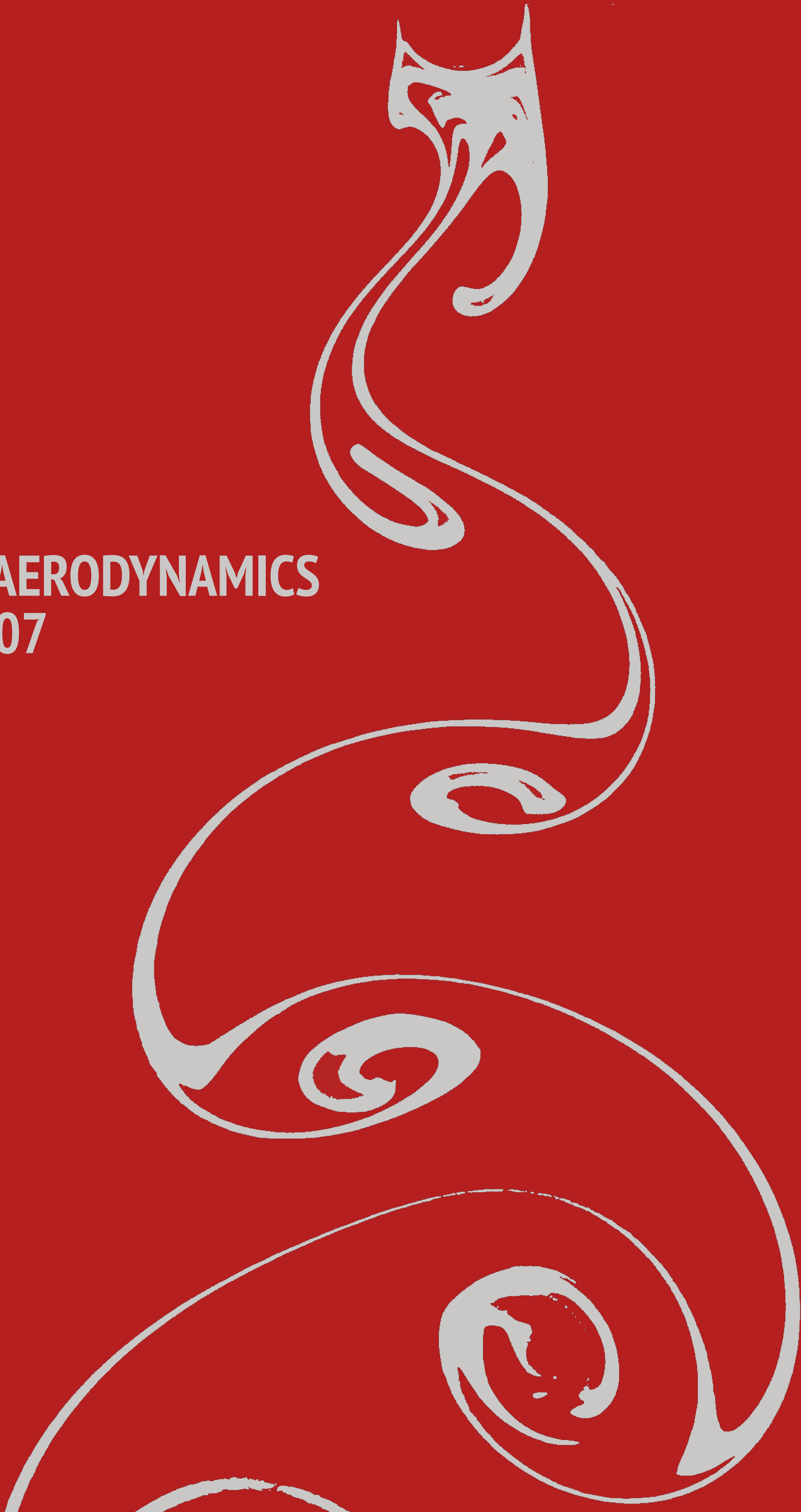
ME-445 AERODYNAMICS
Exercise 07
Week 6



Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0\right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right]\right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $w = \phi + i\psi = U_\infty e^{-i\alpha} z$

Potential vortex in z_0 $w = -\frac{i\gamma}{2\pi}\ln(z - z_0)$

Point source or sink in z_0 $w = \frac{Q}{2\pi}\ln(z - z_0)$

Source-sink doublet in z_0 $w = \frac{\mu}{2\pi(z - z_0)}$

$$\frac{dw}{dz} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\overline{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{dy_c}{dx} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos\theta)}{2}$$

we know:

$$k = 2U_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{dy_c}{dx} d\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dy_c}{dx}\cos n\theta \, d\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0$, $\alpha_i(y) > 0$

if induced velocity points upward: $w < 0$, $\alpha_i < 0$

Prandtl's lifting-line theory

$$U_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{y - y_0} dy$$

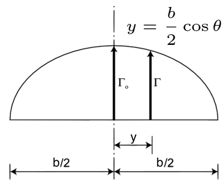
$$\alpha(y_0) = \alpha_{eff}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\frac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi AR}$$



General loading $\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = U_\infty \sum_{n=1}^{\infty} nA_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1 \, AR$$

$$C_{D,i} = \frac{C_L^2}{\pi AR}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\,(A_n/A_1)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \frac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \frac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity	$\nu = 1\times10^{-6}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1000\,\mathrm{kg\,m^{-3}}$

air

kinematic viscosity	$\nu = 1.5\times10^{-5}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1.2\,\mathrm{kg\,m^{-3}}$

$$\sin(x\pm y) = \sin x\cos y \pm \cos x \sin y$$
$$\cos(x\pm y) = \cos x\cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. We want to design a thin airfoil with a specific amount of camber. The camber line is approximated by $\frac{y_c}{c} = a\left[\frac{1}{4} - \left(\frac{x}{c} - \frac{1}{2}\right)^2\right]$, with a a positive constant.

