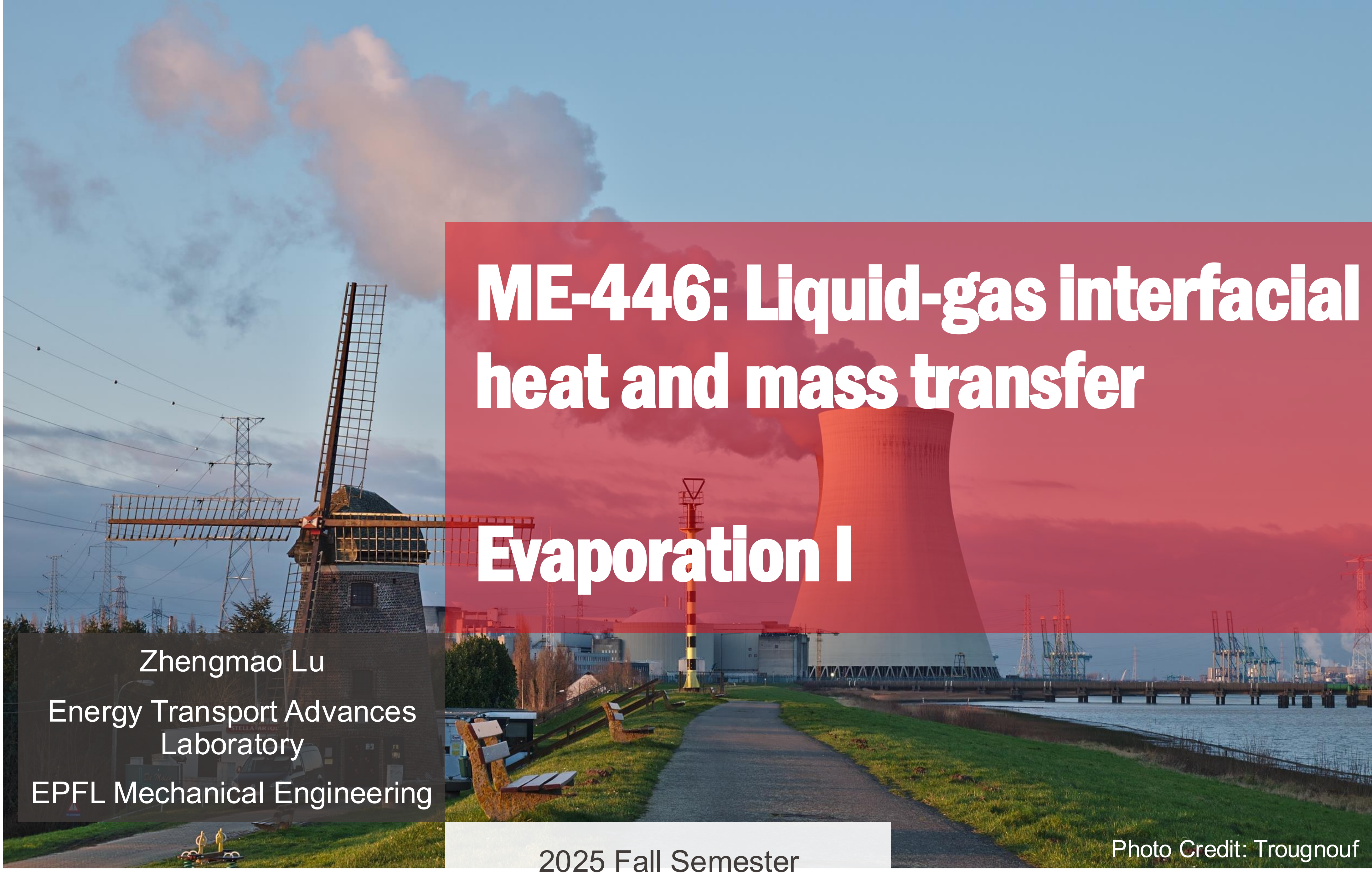
EPFL

ETA-Lab
Energy Transport Advances



ME-446: Liquid-gas interfacial heat and mass transfer

Evaporation I

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2025 Fall Semester

Photo Credit: Trougnouf

Last Time

- Contact angle (definition, Young's equation, hysteresis)
- Wetting on rough surfaces (Cassie-Baxter, Wenzel, Hemispreading)

