

The background image is a composite of two scenes. The left side shows a traditional wooden windmill with a lattice structure, situated in a grassy area with power lines in the background. The right side shows a large industrial cooling tower emitting steam, with a body of water and a pier in the foreground. A semi-transparent red rectangle is overlaid on the right side of the image, containing the main title text.

# ME-446: Liquid-gas interfacial heat and mass transfer

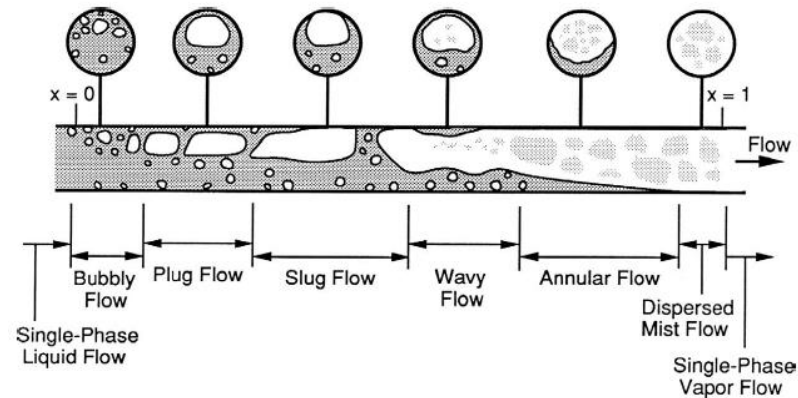
## Condensation

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2025 Fall Semester

Photo Credit: Trougnouf

- How wicking surfaces enhance CHF
- Different flow boiling regimes

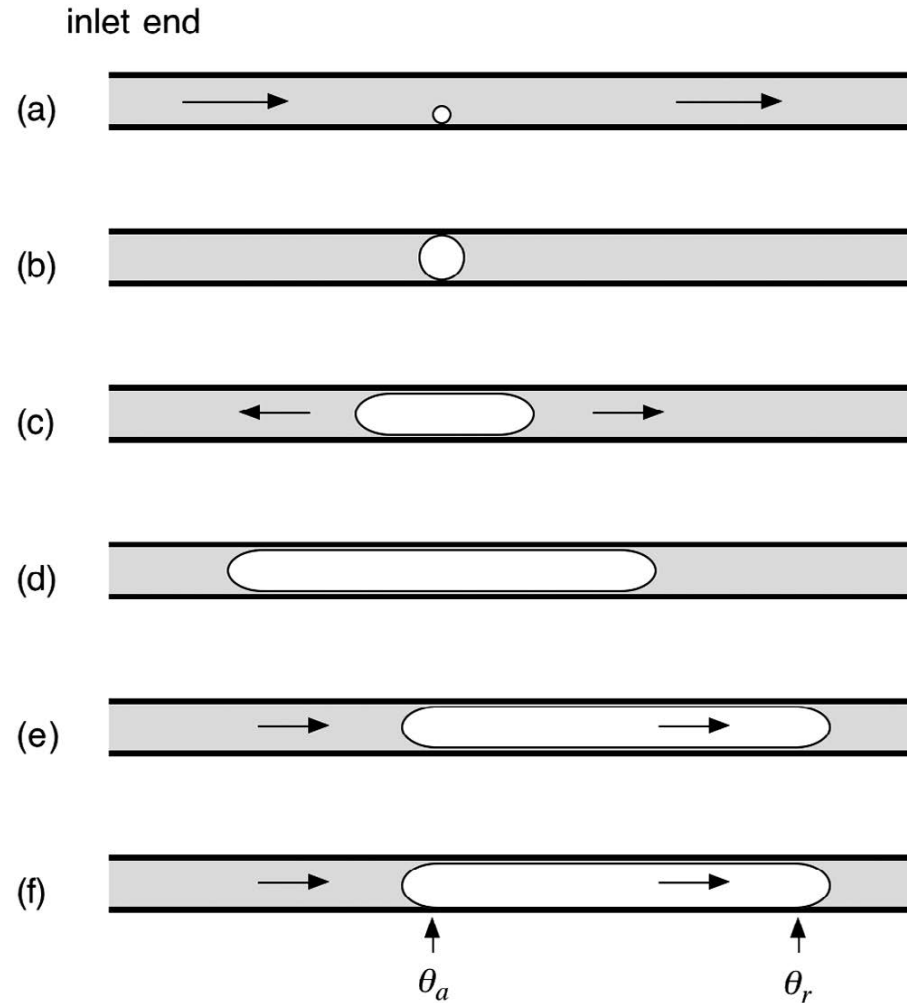


- Evaluate ONB, HTC, and CHF in flow boiling with correlations
  - ONB: Sato and Matsumura
  - HTC: Rohsenow's partition (subcooled), Gungor and Winterton (saturated), Kandlikar (saturated)
  - CHF: Celata (subcooled), Groeneveld lookup table (saturated)

- Miniaturization forces heat dissipation problem into microchannel regime
- Microchannel leads to higher surface-to-volume ratio and lower liquid layer thickness
- How confined the channel needs to be for us to consider it as micro?  
(Kew and Cornwell [https://doi.org/10.1016/S1359-4311\(96\)00071-3](https://doi.org/10.1016/S1359-4311(96)00071-3))

$$Co = \left[ \frac{\sigma}{g(\rho_L - \rho_V)D_h^2} \right]^{1/2} > 0.5$$

$D_h$ : hydraulic diameter



Once confinement is reached:

- Bubble elongation dominates
- Thin-film evaporation becomes the key heat transfer mechanism
- Flow patterns collapse to slug–annular regimes
- Macroscale boiling correlations break down

Fig. 12.31 in Carey

- Liu and Garimella <https://doi.org/10.1115/1.2754944>

$$h = h_{sp}F + h_{nb}S$$

$$h_{sp} = 1.86 \left( \frac{k_l}{D_h} \right) \left[ \frac{\text{Re}_l \text{Pr}_l D}{L} \right]^{1/3} \left( \frac{\mu_l}{\mu_s} \right)^{0.14}$$

$$\text{Re}_l = \frac{G(1-x)D}{\mu_l} \quad \mu_s: \text{liquid viscosity at surface temperature}$$

- Liu and Garimella <https://doi.org/10.1115/1.2754944>

$$h = h_{sp}F + h_{nb}S$$

$$F = 2(\phi_l^2)^{0.25} \left(\frac{\mu_{tp}}{\mu_l}\right)^{0.105} \left(\frac{c_{p,tp}}{c_{p,l}}\right)^{0.25} \left(\frac{k_{tp}}{k_l}\right)^{0.75} \text{Pr}_l^{0.167}$$

$$\phi_l^2 = 1 + \frac{5}{X} + \frac{1}{X^2} \quad X^2 = \left(\frac{1-x}{x}\right) \left(\frac{\rho_v}{\rho_l}\right) \left(\frac{\mu_l}{\mu_v}\right) \quad \text{for laminar flow}$$

Subscript tp: two phase properties  $\Psi_{tp} = x\Psi_v + (1-x)\Psi_l$

- Liu and Garimella <https://doi.org/10.1115/1.2754944>

$$h = h_{sp}F + h_{nb}S$$

$$h_{nb} = h_0 \left( \frac{q''}{q_0''} \right) \left( \frac{R_p}{R_{p0}} \right)^{0.133} \left[ 1.73 p_r^{0.27} + \left( 6.1 + \frac{0.68}{1 - p_r} \right) p_r^2 \right]$$

