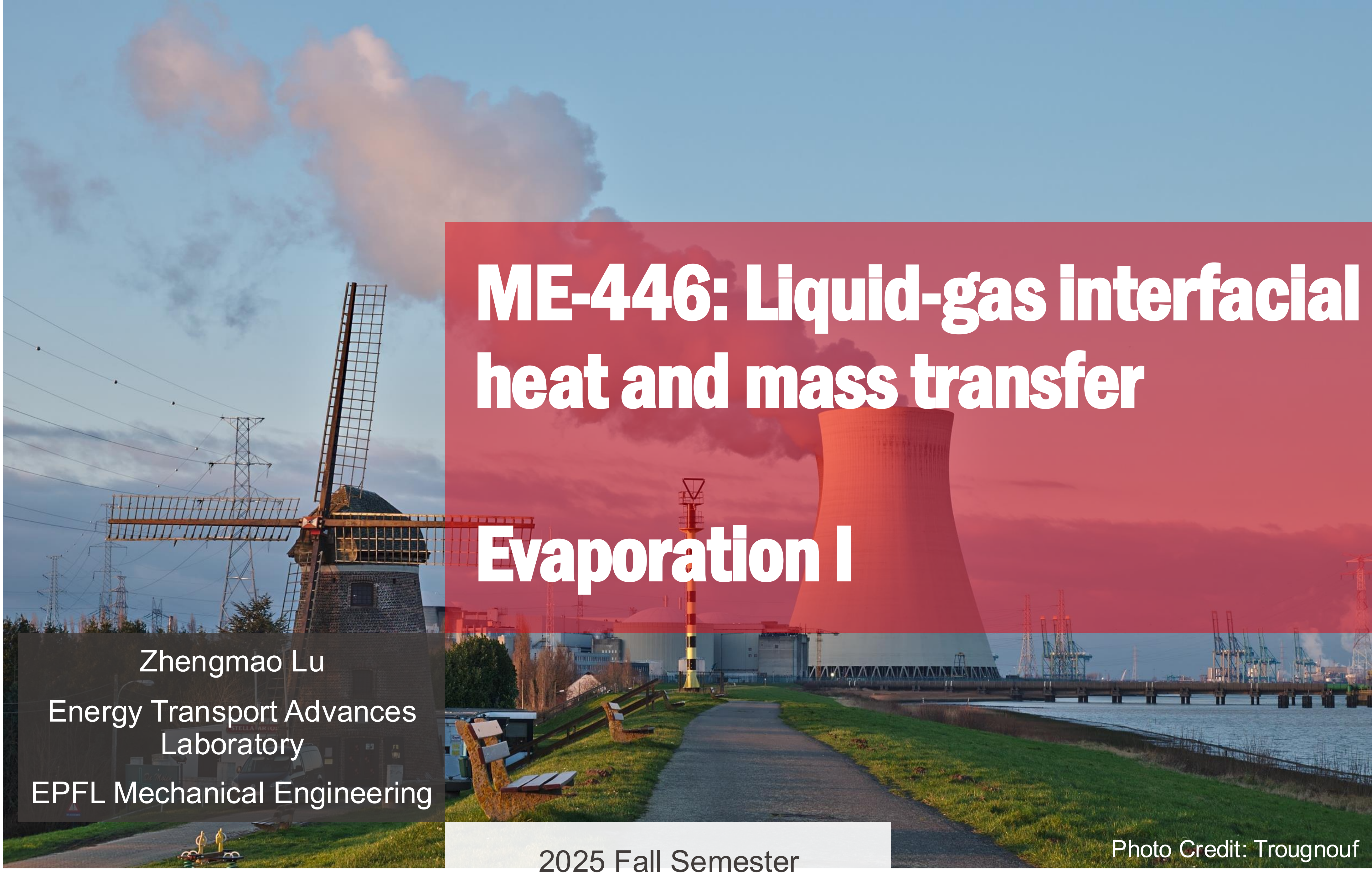
ETA-Lab
Energy Transport Advances



ME-446: Liquid-gas interfacial heat and mass transfer

Evaporation I

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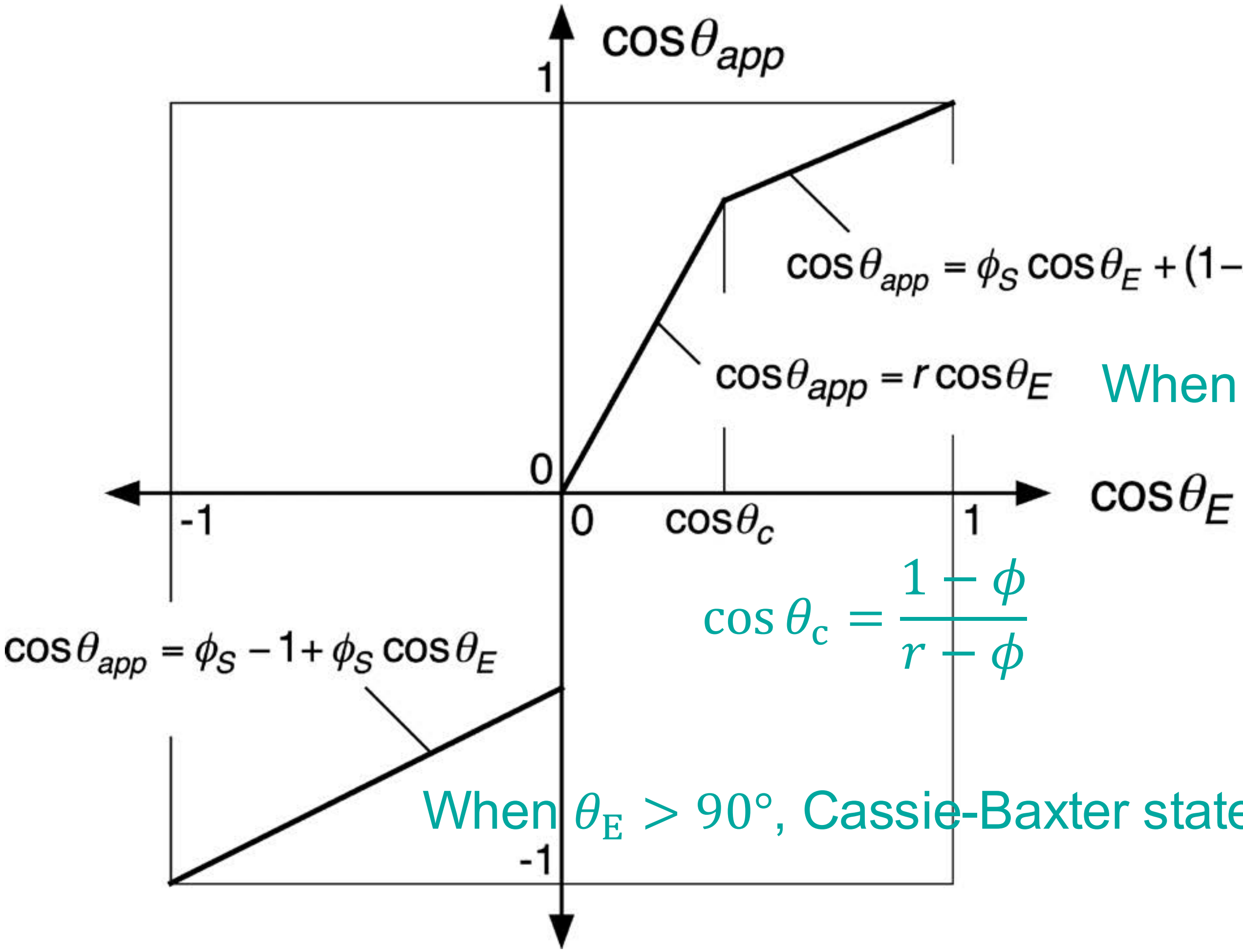
Photo Credit: Trougnouf

Last Time

- Contact angle (definition, Young’s equation, hysteresis)
- Wetting on rough surfaces (Cassie-Baxter, Wenzel, Hemispreading)

Wetting States for Pillar-Structured Surfaces



When $\theta_{\mathrm{E}} < \theta_{\mathrm{c}}$, hemi-spreading state

When $\theta_{\mathrm{c}} < \theta_{\mathrm{E}} < 90°$, Wenzel state

$$\cos\theta_{\mathrm{c}} = \frac{1-\phi}{r-\phi}$$

When $\theta_{\mathrm{E}} > 90°$, Cassie-Baxter state

Figure 3.24 in Carey

Intended Learning Objectives Today

- Explain and apply the **Fick’s Law of Diffusion**
- Apply heat and mass transfer analogy to **convective mass transfer**
- Explain the **coffee ring effect**

