

**Homework 6**

Presentation by Group 6 on Thursday 30th October

**Problem 1: Clausius-Clapeyron relation**

Derive the Clausius-Clapeyron relation following the steps below.

A) For a liquid-vapor phase change process, relate the specific entropy change  $\Delta s$  to the enthalpy of vaporization  $h_{lv}$  and the phase change temperature  $T$ .

*Hint:*  $dh = Tds + vdP$  (the symbols follow the usual thermodynamic definition).

B) Along the liquid-vapor saturation curve in a P-T diagram, we know the Gibbs free energy is the same for the liquid and the vapor. Writing the Gibbs free energy total differential with the liquid and the vapor thermodynamic quantities, respectively.

$$dg = (v_l dP - s_l dT)$$

$$dg = (v_v dP - s_v dT)$$

Combine the above equation with what you obtained in A) to derive

$$\frac{dP}{dT} = \frac{h_{lv}}{T(v_v - v_l)}$$

on the saturation curve, where  $v_v$  and  $v_l$  represent the specific volumes of vapor and liquid, respectively.

---

## Problem 2: Hsu's model

Based on your understanding of Hsu's model, quantitatively describe what will happen to the range of cavity sizes that can be activated for nucleation in your own words when each of the following changes are applied to the system (assuming other conditions are fixed).

1. Thin down the thermal boundary layer.
2. Heat up the substrate.
3. Lower the surface tension of the liquid.

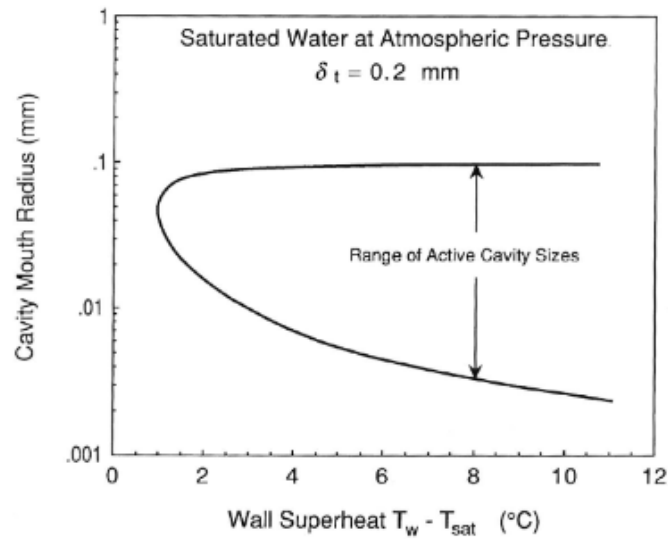


Figure 1: Prediction of the range of active cavity sizes using Hsu's analysis.

---

### Problem 3: Equilibrium vapor pressure for meniscus in a cylindrical pore

Consider a water meniscus in a cylindrical pore with a radius of curvature  $r$ . Assume the whole system (liquid, solid, and vapor) is at the same temperature  $T$ .

A) Take a look at the derivation on slide 18 of the Week 5 lecture again. Write down the equations that will allow you to solve for the equilibrium vapor pressure  $P_v$  on the liquid-vapor interface and the liquid pressure  $P_l$ . (Do NOT assume that  $P_v$  and  $P_{sat}(T)$  are close. An explicit expression of the pressures is not needed).

B) Setting  $T = 300K$ , and using the needed properties of water, write a script that plots the following:

1.  $P_v/P_{sat}(T)$  as a function of the pore radius  $r$ , varying between  $20nm$  and  $0.2mm$ . Use a logarithmic scale for the x-axis (radius).
2. In a separate figure, plot the liquid pressure  $P_l$  as a function of the pore radius  $r$  over the same range ( $20nm$  to  $0.2mm$ ), also using a logarithmic scale for the x-axis.

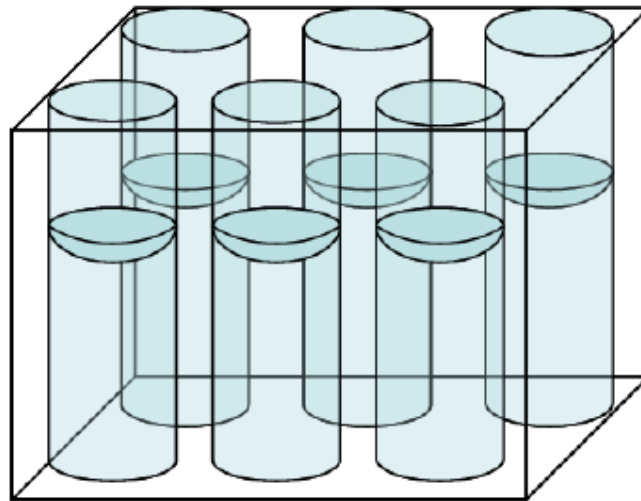


Figure 2: Cylindrical pores with meniscus formed by the liquid in equilibrium with the vapor