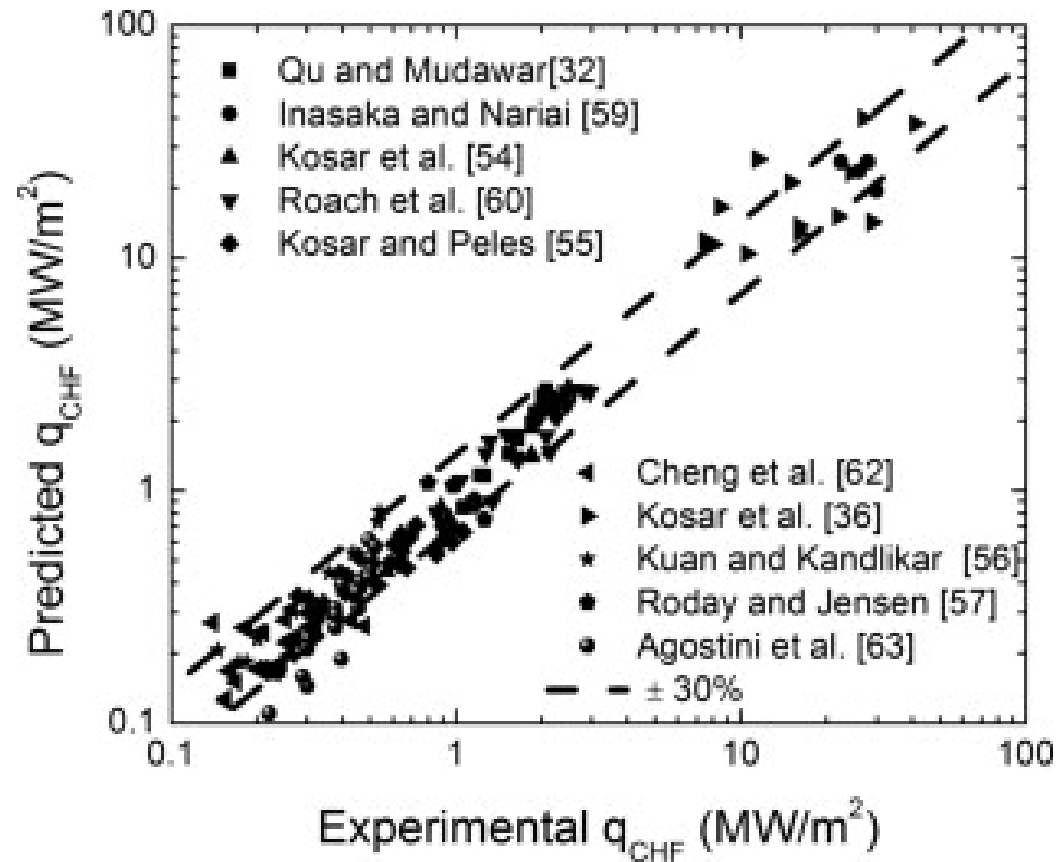
$$S = \exp \left\{ 36.57 - \frac{55746}{\text{Re}_l F^3} - 3.4 \ln(\text{Re}_l F^3) \right\}$$

$$h_0 = 5600 \text{ W/m}^2\text{K}, q_0'' = 20 \text{ kW/m}^2, R_{p0} = 0.4 \text{ }\mu\text{m}$$

$R_p$ : surface roughness (set as 1  $\mu\text{m}$  by Liu and Garimella)

$p_r = P/P_c$  reduced pressure (normalized to critical pressure)