Wetting States for Pillar-Structured Surfaces

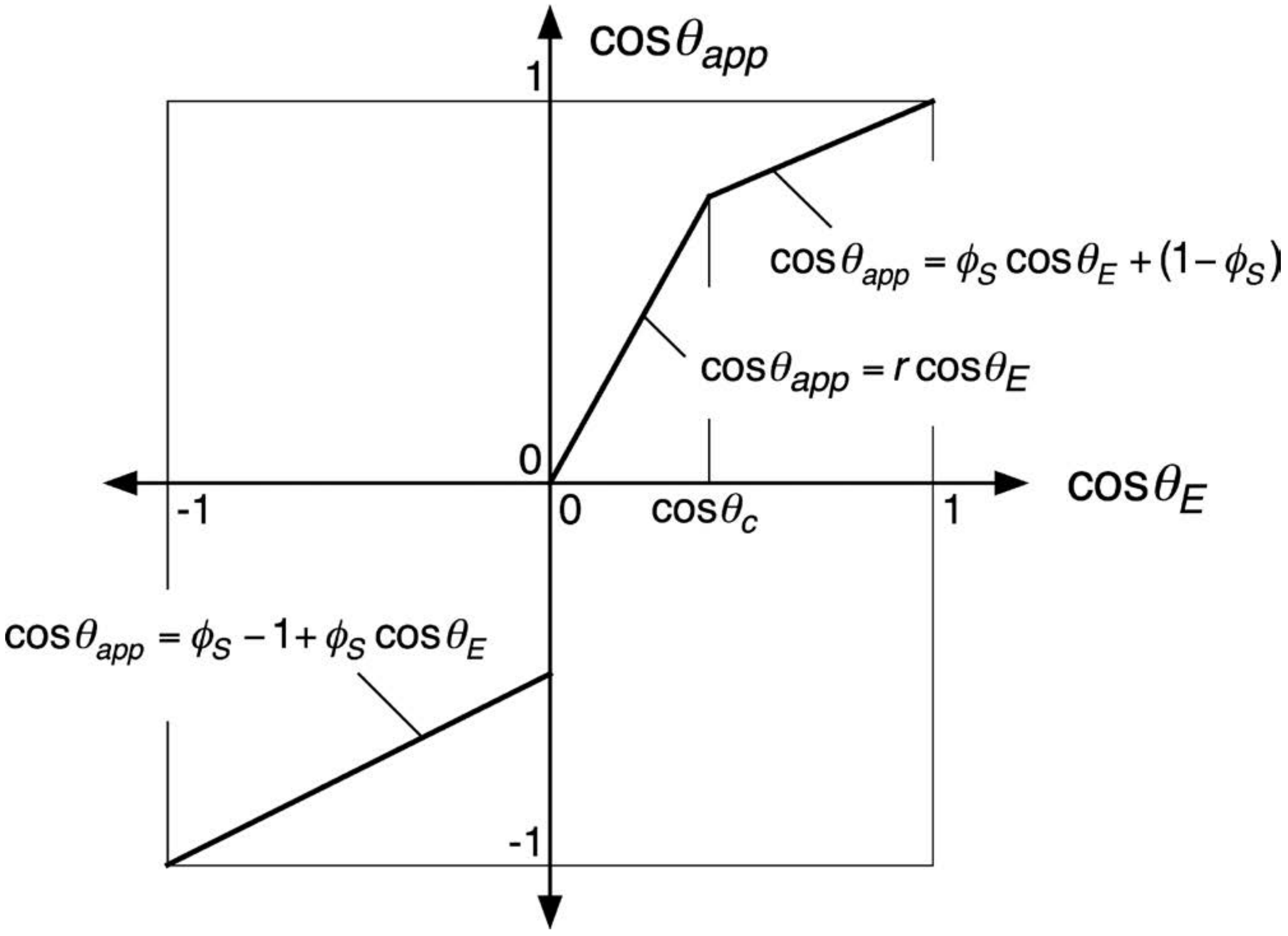


Figure 3.24 in Carey

Intended Learning Objectives Today

- Explain and apply the **Fick's Law of Diffusion**
- Apply heat and mass transfer analogy to **convective mass transfer**
- Explain the **coffee ring effect**