Reading materials: **Lienhard** Chapter 11, **Bird** Chapter 17

Fundamental Picture of Evaporation in Air

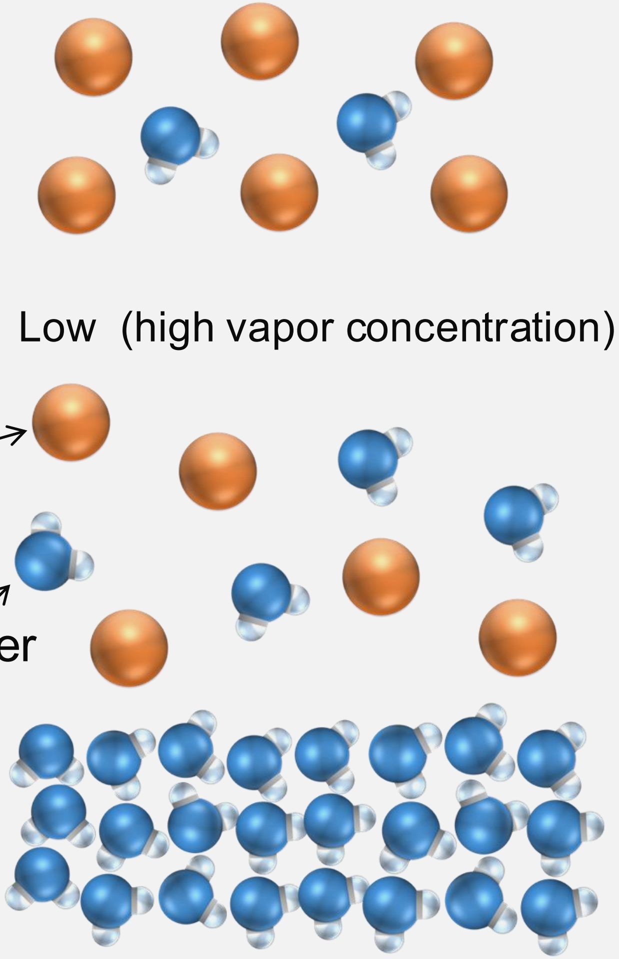
Evaporative cooling tower

Fundamental Picture of Evaporation in Vapor

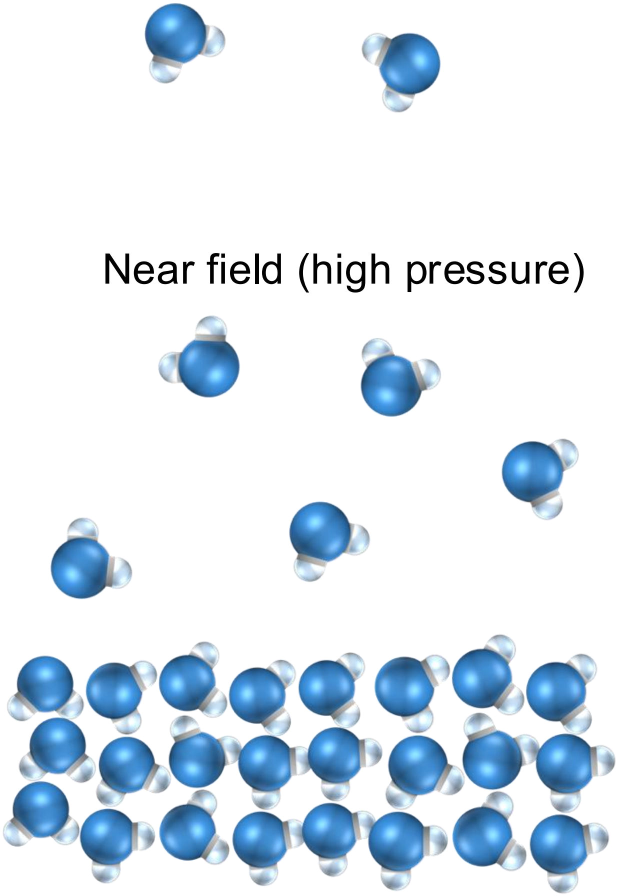
Vapor chamber cooler

Fundamental Picture of Evaporation

Air → Diffusion Limited



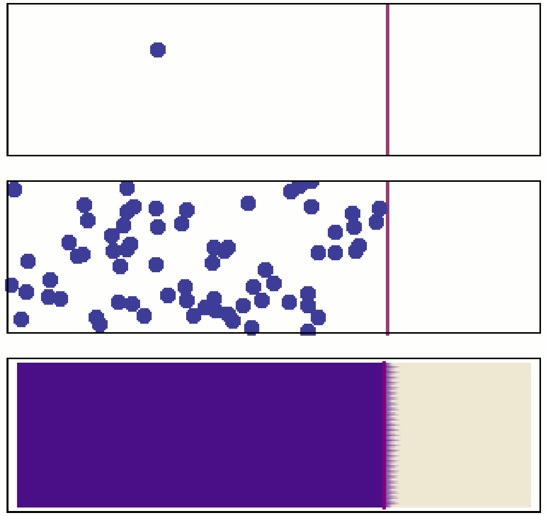
Vapor → Kinetically Limited



Transport of Water Vapor in Air

- Moist air modeled as a binary mixture: **water vapor + dry air**

- **Molecular diffusion**:



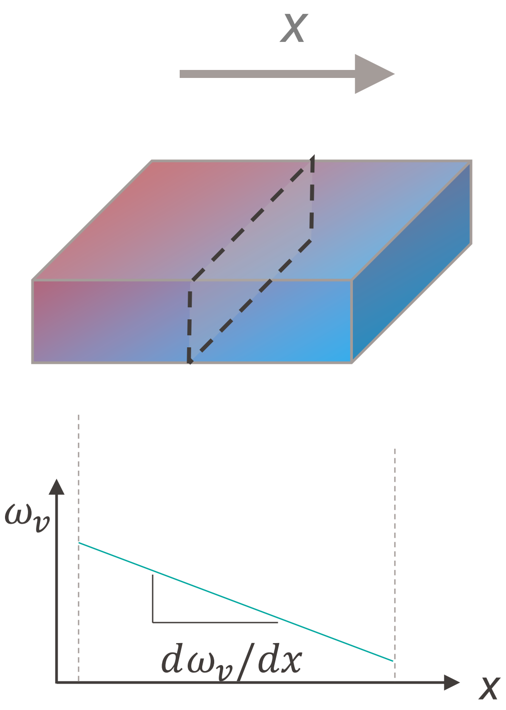
Fick's law wiki page

Molecules in the mixture move around randomly

Spontaneous mass transfer from more concentrated region to less concentrated region

Fick's Law of Diffusion for Moist Air

Empirical law



$$\underset{[\mathrm{kg/m^2s}]}{j_{\mathrm{vd}}} = -\underset{[\mathrm{kg/m^3}]}{\rho}\,\underset{[\mathrm{m^2/s}]}{D_{\mathrm{va}}}\,\frac{d\omega_{\mathrm{v}}}{dx}$$

j_{vd}: mass flux in the mixture reference frame

ρ: mixture mass density

ω_v: vapor mass fraction

D_{va}: the proportionality, diffusion coefficient of vapor in air

Diffusion flux is relative to the mixture frame of reference

Mixture Mass Average Velocity

$$v_m = \omega_v v_v + \omega_a v_a$$

weighted average based on mass fraction

$$j_{vd} = \rho\omega_v(v_v - v_m)$$

diffusion flux of vapor, mixture reference frame

$$j_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \frac{d\omega_{\mathrm{v}}}{dx}$$