CHF model by Kandlikar <https://doi.org/10.1115/1.4001124>



Vapor Backflow Instability

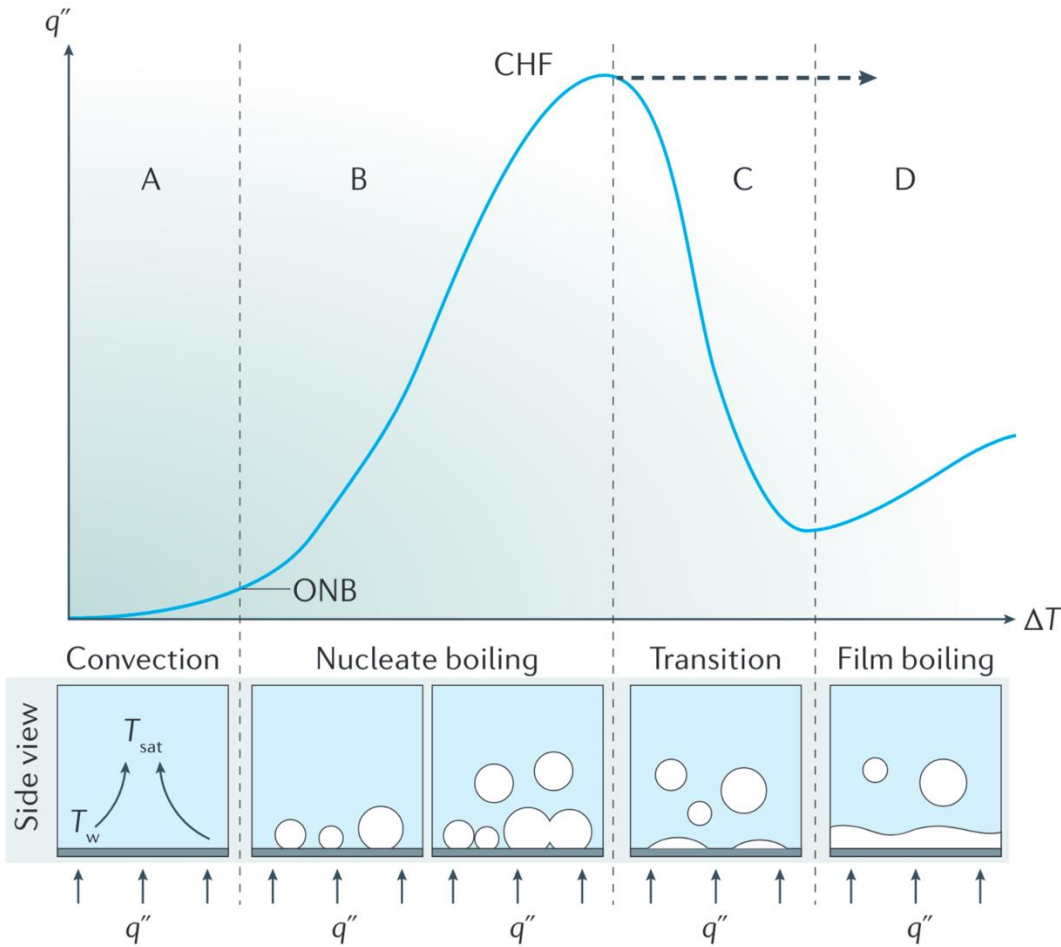


60 fps; real time duration: 210 ms

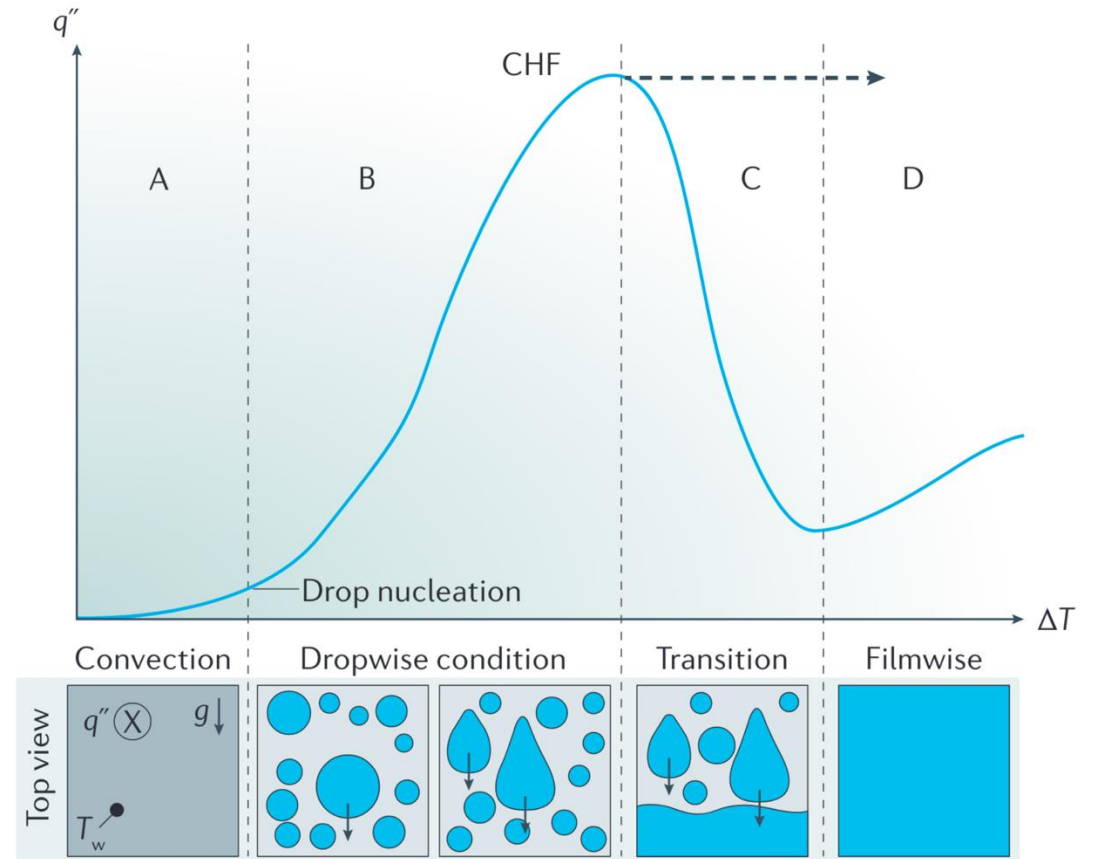
Credit: Mark  
Schepperle

# Boiling vs Condensation

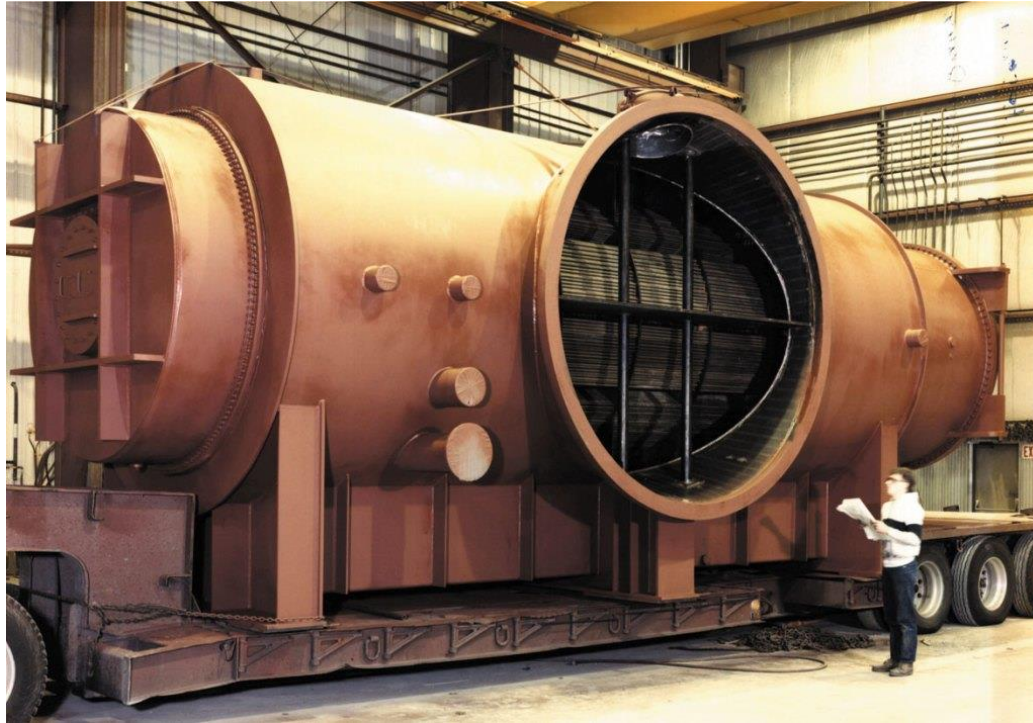
**a Boiling**



**b Condensation**



# Why Condensation Matters



Condenser in Power Plants, Holtec

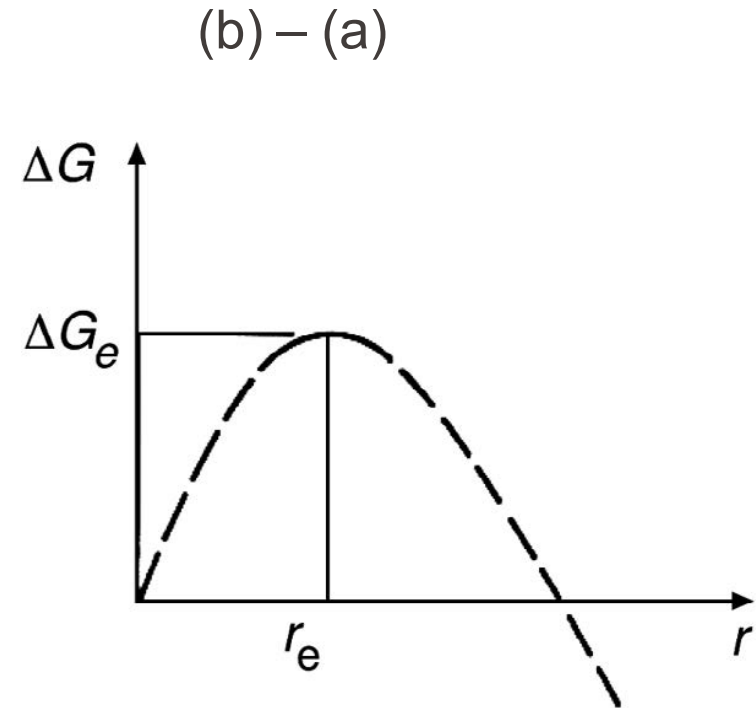
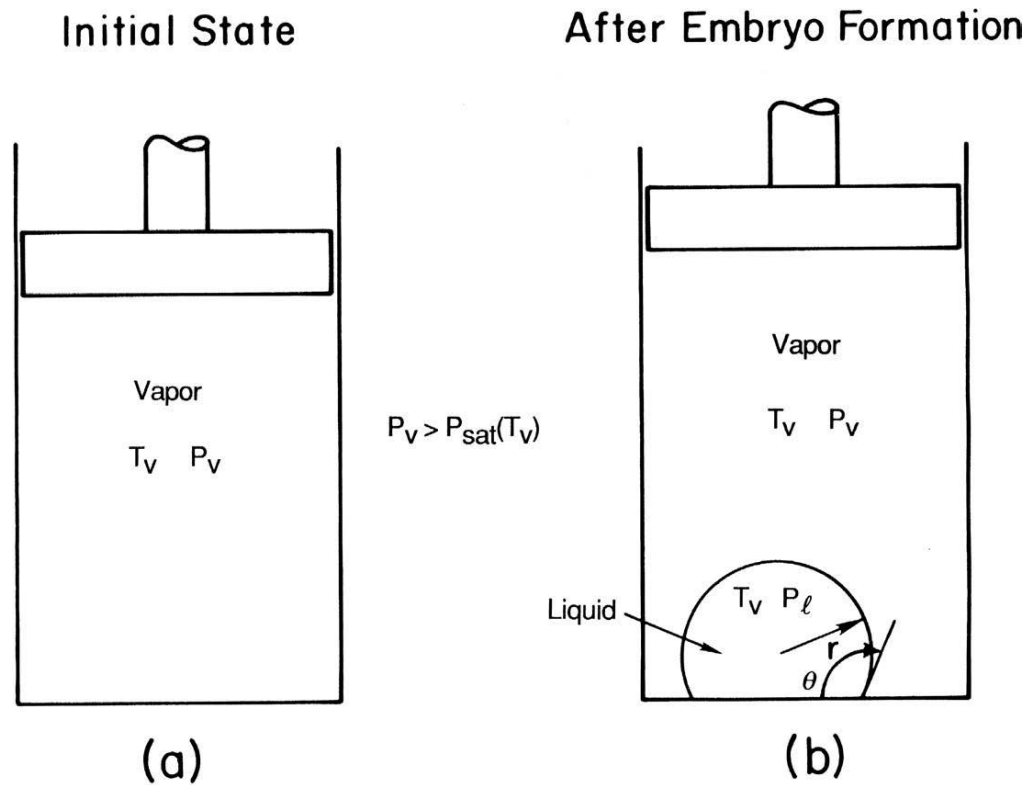


Condenser in Cooling Systems, STACK

# Intended Learning Objectives

- Nucleation in condensation
- Rose's analysis of dropwise condensation
- Nusselt's analysis of filmwise condensation

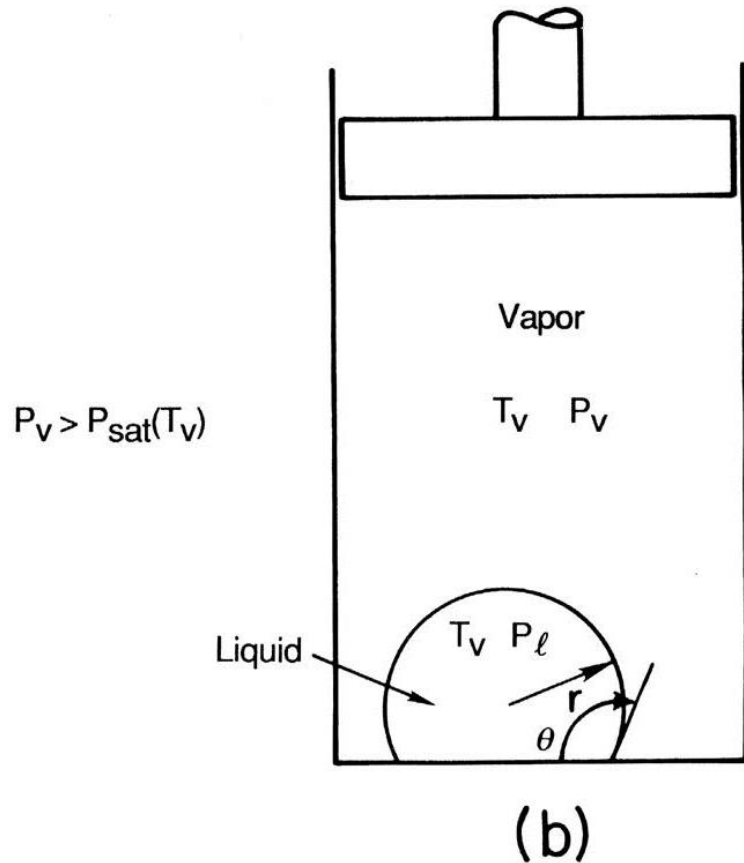
Carey, Chapter 9



If  $r < r_e$ , the droplet collapses  
 If  $r > r_e$ , the droplet grows

Figure 9.2 in Carey

After Embryo Formation



$$g_{sat,l}(T_v, P_{sat}) = g_{sat,v}(T_v, P_{sat}) = g_{sat}$$

$$g_v - g_{sat} = \int_{P_{sat}}^{P_v} v_v dP = \int_{P_{sat}}^{P_v} \frac{RT_v}{P} dP = RT_v \ln\left(\frac{P_v}{P_{sat}}\right)$$