Reading materials: **Lienhard** Chapter 11, **Bird** Chapter 17

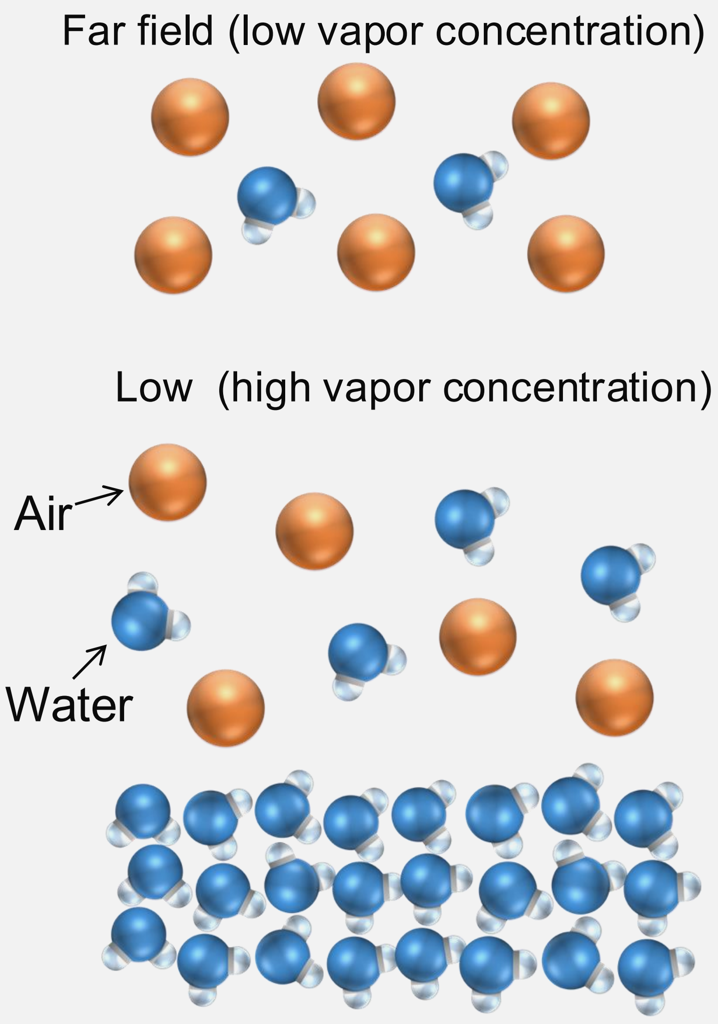
Fundamental Picture of Evaporation in Air


Evaporative cooling tower

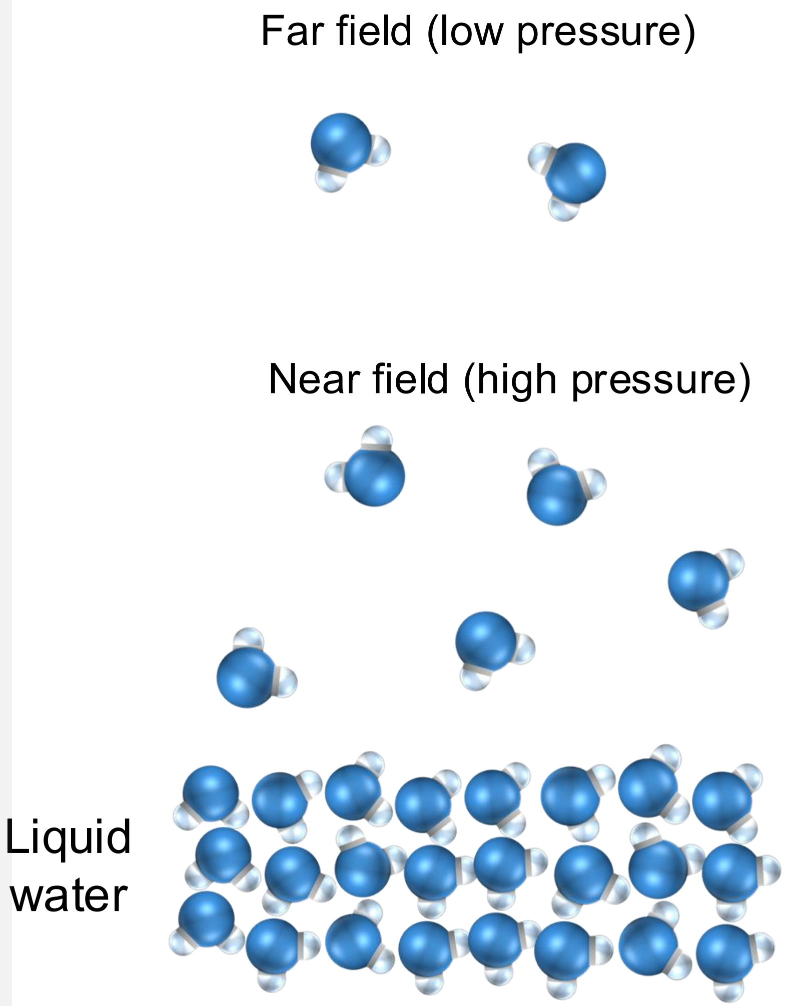
Fundamental Picture of Evaporation in Vapor