Fick’s law 1D

What about diffusion of air in vapor

$$j_{ad} = \rho\omega_a(v_a - v_m)$$

Air diffusion

$$j_{\mathrm{av}} = -\rho D_{\mathrm{av}} \frac{d\omega_{\mathrm{a}}}{dx}$$

$D_{av} = D_{va}$? Exercise

Fick's Law in 3D

$$\vec{J}_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} \qquad \nabla \omega_{\mathrm{v}} = \left(\frac{\partial \omega_{\mathrm{v}}}{\partial x}, \frac{\partial \omega_{\mathrm{v}}}{\partial y}, \frac{\partial \omega_{\mathrm{v}}}{\partial z} \right)$$

$\vec{J}_{\mathrm{vd}}$ is a vector

Applying the gradient operator **∇** onto a scalar field gives you the direction and in which the scalar value increases most quickly

The magnitude determines how fast the increase is in that direction

For example, in heat transfer: $\vec{q} = -k\nabla T$

Special Case

- If mixture density is constant

$$\vec{j}_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} = -D \nabla \rho_{\mathrm{v}}$$

Molar form $\vec{\tilde{j}}_{\mathrm{vd}} = -D \nabla c_{\mathrm{v}}$

c_v: molar concentration [mol/m^3]

What Affects D_{va}

- Temperature
  - High temperature implies higher molecular speeds

- Total gas pressure
  - Lower pressure implies fewer air molecules impeding vapor molecule motion

- Composition of the mixture

- Correlation can be found in literature (Eq. 11.34 in Lienhard)

Convective Mass Transfer

$$\vec{j}_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} \qquad \vec{j}_{\mathrm{vd}} = \rho \omega_v (\vec{v}_v - \vec{v}_m)$$

$$\vec{j}_{\mathrm{v}} = \rho \omega_v \vec{v}_v = \vec{j}_{\mathrm{vd}} + \rho \omega_v \vec{v}_m$$

mass flux of vapor due to bulk
movement of air-vapor mixture

In general, $\vec{v}_m$ is given by fluid mechanics (N-S)

When air is static, $\vec{v}_m = \omega_v \vec{v}_v + \omega_a \vec{v}_a = \omega_v \vec{v}_v$

$$\Rightarrow \rho \omega_v \vec{v}_v = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} + \rho \omega_v^2 \vec{v}_v \Rightarrow \vec{j}_{\mathrm{v}} (1 - \omega_v) = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} = \vec{j}_{\mathrm{vd}}$$

If ω_v is close to zero, $\vec{j}_{\mathrm{v}} \approx -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}} = \vec{j}_{\mathrm{vd}}$

Otherwise, a distinction has to be made between $\vec{j}_{\mathrm{v}}$ and $\vec{j}_{\mathrm{vd}}$ even in static air

Convective Mass Transfer

$$\frac{\partial(\rho\omega_v)}{\partial t} = R - \nabla\cdot\vec{j}_\mathrm{v}$$

Volumetric source term for mass generation/destruction [kg/m3]

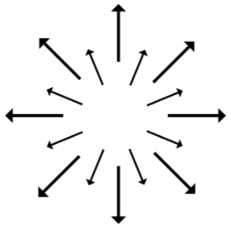
$$\nabla\cdot\vec{j}_\mathrm{v} = \frac{\partial J_x}{\partial x} + \frac{\partial J_\mathrm{y}}{\partial y} + \frac{\partial J_\mathrm{z}}{\partial z}$$

At steady state, with no mass creation/destruction in the bulk mixture

$$\nabla\cdot\vec{j}_\mathrm{v} = 0 \qquad \nabla\cdot(\rho\omega_v\vec{v}_m) + \nabla\cdot(-\rho D_\mathrm{va}\nabla\omega_\mathrm{v}) = 0$$

Applying the divergence operator **∇** · onto a vector field gives you the local sink term for the field flux:

∇ · (**Mass flux**) = local mass “outgoingness”
∇ · (**Energy flux**) = local energy “outgoinness”



Convective Mass Transfer

$$\nabla\cdot(\rho\omega_v\vec{v}_m)+\nabla\cdot(-\rho D_{\mathrm{va}}\nabla\omega_{\mathrm{v}})=0$$

$$\nabla\cdot\left(\psi\vec{A}\right)=\psi\left(\nabla\cdot\vec{A}\right)+(\nabla\psi)\cdot\vec{A}$$

$$\omega_v\left[\cancel{\nabla\cdot(\rho\vec{v}_m)}\right]+\rho\vec{v}_m\cdot\nabla\omega_v+\nabla\cdot(-\rho D_{\mathrm{va}}\nabla\omega_{\mathrm{v}})=0$$

Mass conservation of mixture

$$\rho\vec{v}_m\cdot\nabla\omega_v+(-\rho D_{\mathrm{va}}\nabla^2\omega_{\mathrm{v}})=0$$

Assuming no spatial variation of ρD_{va}

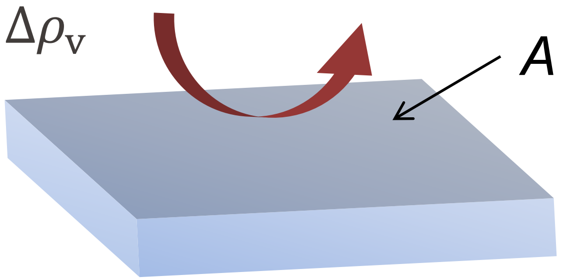
$$\vec{v}_m\cdot\nabla\omega_v-D_{\mathrm{va}}\nabla^2\omega_{\mathrm{v}}=0$$