$$g_l - g_{sat} = \int_{P_{sat}}^{P_l} v_l dP = v_l(P_l - P_{sat})$$

$$P_l - P_v = 2\sigma/r_e$$

$$r_e = \frac{2\sigma}{\frac{RT_v}{v_l} \ln\left(\frac{P_v}{P_{sat}}\right) + P_{sat} - P_v} \approx \frac{2\sigma}{\frac{RT_v}{v_l} \ln\left(\frac{P_v}{P_{sat}}\right)}$$

$r_e \sim 1 \text{ nm}$ , for water vapor with  $T_v = 100^\circ\text{C}$  and  $P_v = P_{sat}(110^\circ\text{C})$

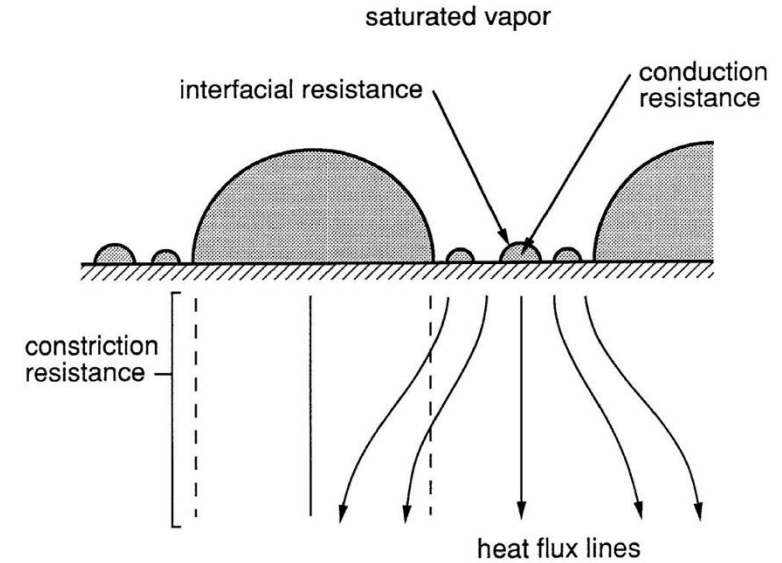


<https://doi.org/10.1021/nl303835d>

- Schrage equation (condensation)

$$q_i'' = \left( \frac{2\hat{\sigma}}{2 - \hat{\sigma}} \right) h_{lv} \sqrt{\frac{1}{2\pi R} \left( \frac{P_v}{\sqrt{T_v}} - \frac{P_l}{\sqrt{T_l}} \right)}$$

$$\approx \left( \frac{2\hat{\sigma}}{2 - \hat{\sigma}} \right) \frac{\rho_v h_{lv}^2}{T_v} \sqrt{\frac{1}{2\pi R T_v} \left( 1 - \frac{P_v}{2\rho_v h_{lv}} \right)} \Delta T_i = h_i \Delta T_i$$



$$\frac{d \left( \frac{P_{sat}}{\sqrt{T}} \right)}{dT} = \frac{\rho_v h_{lv} - \frac{P_v}{2}}{T^{3/2}}$$