Fundamental Picture of Evaporation

Air → Diffusion Limited



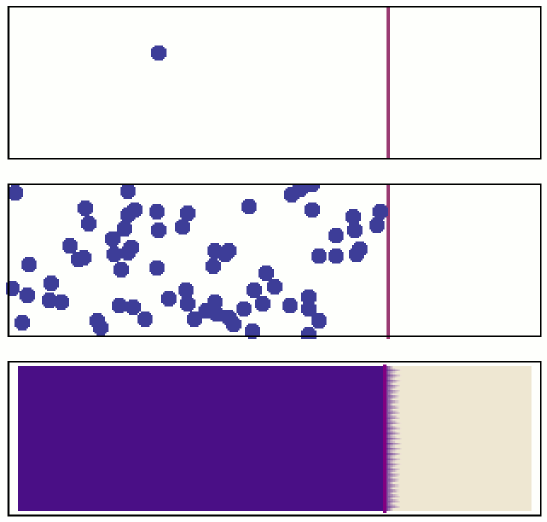
Vapor → Kinetically Limited



Transport of Water Vapor in Air

- Moist air modeled as a binary mixture: **water vapor + dry air**

- **Molecular diffusion**:

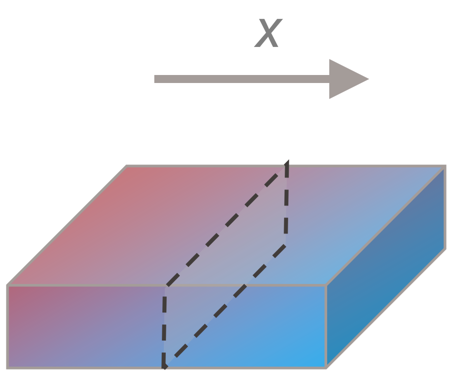


Fick's law wiki page

Molecules in the mixture move around randomly

Spontaneous mass transfer from more concentrated region to less concentrated region

Fick's Law of Diffusion for Moist Air



$$j_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \frac{d\omega_{\mathrm{v}}}{dx}$$

j_{vd}: mass flux in the mixture reference frame

ρ: mixture mass density

ω_v: vapor mass fraction

D_{va}: the proportionality, diffusion coefficient of vapor in air

Mixture Mass Average Velocity

Fick's Law in 3D

Special Case

- If mixture density is constant

What Affects D_{va}

- Temperature
  - High temperature implies higher molecular speeds
- Total gas pressure
  - Lower pressure implies fewer air molecules impeding vapor molecule motion
- Correlation can be found in literature (Eq. 11.34 in Lienhard)

Convective Mass Transfer

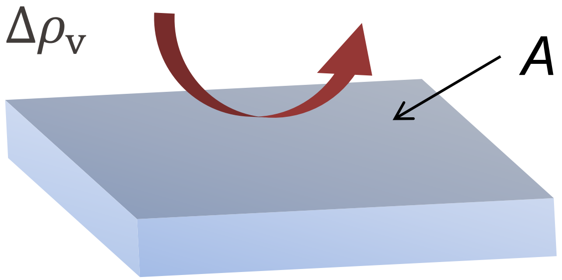
$$\vec{J}_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} \qquad \vec{J}_{\mathrm{vd}} = \rho \omega_v (\vec{v}_v - \vec{v}_m)$$

Convective Mass Transfer

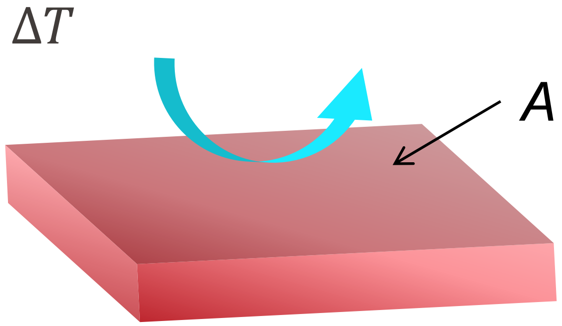
Convective Mass Transfer

$$\nabla \cdot (\rho \omega_v \vec{v}_m) + \nabla \cdot (-\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}}) = 0$$

Heat and Mass Transfer Analogy



Mass transferred due to **bulk movement** of fluids



Energy transferred due to **bulk movement** of fluids

Heat and Mass Transfer Analogy

Fick's law
(diffusive mass transport)

Fourier's law of conduction
(diffusive thermal transport)