Without bulk mixture velocity, in 1D

$$\frac{\partial^2\omega_{\mathrm{v}}}{\partial x^2}=0$$

ω_{v} is linear

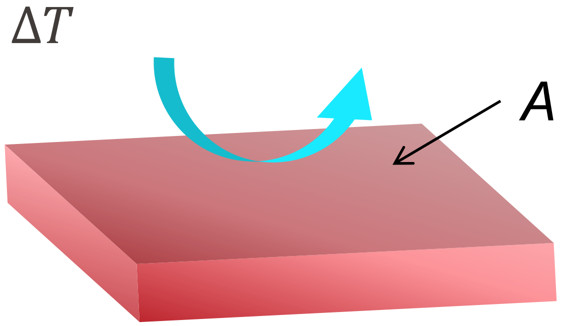
Heat and Mass Transfer Analogy



Mass transferred due to **bulk movement** of fluids

Rate Equation: $\dot{m} = g_m A \Delta\rho_v$

$$\rho_v = \rho\omega_v$$



Energy transferred due to **bulk movement** of fluids

Rate Equation: $Q = hA\Delta T$

Heat and Mass Transfer Analogy

Fick's law
(diffusive mass transport)

$$\vec{j}_{\mathrm{vd}} = -\rho D_{\mathrm{va}} \nabla \omega_{\mathrm{v}}$$

Fourier's law of conduction
(diffusive thermal transport)

$$\vec{q}_{\mathrm{cond}} = -k \nabla T$$

Stead-state mass conservation

$$\vec{v}_m \cdot \nabla \omega_v - D_{\mathrm{va}} \nabla^2 \omega_{\mathrm{v}} = 0$$

Steady-state energy conservation

$$\rho c_p \vec{v}_m \cdot \nabla T - k \nabla^2 T = 0$$

$$\vec{v}_m \cdot \nabla T - \alpha \nabla^2 T = 0$$

Heat and Mass Transfer Analogy

Mass Transfer

$$\vec{v}_m \cdot \nabla\omega_v - D_{\mathrm{va}}\nabla^2\omega_{\mathrm{v}} = 0$$

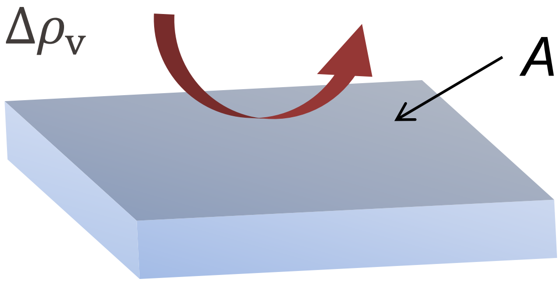
Heat Transfer

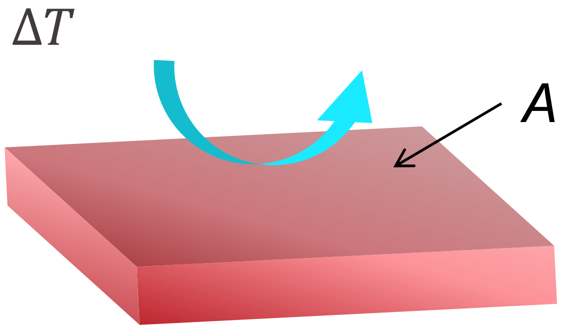
$$\vec{v}_m \cdot \nabla T - \alpha\nabla^2 T = 0$$

ω_{v} ------------------------------------ T

D_{va} ------------------------------------ α

Heat and Mass Transfer Analogy





$$-\rho D_{va}\nabla\omega_v$$

Boundary mass flux assuming vapor flux
is very small and only diffusional

$$-\rho\alpha\nabla T = -\frac{k}{c_p}\nabla T$$

$$= \frac{\text{Boundary heat flux}}{c_p}$$

Mass transfer coefficient g_m

$$\frac{\text{Boundary mass flux}}{\Delta(\rho\omega_v)}$$

$$\frac{\text{Boundary heat flux}}{c_p\Delta(\rho T)} = h/\rho c_p$$

Assuming
constant density

EPFL

Heat and Mass Transfer Analogy

Sherwood number -------------------- Nusselt number

$$\mathrm{Sh} = \frac{g_m L}{D_{\mathrm{va}}}$$

$$\mathrm{Nu} = \frac{hL}{k} = \frac{h}{\rho c_p}L \Big/ \alpha$$

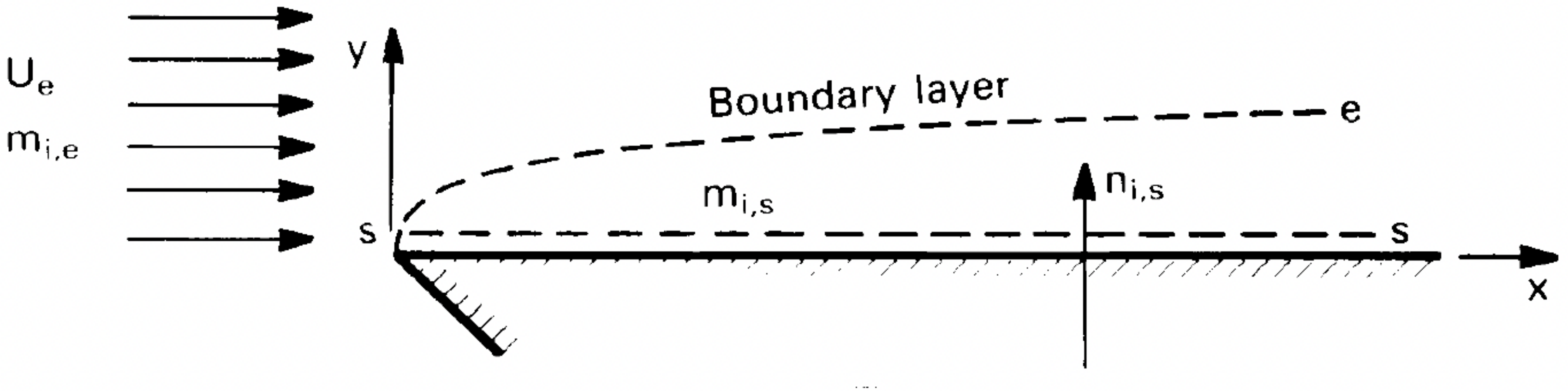
Schmit number $\mathrm{Sc} = \nu / D_{\mathrm{va}}$ ---------- Prandtl number $\mathrm{Pr} = \nu / \alpha$