Clausius-Clapeyron

$$\frac{dP_{sat}}{dT} \approx \frac{\rho_v h_{lv}}{T}$$

Assuming hemispherical droplet

$$q_i'' = \frac{q_d}{2\pi r^2}$$

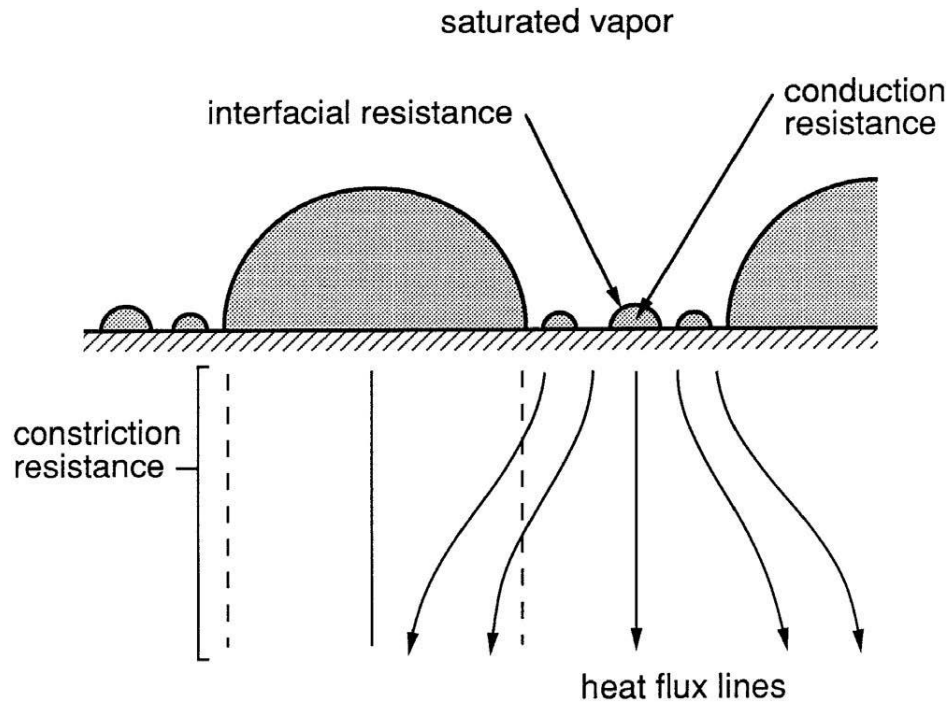
$$\Delta T_i = \frac{q_d}{2\pi r^2 h_i} \quad q_d \text{ [W]}$$

Given a droplet radius  $r$ , the equilibrium droplet temperature at the interface should follow

$$r \approx \frac{2\sigma}{\frac{RT_{eq}}{v_l} \ln\left(\frac{P_v}{P_{sat}(T_{eq})}\right)}$$

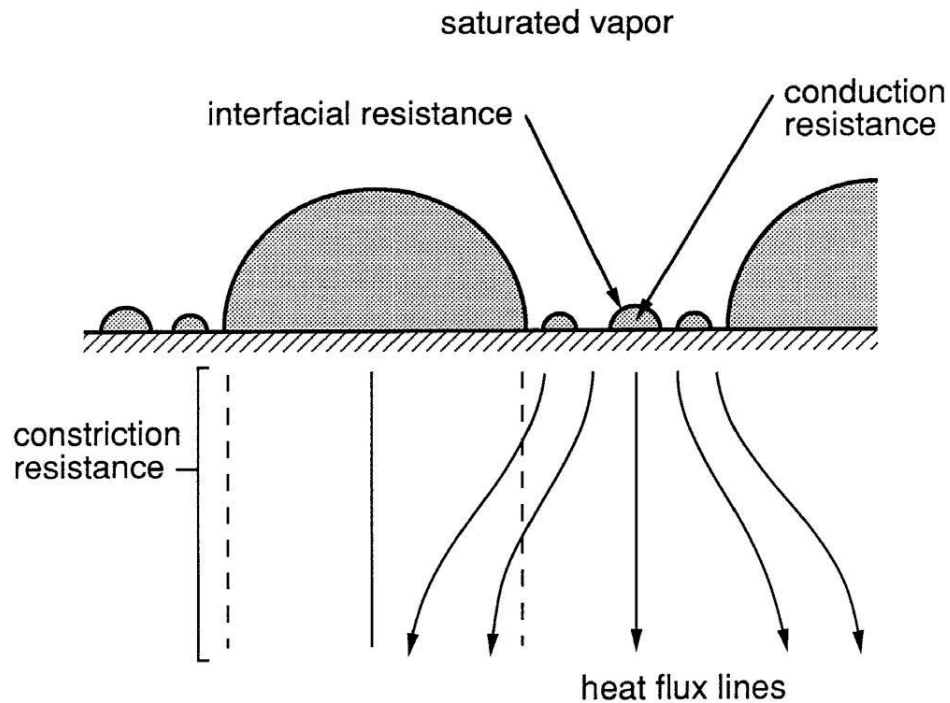
Clausius-Clapeyron  $\frac{dP_{sat}}{dT} = \frac{\rho_v h_{lv}}{T} \approx \frac{P_{sat}}{RT^2} h_{lv}$

$$r \approx \frac{2\sigma v_l T_{sat}(P_v)}{h_{lv}(T_{sat}(P_v) - T_{eq})} \quad \Delta T_{cap} = \frac{2\sigma v_l T_{sat}(P_v)}{h_{lv} r}$$



$$q_d \approx \frac{k_l \Delta T_{con}}{r/2} \cdot 2\pi r^2$$

$$\Delta T_{con} \approx \frac{q_d}{4\pi k_l r}$$



- Interfacial resistance
- Curvature induced resistance
- Conduction through droplet
- Constriction resistance

$$\Delta T_t = \Delta T_i + \Delta T_{cap} + \Delta T_{con} = \frac{q_d}{2\pi r^2 h_i} + \frac{2\sigma T_{sat}(P_v)}{h_{lv} r} + \frac{q_d}{4\pi k_l r}$$

- Cumulative distribution function (postulated form)

$$F(r) = \left(\frac{r}{r_{max}}\right)^{1/3} \text{ for } r_e < r \leq r_{max};$$