Heat and Mass Transfer Analogy

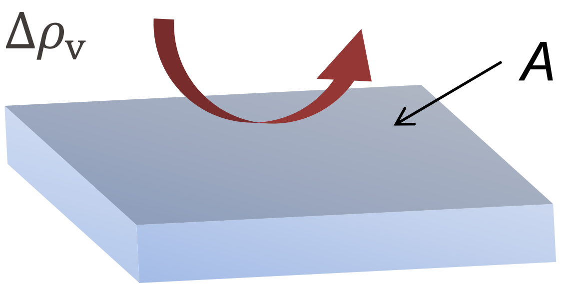
Mass Transfer

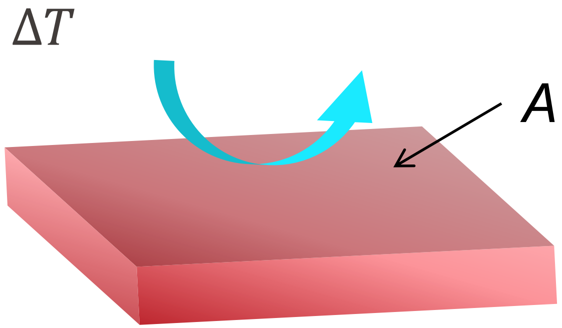
$$\vec{v}_m \cdot \nabla \omega_v - D_{\mathrm{va}} \nabla^2 \omega_{\mathrm{v}} = 0$$