Reynolds number $\mathrm{Re} = \rho U L / \mu$ ---------- Reynolds number $\mathrm{Re} = \rho U L / \mu$

Same functional form

$\mathrm{Sh} = \mathrm{fn}(\mathrm{Re}, \mathrm{Sc})$ ------------------------- $\mathrm{Nu} = \mathrm{fn}(\mathrm{Re}, \mathrm{Pr})$

Example



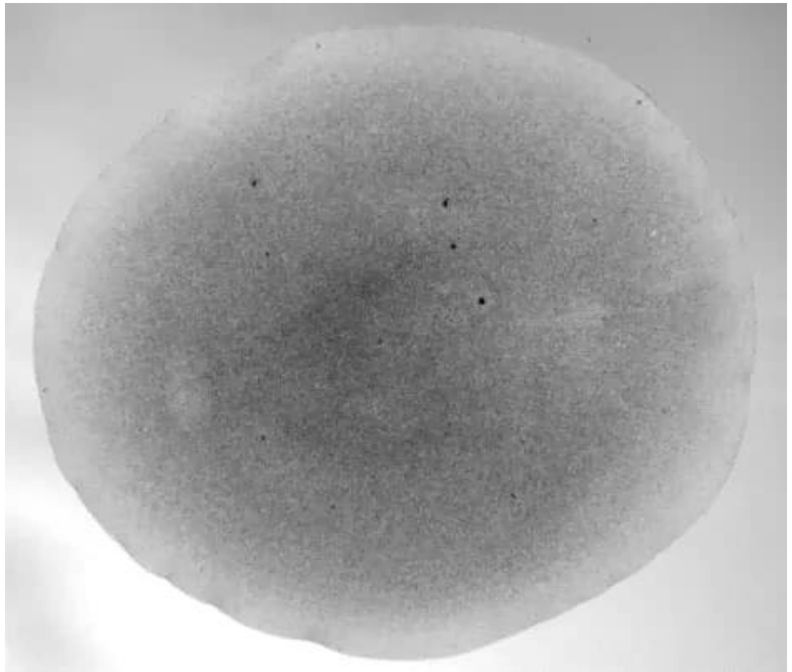
Lienhard, Figure 11.12

$$\mathrm{Nu}_x = 0.332\sqrt{\mathrm{Re}_x}\sqrt[3]{\mathrm{Pr}}$$

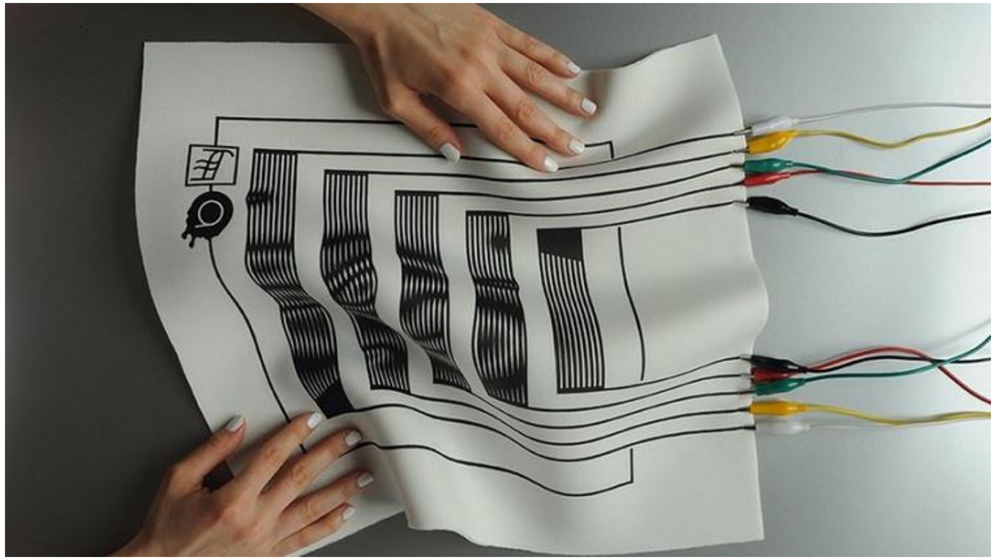
$$\mathrm{Sh}_x = 0.332\sqrt{\mathrm{Re}_x}\sqrt[3]{\mathrm{Sc}}$$

