ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 2

Presentation by Group 2 on Thursday 25th September 2025

Problem 1: Capillary rise/fall in cylindrical tubes

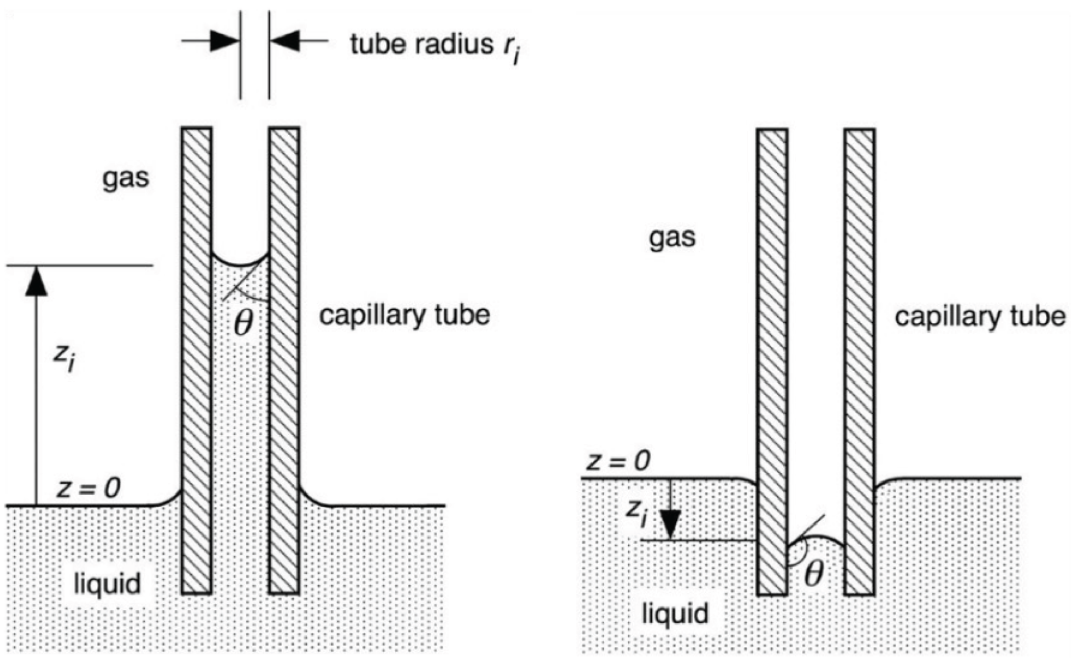
A cylindrical tube is inserted vertically into the surface of a liquid (water) to observe the effects of capillary action.

A. Capillary rise in PVC tubes

Assume the tube is made of PVC, for which the contact angle of water with air θ is less than 90°C (see Figure 1a). Given the tube radius r_i, water surface tension γ, water density ρ, and gravitational acceleration g, derive an expression for the capillary rise z_i inside the tube.

B. Capillary fall in Teflon tubes

Now consider the tube is made of Teflon, where the contact angle of water with air θ exceeds 90° (see Figure 1b). Using the same parameters - tube radius ri, water surface tension γ, water density ρ and gravitational constant g - derive an expression for the capillary fall z_i inside the tube.



(a) (b)

Figure 1: (a) Capillary rise in PVC tube. (b) Capillary fall in Teflon tube.

Problem 2: Advancing/receding droplet

Consider a static liquid droplet on a substrate composed of two materials (see Figure 2). The contact line of the droplet intersects at the boundary between a *wetting Material 1*, where the Young's contact angle of the liquid on the material is 30° and a *non-wetting Material 2*, where the Young's contact angle of the liquid on the material is 120°. The droplet currently has a contact angle θ with respect to the substrate.

A. Advancement into the non-wetting material

Consider a virtual advancement of the contact line into the material 2 section. Use an energy analysis to argue that if $\theta < 120°$, this advancement will not spontaneously occur.

B. Receding into the wetting material

Consider a virtual receding of the contact line into the material 1 section. Use an energy analysis to argue that if $\theta > 30°$, this receding will not spontaneously occur.

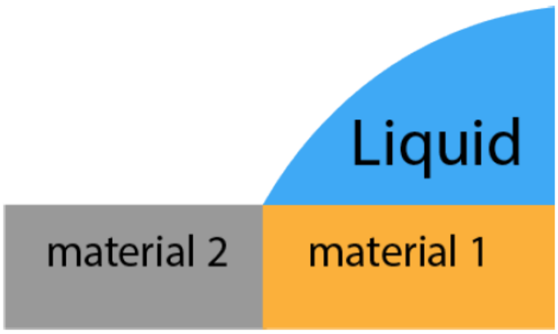


Figure 2: Schematic representation of a sessile droplet at the boundary between two materials with different wetting properties.

From this example, one should be able to see that a static sessile droplet on a substrate can have a contact angle which takes an arbitrary value between the advancing angle and the receding angle.

Problem 3: Micropillared surface

Consider a substrate structured with a micropillar array. The micropillars have the same height $h = 0.1mm$, the same cross-section (a square of size $a \times a$, with $a = 0.05mm$), and the same edge-to-edge spacing $b = 0.1mm$ (see Figure 3).

Quantitatively plot the apparent contact angle θ_{app} on the substrate as a function of the intrinsic Young's contact angle θ_Y on the flat homogeneous surface of the same material assuming θ_Y varies between 0° and 180°.

Use your preferred programming language for this task.

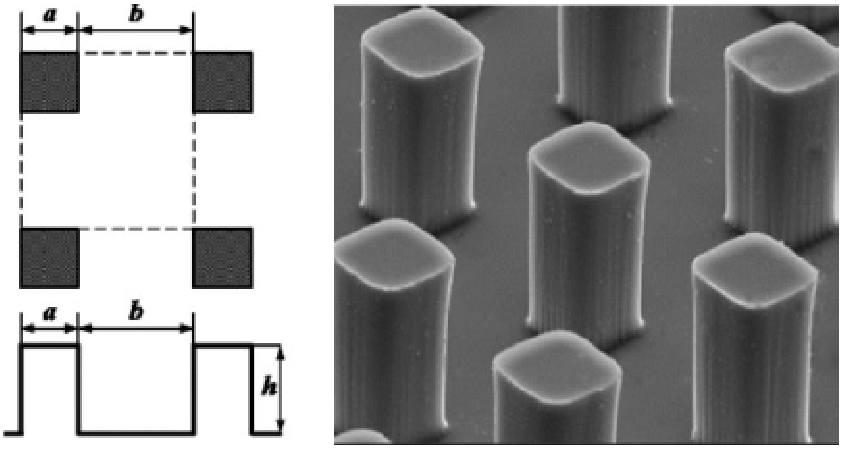


Figure 3: (left) Schematic and (right) SEM image of the micropillar array with a square cross-section pillars (Colloid Polym Sci 294, 833–840 (2016))