- Probability distribution function

$$A(r) = \frac{dF}{dr} = \frac{r^{-2/3}}{3r_{max}^{1/3}}$$

$$r_{max} = K_3 \sqrt{\frac{\sigma}{\rho g}}$$

# Dropwise Condensation Heat Transfer

$$q'' = \int_{r_e}^{r_{max}} \frac{q_d}{2\pi r^2} A(r) dr$$

$$h_{dc} = \frac{1}{\Delta T_t} \int_{r_e}^{r_{max}} \frac{q_d}{2\pi r^2} A(r) dr$$

For dropwise condensation of steam at pressures below 1 atm, Rose et al. recommended the following empirical correlation for the heat transfer coefficient

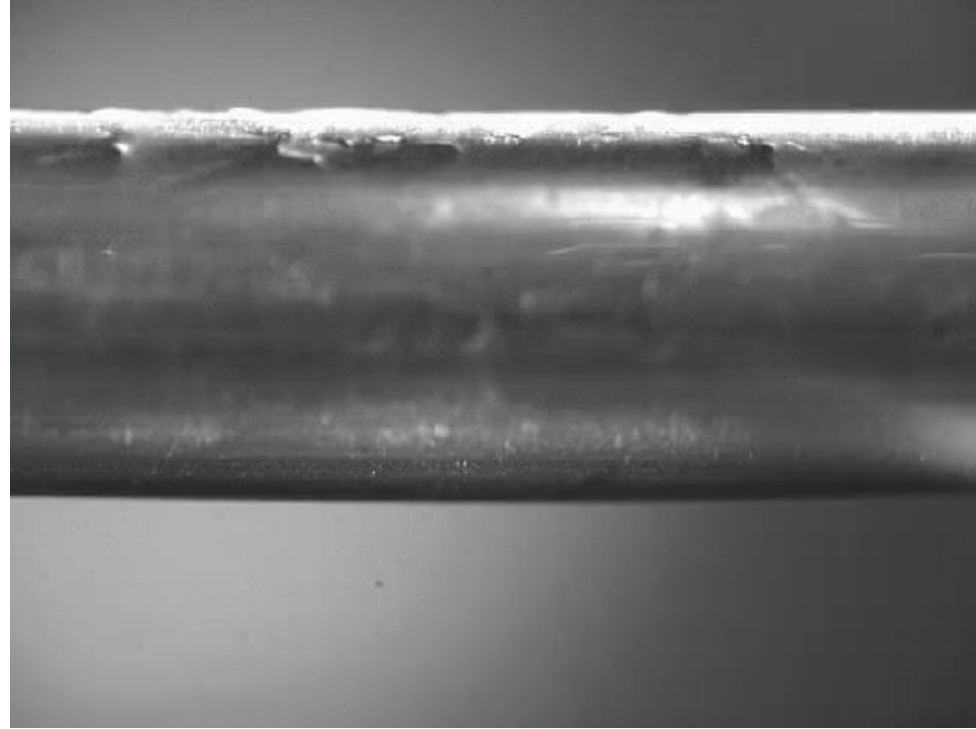
$$h_{dc} = T_v^{0.8} [5 + 0.3(T_{sat} - T_w)] \quad \text{Eq. 9.42 in Carey}$$

Temperature in Celsius and HTC in kW/m<sup>2</sup>K

# Dropwise vs Filmwise Condensation



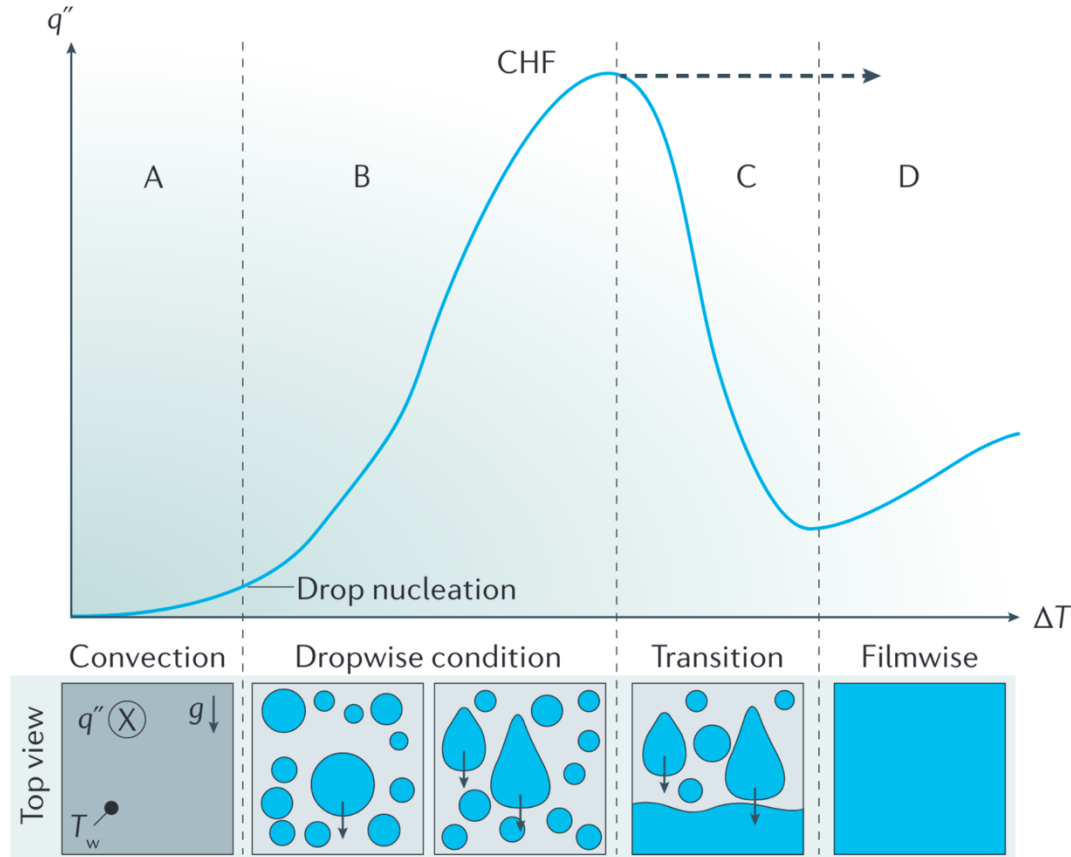
Dropwise condensation



Filmwise condensation

# Dropwise vs Filmwise Condensation

## b Condensation



Data obtained by Takeyama and Shimizu [9.40] for condensation of steam on a short vertical copper surface.