Coffee Ring Effect

Coffee Ring Effect



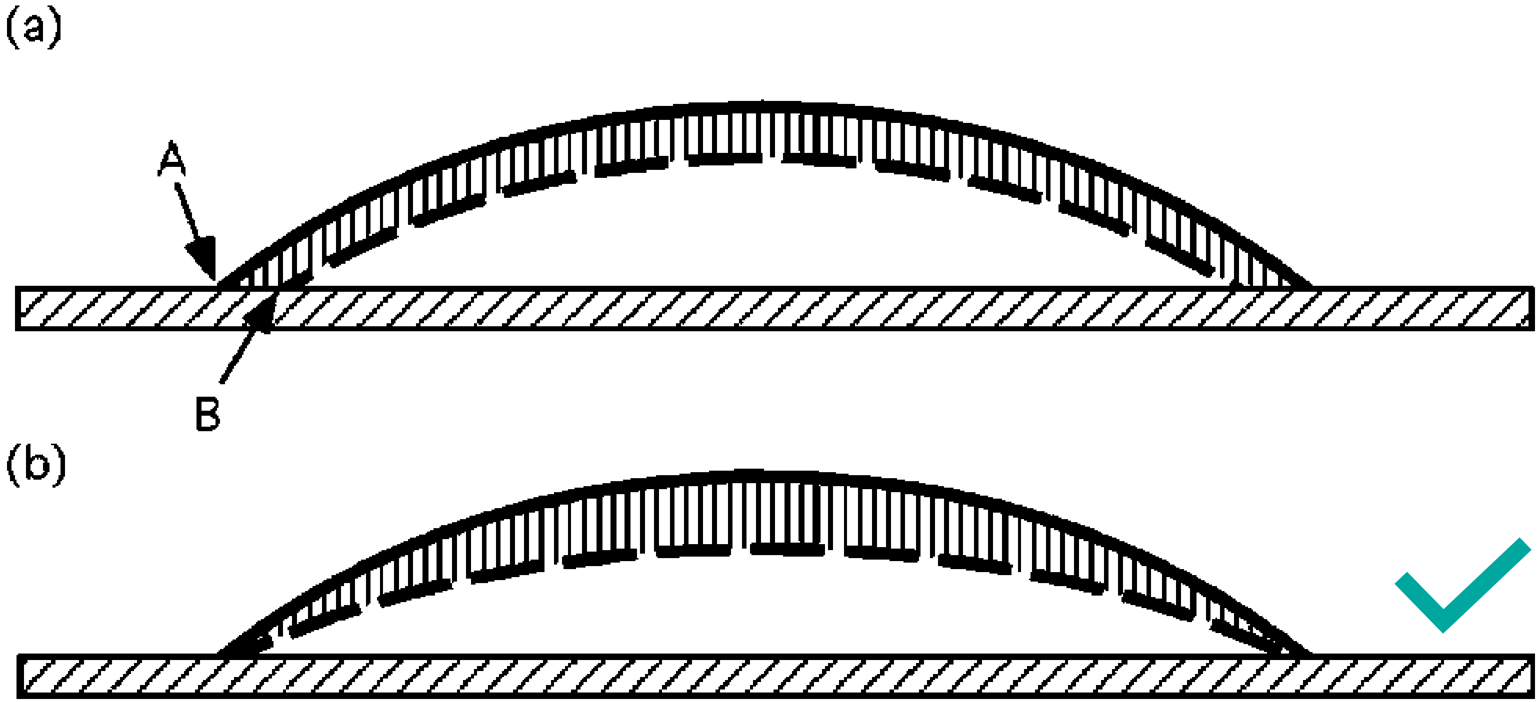
Printed electronics



Yunker *et al.*, *Nature* (2011)

Karim *et al.*, *Scientific Reports* (2019)

Contact Line Pinning



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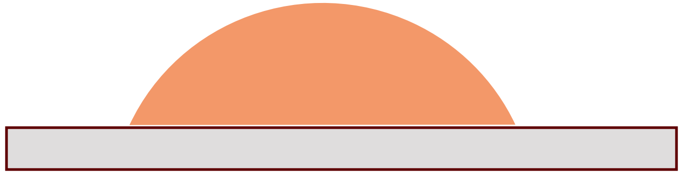
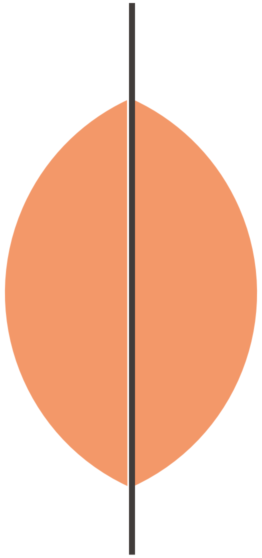
Deegan *et al.*, *Physical Review E* (2000)

Evaporative Transport

$$\cancel{\vec{v}_m \cdot \nabla\omega_v} - D_{\mathrm{va}}\nabla^2\omega_{\mathrm{v}} = 0$$