Heat Transfer

$$\vec{v}_m \cdot \nabla T - \alpha \nabla^2 T = 0$$

Heat and Mass Transfer Analogy





Boundary mass flux assuming vapor flux is very small and only diffusional

Mass transfer coefficient g_m

Heat and Mass Transfer Analogy

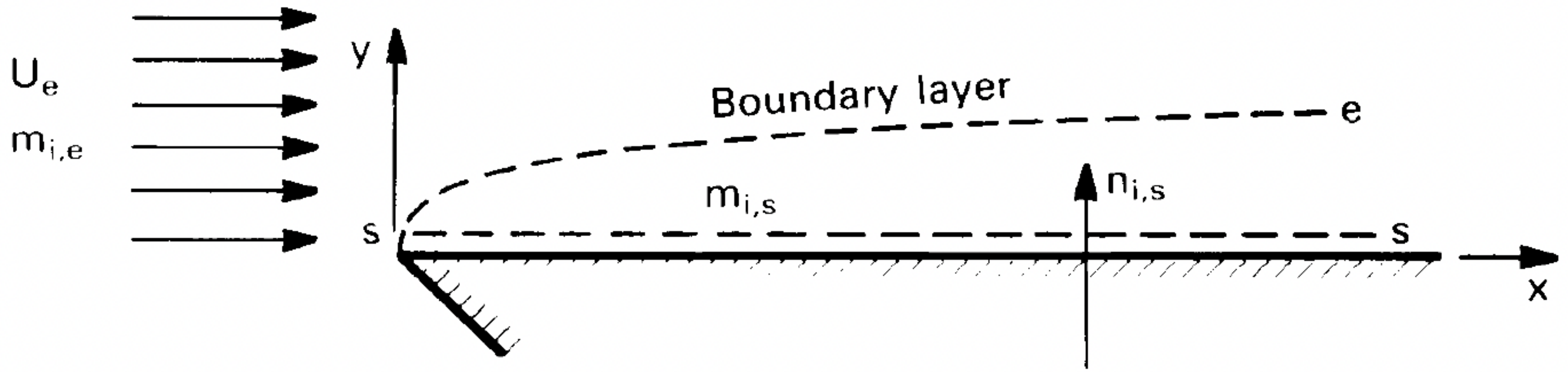
Sherwood number

$$\mathrm{Sh} = \frac{g_m L}{D_{\mathrm{va}}}$$

Schmit number $\mathrm{Sc} = \nu / D_{\mathrm{va}}$

Reynolds number $\mathrm{Re} = \rho U L / \mu$

Example



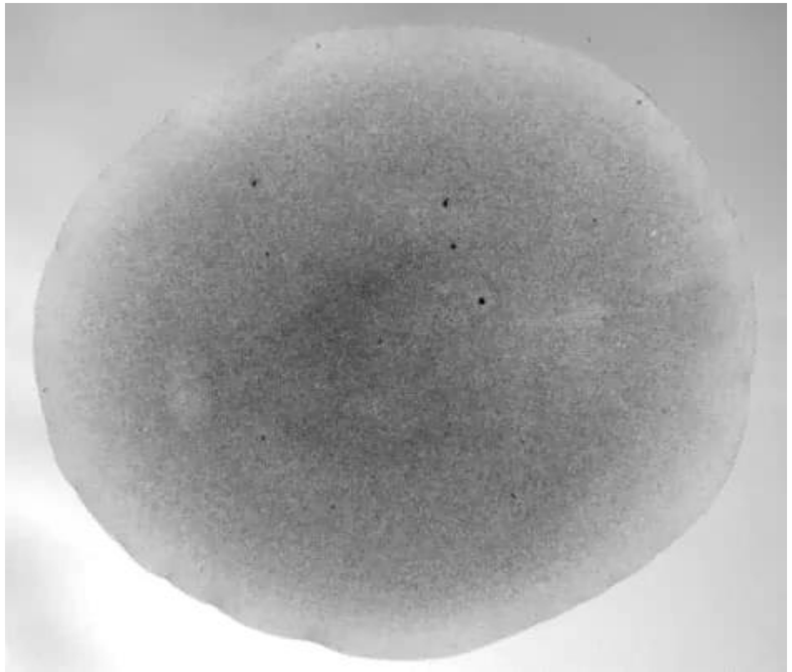
Lienhard, Figure 11.12

$$\mathrm{Nu}_x = 0.332\sqrt{\mathrm{Re}_x}\sqrt[3]{\mathrm{Pr}}$$

$$\mathrm{Sh}_x = 0.332\sqrt{\mathrm{Re}_x}\sqrt[3]{\mathrm{Sc}}$$

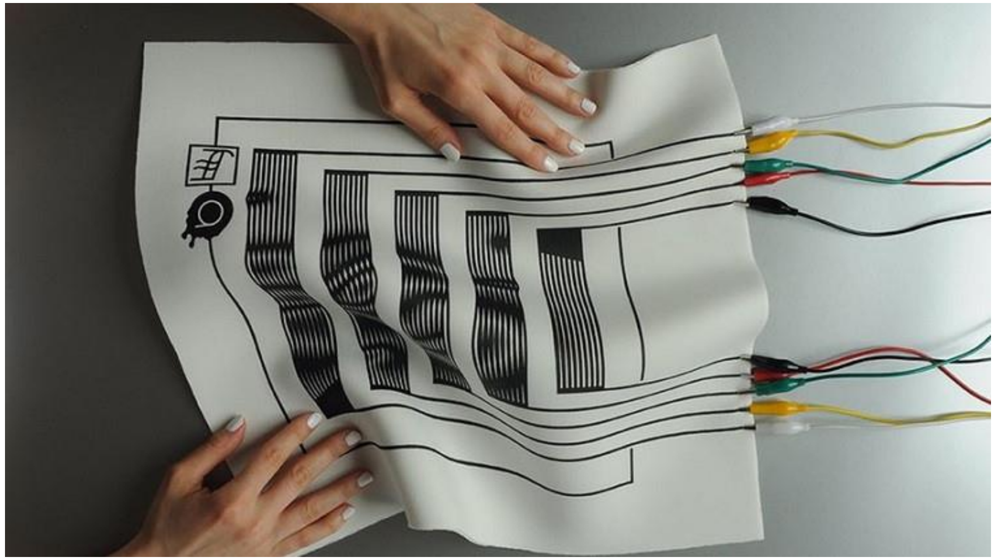
Coffee Ring Effect

Coffee Ring Effect



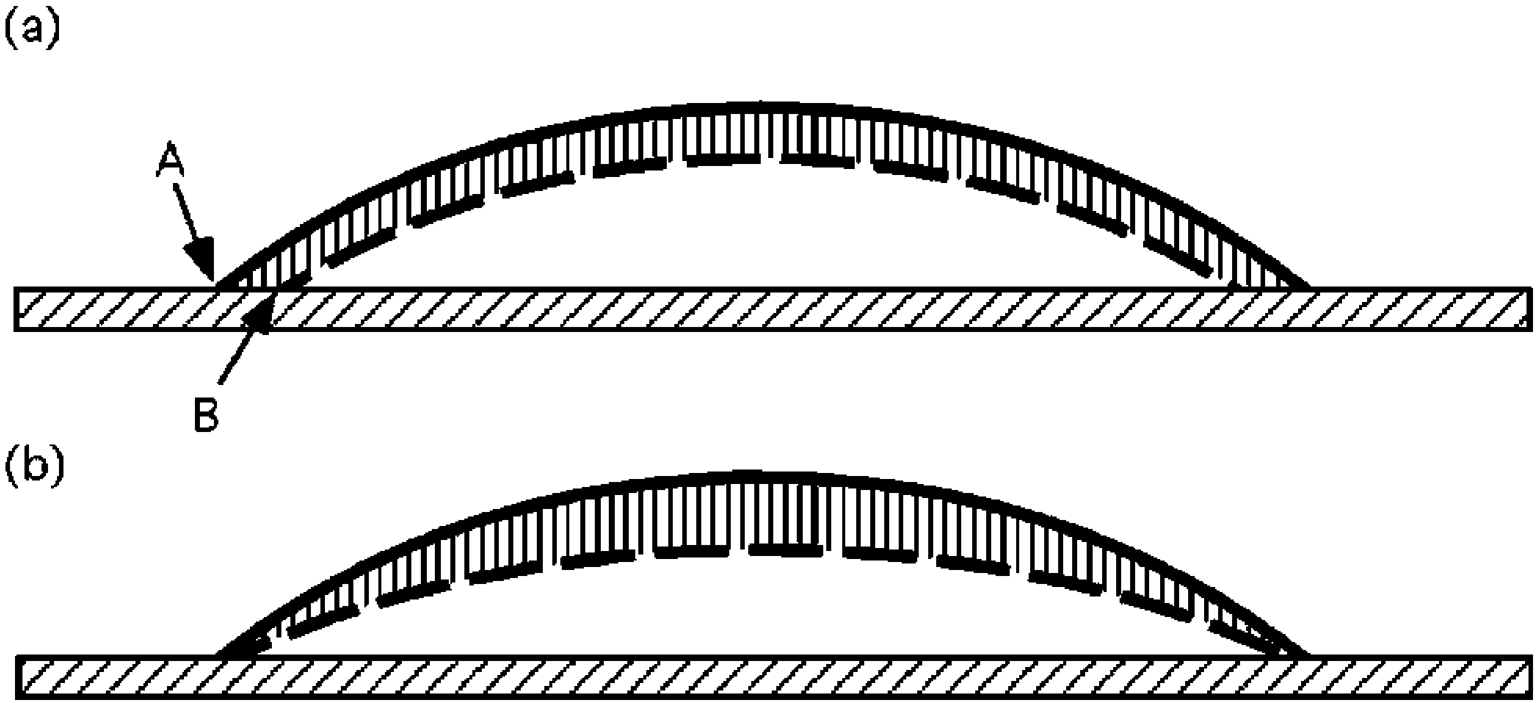
Yunker *et al.*, *Nature* (2011)