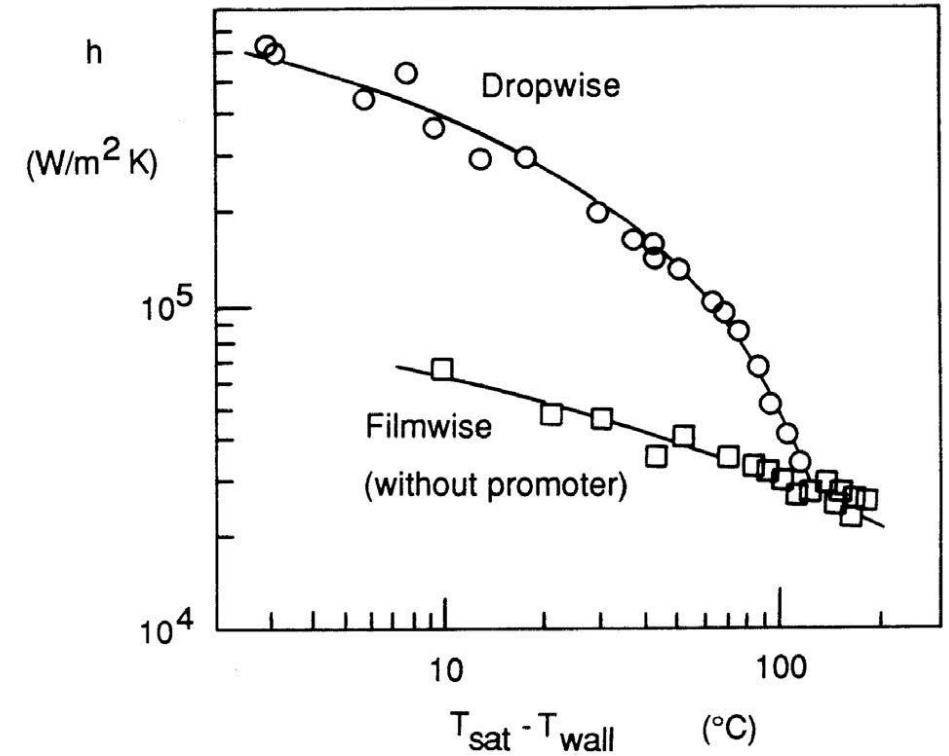
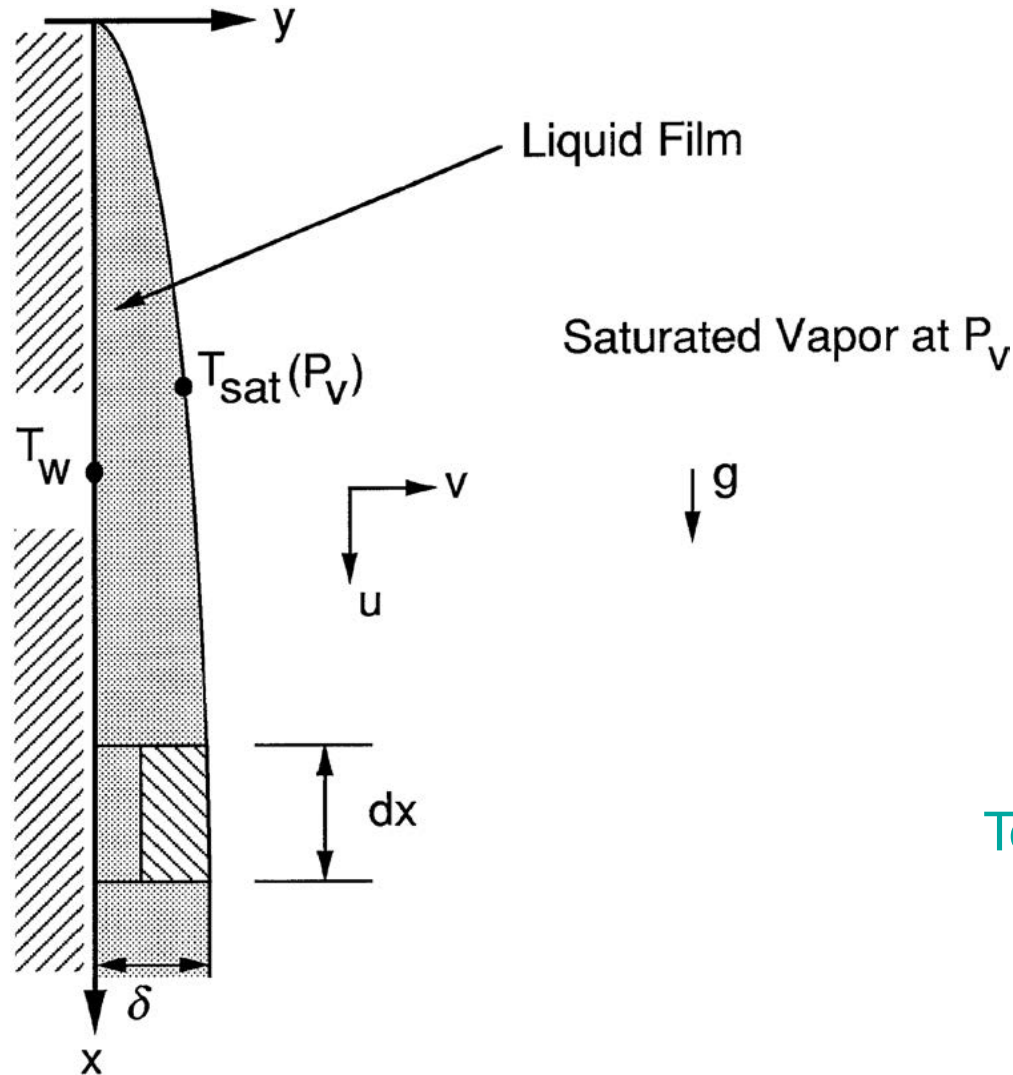


Figure 9.6 in Carey



Force balance on the shaded film element

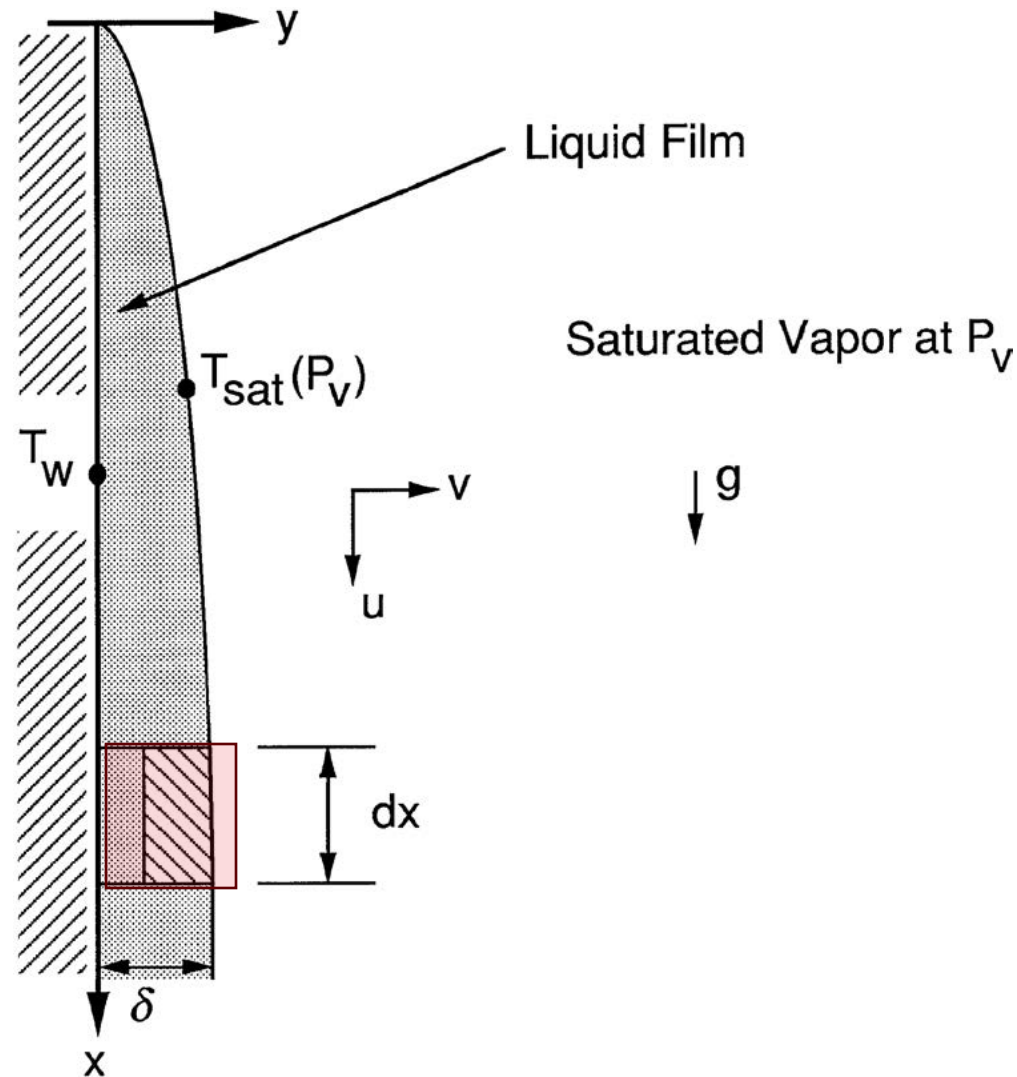
$$(\delta - y)dx(\rho_l - \rho_v)g = \mu_l \left( \frac{du}{dy} \right) dx$$