   (a) Draw a sketch of this airfoil

   (b) What is the value of the parameter a if we want the airfoil to have $2.5\,\%$ camber?

   (c) Determine the coefficients A_0, A_1 and A_2.

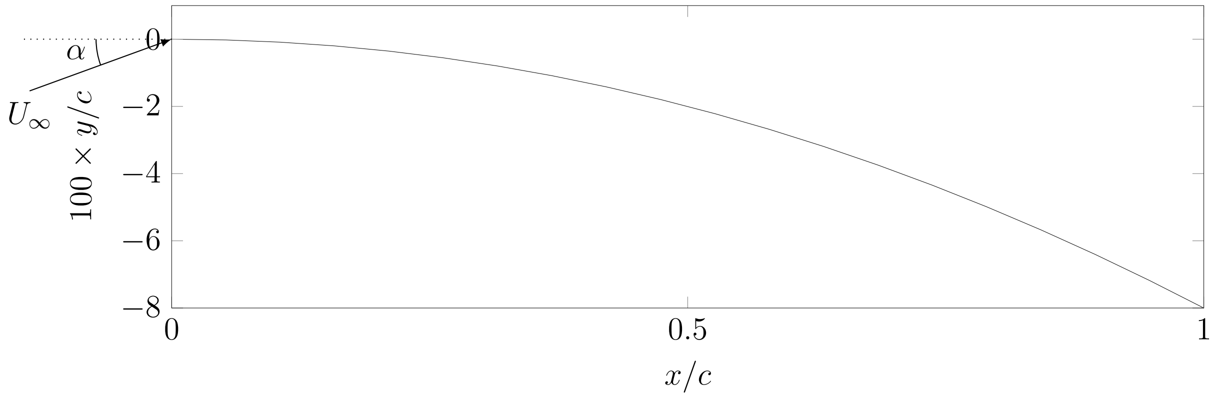
   (d) Draw the lift and quarter chord moment coefficients, C_L and $C_{M,1/4}$ in function of α.

(e) What is the angle of attack for zero lift? And what are C_l and $C_{m,1/4}$ at $\alpha = 0$?

(f) Define and calculate the aerodynamic centre and the centre of pressure for this airfoil at zero angle of attack?

(g) Can you think of a way to change the airfoil to reduce the moment while keeping the maximum camber the same?

2. Thin airfoil theory is used to describe the two-dimensional potential flow around a parabolically curved thin plate of length L place in a uniform free stream with velocity U_∞ at an angle of attack α, as shown in the figure below.



The plate shape is given by:

$$\frac{y_c}{c} = -0.08\left(\frac{x}{c}\right)^2$$

(a) Calculate the Fourier coefficients A_0, A_1 and A_2 for this camber line.

(b) Determine the lift coefficient and position of the center of pressure for $\alpha = 0$.

(c) Determine the value of α for which the pressure difference between the upper and lower surface of the plate is zero at the leading edge (no suction at the leading edge).

3. In this problem, we will investigate the aerodynamic performance of a thin airfoil that has a camber line defined by a third order polynomial $y_c = bc(\xi - a_1)(\xi - a_2)(\xi - a_3)$, where ξ is the dimensionless chord position.

   (a) Determine the values of a_1 and a_2 so that the polynomial function describes a real camberline ($y_c = 0$ at $x = 0$ and $x = c$) and thus show that it can be described by the following equation: $y_c = bc\xi(\xi - 1)(\xi - a)$. What do the remaining parameters b and a represent in terms of airfoil geometry?

   (b) Making use of the change in variable $\xi = \dfrac{x}{c} = \dfrac{1 - \cos\theta}{2}$, show that the camberline derivative can be written as:

$$\frac{dy_c}{dx} = b\left[\frac{1}{8} + \left(a - \frac{1}{2}\right)\cos\theta + \frac{3}{8}\cos 2\theta\right]$$

(c) Show that the Fourier coefficients for a third order polynomial camberline are given by:

$$\begin{cases} A_0 = \frac{b}{8} \\ A_1 = \left(a - \frac{1}{2}\right)b \\ A_2 = \frac{3b}{8} \end{cases}$$

(d) Show that the coefficients of lift and pitching moment (C_l and $C_m|_{AC}$) for an airfoil whose camber line is defined by a third order polynomial at an angle of attack α are given by:

$$\begin{cases} C_l = 2\pi\alpha + \pi b\left(a - \dfrac{3}{4}\right) \\ C_m|_{1/4} = -\dfrac{\pi}{4} b\left(a - \dfrac{7}{8}\right) \end{cases}$$

(e) For an airfoil with $a = 2$ and a maximum camber of 2%, show that $b = 0.052$ and determine the coefficients of lift and pitching moment (C_l and $C_{m,1/4}$) at a three degree angle of attack.