Printed electronics



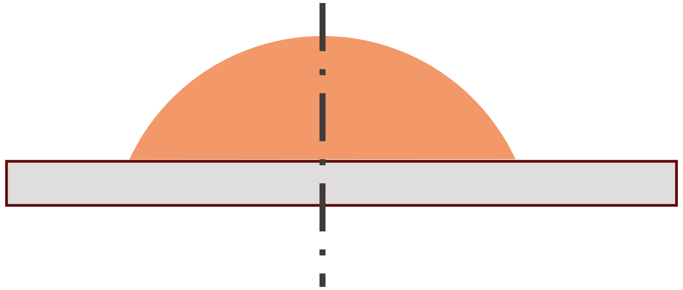
Karim *et al.*, *Scientific Reports* (2019)

Contact Line Pinning



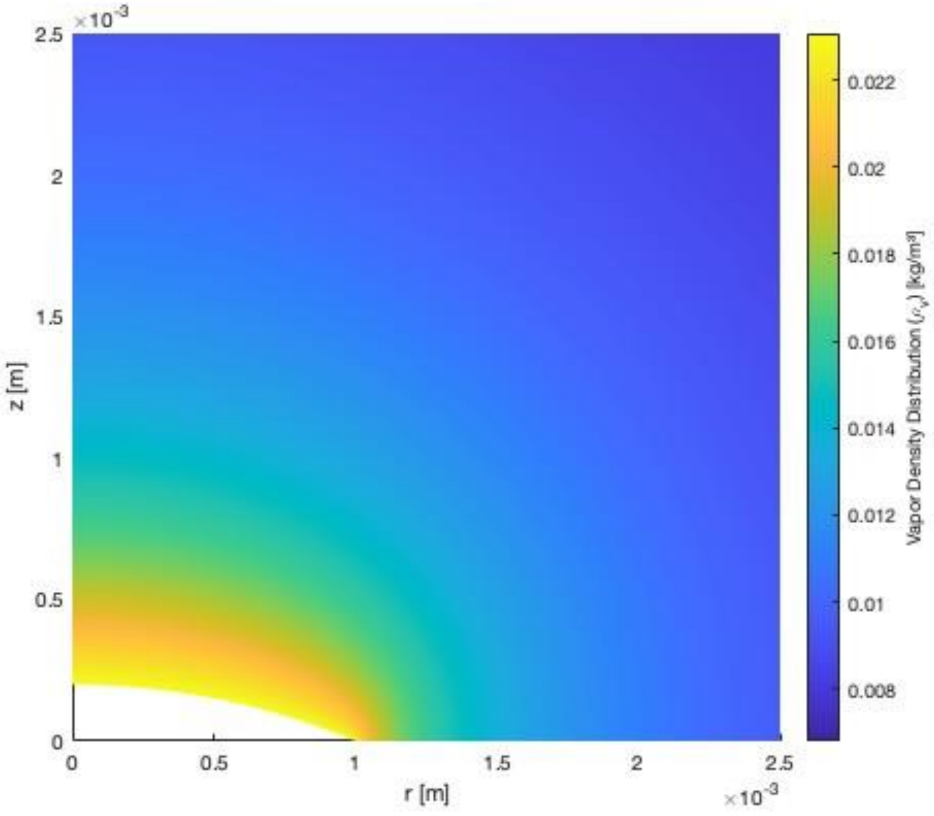
Deegan *et al.*, *Physical Review E* (2000)