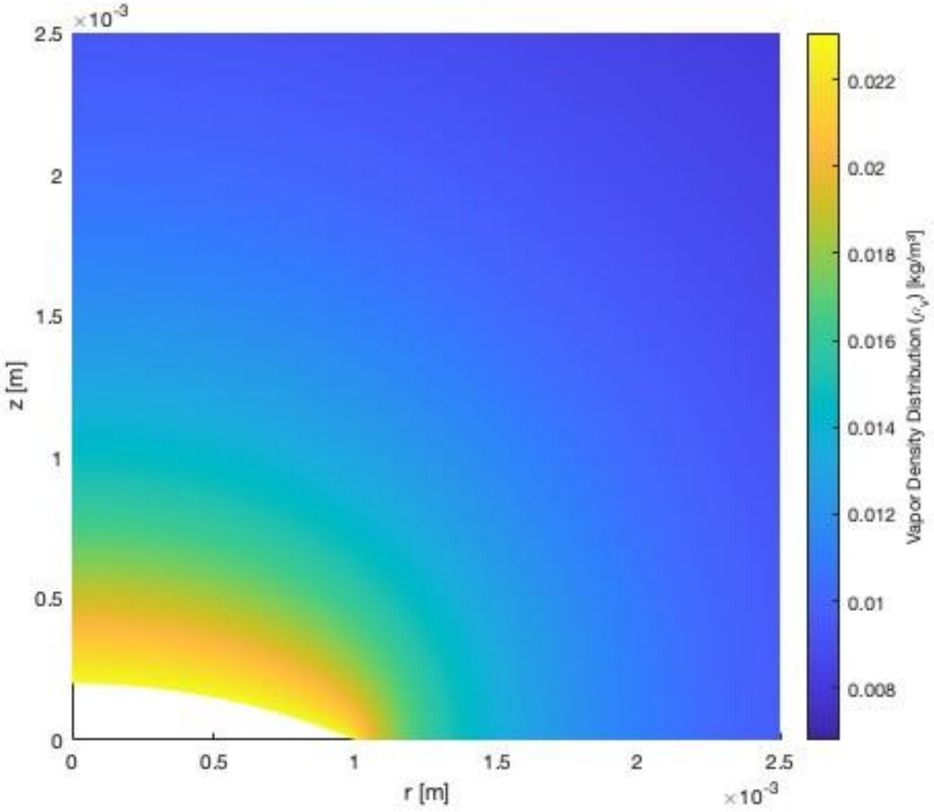
Assuming steady state in the vapor achieved rapidly





Lightening rod

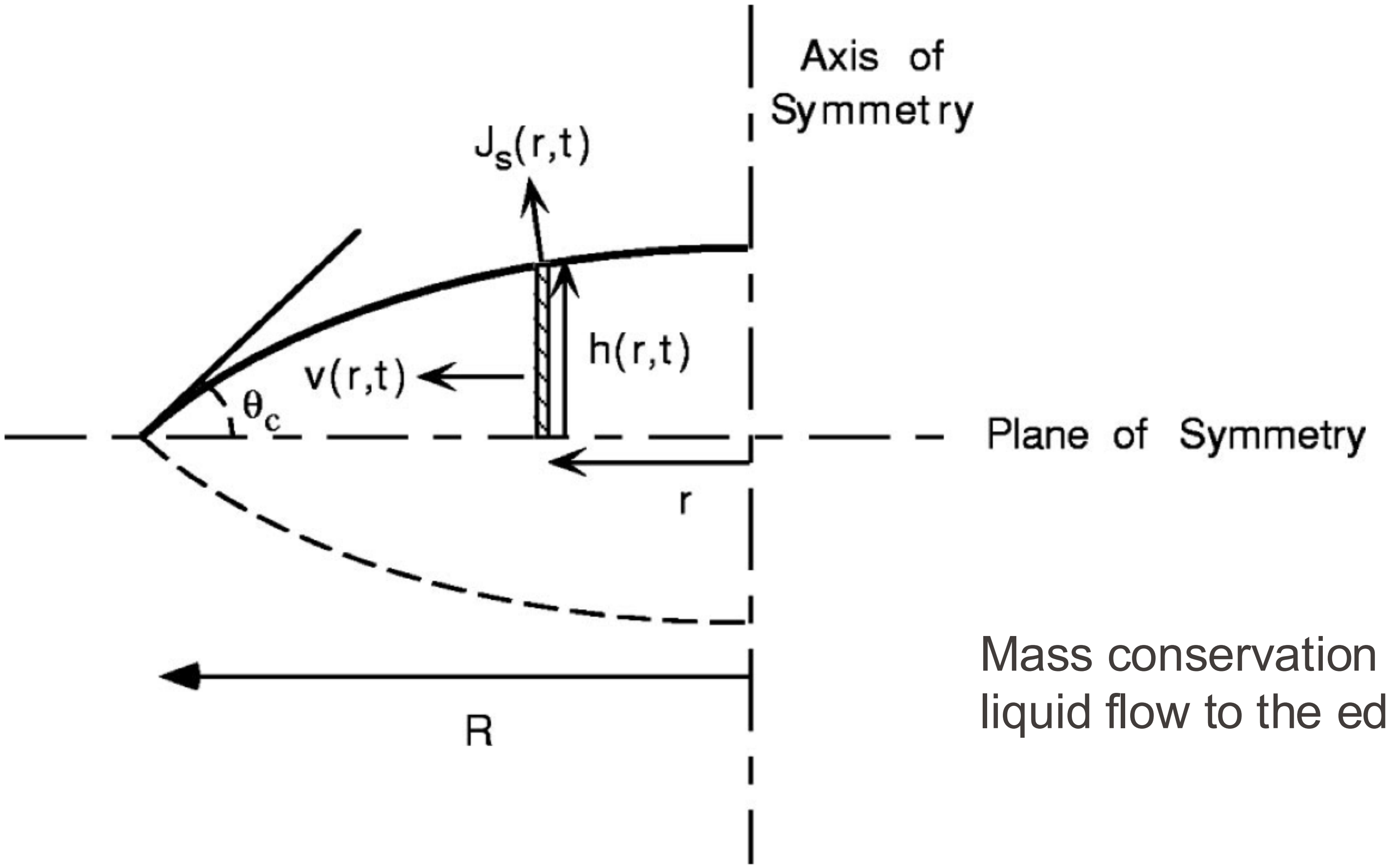
Evaporative Transport



Lightening rod

Higher concentration gradient at the edge
=> Higer evaporation flux at the edge

Mass Conservation



Mass conservation requires an internal liquid flow to the edge

What We Learned Today

- Fick’s Law of Diffusion
- Heat and mass transfer analogy
- Coffee ring effect