Integrating this equation w.r.t.  $y$

$$u = \frac{(\rho_l - \rho_v)g}{\mu_l} \left( y\delta - \frac{y^2}{2} \right)$$

Total mass flow rate per unit depth in  $y$ -direction

$$\dot{m}' = \rho_l \int_0^\delta u dy = \frac{\rho_l(\rho_l - \rho_v)g\delta^3}{3\mu_l}$$



$$dq = \frac{k_l \Delta T}{\delta} dx \quad \Delta T = T_{sat} - T_w$$

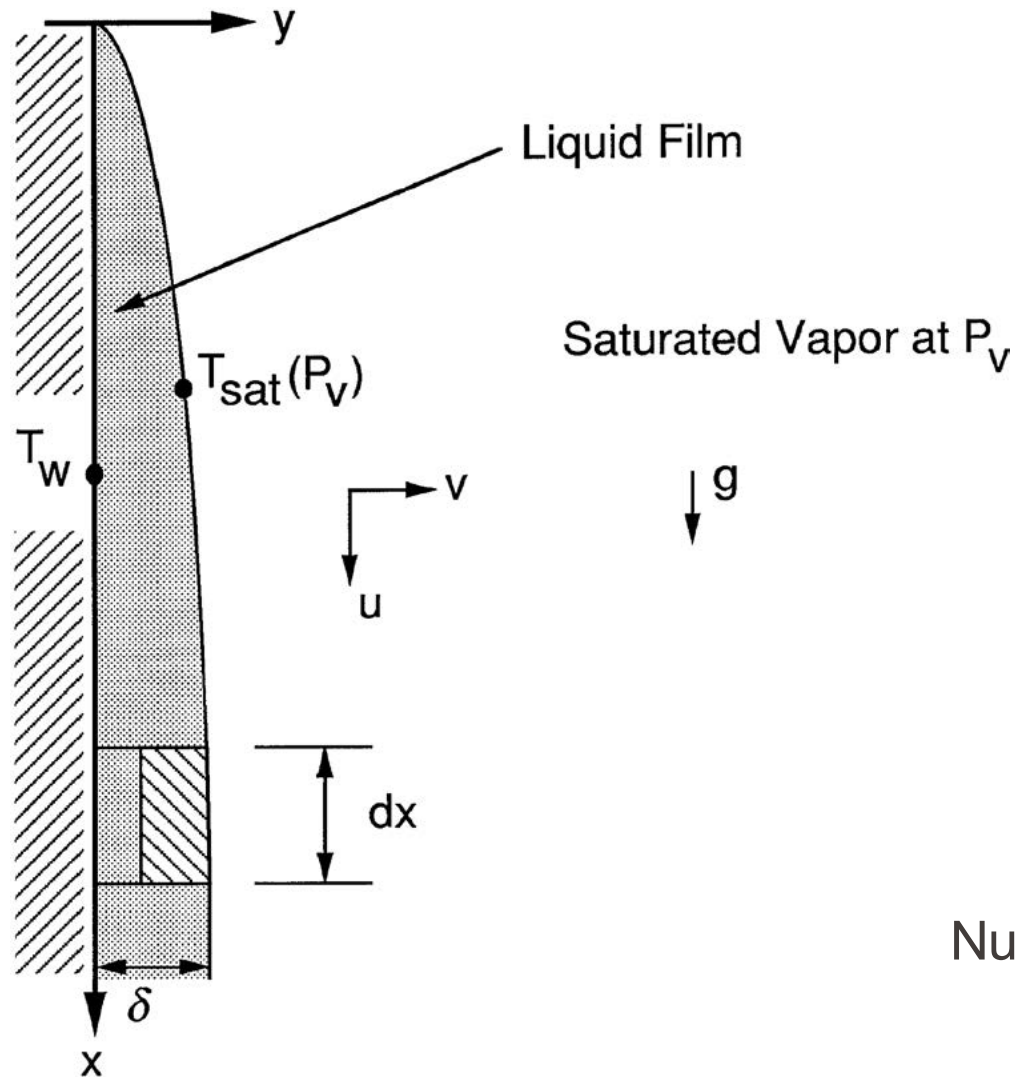
$$dq = h_{lv} d\dot{m}'$$

$$\frac{k_l \Delta T}{\delta} = \frac{h_{lv} d\dot{m}'}{dx}$$

$$\frac{d\delta}{dx} = \frac{k_l \mu_l \Delta T}{\rho_l (\rho_l - \rho_v) g h_{lv} \delta^3}$$

Using  $\delta = 0$  at  $x = 0$

$$\delta = \left[ \frac{4k_l \mu_l x \Delta T}{\rho_l (\rho_l - \rho_v) g h_{lv}} \right]^{1/4}$$



$$h_l = \frac{k_l}{\delta} = k_l \left[ \frac{\rho_l(\rho_l - \rho_v)gh_{lv}}{4k_l\mu_l x\Delta T} \right]^{1/4}$$

$$\bar{h}_l = \frac{1}{x_e} \int_0^{x_e} \frac{k_l}{\delta} dx = \frac{4}{3} k_l \left[ \frac{\rho_l(\rho_l - \rho_v)gh_{lv}}{4k_l\mu_l x\Delta T} \right]^{1/4}$$

$$\overline{Nu}_{x_e} = \frac{\bar{h}_l x_e}{k_l} = \frac{4}{3} \left[ \frac{\rho_l(\rho_l - \rho_v)gh_{lv} x^3}{4k_l\mu_l \Delta T} \right]^{1/4}$$

Nusselt equation for filmwise condensation

- Filmwise condensation still the most common operation mode in industry
  - Not as bad as film boiling as liquid is much more conductive than vapor
  
- Dropwise condensation requires strong hydrophobicity at high supersaturation (droplets must roll off the surface quickly before flooding occurs). However, maintaining strong hydrophobicity over extended period is challenging.