Evaporative Transport



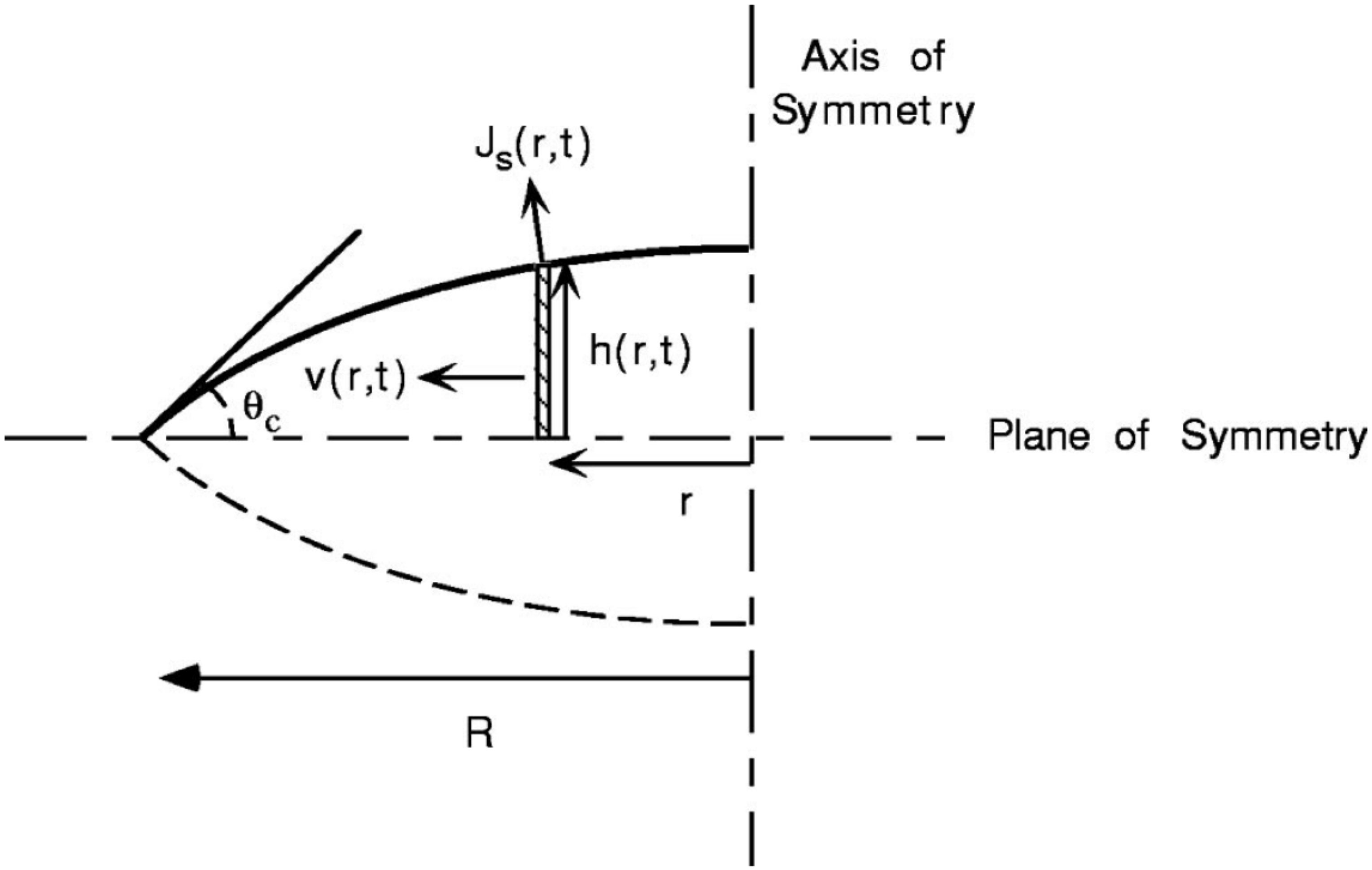
Lightening rod

Evaporative Transport



Lightening rod

Mass Conservation



What We Learned Today

- Fick’s Law of Diffusion
- Heat and mass transfer analogy
- Coffee ring effect